

Problems to prepare for eigenvalues

Note λ is a constant in all these problems.

1. Find the determinant of these matrices. If the determinant is 0, which axis is all the data being squished onto?

$$A = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} -\lambda & -1 \\ 2 & -\lambda \end{bmatrix}$$

2. Find the roots of the following equations (find λ):

$$\lambda^2 + 2 = 0$$

$$\lambda^4 - 1 = 0$$

3. Compute the matrix-vector multiplications

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} =$$

Do you notice anything special about the results?

4. Compute the following derivatives

$$\frac{d}{dt} (e^{\lambda t}) =$$

$$\frac{d}{dx} (\log(x)) =$$

5. Compute the following integrals (anti-derivatives)

$$\int \frac{dx}{x} =$$

$$\int \lambda \, dt =$$

6. BONUS:

Find $x(0.1)$ given $x(0) = 5$ using Euler's method:

$$\frac{dx(t)}{dt} = -x$$