

Robotics Tutorial - Assignment #2

6. Nov 2019

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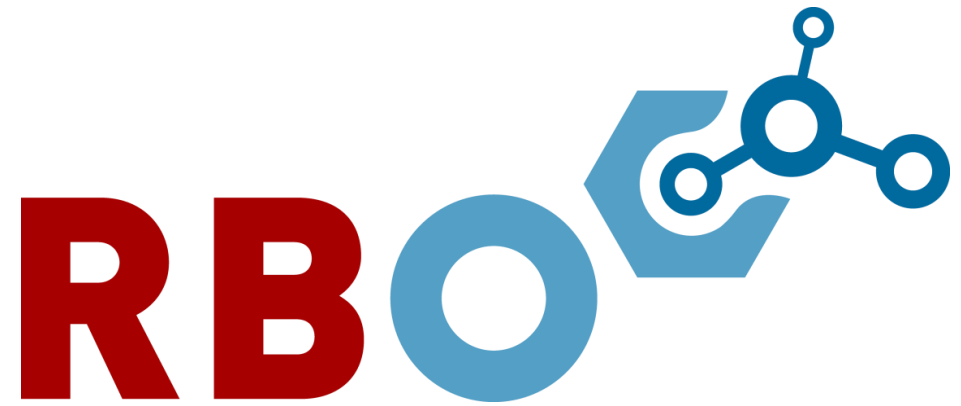
Assignment #2

Topics

- ▶ Forward kinematics
 - Frame transformations
 - Understanding Jacobians

- ▶ Joint Space Trajectories
 - Polynomial splines
 - PD trajectory controller

- ▶ Operational Space Trajectories
 - Jacobian control
 - Parabolic blends

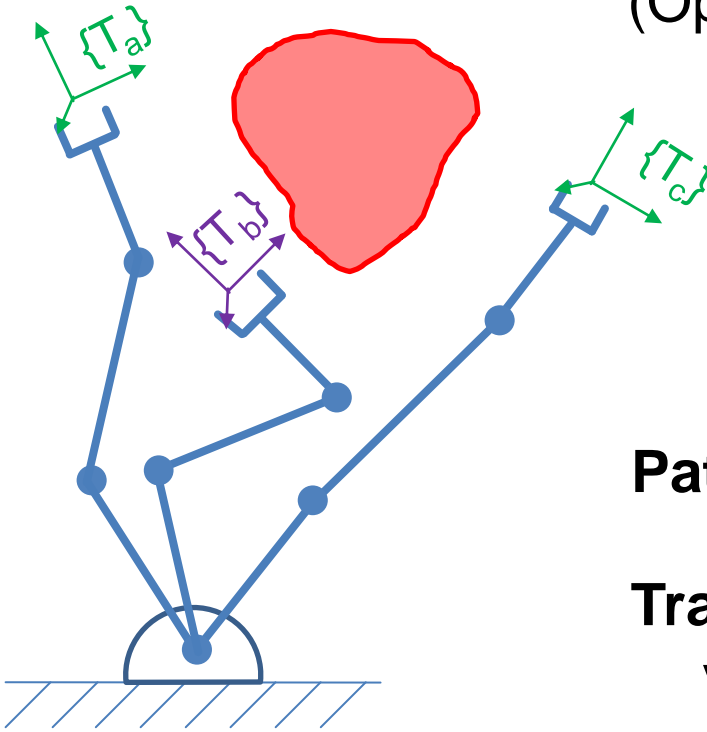


Trajectory Generation

The Problem

Move the manipulator from an initial position $\{T_a\}$ to a desired final position $\{T_c\}$

(Optional: through some via point $\{T_b\}$)



Path points: initial, final and via points

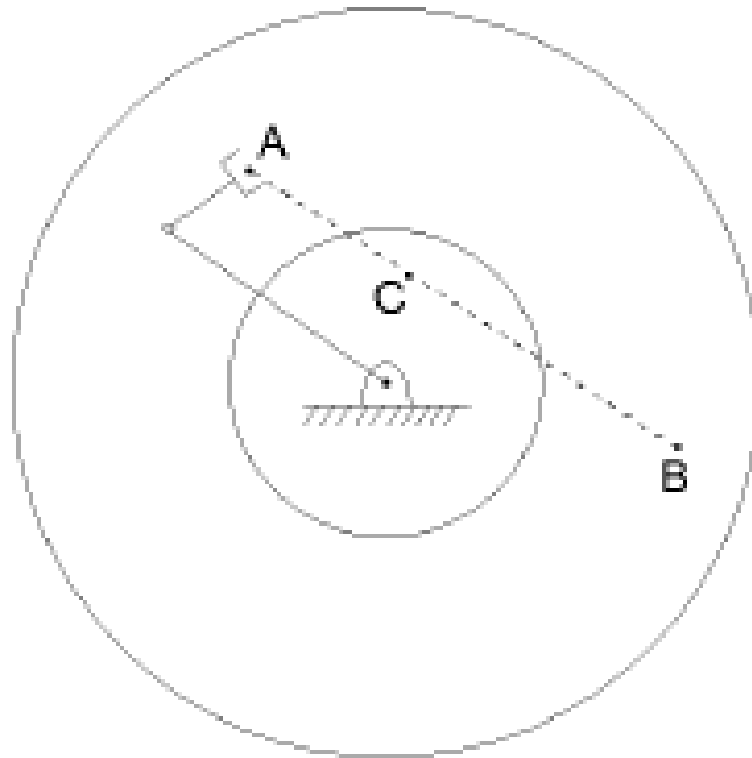
Trajectory: time history of position, velocity and acceleration for each DOF

State Spaces

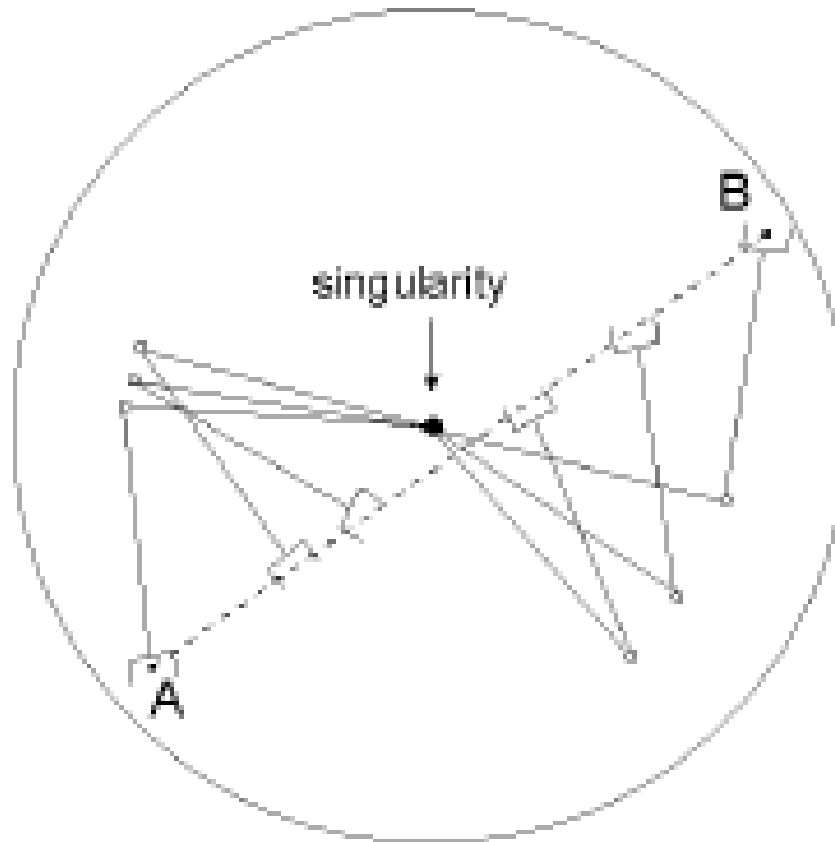
- ▶ Joint/Configuration Space
 - No problems with kinematic singularities
 - Less calculations
 - Cannot track shapes (e.g., a straight line)

- ▶ Operational/Cartesian Space
 - Can track shapes
 - *BUT*: singularities, more expensive at run time, ...

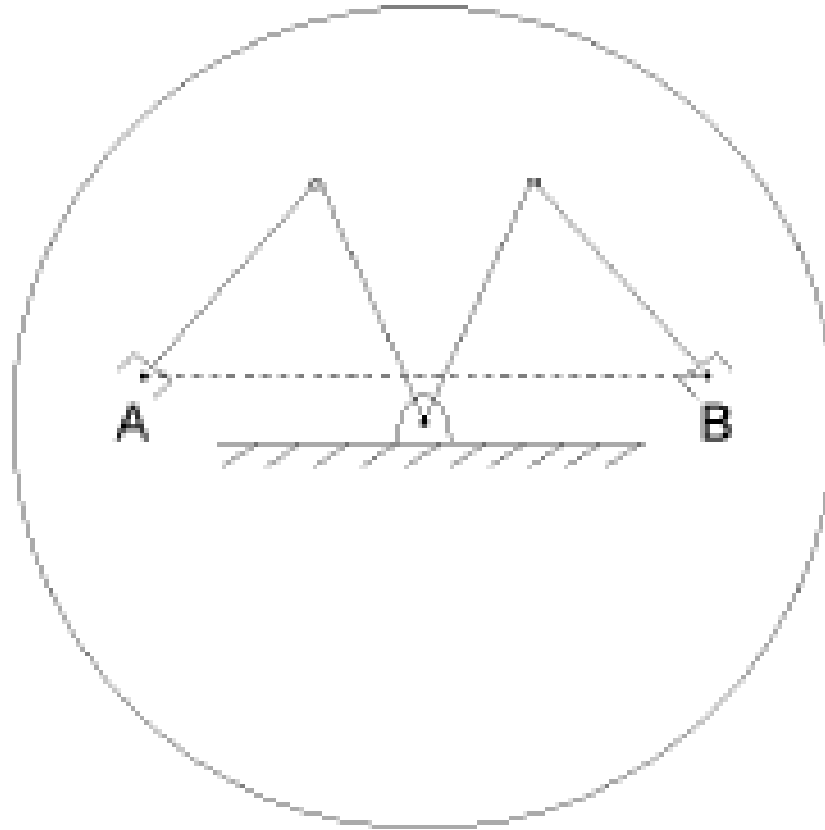
Unreachable Intermediate Points



Kinematic Singularities

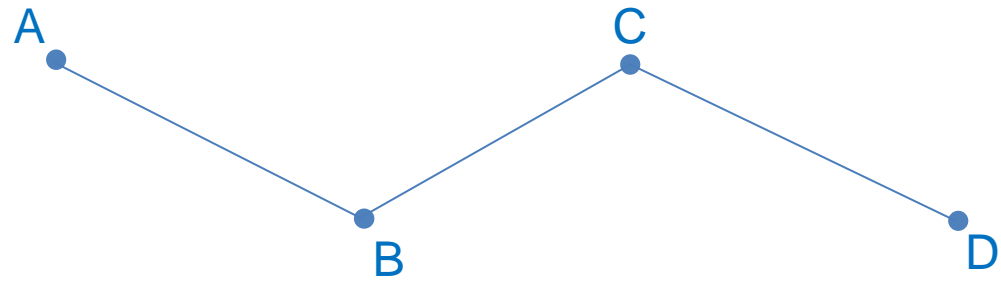


Different Joint Space Solutions

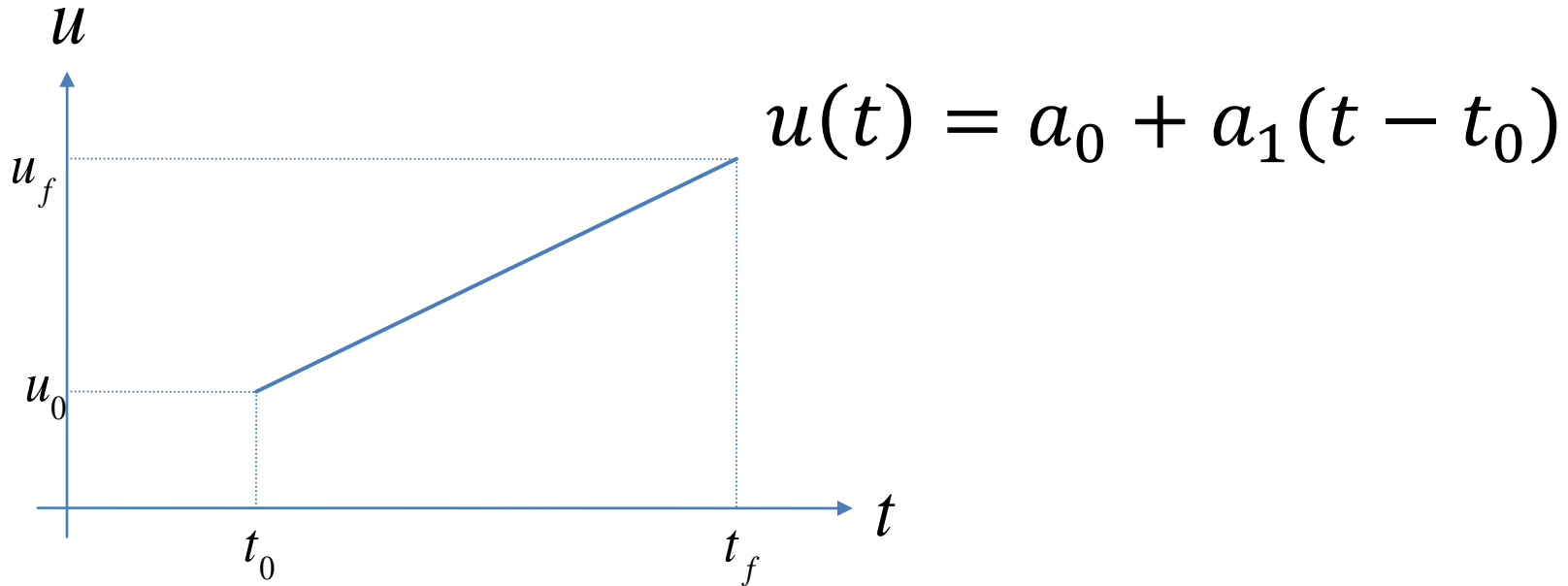


Candidate Curves

Linear



Linear Interpolation

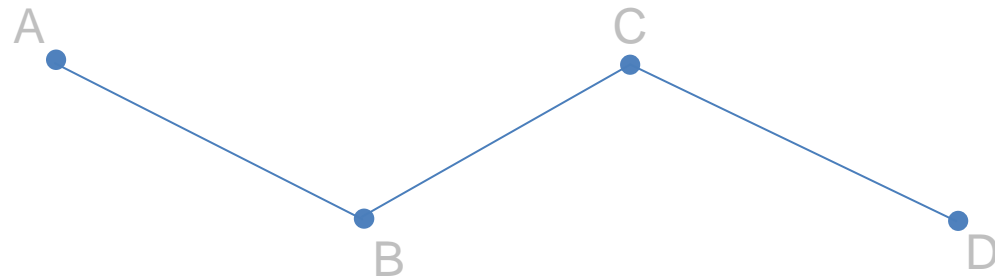


Two conditions: $u(t_0) = u_0$, $u(t_f) = u_f$

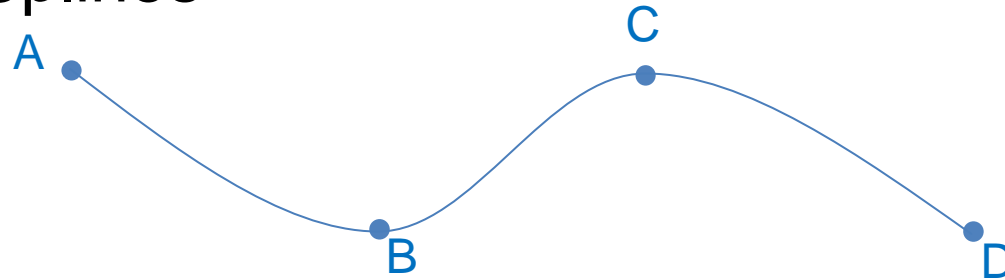
→ No control over velocities: discontinuities at beginning and end of motion require infinite acceleration!

Candidate Curves

Linear

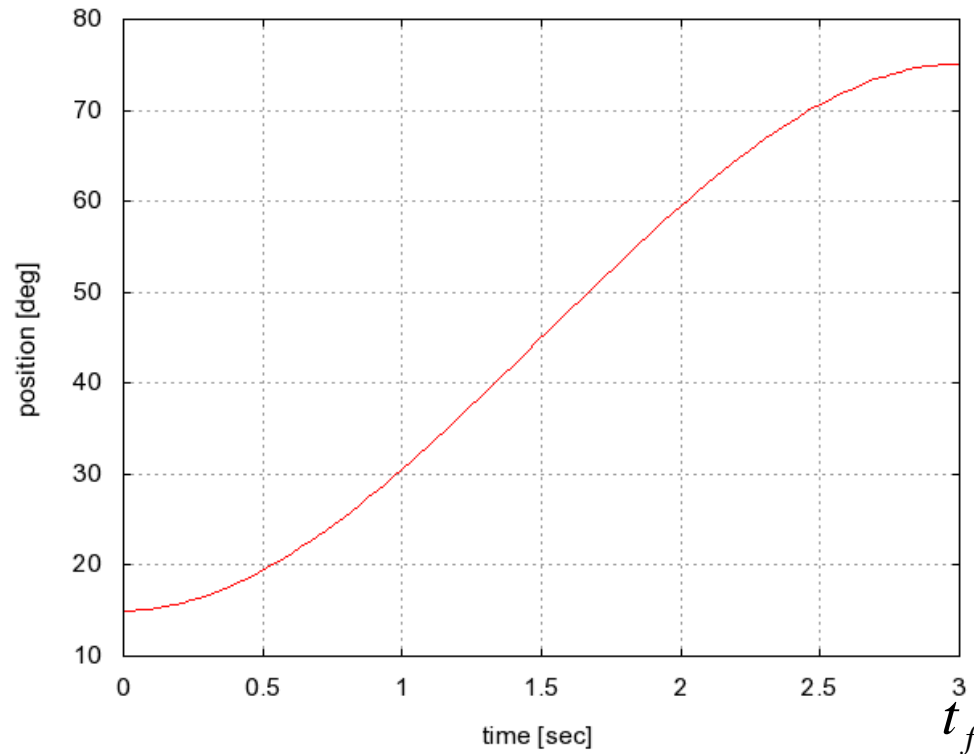


Polynomials/Splines



Cubic Splines

$$u(t) = a_0 + a_1t + a_2t^2 + a_3t^3$$



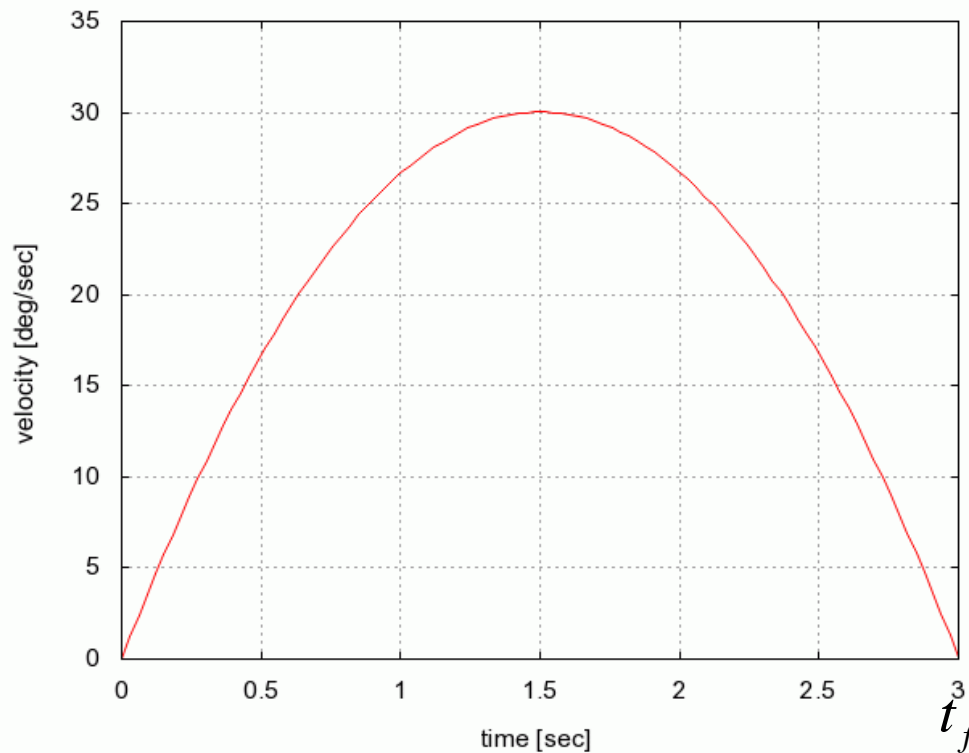
Initial Conditions:

$$u(0) = u_0$$

$$u(t_f) = u_f$$

Cubic Splines

$$\dot{u}(t) = a_1 + 2a_2t + 3a_3t^2$$



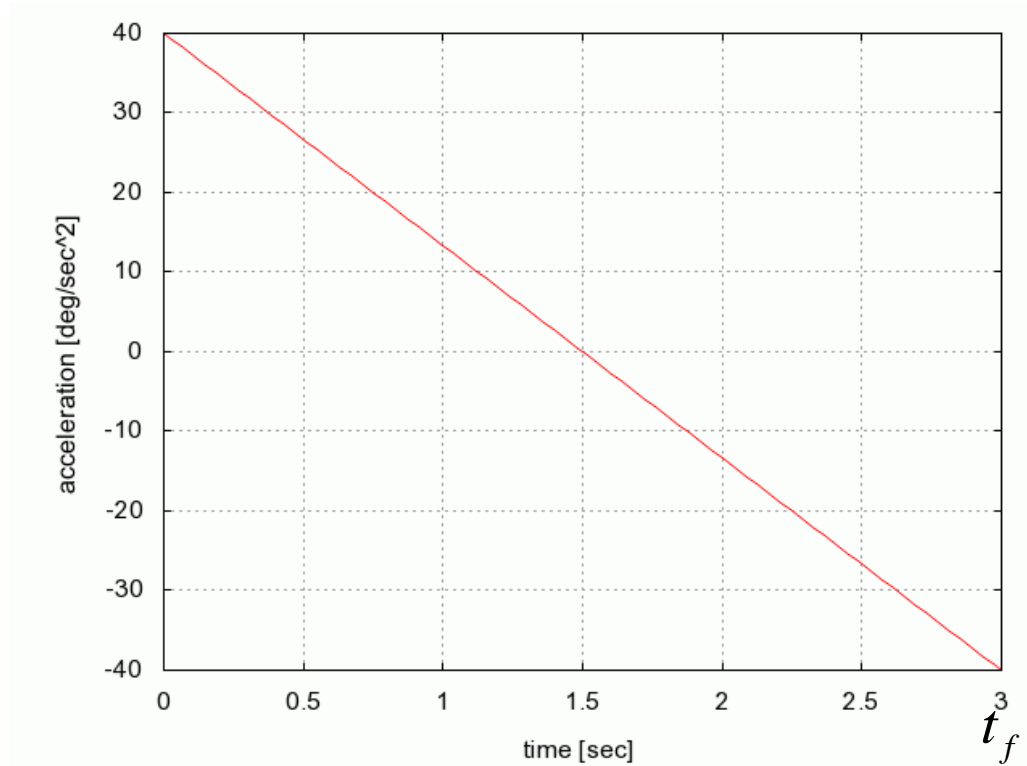
Initial Conditions:

$$\dot{u}(0) = 0$$

$$\dot{u}(t_f) = 0$$

Cubic Splines

$$\ddot{u}(t) = 2a_2 + 6a_3t$$



→ No control over acceleration

(use higher order polynomials (Quintics, Septics, ...))

Cubic Splines

$$u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Four equations: $u(0) = u_0$ $u(t_f) = u_f$ $\dot{u}(0) = 0$ $\dot{u}(t_f) = 0$

Four unknowns: a_0, a_1, a_2, a_3

$$\Rightarrow u(t) = \underbrace{u_0}_{a_0} + \underbrace{\left(\frac{3}{t_f^2}\right)(u_f - u_0)t^2}_{a_2} - \underbrace{\left(\frac{2}{t_f^3}\right)(u_f - u_0)t^3}_{a_3}$$

Including Via Points (1)

Concatenate cubic splines, e.g. for including one via point:

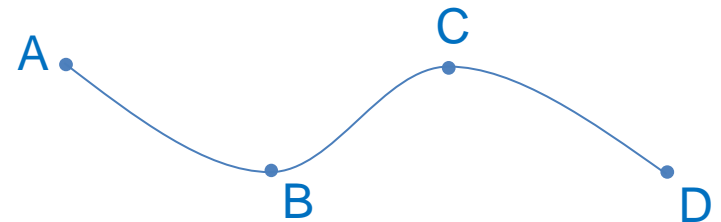
$$u_1(t) = a_0 + a_1t + a_2t^2 + a_3t^3$$

$$u_1(0) = u_0$$

$$u_1(t_{via}) = u_{via}$$

$$\dot{u}_1(0) = \dot{u}_0$$

$$\dot{u}_1(t_{via}) = \dot{u}_{via}$$



$$u_2(t) = b_0 + b_1(t - t_{via}) + b_2(t - t_{via})^2 + b_3(t - t_{via})^3$$

$$u_2(t_{via}) = u_{via}$$

$$u_2(t_f) = u_f$$

$$\dot{u}_2(t_{via}) = \dot{u}_{via}$$

$$\dot{u}_2(t_f) = \dot{u}_f$$

Including Via Points (2)

$$u_1(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$u_1(0) = u_0 \quad u_1(t_{via}) = u_{via}$$

$$\dot{u}_1(0) = \dot{u}_0 \quad \dot{u}_1(t_{via}) = \dot{u}_{via}$$

$$\Rightarrow a_0 = u_0 \quad a_2 = \frac{3}{t_{via}^2} (u_{via} - u_0) - \frac{2}{t_{via}} \dot{u}_0 - \frac{1}{t_{via}} \dot{u}_{via}$$

$$a_1 = \dot{u}_0 \quad a_3 = -\frac{2}{t_{via}^3} (u_{via} - u_0) + \frac{1}{t_{via}^2} (\dot{u}_0 + \dot{u}_{via})$$

Including Via Points (3)

$$u_2(t) = b_0 + b_1(t - t_{\text{via}}) + b_2(t - t_{\text{via}})^2 + b_3(t - t_{\text{via}})^3$$

$$u_2(t_{\text{via}}) = u_{\text{via}} \quad u_2(t_f) = u_f$$

$$\dot{u}_2(t_{\text{via}}) = \dot{u}_{\text{via}} \quad \dot{u}_2(t_f) = \dot{u}_f$$

$$\Rightarrow b_0 = u_{\text{via}} \quad b_2 = \frac{3}{(t_f - t_{\text{via}})^2} (u_f - u_{\text{via}}) - \frac{2}{t_f - t_{\text{via}}} \dot{u}_{\text{via}} - \frac{1}{t_f - t_{\text{via}}} \dot{u}_f$$

$$b_1 = \dot{u}_{\text{via}} \quad b_3 = -\frac{2}{(t_f - t_{\text{via}})^3} (u_f - u_{\text{via}}) + \frac{1}{(t_f - t_{\text{via}})^2} (\dot{u}_{\text{via}} + \dot{u}_f)$$

How to choose the velocity at the via point?

How To Choose Velocities At Via Points

$$u_2(t) = b_0 + b_1(t - t_{\text{via}}) + b_2(t - t_{\text{via}})^2 + b_3(t - t_{\text{via}})^3$$

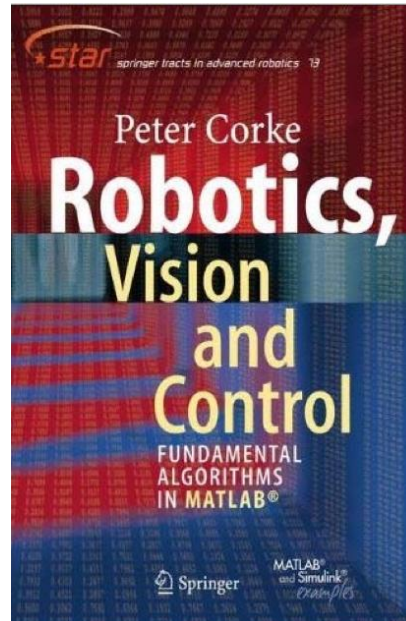
$$u_2(t_{\text{via}}) = u_{\text{via}} \quad u_2(t_f) = u_f$$

$$\dot{u}_2(t_{\text{via}}) = \dot{u}_{\text{via}} \quad \dot{u}_2(t_f) = 0$$

1. Let user specify
2. Use a heuristic (see assignment)
3. Alter the boundary conditions:
Remove velocity constraints and
force acceleration and velocity to be continuous

Recommendation For Reading

- ▶ You find a brief tutorial on how to solve equation systems with Matlab in this textbook:

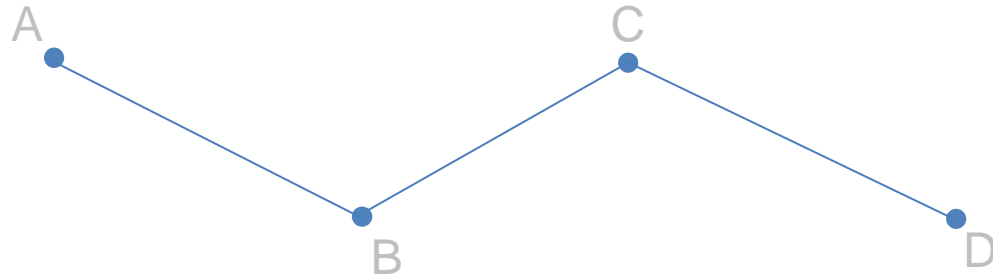


Peter Corke:
Robotics, Vision and Control

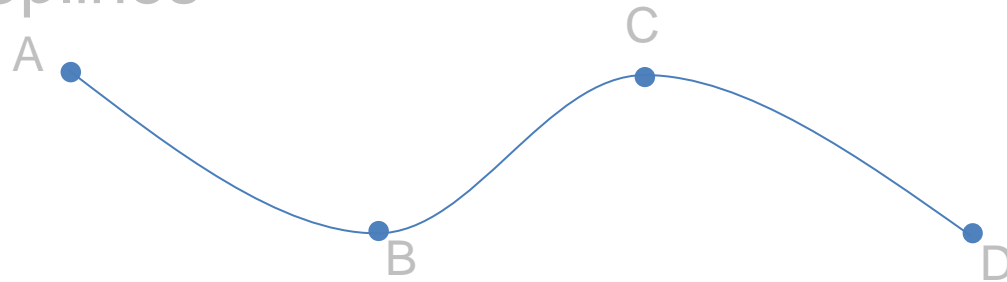
(All the topics of this class are also covered in the Craig textbook)

Candidate Curves

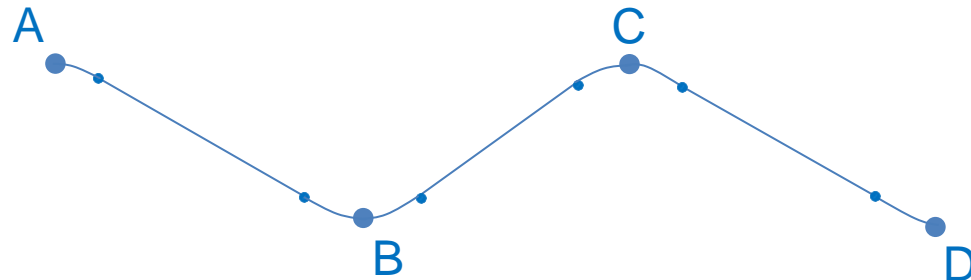
Linear



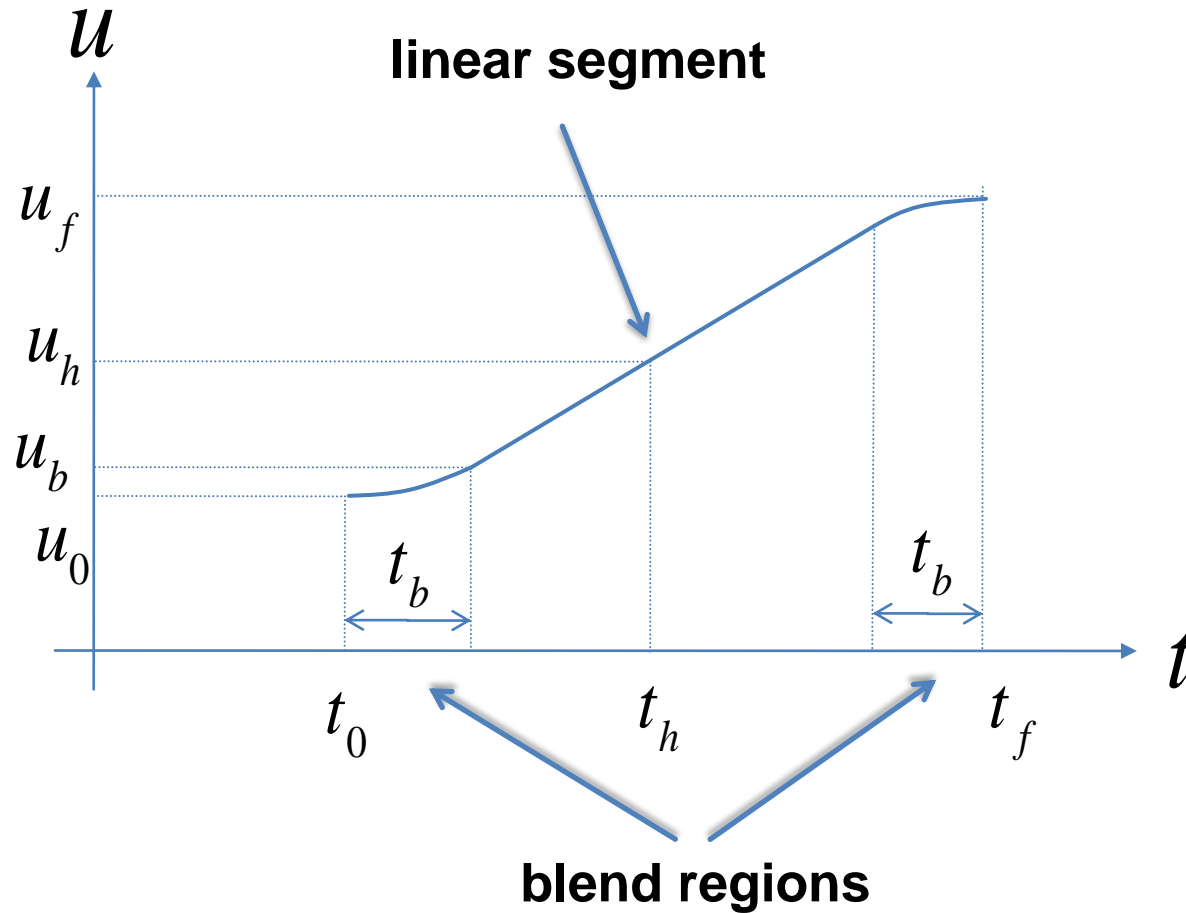
Polynomials/Splines



Linear with polynomial blends



Linear with Parabolic Blends (1)



Linear with Parabolic Blends (2)

- Idea: blends with constant acceleration

$$u(t) = \frac{1}{2} \ddot{u} t^2 + u_0$$

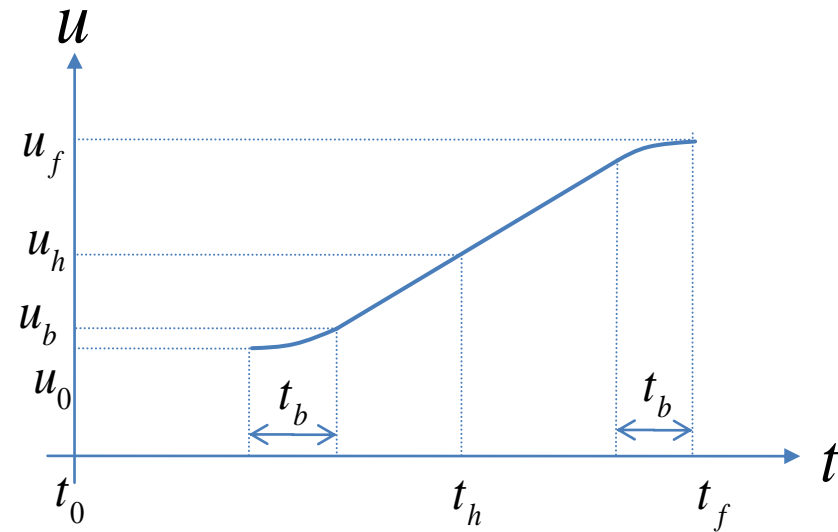
- Velocity at transition t_b :

$$\ddot{u} t_b = \frac{u_h - u_b}{t_h - t_b}$$

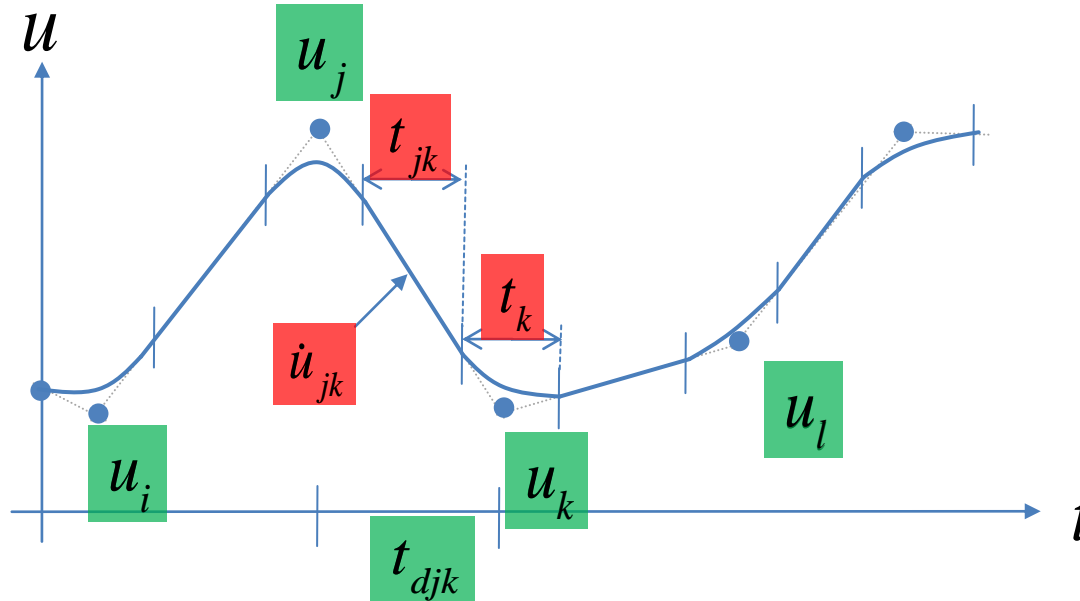
- Distance u_b :

$$u_b = u_0 + \frac{1}{2} \ddot{u} t_b^2$$

- Solve for unknowns



Optional: Including Via Points



Given:

$$u, t_{d^{**}}, |\ddot{u}|$$

Calculate:

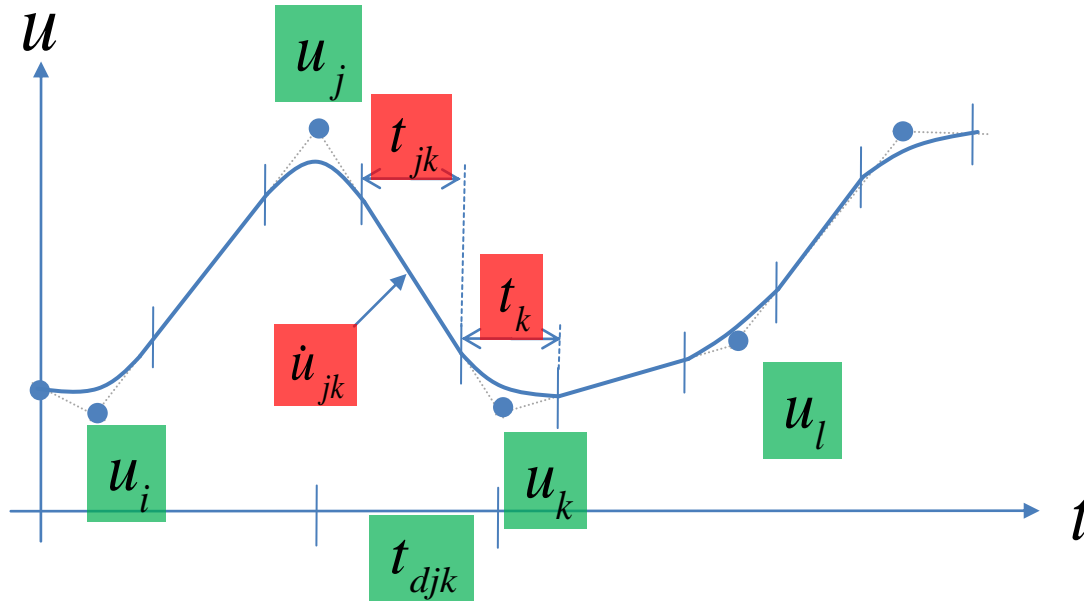
$$\dot{u}_{jk} = \frac{u_k - u_j}{t_{djk}}$$

$$t_k = \frac{\dot{u}_{kl} - \dot{u}_{jk}}{\ddot{u}_k}$$

$$\ddot{u}_k = \text{SGN}(\dot{u}_{kl} - \dot{u}_{jk}) |\ddot{u}_k|$$

$$t_{jk} = t_{djk} - \frac{1}{2} t_j - \frac{1}{2} t_k$$

Optional: Including Via Points



Given:

$$u, t_{d^{**}}, |\ddot{u}|$$

Calculate first segment (similar for last):

$$\dot{u}_{12} = \frac{u_2 - u_1}{t_{d12} - \frac{1}{2}t_1}$$

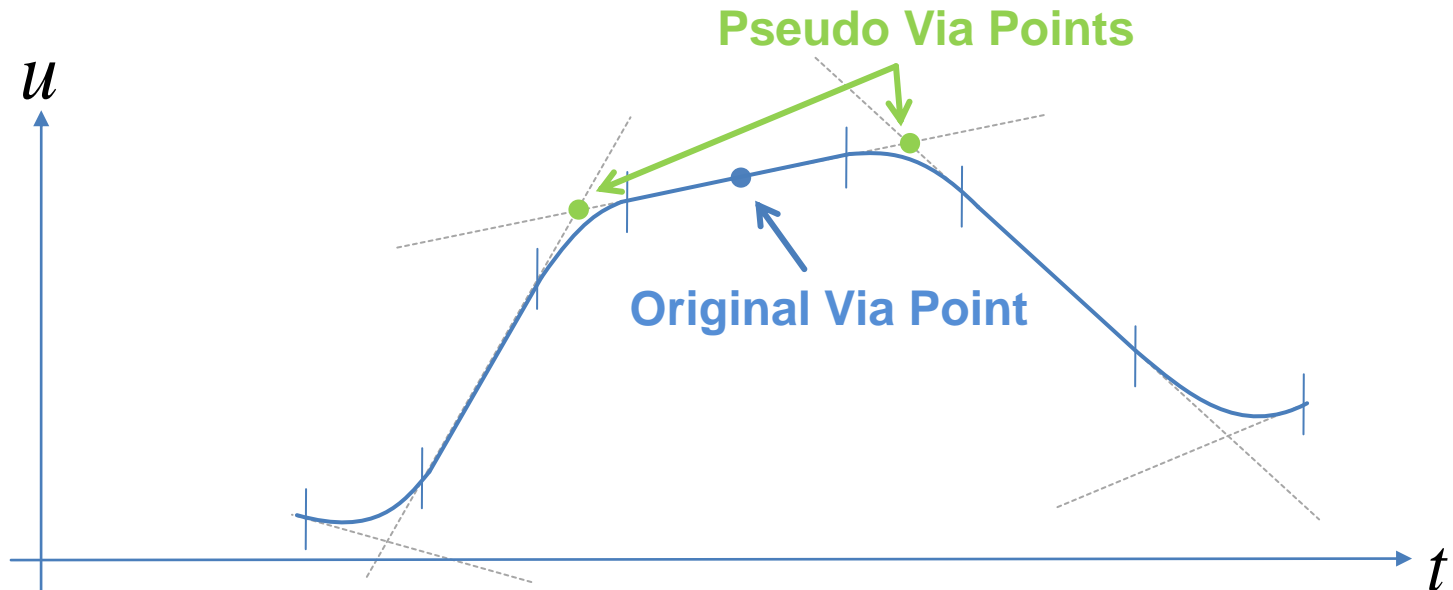
$$t_1 = t_{d12} - \sqrt{t_{d12}^2 - \frac{2(u_2 - u_1)}{\ddot{u}_1}}$$

$$\ddot{u}_k = \text{SGN}(u_2 - u_1) |\ddot{u}_1|$$

$$t_{12} = t_{d12} - t_1 - \frac{1}{2}t_2$$

Optional: How to pass *exactly* through a via point?

- Replace it by two pseudo via points



- Use high acceleration
- Repeat the via point (if we want to stop there)