Example:

2-dimensional vectors $\mathbf{u}=[u_1 \;\; u_2]$ and $\mathbf{v}=[v_1 \;\; v_2]$; let $K(\mathbf{u},\mathbf{v})=(1 + \mathbf{u}\bullet\mathbf{v})^2$,

Need to show that $K(\mathbf{x}_i,\mathbf{x}_j)= \boldsymbol{\phi}(\mathbf{x}_i) \bullet\boldsymbol{\phi}(\mathbf{x}_j)$:

$$K(\mathbf{u},\mathbf{v})=(1 + \mathbf{u}\bullet\mathbf{v})^2 = 1+ u_1^2v_1^2 + 2\, u_1v_1\, u_2v_2 + u_2^2v_2^2 + 2u_1v_1 + 2u_2v_2 =$$

$$= [1,\; u_1^2,\; \sqrt{2}\, u_1u_2,\;\; u_2^2,\; \sqrt{2}u_1,\; \sqrt{2}u_2] \bullet [1,\; v_1^2,\; \sqrt{2}v_1v_2,\; v_2^2,\; \sqrt{2}v_1,\; \sqrt{2}v_2] =$$

$$= \boldsymbol{\phi}(\mathbf{u}) \bullet\boldsymbol{\phi}(\mathbf{v}), \quad \text{where } \boldsymbol{\phi}(\mathbf{x}) = [1,\; x_1^2,\; \sqrt{2}\, x_1x_2,\;\; x_2^2,\; \sqrt{2}x_1,\; \sqrt{2}x_2]$$