Approximate Bayesian inference with Laplace and VB

Janet van Niekerk and Håvard Rue
Janet.vanNiekerk@kaust.edu.sa

جامعة الملك عبدالله للعلوم والتقنية
King Abdullah University of Science and Technology

September 2024

QR code for the slides

https://github.com/JanetVN1201/Geomed_24

Outline

BayesComp group at KAUST

Bayesian inference

Data $\boldsymbol{y}$ (with covariates $\boldsymbol{Z}$), depend on $\boldsymbol{X}$ and $\boldsymbol{\theta}$ such that, $E[Y] = h(\boldsymbol{A}(\boldsymbol{Z})\boldsymbol{X})$.

Bayes' theorem:

$$\begin{aligned} q(\boldsymbol{X}, \boldsymbol{\theta} | \boldsymbol{y}) &\propto L(\boldsymbol{y} | \boldsymbol{X}, \boldsymbol{\theta}) p(\boldsymbol{X}, \boldsymbol{\theta}) \\ \text{Posterior} &\propto \text{Likelihood} \times \text{Prior} \end{aligned}$$

Computational aspects

- Analytical methods - conjugacy (pre-computer era)
- Approximate methods - Laplace (can be inaccurate)
- Exact methods - MCMC (very slow for complex models or large data)

Now, due to complexity and data size approximate methods are gaining popularity - INLA, VB, EP etc.

INLA - 2009 [Rue et al., 2009]
2021+ [Van Niekerk et al., 2023]

What is INLA?

INLA - Integrated Nested Laplace Approximations

- Determinisitic approximations instead of sampling
- LGM - Latent Gaussian models
- Three internal strategies - Gaussian, simplified Laplace, Laplace (pre 2021)
- R package "INLA"

This work proposes a fourth strategy that is now the default in INLA.

Outline

Gaussian approximation from the Laplace method I

Suppose we have a twice differentiable function $\log f(\boldsymbol{X})$, then the Gaussian approximation of $\log f(\boldsymbol{X})$ from the Laplace method is then derived from

$$\log f(\boldsymbol{X}) = \log f(\boldsymbol{X}_0) - \frac{1}{2}(\boldsymbol{X} - \boldsymbol{X}_0)^\top \boldsymbol{H}|_{\boldsymbol{X}=\boldsymbol{X}_0}(\boldsymbol{X} - \boldsymbol{X}_0) + \text{higher order terms},$$

where $\boldsymbol{X}_0$ is the mode of $\log f(\boldsymbol{X})$ and $\boldsymbol{H}$ is the negative Hessian matrix of $\log f(\boldsymbol{X})$. Then

$$\tilde{f}(\boldsymbol{X}) \propto \exp\left(-\frac{1}{2}(\boldsymbol{X} - \boldsymbol{X}_0)^\top \boldsymbol{H}|_{\boldsymbol{X}=\boldsymbol{X}_0}(\boldsymbol{X} - \boldsymbol{X}_0)\right), \tag{1}$$

so that $\boldsymbol{X} \dot{\sim} N(\boldsymbol{X}_0, \boldsymbol{H}^{-1}|_{\boldsymbol{X}=\boldsymbol{X}_0})$.

Gaussian approximation from the Laplace method II

To find the mode we solve for $\boldsymbol{X}_0$ in the system

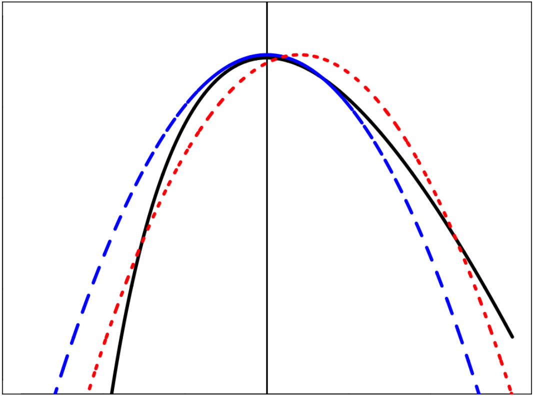
$$\boldsymbol{H}|_{\boldsymbol{X}=\boldsymbol{X}_0}\boldsymbol{X}_0 = \gamma|_{\boldsymbol{X}=\boldsymbol{X}_0} + \boldsymbol{H}|_{\boldsymbol{X}=\boldsymbol{X}_0}\boldsymbol{X}_0, \tag{2}$$

where $\gamma|_{\boldsymbol{X}=\boldsymbol{X}_0}$ is the gradient of $\log f(\boldsymbol{X})$ evaluated at $\boldsymbol{X} = \boldsymbol{X}_0$. Now let $\boldsymbol{Q}_0 = \boldsymbol{H}|_{\boldsymbol{X}=\boldsymbol{X}_0}$ and $\boldsymbol{b}_0 = \gamma|_{\boldsymbol{X}=\boldsymbol{X}_0} + \boldsymbol{H}|_{\boldsymbol{X}=\boldsymbol{X}_0}\boldsymbol{X}_0$, then the system can be written as

$$\boldsymbol{Q}_0\boldsymbol{X}_0 = \boldsymbol{b}_0. \tag{3}$$

Gaussian approximation from the Laplace method III

Maybe the approximation at the mode does not give the best approximation?



Variational Inference

Optimization problem - minimize the KLD between the prior and a family of posteriors.
For a Gaussian approximation to $f(\boldsymbol{X})$, we can solve for $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ in

$$\arg_{\boldsymbol{\mu},\boldsymbol{\Sigma}} \min \text{KLD}(N(\boldsymbol{\mu},\boldsymbol{\Sigma})||f(\boldsymbol{X}))$$

The ELBO is then derived from this KLD as the optimization target that should be maximized. Various ways to "do" the optimization has been developed and still ongoing.
Mean-field VI is a simplification...

Model definition - GAMM

Suppose we have response data $\boldsymbol{y}_{n\times 1}$ (conditionally independent) with density function $\pi(y|\boldsymbol{X},\boldsymbol{\theta})$ and link function $h(.)$, that is linked to some covariates $\boldsymbol{Z}$ through linear predictors

$$\boldsymbol{\eta}_n = \beta_0 + \boldsymbol{Z}_\beta\boldsymbol{\beta} + \sum f^k(\boldsymbol{Z}_f) = \boldsymbol{A}\boldsymbol{X}$$

The inferential aim is to estimate the latent field $\boldsymbol{X}_m = \{\beta_0, \boldsymbol{\beta}, \boldsymbol{f}\}$, and the hyperparameters $\boldsymbol{\theta}$.

Information theoretic point of view - Zellner (1988)[1]

Based on prior information $\mathcal{I}$, data $\boldsymbol{y}$ and parameters $\boldsymbol{P} = \{\boldsymbol{X}, \boldsymbol{\theta}\}$, define the following:

1. $\pi(\boldsymbol{P}|\mathcal{I})$ is the prior model
2. $q(\boldsymbol{P}|\mathcal{D})$ is the learned model from the prior information and the data where $\mathcal{D} = \{\mathcal{I}, \boldsymbol{y}\}$
3. $l(\boldsymbol{P}|\boldsymbol{y}) = f(\boldsymbol{y}|P)$ is the likelihood
4. $p(\boldsymbol{y}|\mathcal{I})$ is the marginal model for the data where $p(\boldsymbol{y}|\mathcal{I}) = \int f(\boldsymbol{y}|\boldsymbol{P})\pi(\boldsymbol{P}|\mathcal{I})d\boldsymbol{P}$

The input information in the learning of $\boldsymbol{P}$ is given by $\pi(\boldsymbol{P}|\mathcal{I})$ and $l(\boldsymbol{P}|\boldsymbol{y})$. An information processing rule (IPR) then delivers $q(\boldsymbol{P}|\mathcal{D})$ and $p(\boldsymbol{y}|\mathcal{I})$ as output information.

[1] Zellner, A., 1988. Optimal information processing and Bayes's theorem. The American Statistician, 42(4), pp.278-280.

Bayes Rule as an efficient IPR

A stable and efficient IPR would provide the same amount of output information than received through the input information, thus being information conservative. Zellner shows that $q(\boldsymbol{P}|\mathcal{D})$ minimizes

$$
\begin{aligned}
&- \int \left[\log \pi(\boldsymbol{P}|\mathcal{I}) + \log l(\boldsymbol{P}|\boldsymbol{y})\right] q(\boldsymbol{P}|\mathcal{D}) d\boldsymbol{P} + \int \left[\log q(\boldsymbol{P}|\mathcal{D}) + \log p(\boldsymbol{y}|\mathcal{I})\right] q(\boldsymbol{P}|\mathcal{D}) d\boldsymbol{P} \\
&= E_{q(\boldsymbol{P}|\mathcal{D})}\left[-\log l(\boldsymbol{P}|\boldsymbol{y})\right] + \text{KLD}\left[q(\boldsymbol{P}|\mathcal{D}) || \pi(\boldsymbol{P}|\mathcal{I})\right]. \qquad (4)
\end{aligned}
$$

Variational form of Bayes' theorem

Finding the best fit from a certain family $Q = \{q(\boldsymbol{P})\}$, for prior $\pi(\boldsymbol{P})$,

$$\arg\min_{p \in Q} \left(\mathrm{E}_{p(\boldsymbol{P})} \left[-\sum_{i=1}^{n} \log f(y_i | \boldsymbol{P}) \right] + \mathrm{KLD}(p || \pi) \right) \tag{5}$$

This enables us to do Variational Inference without the need for an ELBO or other simplifying assumptions.

Laplace method with low-rank Variational Bayes correction I

For a GAMM, data $\boldsymbol{y}$, model parameters $\boldsymbol{X}$ and prior $\pi(\boldsymbol{X})$,

- Gaussian approximation using Laplace method to $q(\boldsymbol{X}|\boldsymbol{y})$

$$\boldsymbol{Q}_0\boldsymbol{X}_0 = \boldsymbol{b}_0$$

- Correct the Laplace method's mean with VB[2], $\boldsymbol{X}_0^*$ such that

$$\boldsymbol{Q}_0(\boldsymbol{X}_0 + \boldsymbol{\delta}) = \boldsymbol{b}_0^*$$

But what if the dimension of $\boldsymbol{X}$ is large?

Laplace method with low-rank Variational Bayes correction II

Do an implicit correction of the mean, solve for $\boldsymbol{\delta}$ such that

$$\boldsymbol{Q}_0\boldsymbol{X}_0^* = \boldsymbol{b}_0 + \boldsymbol{\delta}$$

This means that there is a map for the change the i^{th} element of δ will cause in the j^{th} element of $\boldsymbol{X}_0$. This map is constructed from $\boldsymbol{Q}_0$.

If $dim(X) = m$ then we can have that the non-zero entries in δ is at most p, $p << m$. The optimization is then in p dimensions and not in m dimensions.

Who? What? Why? When? etc...

[2] van Niekerk, J. and Rue, H., 2024. Low-rank variational Bayes correction to the Laplace method. Journal of Machine Learning Research, 25(62), pp.1-25.

Example I

$$y_i \sim \text{Poisson}(\exp(\eta_i)), \quad \eta_i = \beta_0 + \beta_1 x_i + u_i,$$

with a sum-to-zero constraint on $\boldsymbol{u}$, to ensure identifiability of β_0. We want to perform full Bayesian inference for the latent field $\boldsymbol{\psi} = \{\beta_0, \beta_1, \boldsymbol{u}\}$, and the linear predictors $\boldsymbol{\eta} = \{\eta_1, \eta_2, \dots, \eta_n\}$. We assume the following illustrative priors,

$$\beta_0 \sim t(5), \quad \beta_1 \sim U(-3, 3) \quad \text{and} \quad \boldsymbol{u} \sim N(\mathbf{0}, 0.25\boldsymbol{I})$$

Example II

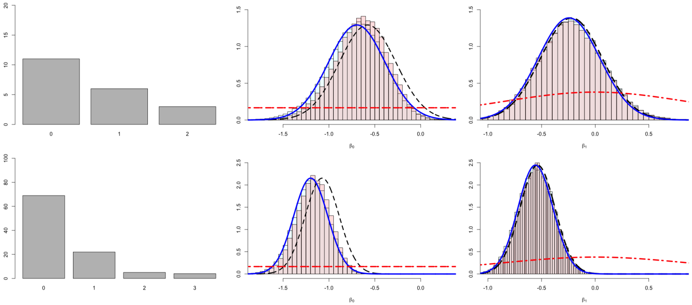


Figure: Poisson counts simulated from (19) (left) and the marginal posterior of β_0 (center) and β_1 (right) from MCMC (blue histogram), HMC (red histogram), the Laplace method (dashed line) and VBC (solid line) based on the prior (broken line) for $n = 20$ (top) and $n = 100$ (bottom)

Example III

	n=100			
	LM	VBC	MCMC	HMC
β_0	-1.073	-1.199	-1.196	-1.180
β_1	-0.538	-0.567	-0.552	-0.552
u_1	0.177	0.174	0.175	0.174
u_8	-0.046	-0.052	-0.049	-0.044
u_{15}	-0.074	-0.079	-0.077	-0.073
Time(s)	9.48	17.36	384.12	169.57

Table: Posterior means from the Laplace method, VBC, MCMC and HMC

Outline

GAMM $\rightarrow$ LGM

Assume

$$\boldsymbol{X}|\boldsymbol{\theta} \sim N(\boldsymbol{0}, \boldsymbol{Q}(\boldsymbol{\theta})^{-1})$$

where $\boldsymbol{Q}(\boldsymbol{\theta})$ is a sparse matrix ($\boldsymbol{X}$ is a GMRF).

$p(\boldsymbol{X}, \boldsymbol{\theta}) = p(\boldsymbol{X}|\boldsymbol{\theta})p(\boldsymbol{\theta})$ and $p(\boldsymbol{\theta})$ can be non-Gaussian.

Ingredients

Likelihood - $\prod_{i=1}^{n} \pi(y_i|\boldsymbol{X}, \boldsymbol{\theta}), \eta = \boldsymbol{A}\boldsymbol{X}$
Prior for the latent - $\pi(\boldsymbol{X}|\boldsymbol{\theta})$
Prior for the hyperparameters - $\pi(\boldsymbol{\theta})$
Goal:

- $q(X_j|\boldsymbol{y})$
- $q(\theta_k|\boldsymbol{y})$

Posterior approximations by INLA

For

$$\pi(\boldsymbol{X},\boldsymbol{\theta},\boldsymbol{y}) = \pi(\boldsymbol{\theta})\pi(\boldsymbol{X}|\boldsymbol{\theta})\prod_{i=1}^{n}\pi(y_i|(\boldsymbol{A}\boldsymbol{X})_i,\boldsymbol{\theta})$$

1. $$\tilde{q}(\boldsymbol{\theta}|\boldsymbol{y}) \propto \left.\frac{\pi(\boldsymbol{X},\boldsymbol{\theta},\boldsymbol{y})}{\pi_G(\boldsymbol{X}|\boldsymbol{\theta},\boldsymbol{y})}\right|_{\boldsymbol{X}=\boldsymbol{\mu}(\boldsymbol{\theta})}$$
2. $$\tilde{q}(\theta_j|\boldsymbol{y}) = \int \tilde{\pi}(\boldsymbol{\theta}|\boldsymbol{y})d\boldsymbol{\theta}_{-j}$$
3. $$\tilde{q}(\boldsymbol{X}_j|\boldsymbol{y}) = \int \tilde{q}(\boldsymbol{X}_j|\boldsymbol{\theta},\boldsymbol{y})\tilde{q}(\boldsymbol{\theta}|\boldsymbol{y})d\boldsymbol{\theta},$$

$\tilde{q}(\boldsymbol{X}_j|\boldsymbol{\theta},\boldsymbol{y})$ depends on the approximation used in Stage 1.

Skip

INLA 1.0

- $\boldsymbol{X} = \{\boldsymbol{\eta}, \beta_0, \boldsymbol{\beta}, \boldsymbol{f}\}$
- Laplace strategy: For each j,

$$\tilde{q}(X_j|\boldsymbol{\theta}, \boldsymbol{y}) = \left.\frac{\pi(\boldsymbol{X}, \boldsymbol{\theta}, \boldsymbol{y})}{\pi_G(\boldsymbol{X}_{-j}|X_j, \boldsymbol{\theta}, \boldsymbol{y})}\right|_{\boldsymbol{X}_{-j}=\boldsymbol{\mu}_{-j}}$$

Wishes and dreams

1. How can we get a good and cheap approximation $\tilde{q}(\boldsymbol{X}_j|\boldsymbol{\theta}, \boldsymbol{y})$ using $\pi_G(\boldsymbol{X}|\boldsymbol{\theta}, \boldsymbol{y})$?
   Non-Gaussian? Other family?
2. How can we remove $\boldsymbol{\eta}$ from $\boldsymbol{X}$ and still produce full posteriors of $\boldsymbol{\eta}$?
   Huge data? Prediction? Stability?

Gaussian approximation $\pi_G(\boldsymbol{X}|\boldsymbol{\theta}, \boldsymbol{y})$

Laplace method - fit the best Gaussian at the mode of a curve where the variance is derived from the inverse Hessian at the mode.

$$
\begin{aligned}
\log\left(\pi(\boldsymbol{X}|\boldsymbol{\theta}, \boldsymbol{y})\right) &\propto -\frac{1}{2}\boldsymbol{X}^\top \boldsymbol{Q}(\boldsymbol{\theta})\boldsymbol{X} + \sum_{i=1}^{n}\left(b_i\boldsymbol{X}_i - \frac{1}{2}c_i\boldsymbol{X}_i^2\right) \\
&= -\frac{1}{2}\boldsymbol{X}^\top\left(\boldsymbol{Q}(\boldsymbol{\theta}) + \boldsymbol{D}\right)\boldsymbol{X} - \boldsymbol{b}^\top\boldsymbol{X}
\end{aligned}
$$

hence

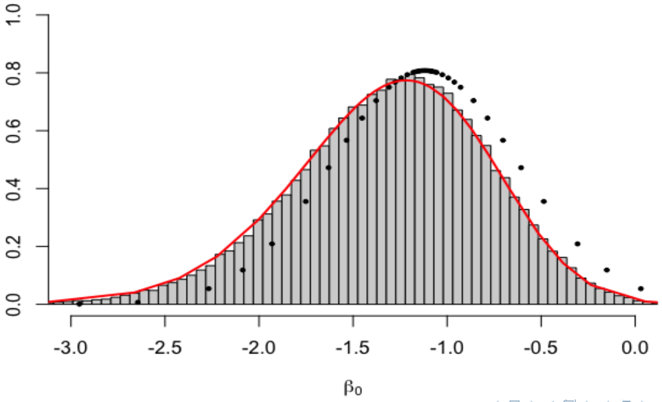
$$
\boldsymbol{X}|\boldsymbol{\theta}, \boldsymbol{y} \sim N(\boldsymbol{\mu}, (\boldsymbol{Q}(\boldsymbol{\theta}) + diag(\boldsymbol{c}))^{-1}) \tag{6}
$$

with

$$
(\boldsymbol{Q}(\boldsymbol{\theta}) + \boldsymbol{D})\boldsymbol{\mu} = \boldsymbol{b}
$$

Poisson example

$$Y_i|\beta_0 = -1, \beta_1 = -0.5 \quad \sim \quad \text{Poisson}(\exp(\beta_0 + \beta_1 X_i))$$



How can we use this?

We apply this to the Gaussian approximation in the denominator of Stage 1 ▸▸ Stage 1.

Recall that $(\boldsymbol{Q}(\boldsymbol{\theta}) + \boldsymbol{D})\boldsymbol{\mu} = \boldsymbol{Q}\boldsymbol{\mu} = \boldsymbol{b}$.

Now let's formulate $\boldsymbol{Q}\boldsymbol{\mu}^* = \boldsymbol{b} + \boldsymbol{\lambda}$, so that

$$\boldsymbol{\mu}^* = \boldsymbol{\mu} + \boldsymbol{M}\boldsymbol{\lambda}$$

So now we solve for,

$$\arg_{\boldsymbol{\lambda}} \min_{p(\boldsymbol{X}|\boldsymbol{y},\boldsymbol{\theta})} \left(\mathrm{E}_{p(\boldsymbol{X}|\boldsymbol{y},\boldsymbol{\theta})} \left[-\sum_{i=1}^{n} \log f(y_i|\boldsymbol{X}_i, \boldsymbol{\theta}) \right] + \mathrm{KLD}(p||\pi) \right)$$

where $\boldsymbol{X}|\boldsymbol{y}, \boldsymbol{\theta} \sim N(\boldsymbol{\mu}^*, \boldsymbol{Q}^{-1})$.

Low-rank correction $\rightarrow$ Only correct some $b's$, change to all $\mu's$.

VB corrected marginal posterior of η_i

$$
\begin{aligned}
\eta_j|\boldsymbol{\theta},\boldsymbol{y} &\sim N(\mu_j(\boldsymbol{\theta}),\sigma_j^2(\boldsymbol{\theta}))\\
\mu_j(\boldsymbol{\theta}) &= (\boldsymbol{A}\boldsymbol{\mu}^*(\boldsymbol{\theta}))_j\\
\mathrm{Cov}(\boldsymbol{\eta}|\boldsymbol{\theta},\boldsymbol{y}) &= \boldsymbol{A}\mathrm{Cov}(\boldsymbol{X}|\boldsymbol{\theta},\boldsymbol{y})\boldsymbol{A}^\top\\
\tilde{\pi}(\eta_j|\boldsymbol{y}) &\approx \sum_{k=1}^{K}\pi_G(\eta_j|\boldsymbol{\theta}_k,\boldsymbol{y})\tilde{\pi}(\boldsymbol{\theta}_k|\boldsymbol{y})\delta_k.
\end{aligned}
$$

Overdispersed Poisson regression I

$$y_i \sim \text{Poisson}(\exp(\eta_i)), \quad \eta_i = \beta_0 + \beta_1 x_i + u_i, \tag{7}$$

for $i = 1, 2, ..., n$, where $u_i|\tau \sim N(0, \tau^{-1})$, $\log \tau \sim \text{loggamma}(1, 5 \times 10^{-5})$, $\beta_0 \sim N(0, 1)$ and $\beta_1 \sim N(0, 1)$. The data is simulated based on $\beta_0 = -1, \beta_1 = -0.5, \tau = 1, n = 1000$ and the parameters to infer are $\boldsymbol{\psi} = \{\beta_0, \beta_1, u_1, u_2, ..., u_n\}$, the linear predictors $\{\eta_1, \eta_2, ..., \eta_n\}$ i.e. $\boldsymbol{X} = \{\boldsymbol{\psi}, \boldsymbol{\eta}\}$, and the set of hyperparameters $\boldsymbol{\theta} = \{\tau\}$.

Overdispersed Poisson regression II

	GA	INLA	INLA-VBC	HMC
β_0	-0.972	-0.664	-0.972	-0.934
β_1	-0.484	-0.532	-0.531	-0.529
τ	1.056	1.056	1.056	1.037
Time(s)	5.067	18.299	5.718	207.445

Table: Posterior means from the Gaussian strategy (GA), Laplace strategy (INLA), INLA-VBC and MCMC

Tokyo example

The Tokyo dataset in the R INLA library contains information on the number of times the daily rainfall measurements in Tokyo was more than 1*mm* on a specific day *t* for two consecutive years. In order to model the annual rainfall pattern, a stochastic spline model with fixed precision is used to smooth the data.

$$
\begin{aligned}
y_i|\mathcal{X} &\sim Bin\left(n_i, p_i = \frac{\exp(\alpha_i)}{1+\exp(\alpha_i)}\right) \\
(\alpha_{i+1} - 2\alpha_i + \alpha_{i-1})|\tau &\overset{\text{iid}}{\sim} N(0, \tau^{-1}),
\end{aligned}
$$

where $i = 1, 2, ..., 366$ on a torus, and $n_{60} = 1$ else $n_i = 2$.

Results

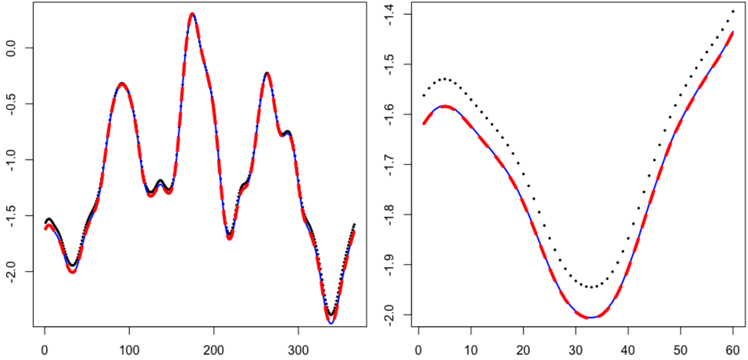


Figure: Posterior mean of α (left) (zoomed for the first two months (right)) from the Laplace method (points), VBC (solid line) and INLA (broken line)

Spatial survival example I

Consider the R dataset `Leuk` that features the survival times of 1043 patients with acute myeloid leukemia (AML) in Northwest England between 1982 to 1998.

$$h(t, \boldsymbol{s}) = h_0(t)\exp(\beta \boldsymbol{X} + \boldsymbol{u}(\boldsymbol{s})),$$

with

$$\eta_i(s) = \beta_0 + \beta_1 \text{Age}_i + \beta_2 \text{WBC}_i + \beta_3 \text{TPI}_i + u(s).$$

which implies a latent field of size $m = 39158$.

Spatial survival example II

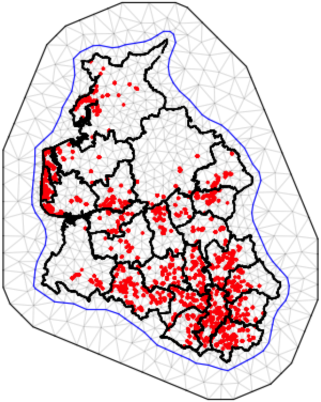


Figure: Exact residential locations of patients with AML

Results I

	GA	INLA	INLA-VBC
β_0	-2.023	-2.189	-2.189
β_1	0.596	0.597	0.597
β_2	0.242	0.241	0.241
β_3	0.108	0.108	0.108
τ	0.340	0.340	0.340
σ_u	0.223	0.223	0.223
r	0.202	0.202	0.202
Time(s)	25.9	1276	26.3

Table: Posterior means from the Gaussian strategy, INLA and INLA-VBC - all fixed effects are significant

Results II

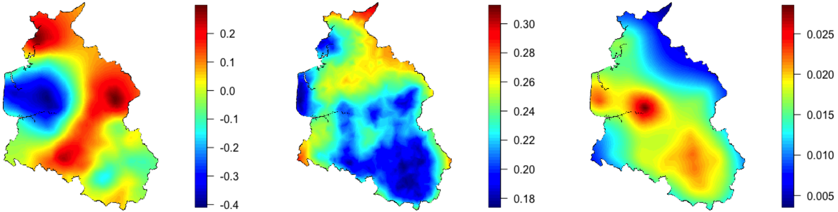


Figure: Posterior mean (left) and posterior standard deviation (center) of $\boldsymbol{u}(\boldsymbol{s})$ from INLA-VBC with the absolute difference between the posterior means of $\boldsymbol{u}(\boldsymbol{s})$ from the Gaussian strategy and INLA-VBC (right)

cs-fMRI model

Functional magnetic resonance imaging (fMRI) is a noninvasive neuro-imaging technique used to localize regions of specific brain activity during certain tasks. For T timepoints and N vertices per hemisphere resulting in data $\boldsymbol{y}_{TN\times 1}$ with the latent Gaussian model as follows:

$$
\begin{aligned}
\boldsymbol{y}|\beta,\boldsymbol{b},\boldsymbol{\theta} &\sim N(\boldsymbol{\mu}_y,\boldsymbol{V}), \quad \boldsymbol{\mu}_y = \sum_{k=0}^{K}\boldsymbol{X}_k\beta_k + \sum_{j=1}^{J}\boldsymbol{Z}_j\boldsymbol{b}_j \\
\beta_k &= \boldsymbol{\Psi}_k\boldsymbol{w}_k \quad \text{(SPDE prior on } \beta_k\text{)} \\
\boldsymbol{w}_k|\boldsymbol{\theta} &\sim N(\boldsymbol{0},\boldsymbol{Q}^{-1}_{\tau_k,\kappa_k}) \\
\boldsymbol{b}_j &\sim N(\boldsymbol{0},\delta\boldsymbol{I}) \quad \text{(Diffuse priors for } \boldsymbol{b}_j\text{)} \\
\boldsymbol{\theta} &\sim \pi(\boldsymbol{\theta}),
\end{aligned}
$$

where we have K task signals and J nuisance signals.[3]

[3] Van Niekerk, J., Krainski, E., Rustand, D. and Rue, H., 2023. A new avenue for Bayesian inference with INLA. Computational Statistics & Data Analysis, 181, p.107692.

cs-fMRI model

The data consists of a 3.5-min fMRI for each subject, consisting of 284 volumes, where each subject performs 5 different motor tasks interceded with a 3 second visual cue. Each hemisphere of the brain contained 32492 surface vertices. From these, 5000 are resampled to use for the analysis. This results in a response data vector $\boldsymbol{y}$ of size **2 523 624**, with an SPDE model defined on a mesh with 8795 triangles.

The inference based on the modern formulation of INLA was computed in 148 seconds.

cs-fMRI model

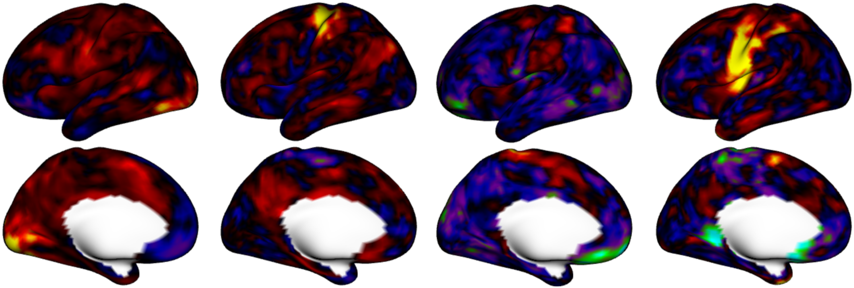


Figure: Activation areas for the different tasks in the left hemisphere - visual cue, right hand motor, right foot motor, tongue motor task (from left to right)

Further details

`www.r-inla.org`

New default setting in INLA (VB) (previously inla.mode = "experimental")

- INLA can fit many different statistical models and complex models can be built using multiple "building blocks"/random effects.
- Remove the linear predictors from the latent field $\rightarrow$ accurate posterior inference with VB correction (I - VB - LA)
- New applications that aren't feasible with INLA 1.0
- Variance and skewness correction - Coming soon!

Rue, H., Martino, S., and Chopin, N. (2009).
Approximate Bayesian inference for latent Gaussian models by using integrated nested laplace approximations.
Journal of the Royal Statistical Society: Series B (Statistical Methodology), 71(2):319–392.

Van Niekerk, J., Krainski, E., Rustand, D., and Rue, H. (2023).
A new avenue for Bayesian inference with INLA.
Computational Statistics & Data Analysis, 181:107692.

Thank you • شكرا

جامعة الملك عبدالله
للعلوم والتقنية
King Abdullah University of
Science and Technology