Agent-Based Modeling of Firm Networks and Innovation

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1 Research Question

Given the findings by Aghion et al. (2005) that illustrate an inverted-U shaped relationship between competition and innovation, this research aims to simulate the second half, downwarding trend, of this relationship, specifically, how competitiveness and network effects can influence firms' innovation decisions in excessive competitive market using agent-based modeling. Specifically, the study examines how market innovation competitiveness, innovation gap between leader firms (with high technological advancement) and follower firms (with lower technological advancement), and technological spillover impacts their likelihood to innovate.

2 Background

Previous studies, including the influential work by Aghion et al. (2005), indicate that moderate competition can spur innovation, but excessive competition or a significant lead by the market leader can stifle the incentive to innovate. Aghion et al. developed a model where competition encourages "neck-and-neck" firms (leader firms) to innovate to escape competition, while it discourages laggard firms (follower firms) from innovating due to reduced post-innovation rents, leading to an inverted-U relationship between competition and innovation. Their model includes both leader and follower firms, where the innovation incentives depend on the difference between post-innovation and pre-innovation rents.

A key element in this study is the incorporation of network elements, specifically technological spillovers, into the modeling equations. The model introduces network effects by considering how technological advancements in leader firms can spill over to follower firms, thereby influencing the followers' innovation intensity. This study uses agent- based modeling to explore these dynamics, incorporating the influence of network effects on firms' innovation success probability, and Innovation gap to model follower firms' ability to catch up / leader firms ability to level off.

3 Initialization/Parameter Set-Up

The Parameter set up and variable explanation is shown in table 1 .

Variable	Explanation	Mean (Std)
Technology Advancement (TAR)	Firm i's technology advancement. Initial TAR is assigned randomly within range of 1 to 100	569.143 (537.332)
TAR Distribution	Initial TAR distributions can be chosen as left-skewed (more firms with low TA), normal, or right-skewed. It simulates initial market condition.	-
Leader Firm	Firms with a TAR higher than the market median (above the 50th percentile).	-
Follower Firm	Firms with a technological lag, trying to catch up with the leader in innovation (TAR at or below the 50th percentile).	-
Innovation Gap	According to Aghion et al. (2005), leader firms may stop innovating if the gap between follower firm i and leader firms' innovations exceeds a predetermined threshold. Adjustable by slider, ranging from 1 to 100 with an interval of 20. The innovation gap sets a threshold for the technological advancement difference ($\lvert \text{TAR}_{\text{median}} - \text{TAR}_i \rvert$) between firms. If the difference exceeds the gap, firms will stop innovating. Ranging from 1 to 60 with increment 1. Parameter sweeps from 10 to 60 with increments 10.	35.5 (17.42)
Network Effect	Adjustable by slider, ranging from 0 to 1 with an interval of 0.1. Higher network effect increases the ability of firms to benefit from the R&D of their network connections. Ranging from 0 to 0.05 with increment 0.005. Parameter sweeps from 0 to 0.05 with increments 0.01.	0.025 (0.0177)
Average Node Degree	Mean number of connections each firm has. Ranging from 3 to 8 with increment 1. Parameter sweeps in 3 and 8 in batch run.	5.5 (1.863)
Network Influence	Generated in analysis section to better assess network influence, and calculated based on the intensity (network effect) and quantity (average node degree), which measures the average number of connection firms have.	0.1375 (0.1127)
Number of Firms	Ranging from 10 to 100 with increment 1. In batch run, it fixed at 50 to provide a fair number that does not significantly influence the simulation.	50
Baseline Success Probability	Industry-level innovation difficulty, adjustable by slider. Ranging from 0 to 1 with increment 0.01. In batch run, this parameter is fixed at 0.5.	0.5 (0.0)
Success Probability Adjustment	Simulates the change in resources available due to the previous step's outcome. Innovation costs are incorporated into success probability as a risk factor. Positive if firm successfully innovated and negative otherwise. For example, if firm i failed in innovation, Small firms may lack funds to invest further, while large firms may see failed innovation as a negative signal, reducing future investments. Ranging from 0.02 to 0.15 with increment 0.01. In batch run, it sweeps in 0.02 , 0.085, 0.15.	0.085 (0.0531)
TAR Gain	The increment in TAR after a successful innovation. Ranging from 5 to 10 with increment 1. In batch run, it sweeps from 2 to 10 with increments 3.	6 (3.266)
Simulation Step	The current step of the simulation.	425.253 (379.791)

The above table also show mean and standard deviation of some parameters in data collected from batch run. Besides the research of interest parameters (Innovation Gap, network effects), this study also incorporates elements of innovation success probability, innovation outcomes, gains and costs, market conditions, innovation distribution, and the number of firms, to better simulate various conditions. The parameter sweeps in the batch run are selected to cover sufficiently refined situations for the research of interest. For other elements, the sweeps mainly cover high, average, and low values, but are also constrained by computational power limitations.The parameter values for the network effect are selected to ensure that the updated success probability calculated in the equation 4 does not exceed 1. Values of other innovation success probability-related parameters are chosen to provide a roughly equivalent impact over the simulation as the network effect.

4 Simulation Explanation

The simulation details for each step are illustrated in the flowchart (Figure 1).

- Setup in the first step of simulation: Firms are assigned initial TAR values randomly from 1 to 100 based on TAR distribution selected.
- Decision to Innovate: Firms decide to innovate if the absolute value of (their current TAR - market TAR median [1]) is smaller parameter innovation gap. They decide not to innovate otherwise.
- Firm Inactive: If a firm decides not to innovate for more than 5 consecutive steps, it becomes inactive. This simulates the real-world scenario in most cases where firms that fail to innovate gradually exit the market due to their inability to keep up with technological advancements and competitive pressures.
- Calculation of Innovation Success Probability: The innovation success probability (P_s) is calculated based on the baseline probability (P_{pre}) and the network effect. The equation for P_s is:

$$P_s = P_{\text{pre}} \left(\left(\sum_{j=1}^{n} 1(N_{i,j} = 1) \cdot 1(TAR_j > TAR_i) \cdot \frac{(TAR_j - TAR_i)}{TAR_{\text{median}}} \right) \times (NE + 1) \right) \tag{1}$$

For each firm i, its innovation success probability (P_s) is determined by the baseline success probability (P_{pre}) multiplied by the network influence. Firm i benefits from

[1] Using the median in innovation decision-making has several benefits. By connecting with the Technology Advancement (TAR), firms with higher value are more likely to level off to reach a "monopoly" status. Additionally, as firms with low TAR exit the market and become inactive, the median TAR of the market increases. This forces remaining firms to keep innovating to continue staying competitive in the market. This approach effectively simulates real business situations, where continuous innovation is necessary to maintain market presence.

firm j if there is a connection between them, and j has a higher TAR (characteristic functions, equal 1 if conditions meet). Then the network influence is calculated by summing the differences between these connected firms' TARs and firm i's TAR (the difference is divided by market TAR median to scale the impact of network), then multiplying by the network effect parameter (NE). Therefore, the wider the difference between firm i and its connected firms j and the higher the network effect parameter (NE), the higher the probability of success.

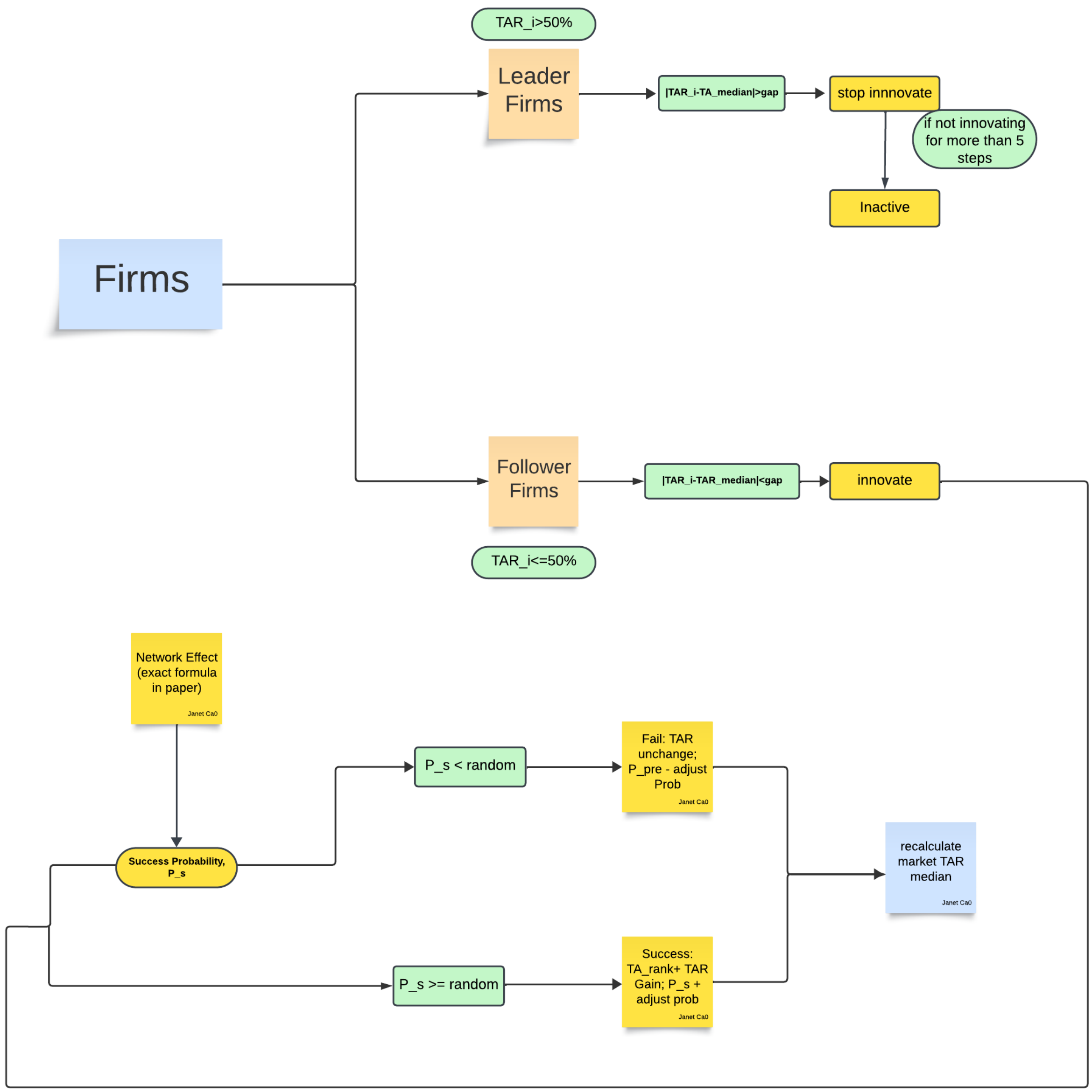

Figure 1: Flowchart Describing the Innovation Decision Process

- Outcome of innovation: it is determined by comparing P_s with a random generated number. If innovation is successful, the firm's TAR increase by parameter TAR Gain and P_s would be increased by success probability adjustment parameter. If fails, the TAR will remain unchange and the P_s for next step would decrease by success probability adjustment parameter.

- Recalculation of market TAR median: The TAR of firms are updated based on the current firms TAR for preparing for next step.
- Stop condition: the simulation stop if there is less than or equal to (`initial total number of firms / 5`) active firms in the market. It mark the formation of a market occupy by few leader firms.

5 Data Collection

Data are collected based on the following parameter sweeps: Average node degree, Innovation Gap, Baseline success probability, TAR gain, Success probability adjustment, and Network Effect. The detailed parameter sweeps are illustrated in Section 3. Data is collected at the last step of each run and is recorded at the individual firm level for TAR, while the other variables are recorded at the simulation level.

The model is run 50 iterations to capture enough variation.

6 Analysis

This section is also displayed in `Competition_network_innovation.git > competition > batch_run.ipynb`.

To assess the effects of networks and innovation gap(firms' ability of catch up or leveling off), on market innovation intensity and time taken to form a market occupied by few companies, I have data visualization using box plots and regression analysis.

6.1 Box Plots

Simulation time taken("step")

Network Influence: The network_influence parameter does not seem to obviously affect the simulation steps, as indicated by the similar distributions across its values in plot 5. This could mean that within the studied range, network_influence does not play a critical role in determining the time to reach the event of interest.

Innovation Gap: Innovation_gap has a obvious positive impact on the number of steps, as shown in plot 2. Higher values of innovation_gap lead to more steps, implying that firms with greater ability to catch up and greater tolerance with innovation gap are more likely to keep innovating and stay in the market. Also, the whole market would stay at competitive state in a longer time.

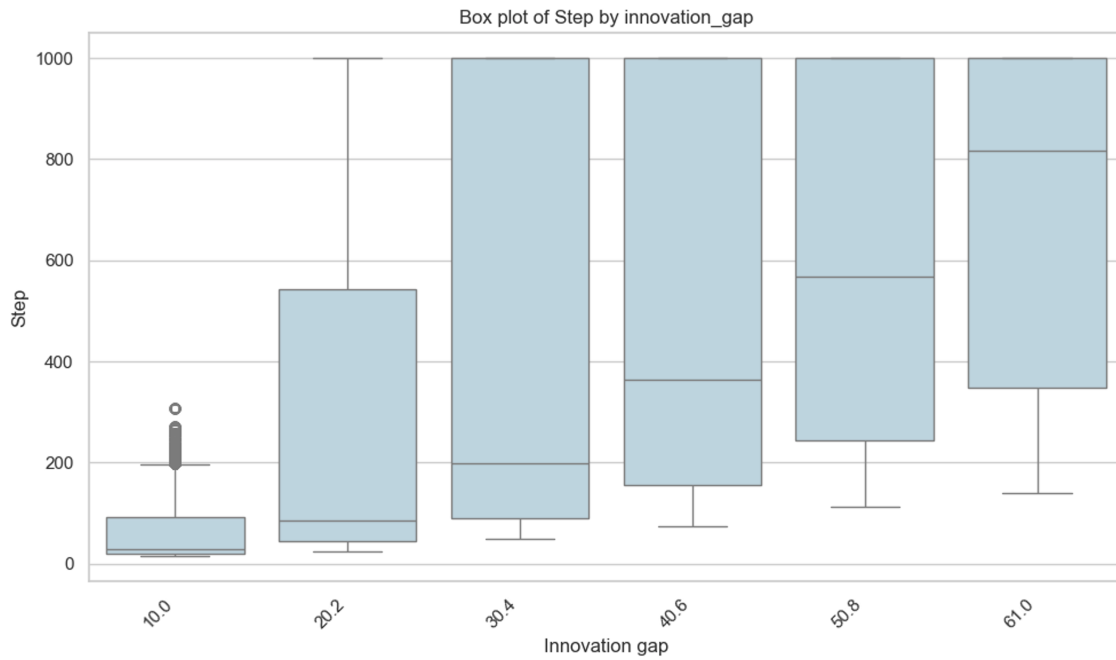

Figure 2: Box Plot of Steps by Innovation Gap

Tech Advancement ("TAR")

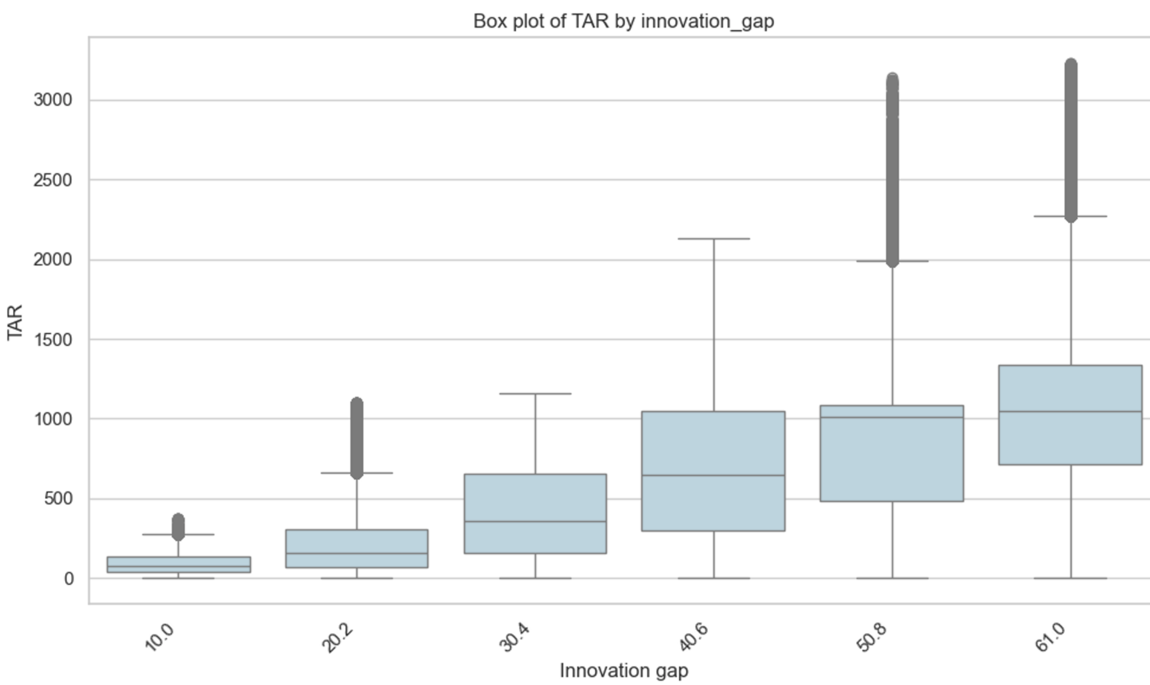

Figure 3: Box Plot of TAR by Innovation Gap

Network Influence: From the box plot of TAR by network influence,as shown in plot 6 there is no obvious effect on the TAR, as indicated by the similar distributions across its values.

Innovation Gap: As shown in plot 3, there is a clear positive relationship between the Innovation Gap and the number of steps. Larger gaps lead to higher TAR and greater variability, suggesting that market with higher innovation gaps value allow firms more time to adjust and catch up, resulting in higher and more diverse tech advancement.

6.2 Regression Analysis

In the previous plots, I observed that there is no obvious impact of network influence on the simulation outcomes. It is possible that the effect of network influence is being confounded by other factors within the model. To better understand the impact of network influence on the TAR, I conducted a regression analysis.

$$\begin{aligned} \ln(\text{TAR}) = \beta_0 &+ \beta_1 \cdot \text{network\_influence} + \beta_2 \cdot \text{innovation\_gap} \\ &+ \beta_3 \cdot \text{tar\_gain} + \beta_4 \cdot \text{Step\_count} \\ &+ \beta_5 \cdot \text{success\_prob\_adjustment} + \epsilon \end{aligned} \tag{2}$$

By regressing the natural logarithm of TAR (lnTAR) on network influence while controlling for other relevant factors, I can isolate the effect of network influence. This approach allows me to account for the multiplicative nature of the relationships in the data and mitigate the impact of outliers or skewed distributions. The control variables included in the regression are innovation gap, tar gain, success probability adjustment, and simulation step. This analysis aims to provide a clearer understanding of how network influence affects TAR when other variables are held constant.

Before conducting the regression, I aggregate the data by RunId to sum TAR, obtaining the market-level TAR for a single run. This approach is necessary because TAR varies for each firm, whereas the other variables are collected at the end of the simulation and remain constant for all firms within a given simulation run.

Regression Result: As shown in regression output table 4.

Network Influence: The coefficient for network influence is 0.1907, which is positive and highly significant (p-value = 0.000). This indicates that for each unit increase in network influence, lnTAR increases by approximately 0.1907 units, holding other factors constant. This result is consistent with the expectation that network influence positively impacts technological advancement and innovation. The positive sign makes sense as network influence can facilitate the copying and sharing of technology among firms, leading to higher success probabilities in innovation and, consequently, greater technological advancements.

Innovation Gap, TAR Gain, Step Count: As expected, they all have positive significant coefficient, indicating that they are sinificantly contributing to tech advancement in the simulation.

Success Probability Adjustment: The coefficient for success probability adjustment is -0.0024, which is not statistically significant (p-value = 0.754). This indicates that variations in the adjustment of success probability do not have a significant impact on lnTAR in this model. This make sense as this adjustment is postive when firm successfully innovate but negative when fail to innovate.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  lnTAR   R-squared:                       0.934
Model:                            OLS   Adj. R-squared:                  0.934
Method:                 Least Squares   F-statistic:                 2.861e+06
Date:                Wed, 22 May 2024   Prob (F-statistic):          1.23e-132
Time:                        18:02:35   Log-Likelihood:                 734.25
No. Observations:               54000   AIC:                            -1457.
Df Residuals:                   53994   BIC:                            -1403.
Df Model:                           5
Covariance Type:              cluster
==========================================================================================
                             coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------------------
const                      8.0914      0.002   4025.192      0.000       8.087       8.095
network_influence          0.1907      0.003     62.431      0.000       0.185       0.197
innovation_gap             0.0358   4.39e-05    815.803      0.000       0.036       0.036
tar_gain                   0.0116      0.000     42.865      0.000       0.011       0.012
Step_count                 0.0011   3.24e-06    325.149      0.000       0.001       0.001
success_prob_adjustment   -0.0024      0.008     -0.314      0.754      -0.017       0.012
==============================================================================
Omnibus:                     1619.305   Durbin-Watson:                   0.791
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1385.371
Skew:                          -0.327   Prob(JB):                    1.48e-301
Kurtosis:                       2.567   Cond. No.                     1.08e+04
==============================================================================
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Figure 4: Regression output of Ln(TAR) on Network influence

7 Conclusion & Discussion

In this study, I aimed to simulate the downward trend of the inverted-U shaped relationship between competition and innovation, focusing on how competitiveness and network effects influence firms' innovation decisions. Utilizing agent-based modeling, I examined the impacts of market innovation competitiveness, the innovation gap between leader and follower firms, and technological spillovers on firms' likelihood to innovate.

My findings suggest that the innovation gap significantly affects the time taken for market convergence and the technological advancement of firms. Larger innovation gaps result in extended simulation steps, indicating that firms with a greater ability to catch up are more likely to continue innovating and remain active in the market. Additionally, the innovation gap positively correlates with higher and more diverse technological advancements (TAR), demonstrating that firms with larger gaps have more time to adjust and innovate, leading to greater variability and higher overall technological progress.

The network effect parameter, while theoretically expected to influence innovation success probability, did not show a significant impact on the simulation steps or TAR in the initial analysis. However, through regression analysis, I found that network influence positively and significantly affects technological advancement when controlling for other variables. This highlights the importance of network connections in facilitating technological sharing and copying, ultimately enhancing firms' innovation success rates and technological progress.

Overall, my study confirms that the dynamics of competition and innovation are complex and multifaceted. The ability of follower firms to catch up with leader firms (innovation gap) plays a crucial role in sustaining innovation within the market. Additionally, network effects, while not immediately evident in simple analysis, are crucial in driving technological advancement when considering their interactions with other factors. These insights can help policymakers and business strategists better understand the factors that drive or hinder innovation in competitive markets.

Future research could expand on this study by exploring different market conditions, varying the number of firms, and incorporating additional factors such as market regulations and global competition. Further research could focus on establishing a more realistic ABM model by having a function accounting for innovation cost calculation instead of incorporating it as a success probability adjustment in this model. Additionally, further study could utilize supercomputers to conduct more comprehensive batch runs so that a more refined condition could be analyzed.

Appendix

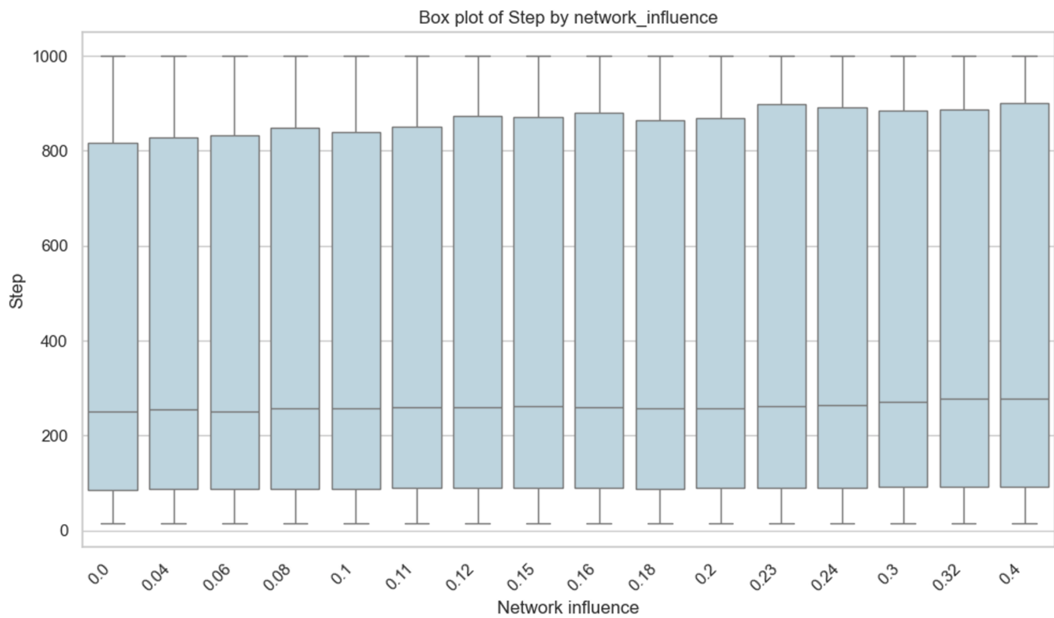

Figure 5: Box plot of Steps by Network Influence

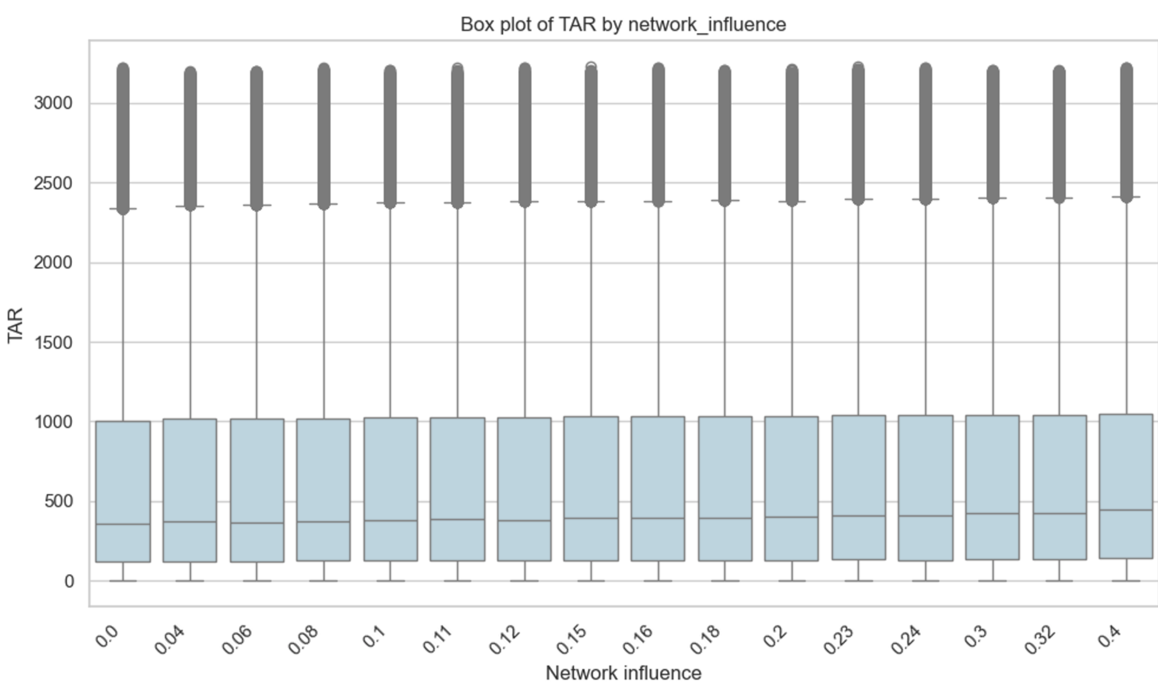

Figure 6: Box plot of TAR by Network Influence

References

[1] Aghion, P., Bloom, N., Blundell, R., Griffith, R., & Howitt, P. (2005). Competition and Innovation: An Inverted-U Relationship. *Quarterly Journal of Economics*, 120(2), 701-728.