Ecosystem modeling

II. From biological model to ecosystem model

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Recap

The governing equation for many biological/chemical subjects are given by

$$\underbrace{\boxed{\frac{dP}{dt}}}_{\text{Temporal change of subject } P} = \sum \underbrace{\boxed{S_o}}_{\text{Source}} - \sum \underbrace{\boxed{S_i}}_{\text{Sink}} \tag{1}$$

For example, phytoplankton concentration P considering growth and mortality can be modeled by

$$\frac{dP}{dt} = UP - \sigma P. \tag{2}$$

Recap

For constant U and σ, it can be (analytically) solved by hand and yields $P = P_0 e^{U_{net} t}$ where $U_{net} = U - \sigma$.

How to use the model (equations and its solution)?

Forward problem

For given parameters and initial condition, P at any time can be calculated and predicted.

Inverse problem

For given $P(t)$, quantified characteristics (parameters) can be estimated (e.g., regression).

Better growth rate model

In most of cases, the growth rate U is not constant but function of environmental factors (e.g., nutrient concentration N, light intensity I, and so on).

$$\frac{dP}{dt} = U(N, I, \cdots)P - \sigma P \tag{3}$$

So what kind of formulation we can use to describe dependency for N and I?

There is no strict rule but good start point is

$$\boxed{U = U_{max} f_I(I) f_N(N) \cdots} \tag{4}$$

where U_{max} is constant maximum growth rate and f_x is limiting function that depends on a factor x.

Better growth rate model

For f_I and f_N, many people "usually" use functions satisfying

$$f_x(0) = 0 \tag{5a}$$

$$0 \leq f_x \leq 1 \tag{5b}$$

$$\lim_{x \to \infty} f_x = 1 \tag{5c}$$

but the exceptions can be easily found (e.g., model considering photoinhibition).

One of the very commonly used functions for f_x is the Monod function:

$$f_x = \frac{x}{x + k_x} \tag{6}$$

where k_x is referred to as half saturation constant because it yields $U = U_{max}/2$ when $x = k_x$.

Better growth rate model

There are many other options such as

$$f_x = \frac{x}{\sqrt{x^2 + {k_x}^2}} \tag{7}$$

$$f_x = (1 - e^{-x/k_x}) \tag{8}$$

where k_x is the saturation (but not half saturation) constant.

Examples:
Powell et al. (2006):

$$U = U_{max} \frac{I}{\sqrt{I^2 + (U_{max}/\alpha)^2}} \frac{N}{N + k_N} \tag{9}$$

Better growth rate model

Temperature (T) dependency

Q10 rule

$$f_T = {Q_{10}}^{T/10} = e^{k_T T} \tag{10}$$

where $k_T = (\ln Q_{10})/10$. This can be used for response of bulk phytoplankton groups, not single phytoplankton group (Eppley, 1972).

Multiple-nutrient (N_1, N_2, ...) dependency

Liebig's law of minimum

$$f_N = \min(f_{N_1}, f_{N_2}, \cdots) \tag{11}$$

Considering changes in environments

Note that (3) is a biological model for P, not an ecosystem model, because it does not consider its influence on surrounding environment (e.g., nutrient consumption through uptake).

It can be considered by additional governing equation for N:

$$\frac{dP}{dt} = U(N)P - \sigma P \tag{12a}$$

$$\boxed{\frac{dN}{dt} = -U(N)P + \sigma P} \tag{12b}$$

Summation of (12a) and (12b) yields

$$\frac{d(N+P)}{dt} = 0, \qquad \therefore N + P = N_0 + P_0 = N_T \tag{13}$$

representing mass conservation. N_T indicates total mass determined by the initial condition.

Lab 1

Build Nutrient-Phytoplankton (NP) model of which governing equation given by

$$\frac{dP}{dt} = UP - \sigma P \tag{14a}$$

$$\frac{dN}{dt} = -UP + \sigma P \tag{14b}$$

where

$$U = U_{max}\frac{N}{N + k_N} \tag{14c}$$

1. Run the model (solve the equations) with parameters $U_{max} \approx 0.5\,d^{-1}$, $k_N \approx 0.1\,\mu M$, $\sigma \approx 0.1\,d^{-1}$, $N|_{t=0} \approx 5.0\,\mu M,\ P|_{t=0} \approx 0.1\,\mu M$.
2. Run the model with changed initial condition $N|_{t=0} \approx 0.005\,\mu M,\ P|_{t=0} \approx 0.015\,\mu M$

Lab 2

Model with two different phytoplankton group can be formulated by

$$\frac{dP_1}{dt} = U_1 P_1 - \sigma_1 P_1, \qquad U_1 = U_{max1} \frac{N}{N + k_{N1}} \tag{15a}$$

$$\frac{dP_2}{dt} = U_2 P_2 - \sigma_2 P_2, \qquad U_2 = U_{max2} \frac{N}{N + k_{N2}} \tag{15b}$$

1. In the same manner with (12), what is proper governing equation for N?
2. With the equation for N, solve the model with arbitrary parameters. See that phytoplankton groups are seldomly coexist.

References I

Eppley, R. W. (1972). "Temperature and phytoplankton growth in the sea". In: *Fishery bulletin* 70.4.

Powell, T. M. et al. (2006). "Results from a three-dimensional, nested biological-physical model of the California Current System and comparisons with statistics from satellite imagery". In: *Journal of Geophysical Research: Oceans* 111.C7.