Ecosystem modeling

III. Adding zooplankton and biological oscillation

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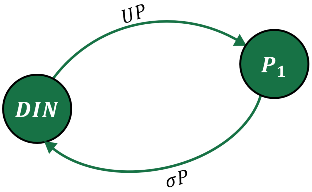
Recap

Your first ecosystem model considering nutrient-phytoplankton interaction was given by

$$\frac{dN}{dt} = -UP + \sigma P \qquad (1a)$$

$$\frac{dP}{dt} = UP - \sigma P \qquad (1b)$$

$$U = U_{max} \frac{N}{N + k_N} \qquad (1c)$$



Adding zooplankton

We can add additional variables representing zooplankton (Z) group:

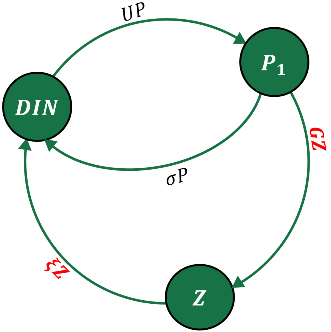
$$\frac{dN}{dt} = -UP + \sigma P + \xi Z \tag{2a}$$

$$\frac{dP}{dt} = UP - \sigma P - GZ \tag{2b}$$

$$\frac{dZ}{dt} = GZ - \xi Z \tag{2c}$$

$$U = U_{max} \frac{N}{N + k_N} \tag{2d}$$

$$G = R_m \left(1 - e^{-\Lambda P}\right) \tag{2e}$$



where G and ξ are zooplankton growth rate, depending on phytoplankton concentration P, and mortality rate, respectively.

Adding zooplankton

We can add additional variables representing zooplankton (Z) group with inefficiency:

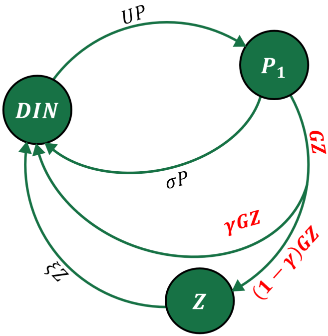
$$\frac{dN}{dt} = -UP + \sigma P + \xi Z + \gamma GZ \quad (3a)$$

$$\frac{dP}{dt} = UP - \sigma P - GZ \quad (3b)$$

$$\frac{dZ}{dt} = (1-\gamma)GZ - \xi Z \quad (3c)$$

$$U = U_{max}\frac{N}{N+k_N} \quad (3d)$$

$$G = R_m\left(1 - e^{-\Lambda P}\right) \quad (3e)$$



where G and ξ are zooplankton growth rate, depending on phytoplankton concentration P, and mortality rate, respectively. Formulation is from Powell et al., 2006.

Lab 1

Develop Nutrient-Phytoplankton-Zooplankton (NPZ) ODE model formulated by

$$\frac{dN}{dt} = -UP + \sigma P + \xi Z + \gamma GZ \quad (4a)$$

$$\frac{dP}{dt} = UP - \sigma P - GZ \quad (4b)$$

$$\frac{dZ}{dt} = (1-\gamma)GZ - \xi Z \quad (4c)$$

$$U = U_{max}\frac{N}{N + k_N} \quad (4d)$$

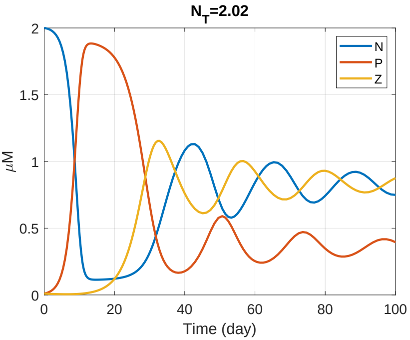
$$G = R_m\left(1 - e^{-\Lambda P}\right) \quad (4e)$$

Symbol	Value	Unit
U_{max}	1.0	d^{-1}
k_N	1.0	μM
σ	0.1	d^{-1}
R_m	0.5	d^{-1}
ξ	0.15	d^{-1}
Λ	1.0	μM^{-1}
γ	0.0	-

Run model to $t = 100\,d^{-1}$ with initial conditions: $P = 0.01\,\mu M$, $Z = 0.01\,\mu M$ under different initial nutrient concentrations $N = 0.01\,\mu M$, $N = 0.2\,\mu M$, $N = 1.0\,\mu M$, and $N = 2.0\,\mu M$.

Biological oscillation

Once total nitrogen in the system exceeds a threshold (eutrophic environment), equilibrium becomes unstable (Busenberg et al., 1990) due to prey-predator interaction(Lotka, 1920; Volterra, 1928).



Lab 2

Develop ecosystem model[1] formulated by:

$$\frac{dN}{dt} = -U_1 P_1 - U_2 P_2 + \sigma_1 P_1 + \sigma_2 P_2 + \xi Z \tag{5a}$$

$$\frac{dP_1}{dt} = U_1 P_1 - \sigma P_1 - GZ, \quad U_1 = U_{max1} \frac{N}{N + k_{N1}} \tag{5b}$$

$$\frac{dP_2}{dt} = U_2 P_2 - \sigma P_2, \quad U_2 = U_{max2} \frac{N}{N + k_{N2}} \tag{5c}$$

$$\frac{dZ}{dt} = GZ - \xi Z, \quad G = R_m \left(1 - e^{-\Lambda P}\right) \tag{5d}$$

1. What's the key difference between equation for P_1 and P_2?
2. Conduct sensitivity experiment for total nitrogen and shows that both phytoplankton groups can coexist.

[1] Choi, Lippmann, and Harvey, 2023

References I

Busenberg, Stavros et al. (1990). “The dynamics of a model of a plankton-nutrient interaction”. In: *Bulletin of Mathematical Biology* 52.5, pp. 677–696.

Choi, Jang-Geun, Thomas C Lippmann, and Elizabeth L Harvey (2023). “Analytical population dynamics underlying harmful algal blooms triggered by prey avoidance”. In: *Ecological Modelling* 481, p. 110366.

Lotka, Alfred J (1920). “Analytical note on certain rhythmic relations in organic systems”. In: *Proceedings of the National Academy of Sciences* 6.7, pp. 410–415.

Powell, T. M. et al. (2006). “Results from a three-dimensional, nested biological-physical model of the California Current System and comparisons with statistics from satellite imagery”. In: *Journal of Geophysical Research: Oceans* 111.C7.

References II

Volterra, Vito (1928). "Variations and fluctuations of the number of individuals in animal species living together". In: *ICES Journal of marine science* 3.1, pp. 3–51.