

# Ecosystem modeling

## III. Adding zooplankton and biological oscillation

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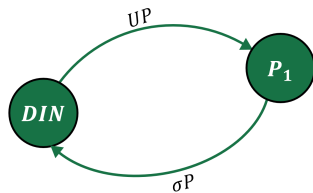
# Recap

Your first ecosystem model considering nutrient-phytoplankton interaction was given by

$$\frac{dN}{dt} = -UP + \sigma P \quad (1a)$$

$$\frac{dP}{dt} = UP - \sigma P \quad (1b)$$

$$U = U_{max} \frac{N}{N + k_N} \quad (1c)$$



## Adding zooplankton

We can add additional variables representing zooplankton ( $Z$ ) group:

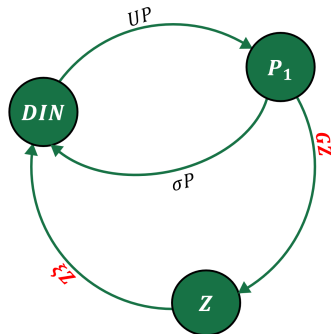
$$\frac{dN}{dt} = -UP + \sigma P + \xi Z \quad (2a)$$

$$\frac{dP}{dt} = UP - \sigma P - GZ \quad (2b)$$

$$\frac{dZ}{dt} = GZ - \xi Z \quad (2c)$$

$$U = U_{max} \frac{N}{N + k_N} \quad (2d)$$

$$G = R_m (1 - e^{\Lambda P}) \quad (2e)$$



where  $G$  and  $\xi$  are zooplankton growth rate, depending on phytoplankton concentration  $P$ , and mortality rate, respectively.

## Adding zooplankton

We can add additional variables representing zooplankton ( $Z$ ) group with inefficiency:

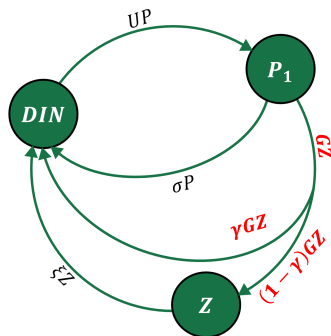
$$\frac{dN}{dt} = -UP + \sigma P + \xi Z + \gamma GZ \quad (3a)$$

$$\frac{dP}{dt} = UP - \sigma P - GZ \quad (3b)$$

$$\frac{dZ}{dt} = (1 - \gamma)GZ - \xi Z \quad (3c)$$

$$U = U_{max} \frac{N}{N + k_N} \quad (3d)$$

$$G = R_m (1 - e^{\Lambda P}) \quad (3e)$$



where  $G$  and  $\xi$  are zooplankton growth rate, depending on phytoplankton concentration  $P$ , and mortality rate, respectively. Formulation is from Powell et al., 2006.

# Lab 1

Develop Nutrient-Phytoplankton-Zooplankton (NPZ) ODE model formulated by

$$\frac{dN}{dt} = -UP + \sigma P + \xi Z + \gamma GZ \quad (4a)$$

$$\frac{dP}{dt} = UP - \sigma P - GZ \quad (4b)$$

$$\frac{dZ}{dt} = (1 - \gamma)GZ - \xi Z \quad (4c)$$

$$U = U_{max} \frac{N}{N + k_N} \quad (4d)$$

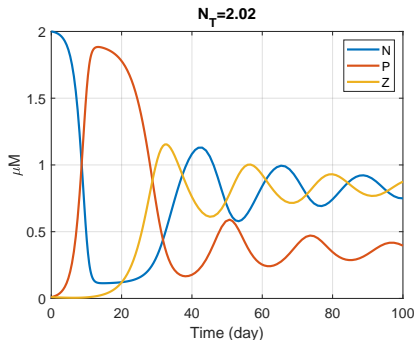
$$G = R_m (1 - e^{\Lambda P}) \quad (4e)$$

Symbol	Value	Unit
$U_{max}$	1.0	$d^{-1}$
$k_N$	1.0	$\mu M$
$\sigma$	0.1	$d^{-1}$
$R_m$	0.5	$d^{-1}$
$\xi$	0.15	$d^{-1}$
$\Lambda$	1.0	$\mu M^{-1}$

Run model with initial conditions:  $P = 0.01 \mu M$ ,  $Z = 0.01 \mu M$  under different initial nutrient concentrations  $N = 0.01 \mu M$ ,  $N = 0.2 \mu M$ ,  $N = 1.0 \mu M$ , and  $N = 2.0 \mu M$ .

# Biological oscillation

Once total nitrogen in the system exceeds a threshold (eutrophic environment), equilibrium becomes unstable (Busenberg et al., 1990) due to prey-predator interaction (Lotka, 1920; Volterra, 1928).



## Lab 2

Develop ecosystem model<sup>1</sup> formulated by:

$$\frac{dN}{dt} = -U_1P_1 - U_2P_2 + \sigma_1P_1 + \sigma_2P_2 + \xi Z \quad (5a)$$

$$\frac{dP_1}{dt} = U_1P_1 - \sigma P_1 - GZ, \quad U_1 = U_{max1} \frac{N}{N + k_{N1}} \quad (5b)$$

$$\frac{dP_2}{dt} = U_2P_2 - \sigma P_2, \quad U_2 = U_{max2} \frac{N}{N + k_{N2}} \quad (5c)$$





$$\frac{dZ}{dt} = GZ - \xi Z, \quad G = R_m (1 - e^{\Lambda P}) \quad (5d)$$

1. What's the key difference between equation for  $P_1$  and  $P_2$ ?
2. Conduct sensitivity experiment for total nitrogen and shows that both phytoplankton groups can coexist.

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<sup>1</sup>Choi, Lippmann, and Harvey, 2023

# References I

-  Busenberg, Stavros et al. (1990). “The dynamics of a model of a plankton-nutrient interaction”. In: *Bulletin of Mathematical Biology* 52.5, pp. 677–696.
-  Choi, Jang-Geun, Thomas C Lippmann, and Elizabeth L Harvey (2023). “Analytical population dynamics underlying harmful algal blooms triggered by prey avoidance”. In: *Ecological Modelling* 481, p. 110366.
-  Lotka, Alfred J (1920). “Analytical note on certain rhythmic relations in organic systems”. In: *Proceedings of the National Academy of Sciences* 6.7, pp. 410–415.
-  Powell, T. M. et al. (2006). “Results from a three-dimensional, nested biological-physical model of the California Current System and comparisons with statistics from satellite imagery”. In: *Journal of Geophysical Research: Oceans* 111.C7.



# References II



Volterra, Vito (1928). “Variations and fluctuations of the number of individuals in animal species living together”. In: *ICES Journal of marine science* 3.1, pp. 3–51.