Ecosystem modeling

IV. Adding detritus and dissipative system

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Recap

We talked about Nutrient-Phytoplankton-Zooplankton (NPZ) model, assuming that dead bodies of plankton instantaneously becomes nutrient:

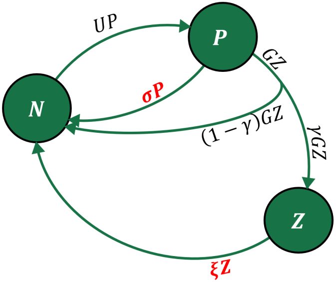
$$\frac{dN}{dt} = -UP + \sigma P + \xi Z + \gamma GZ \quad (1a)$$

$$\frac{dP}{dt} = UP - \sigma P - GZ \quad (1b)$$

$$\frac{dZ}{dt} = (1-\gamma)GZ - \xi Z \quad (1c)$$

$$U = U_{max}\frac{N}{N + k_N} \quad (1d)$$

$$G = R_m\left(1 - e^{-\Lambda P}\right) \quad (1e)$$



Adding detritus

This can be revised by considering additional variable "detritus D" representing the intermediate form between dead body and dissolved nutrient.

$$\frac{dN}{dt} = -UP + \gamma GZ + \delta D \quad \text{(2a)}$$

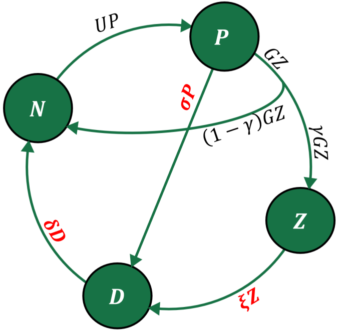
$$\frac{dP}{dt} = UP - \sigma P - GZ \quad \text{(2b)}$$

$$\frac{dZ}{dt} = (1-\gamma)GZ - \xi Z \quad \text{(2c)}$$

$$\frac{dD}{dt} = \sigma P + \xi Z - \delta D \quad \text{(2d)}$$

$$U = U_{max}\frac{N}{N + k_N} \quad \text{(2e)}$$

$$G = R_m\left(1 - e^{-\Lambda P}\right) \quad \text{(2f)}$$



Adding detritus

In this Nutrient-Phytoplankton-Zooplankton-Detritus (NPZD) model, you two options: mortality to detritus and nutrient pools. This is up to the choice[1].

$$\frac{dN}{dt} = -UP + \gamma GZ + \delta D + \sigma_N P + \xi_N Z \tag{3a}$$

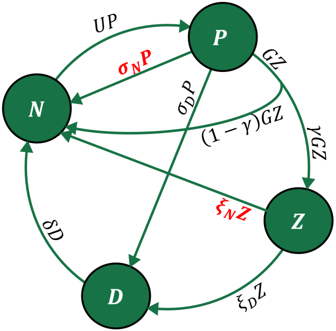
$$\frac{dP}{dt} = UP - (\sigma_N + \sigma_D)P - GZ \tag{3b}$$

$$\frac{dZ}{dt} = (1-\gamma)GZ - (\xi_N + \xi_D)Z \tag{3c}$$

$$\frac{dD}{dt} = \sigma_D P + \xi_D Z - \delta D \tag{3d}$$

$$U = U_{max}\frac{N}{N + k_N} \tag{3e}$$

$$G = R_m\left(1 - e^{-\Lambda P}\right) \tag{3f}$$



[1] Powell et al., 2006; Heinle and Slawig, 2013; Fennel and Neumann, 2014

Adding detritus

Note that the detritus is subject to the vertical sinking (marine snow), that can be described by an advection term.

$$\frac{dN}{dt} = -UP + \gamma GZ + \delta D + \sigma_N P + \xi_N Z \tag{4a}$$

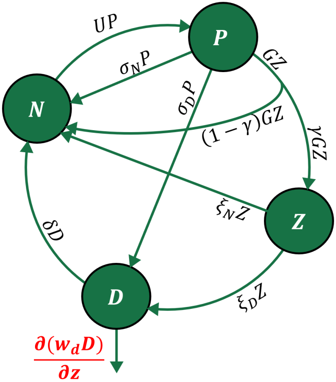
$$\frac{dP}{dt} = UP - (\sigma_N + \sigma_D)P - GZ \tag{4b}$$

$$\frac{dZ}{dt} = (1-\gamma)GZ - (\xi_N + \xi_D)Z \tag{4c}$$

$$\frac{dD}{dt} - \frac{\partial(w_d D)}{\partial z} = \sigma_D P + \xi_D Z - \delta D \tag{4d}$$

$$U = U_{max}\frac{N}{N + k_N} \tag{4e}$$

$$G = R_m\left(1 - e^{-\Lambda P}\right) \tag{4f}$$



Lab 1

Develop NPZD model formulated by Powell et al., 2006 with approximated detritus sinking term (Choi and Lippmann, 2024):

$$\frac{dN}{dt} = -UP + \gamma GZ + \delta D \quad (5a)$$

$$\frac{dP}{dt} = UP - \sigma P - GZ \quad (5b)$$

$$\frac{dZ}{dt} = (1-\gamma)GZ - \xi Z \quad (5c)$$

$$\frac{dD}{dt} = \sigma P + \xi Z - \delta D \boxed{- \frac{w_d}{H_s} D} \quad (5d)$$

Approximated sinking term

$$U = U_{max} \frac{N}{N + k_N} \quad (5e)$$

$$G = R_m \left(1 - e^{-\Lambda P}\right). \quad (5f)$$

Symbol	Value	Unit
U_{max}	1.0	d^{-1}
k_N	1.0	μM
σ	0.1	d^{-1}
R_m	0.5	d^{-1}
ξ	0.15	d^{-1}
Λ	1.0	μM^{-1}
γ	0.0	-
δ	1.0	d^{-1}
w_d/H_s	0.8	d^{-1}

Run model with and without $(w_d/H_s = 0)$ detritus sinking term.

Dissipative system

Mass is no more conservative. We need additional supply term (e.g., nutrient supply via vertical mixing) to sustain ecosystem.

$$\frac{dN}{dt} = -UP + \gamma GZ + \delta D + \sigma_N P + \xi_N Z + F(t) \quad (6a)$$

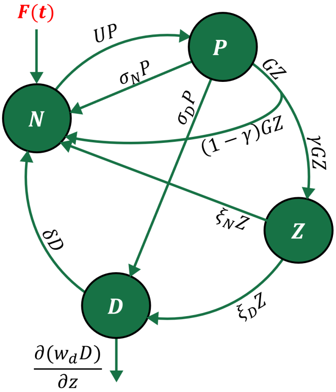
$$\frac{dP}{dt} = UP - (\sigma_N + \sigma_D)P - GZ \quad (6b)$$

$$\frac{dZ}{dt} = (1-\gamma)GZ - (\xi_N + \xi_D)Z \quad (6c)$$

$$\frac{dD}{dt} - \frac{\partial (w_d D)}{\partial z} = \sigma_D P + \xi_D Z - \delta D \quad (6d)$$

$$U = U_{max}\frac{N}{N + k_N} \quad (6e)$$

$$G = R_m\left(1 - e^{-\Lambda P}\right) \quad (6f)$$



Box model approach

$$\frac{dN}{dt} = -UP + \gamma GZ + \delta D + \sigma_N P + \xi_N Z - A(N - N^{(2)}) \quad (7a)$$

$$\frac{dP}{dt} = UP - (\sigma_N + \sigma_D)P - GZ \quad (7b)$$

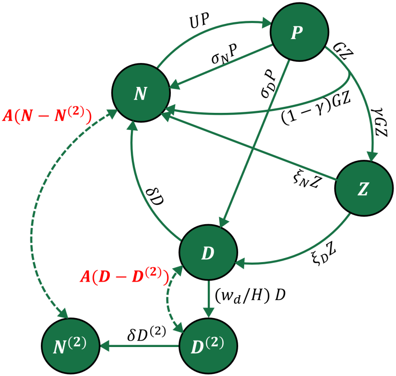
$$\frac{dZ}{dt} = (1-\gamma)GZ - (\xi_N + \xi_D)Z \quad (7c)$$

$$\frac{dD}{dt} = \sigma_D P + \xi_D Z - \delta D - \frac{w_d}{H_s} D - A(D - D^{(2)}) \quad (7d)$$

$$\frac{dN^{(2)}}{dt} = \delta D^{(2)} + A(N - N^{(2)}) \quad (7e)$$

$$\frac{dD^{(2)}}{dt} = \frac{w_d}{H_s} D - \delta D^{(2)} + A(D - D^{(2)}) \quad (7f)$$

We can model subsurface variables and their interaction with surface variables (simple box model example from Fennel and Neumann, 2014).



References I

Choi, Jang-Geun and Thomas C Lippmann (2024). "Biogeochemical dynamics underlying equilibrium between nitrogen fixation and denitrification and its impact on a coastal marine ecosystem model". In: *Ecological Modelling* 494, p. 110767.

Fennel, Wolfgang and Thomas Neumann (2014). *Introduction to the modelling of marine ecosystems*. Vol. 72. Elsevier.

Heinle, Anna and Thomas Slawig (2013). "Internal dynamics of NPZD type ecosystem models". In: *Ecological modelling* 254, pp. 33–42.

Powell, T. M. et al. (2006). "Results from a three-dimensional, nested biological-physical model of the California Current System and comparisons with statistics from satellite imagery". In: *Journal of Geophysical Research: Oceans* 111.C7.