Ecosystem modeling

III. Considering hydrodynamics: advection and diffusion

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Recap

An example of Nutrient-Phytoplankton-Zooplankton-Detritus (NPZD) model is given by

$$\frac{dN}{dt} = -UP + \gamma GZ + \delta D \quad (1a)$$

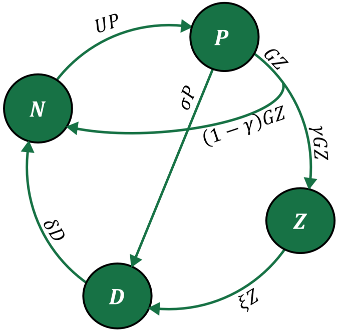
$$\frac{dP}{dt} = UP - \sigma P - GZ \quad (1b)$$

$$\frac{dZ}{dt} = (1-\gamma)GZ - \xi Z \quad (1c)$$

$$\frac{dD}{dt} = \sigma P + \xi Z - \delta D \quad (1d)$$

$$U = U_{max}\frac{N}{N+k_N} \quad (1e)$$

$$G = R_m\left(1 - e^{-\Lambda P}\right) \quad (1f)$$



that do not consider physical transport mechanism (advection and mixing).

Considering physical transport

The governing equations for biological/chemical subjects can be generalized to

Temporal change of subject N

$$\boxed{\frac{dN}{dt}} = \sum \boxed{S_o^N} - \sum \boxed{S_i^N} \tag{2}$$

Source of N ⋮ Sink of N

The physical transport terms can be considered by additional terms:

“**Advection**” $\equiv \nabla \cdot (\vec{u}N)$

$$\begin{aligned} \frac{\partial N}{\partial t} + \boxed{\frac{\partial (uN)}{\partial x} + \frac{\partial (vN)}{\partial y} + \frac{\partial (wN)}{\partial z}} &= \sum S_o^N - \sum S_i^N \\ &+ \boxed{\frac{\partial}{\partial x}\left(K_h \frac{\partial N}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_h \frac{\partial N}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial N}{\partial z}\right)} \end{aligned} \tag{3}$$

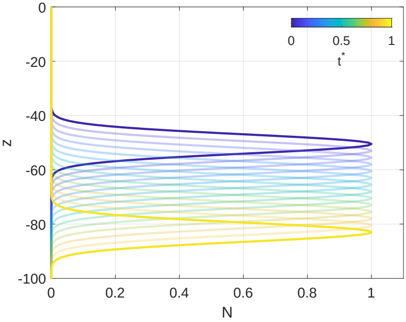
“**Diffusion (mixing)**” $\equiv \nabla \cdot (\vec{K}\nabla N)$

Advection and diffusion

Advection equation

$$\frac{\partial N}{\partial t} + \frac{\partial (uN)}{\partial x} + \frac{\partial (vN)}{\partial y} + \frac{\partial (wN)}{\partial z} = 0 \tag{4}$$

where u, v, and w are velocity component in x-, y-, and z-directions, respectively.

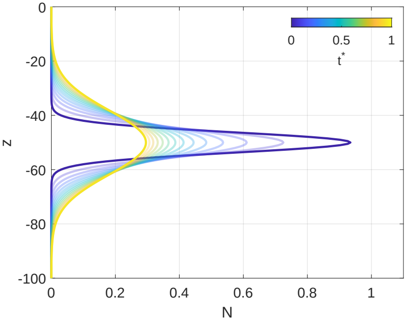


Advection and diffusion

Diffusion equation

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x}\left(K_h \frac{\partial N}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_h \frac{\partial N}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial N}{\partial z}\right) \qquad (5)$$

where K_h and K_z are the horizontal and vertical diffusion coefficients, in m^2/s.



NPZD model coupled with hydrodynamics

$$\frac{\partial N}{\partial t} + \underbrace{\nabla \cdot (\vec{u}N)}_{\text{Transport}} = \underbrace{\nabla \cdot (\vec{K}\nabla N)}_{\text{(advection \& diffusion)}} \underbrace{- UP + \gamma GZ + \delta D}_{\text{Reaction (biogeochemical processes)}} \tag{6a}$$

$$\frac{\partial P}{\partial t} + \nabla \cdot (\vec{u}P) = \nabla \cdot (\vec{K}\nabla P) + UP - \sigma P - GZ \tag{6b}$$

$$\frac{\partial Z}{\partial t} + \nabla \cdot (\vec{u}Z) = \nabla \cdot (\vec{K}\nabla Z) + (1-\gamma)GZ - \xi Z \tag{6c}$$

$$\frac{\partial D}{\partial t} + \nabla \cdot (\vec{u}D) \underbrace{- \frac{\partial (w_d D)}{\partial z}}_{\text{Detritus sinking}} = \nabla \cdot (\vec{K}\nabla D) + \sigma P + \xi Z - \delta D \tag{6d}$$

where velocity field $\vec{u}$ and diffusion coefficient $\vec{K}$ can be provided by hydrodynamics models (e.g., Regional Ocean Modeling System; ROMS) as Powell et al. (2006) did.

Simple 1D (vertical) model example

Note that the 1D (vertical) model is good enough in many cases and extremely useful in some cases (Choi et al., 2024). Below is a simple example of the 1D model.

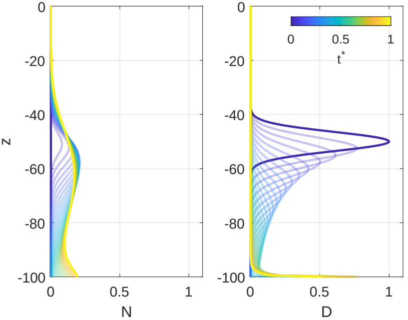
Transport (diffusion)

$$\frac{\partial N}{\partial t} = A_z \frac{\partial^2 N}{\partial z^2} + \delta D \qquad \text{(7a)}$$

$$\frac{\partial D}{\partial t} - \frac{\partial (w_d D)}{\partial z} = A_z \frac{\partial^2 D}{\partial z^2} - \delta D \qquad \text{(7b)}$$

Detritus sinking

Reaction
(detritus remineralization)



References I

Choi, Jang-Geun et al. (2024). "A new ecosystem model for Arctic phytoplankton phenology from ice-covered to open-water periods: Implications for future sea ice retreat scenarios". In: *Geophysical Research Letters* 51.19, e2024GL110155.

Powell, T. M. et al. (2006). "Results from a three-dimensional, nested biological-physical model of the California Current System and comparisons with statistics from satellite imagery". In: *Journal of Geophysical Research: Oceans* 111.C7.