Error at w_{11}^1 $\frac{\partial E_{total}}{\partial w_{11}^1} = \frac{\partial E_{total}}{\partial a_{11}} \frac{\partial a_{11}}{\partial z_{11}} \frac{\partial z_{11}}{\partial w_{11}^1}$

$$\frac{\partial E_{total}}{\partial a_{11}} = \frac{\partial E_1}{\partial a_{11}} + \frac{\partial E_2}{\partial a_{11}}$$

$$= \frac{\partial E_1}{\partial a_{21}} \frac{\partial a_{21}}{\partial a_{11}} + \frac{\partial E_2}{\partial a_{21}} \frac{\partial a_{21}}{\partial a_{11}}$$

$$= \frac{\partial E_1}{\partial a_{21}} \frac{\partial a_{21}}{\partial z_{21}} \frac{\partial z_{21}}{\partial a_{11}} + \frac{\partial E_2}{\partial a_{21}} \frac{\partial a_{21}}{\partial z_{22}} \frac{\partial z_{22}}{\partial a_{11}}$$

$$= \left[\tfrac{1}{2} \cdot 2 (y_1 - \hat{y}_1)(-1)\right]\left[(\hat{y}_1)(1-\hat{y}_1)\right](w_{11}^2)$$

$$+ \left[\tfrac{1}{2} \cdot 2 (y_2 - \hat{y}_2)(-1)\right]\left[(\hat{y}_2)(1-\hat{y}_2)\right](w_{21}^2)$$

$$= (\hat{y}_1 - y_1)(1-\hat{y}_1)(\hat{y}_1)(w_{11}^2) + (\hat{y}_2 - y_2)(1-\hat{y}_2)(\hat{y}_2)(w_{21}^2)$$

$$= (0.107215)(0.392785)(0.607215)(0.4)$$

$$+ (0.268625)(0.368625)(0.631375)(0.5)$$

$$= 0.01022852 + 0.03125997$$

$$= 0.04148849$$

$$\frac{\partial a_{11}}{\partial z_{11}} = a_{11}(1-a_{11}) = (0.512490)(1-0.512490) = 0.24984382$$

$$\frac{\partial z_{11}}{\partial w_{11}'} = x_1 = 0.5$$

$$\frac{\partial E_{total}}{\partial w_{11}'} = 0.00518282$$

$$W_{11}' \leftarrow w_{11}' - \eta \frac{\partial E_{total}}{\partial w_{11}'}$$

$$\leftarrow 0.1 - 0.5\,(0.00518282)$$

$$\leftarrow 0.09740859$$

2020년 4월 3일 금요일 오후 5:21

1번

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Reciprocal Rule

$$\left(\frac{1}{u(x)}\right)' = (u(x)^{-1})'$$

$$= -\left(\frac{y'(x)}{u(x)^2}\right)$$

$$= -u(x)^{-2} \cdot u'(x)$$

$$\sigma'(x) = \frac{d}{dx}\sigma(x) = \frac{d}{dx}\left(\frac{1}{1+e^{-x}}\right) = \frac{d}{dx}(1+e^{-x})^{-1}$$

$$= -(1+e^{-x})^{-2}\frac{d}{dx}(1+e^{-x}) = -(1+e^{-x})^{-2}(0+e^{-x}(-1))$$

$$= (1+e^{-x})^{-2} \cdot e^{-x} = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{(1+e^{-x})} \cdot \frac{e^{-x}+1-1}{(1+e^{-x})}$$

$$= \frac{1}{(1+e^{-x})}\left(\frac{1+e^{-x}-1}{1+e^{-x}}\right) = \frac{1}{1+e^{-x}}\left(1-\frac{1}{1+e^{-x}}\right) \quad \sigma(x) = \frac{1}{1+e^{-x}}$$

$$= \sigma(x)(1-\sigma(x))$$

2번

(1) 임의의 w와 b ⇒ $w_1 = -1$ $w_2 = 1$ $b = 1$

	x_1	x_2	→ z $(w_1x_1 + w_2x_2 + b)$	→ ϕ	→ $\hat{y}$	y (실제)
①	1	1	$-1 \cdot 1 + 1 \cdot 1 + 1 = 1$	$\phi(1)$	1	1
②	1	0	$-1 \cdot 1 + 1 \cdot 0 + 1 = 0$	$\phi(0)$	1	0
③	0	1	$-1 \cdot 0 + 1 \cdot 1 + 1 = 1$	$\phi(1)$	1	1
④	0	0	$-1 \cdot 0 + 1 \cdot 0 + 1 = 1$	$\phi(1)$	1	1

(2) 학습률 $\eta = 0.05$ $\quad W \leftarrow W + \eta(y - \hat{y})x$

① $b \leftarrow b + 0.05(1-1)1$ No Update $b = 1$

$W_1 \leftarrow W_1 + 0.05(0-1)\cdot 1$ $\quad W_1 \leftarrow -1 - 0.05 = -1.05$ $\quad W_1 = -1.05$

$W_2 \leftarrow W_2 + 0.05(0-1)\cdot 1$ $\quad W_2 \leftarrow 1 - 0.05 = 0.95$ $\quad W_2 = 0.95$

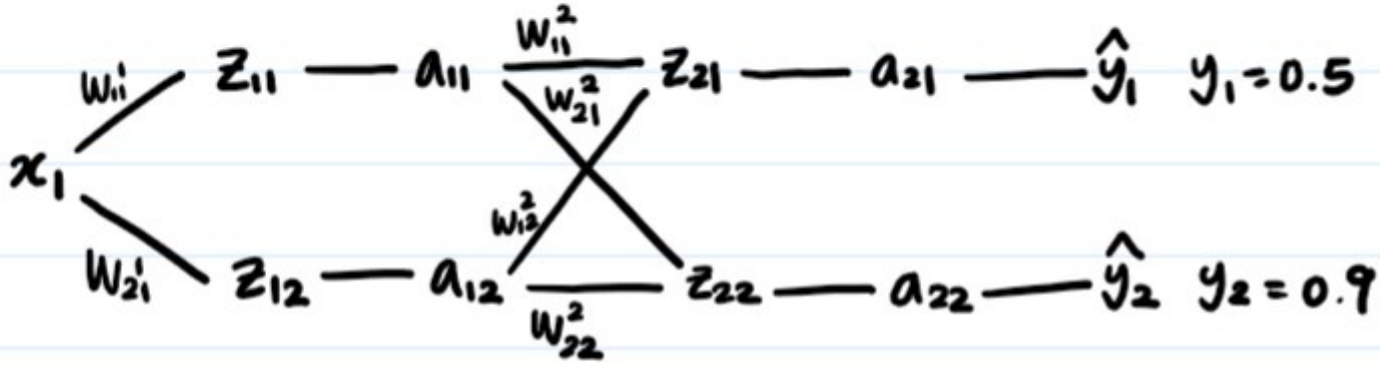
②③④ $y - \hat{y} = 0$으로 No Update !!

다시 분류 ($b = 1$ $\quad W_1 = -1.05$ $\quad W_2 = 0.95$)

	x_1	x_2	$\rightarrow \Sigma (W_1x_1 + W_2x_2 + b)$	$\rightarrow \phi$	$\rightarrow \hat{y}$	y (실제)
①	1	1	$-1.05\cdot 1 + 0.95\cdot 1 + 1$	$\phi(0.9)$	1	1
②	1	0	$-1.05\cdot 1 + 0.95\cdot 0 + 1$	$\phi(-0.05)$	0	0
③	0	1	$-1.05\cdot 0 + 0.95\cdot 1 + 1$	$\phi(1.95)$	1	1
④	0	0	$-1.05\cdot 0 + 0.95\cdot 0 + 1$	$\phi(1)$	1	1

3번

풀이 명확하게 그림 사알짝 수정



$x_1 = 0.5$ $W_{11}^1 = 0.1$ $W_{11}^2 = 0.4$ $W_{12}^2 = 0.45$ $y_1 = 0.5$

$W_{21}^1 = 0.1$ $W_{21}^2 = 0.5$ $W_{22}^2 = 0.55$ $y_2 = 0.9$

3-1번

$z_{11} = W_{11}^1 \cdot x_1 = 0.05$ $\quad a_{11} = \frac{1}{1+e^{-z_{11}}} = \frac{1}{1+e^{-x_1 \cdot W_{11}^1}} = 0.512497$

$z_{12} = W_{21}^1 \cdot x_1 = 0.05$

$a_{12} = \frac{1}{1+e^{-z_{12}}} = \frac{1}{1+e^{-x_1 \cdot W_{21}^1}} = 0.512497$

$z_{21} = a_{11} \cdot W_{11}^2 + a_{12} \cdot W_{12}^2$

$= 0.435622$

$a_{21} = \frac{1}{1+e^{-z_{21}}} = 0.607215 = \hat{y}_1$

$z_{22} = a_{11} \cdot W_{21}^2 + a_{12} \cdot W_{22}^2$

$= 0.538122$

$a_{22} = \frac{1}{1+e^{-z_{22}}} = 0.631375 = \hat{y}_2$

3-24

$$MSE = E_{total} = \sum_{i=1}^{N} \frac{1}{2}(y_i - \hat{y}_i) = \frac{1}{2}\left[(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2\right] = 0$$

$$E_1 = \frac{1}{2}\left[y_1 - \hat{y}_1\right]^2 = \frac{1}{2}(0.5 - 0.607215)^2 = 0.005748$$

$$E_2 = \frac{1}{2}\left[y_2 - \hat{y}_2\right]^2 = \frac{1}{2}(0.9 - 0.631375)^2 = 0.036080$$

$$\Rightarrow E_{total} = E_1 + E_2 = 0.041828$$

3-34

Error at w_{11}^2

$$\frac{\partial E_{total}}{\partial w_{11}^2} = \frac{\partial E_{total}}{\partial a_{21}} \frac{\partial a_{21}}{\partial z_{21}} \frac{\partial z_{21}}{\partial w_{11}^2}$$

$$\frac{\partial E_{total}}{\partial a_{21}} = \frac{1}{2} \cdot 2 (y_1 - \hat{y}_1)(-1) = \hat{y}_1 - y_1 = 0.107215$$

$$\frac{\partial a_{21}}{\partial z_{21}} = \frac{1}{1+e^{-z_{21}}} \left(1 - \frac{1}{1+e^{-z_{21}}}\right) = \hat{y}_1 (1 - \hat{y}_1) = 0.238505$$

$$\frac{\partial z_{21}}{\partial w_{11}^2} = a_{11} = 0.512497$$

$$\frac{\partial E_{total}}{\partial w_{11}^2} = 0.00609889$$

$$\Rightarrow \quad w_{11}^2 \Leftarrow w_{11}^2 - \eta \frac{\partial E_{total}}{\partial w_{11}^2}$$

$$\Leftarrow 0.4 - 0.5\,(0.00609889)$$

$$\Leftarrow 0.396950555$$