Programming Languages

2nd edition

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Chapter 6
Type Systems

I was eventually persuaded of the need to design programming notations so as to maximize the number of errors that cannot be made, or if made, can be reliably detected at compile time.

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Contents

Motivation: Detecting Type Errors

- The detection of type errors, either at compile time or at run time, is called *type checking*.
  - *Type errors occur frequently in programs.*
  - *Type errors can't be prevented/detected by EBNF*
  - *If undetected, type errors can cause severe run-time errors.*
  - *A type system can identify type errors before they occur.*

6.1 Type System for CLite

- Static binding
- Single function: `main`
- Single scope: no nesting, no globals
- Name resolution errors detected at compile time
  - *Each declared variable must have a unique identifier*
  - *Identifier must not be a keyword (syntactically enforced)*
  - *Each variable referenced must be declared.*

Example *Clite* Program (Fig 6.1)

```
// compute the factorial of integer n
void main ( ) {
    int n, i, result;
    n = 8;
    i = 1;
    result = 1;
    while (i < n) {
            i = i + 1;
            result = result * i;
    }
}
```

Designing a Type System

- A set of rules *V* in highly-stylized English
  - *return true or false*
  - *based on abstract syntax*
    - Note: standards use concrete syntax
  - *Mathematically a function:*

    V: AbstractSyntaxClass → *Boolean*
- Facilitates static type checking.
- Implementation throws an exception if invalid

Type Rule 6.1

- *All referenced variables must be declared.*
  - Type map is a set of ordered pairs

    E.g., {<n, int>, <i, int>, <result, int>}
  - Can implement as a hash table
  - Function typing creates a type map
  - Function typeOf retrieves the type of a variable:

    typeOf(id) = type

The *typing* Function creates a type map

```
public static TypeMap typing (Declarations d) {
    TypeMap map = new TypeMap( );
    for (Declaration di : d) {
        map.put (di.v, di.t);
    }
    return map;
}
```

Type Rule 6.2

- *All declared variables must have unique names.*

```
public static void V (Declarations d) {
    for (int i=0; i<d.size() - 1; i++)
        for (int j=i+1; j<d.size(); j++) {
            Declaration di = d.get(i);
            Declaration dj = d.get(j);
            check( ! (di.v.equals(dj.v)),
                "duplicate declaration: " + dj.v);
        }
}
```

Rule 6.2 example

```
// compute the factorial of integer n
void main ( ) {
    int n, i, result;
    n = 8;
    i = 1;
    result = 1;
    while (i < n) {
        i = i + 1;
        result = result * i;
    }
}
```

These must all be unique

Type Rule 6.3

- *A program is valid if*
  - *its Declarations are valid and*
  - *its Block body is valid with respect to the type map for those Declarations*

```
public static void V (Program p) {
    V (p.decpart);
    V (p.body, typing (p.decpart));
}
```

Rule 6.3 Example

```
// compute the factorial of integer n
void main ( ) {
    int n, i, result;
    n = 8;
    i = 1;
    result = 1;
    while (i < n) {
            i = i + 1;
            result = result * i;
    }
}
```

These must be valid.

Type Rule 6.4

- *Validity of a Statement:*
  - *A Skip is always valid*
  - *An Assignment is valid if:*
    - Its target *Variable* is declared
    - Its source *Expression* is valid
    - If the target *Variable* is float, then the type of the source *Expression* must be either float or int
    - Otherwise if the target *Variable* is int, then the type of the source *Expression* must be either int or char
    - Otherwise the target *Variable* must have the same type as the source *Expression*.

Type Rule 6.4 (continued)

- *A Conditional is valid if:*
  - Its test *Expression* is valid and has type bool
  - Its thenbranch and elsebranch *Statements* are valid
- *A Loop is valid if:*
  - Its test *Expression* is valid and has type bool
  - Its Statement body is valid
- *A Block is valid if all its Statements are valid.*

Rule 6.4 Example

```
// compute the factorial of integer n
void main ( ) {
    int n, i, result;
    n = 8;
    i = 1;
    result = 1;
    while (i < n) {
            i = i + 1;
            result = result * i;
    }
}
```

This assignment is valid if:
n is declared,
8 is valid, and
the type of 8 is int or char
(since n is int).

Rule 6.4 Example

```
// compute the factorial of integer n
void main ( ) {
    int n, i, result;
    n = 8;
    i = 1;
    result = 1;
    while (i < n) {
        i = i + 1;
        result = result * i;
    }
}
```

This loop is valid if
- i < n is valid,
- i < n has type bool, and
- the loop body is valid

Type Rule 6.5

- *Validity of an Expression:*
  - *A Value is always valid.*
  - *A Variable is valid if it appears in the type map.*
  - *A Binary is valid if:*
    - Its *Expressions* term1 and term2 are valid
    - If its *Operator* op is arithmetic, then both *Expressions* must be either int or float
    - If op is relational, then both *Expressions* must have the same type
    - If op is && or ||, then both *Expressions* must be bool
  - *A Unary is valid if:*
    - Its *Expression* term is valid,
    - …

Type Rule 6.6

- *The type of an Expression e is:*
  - *If e is a Value, then the type of that Value.*
  - *If e is a Variable, then the type of that Variable.*
  - *If e is a Binary* op term1 term2*, then:*
    - *If* op *is arithmetic, then the (common) type of* term1 *or* term2
    - *If* op *is relational, && or* ||, *then* bool
  - *If e is a Unary* op term*, then:*
    - *If* op *is ! then* bool
    - ...

Rule 6.5 and 6.6 Example

```
// compute the factorial of integer n
void main ( ) {
    int n, i, result;
    n = 8;
    i = 1;
    result = 1;
    while (i < n) {
            i = i + 1;
            result = result * i;
    }
}
```

This *Expression* is valid since:
op is arithmetic (*) and
the types of i and result are int.
Its result type is int since:
the type of i is int.

6.2 Implicit Type Conversion

- Clite Assignment supports implicit widening conversions
- We can transform the abstract syntax tree to insert explicit conversions as needed.
- The types of the target variable and source expression govern what to insert.

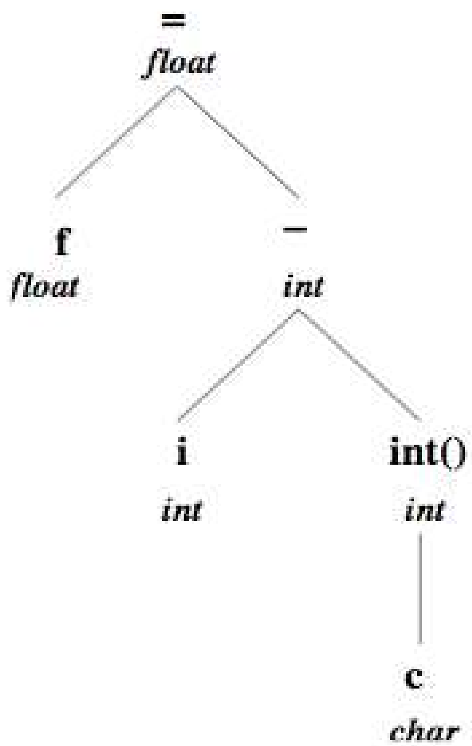
Example: Assignment of int to float

Suppose we have an assignment

f = i - int(c);

(f, i, and c are float, int, and char variables).

The abstract syntax tree is:

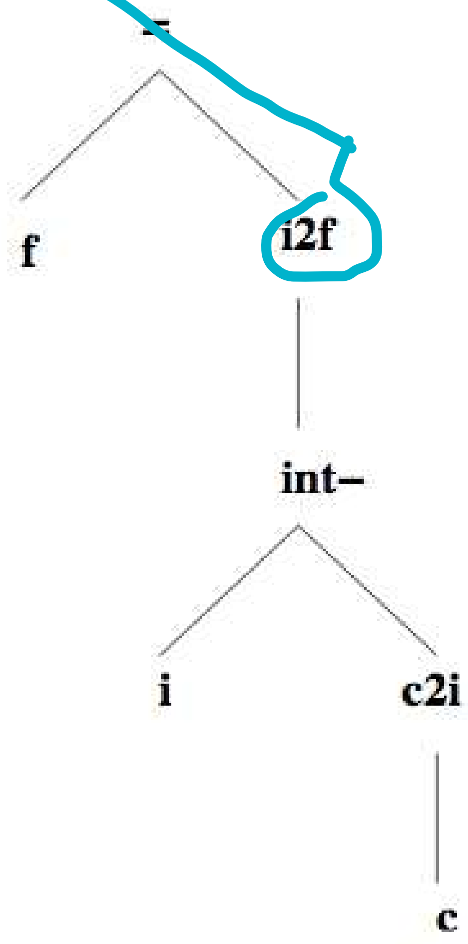


Example (cont'd)

So an implicit widening is inserted to transform the tree to:

Here, **c2i** denotes conversion from char to int, and **itof** denotes conversion from int to float.

Note: **c2i** is an explicit conversion given by the operator int() in the program.



6.3 Formalizing the Clite Type System

Type map: $tm = \{<v_1, t_1>, <v_2, t_2>, \ldots, <v_n, t_n>\}$

Created by:
(Type Rule 6.1)

$$typing : Declarations \to TypeMap$$

$$typing(d) = \bigcup_{i \in \{1,\ldots,n\}} <d_i.v, d_i.t>$$

Validity of Declarations:
(Type Rule 6.2)

$$V : Declarations \to \mathrm{B}$$

$$V(d) = \forall i, j \in \{1,\ldots,n\}(i \neq j \Rightarrow d_i.v \neq d_j.v)$$

Validity of a Clite Program

(Type Rule 6.3)

$$V : Program \rightarrow \mathrm{B}$$
$$V(p) = V(p.\mathrm{decpart}) \wedge V(p.\mathrm{body}, typing(p.\mathrm{decpart}))$$

Validity of a Clite Statement

(Type Rule 6.4, simplified version for an *Assignment*)

$$V : Statement \times TypeMap \rightarrow \mathrm{B}$$

$$V(s, tm) = true \qquad \text{if } s \text{ is a } Skip$$

$$= s.\text{target} \in tm \wedge V(s.\text{source}, tm) \wedge typeOf(s.\text{target}, tm) = typeOf(s.\text{source}, tm) \qquad \text{if } s \text{ is an } Assignment$$

$$= V(s.\text{test}, tm) \wedge typeOf(s.\text{test}, tm) = bool \wedge V(s.\text{thenbranch}, tm) \wedge V(s.\text{elsebranch}, tm) \qquad \text{if } s \text{ is a } Conditional$$

$$= V(s.\text{test}, tm) \wedge typeOf(s.\text{test}, tm) = bool \wedge V(s.\text{body}, tm) \qquad \text{if } s \text{ is a } Loop$$

$$= V(b_1, tm) \wedge V(b_2, tm) \wedge \ldots \wedge V(b_n, tm) \qquad \text{if } s \text{ is a } Block$$

Validity of a Clite Expression

(Type Rule 6.5, abbreviated versions for *Binary* and *Unary*)

Type of a Clite Expression

(Type Rule 6.6, abbreviated version)

그림을 표시할 수 없습니다.