논리식 집합

귀납적 정의

$$
\begin{array}{rcl}
f & \rightarrow & T \mid F \\
 & \mid & \neg f \\
 & \mid & f \wedge f \\
 & \mid & f \vee f \\
 & \mid & f \Rightarrow f
\end{array}
$$

논리식 의미

조립식 정의 compositional definition

$$\begin{array}{rcl} [\![T]\!] & = & \textsf{true} \\ [\![F]\!] & = & \textsf{false} \\ [\![\neg f]\!] & = & \textsf{not}[\![f]\!] \\ [\![f_1 \wedge f_2]\!] & = & [\![f_1]\!]\ \textsf{andalso}\ [\![f_2]\!] \\ [\![f_1 \vee f_2]\!] & = & [\![f_1]\!]\ \textsf{orelse}\ [\![f_2]\!] \\ [\![f_1 \Rightarrow f_2]\!] & = & [\![f_1]\!]\ \textsf{implies}\ [\![f_2]\!] \end{array}$$

임의의 논리식 f의 의미가 정의 된 셈.

$$
\begin{aligned}
&[\![(T \wedge (T \vee F)) \Rightarrow F]\!] \\
= \quad & [\![T \wedge (T \vee F)]\!] \text{ implies } [\![F]\!] \\
= \quad & ([\![T]\!] \text{ andalso } [\![T \vee F]\!]) \text{ implies false} \\
= \quad & (\text{true andalso } ([\![T]\!] \text{ orelse } [\![F]\!])) \text{ implies false} \\
= \quad & (\text{true andalso } (\text{true orelse false})) \text{ implies false} \\
= \quad & \text{false}
\end{aligned}
$$

어떤 집합의 정의

쌍 $(\{g_1, \cdots, g_n\}, f)$들의 집합

$$\frac{}{(\Gamma, T)} \quad \frac{}{(\Gamma, f)}\; f \in \Gamma \qquad \frac{(\Gamma, F)}{(\Gamma, f)} \quad \frac{(\Gamma, \neg\neg f)}{(\Gamma, f)}$$

$$\frac{(\Gamma, f_1) \quad (\Gamma, f_2)}{(\Gamma, f_1 \wedge f_2)} \qquad \frac{(\Gamma, f_1 \wedge f_2)}{(\Gamma, f_1)}$$

$$\frac{(\Gamma, f_1)}{(\Gamma, f_1 \vee f_2)} \qquad \frac{(\Gamma, f_1 \vee f_2) \quad (\Gamma \cup \{f_1\}, f_3) \quad (\Gamma \cup \{f_2\}, f_3)}{(\Gamma, f_3)}$$

$$\frac{(\Gamma \cup \{f_1\}, f_2)}{(\Gamma, f_1 \Rightarrow f_2)} \qquad \frac{(\Gamma, f_1 \Rightarrow f_2) \quad (\Gamma, f_1)}{(\Gamma, f_2)}$$

$$\frac{(\Gamma \cup \{f\}, F)}{(\Gamma, \neg f)} \qquad \frac{(\Gamma, f) \quad (\Gamma, \neg f)}{(\Gamma, F)}$$

형식논리의 표기법으로

$$\frac{}{\Gamma \vdash T} \quad \frac{}{\Gamma \vdash f}\; f \in \Gamma \qquad\qquad \frac{\Gamma \vdash F}{\Gamma \vdash f} \quad \frac{\Gamma \vdash \neg\neg f}{\Gamma \vdash f}$$

$$\frac{\Gamma \vdash f_1 \quad \Gamma \vdash f_2}{\Gamma \vdash f_1 \wedge f_2} \qquad\qquad \frac{\Gamma \vdash f_1 \wedge f_2}{\Gamma \vdash f_1}$$

$$\frac{\Gamma \vdash f_1}{\Gamma \vdash f_1 \vee f_2} \qquad\qquad \frac{\begin{array}{c}\Gamma \vdash f_1 \vee f_2 \\ \Gamma \cup \{f_1\} \vdash f_3 \quad \Gamma \cup \{f_2\} \vdash f_3\end{array}}{\Gamma \vdash f_3}$$

$$\frac{\Gamma \cup \{f_1\} \vdash f_2}{\Gamma \vdash f_1 \Rightarrow f_2} \qquad\qquad \frac{\Gamma \vdash f_1 \Rightarrow f_2 \quad \Gamma \vdash f_1}{\Gamma \vdash f_2}$$

$$\frac{\Gamma \cup \{f\} \vdash F}{\Gamma \vdash \neg f} \qquad\qquad \frac{\Gamma \vdash f \quad \Gamma \vdash \neg f}{\Gamma \vdash F}$$

또 다른 시선: 증명들의 집합을 정의

증명들의 집합을 만드는 귀납규칙 (“증명규칙” inference rules).

- 예를 들어, 증명규칙

$$\frac{\Gamma \vdash f_1 \quad \Gamma \vdash f_2}{\Gamma \vdash f_1 \wedge f_2}$$

  은 증명을 만드는 귀납 규칙
- $\Gamma \vdash f_1$와 $\Gamma \vdash f_2$의 증명들을 가지고 $\Gamma \vdash f_1 \wedge f_2$의 증명을 만든다.

증명 나무

$$
\dfrac{\dfrac{\dfrac{}{\{p \to \neg p, p\} \vdash p} \qquad \dfrac{\dfrac{}{\{p \to \neg p, p\} \vdash p \to \neg p} \qquad \dfrac{}{\{p \to \neg p, p\} \vdash p}}{\{p \to \neg p, p\} \vdash \neg p}}{\{p \to \neg p, p\} \vdash F}}{\{p \to \neg p\} \vdash \neg p}
$$