ECE 414 Homework 3

This homework explores the effect of added zeros and poles to the standard second order transfer function.

$$H(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

where ω_n is the undamped natural frequency and ζ is the damping factor. The response is overdamped for $\zeta>1$ and underdamped for $\zeta<1$, with the dividing line between these responses being critically damped.

In the underdamped case the roots are

$$s_1, s_2=-\zeta\omega_n \pm j\omega_d \quad \text{where} \quad \omega_d=\omega_n\sqrt{1-\zeta^2}$$

where $\zeta\omega_n$ is the damping factor, which is equal to how far left in the s-plane the two poles are, and ω_d is the damped natural frequency.

For this work, let $\omega_n = 100$ and $\zeta = 0.7$. Use the step response to $H(s)$ as given above for comparison purposes in the work below.

1. First add a zero at $-\alpha\zeta\omega_n$ to $H(s)$ so that the DC gain is unchanged. This guarantees that the steady state value of $y(t)$ remains at one. The variable α sets the zero location relative to the negative real part of the poles. The larger α is, the further to the left in the s-plane the zero is relative to the real part of the poles.
(a) let α vary from 0.1 to 10 in a logarithmic fashion and gather the transient specifications to and compare them to those of the original system with no zero. For what values of α does the added zero not have much impact on each of the transient specifications?
(b) repeat part (a) but let α vary from –0.1 to –10 in a logarithmic fashion. This places the zero in the right half s-plane. What is unique or strange about having a zero in the right half plane?

2. Now add a pole at $-\alpha\zeta\omega_n$ to $H(s)$ so that the DC gain is unchanged. This guarantees that the steady state value of $y(t)$ remains at one. The variable α sets the added pole location relative to the negative real part of the two primary poles. The larger α is, the further to the left in the s-plane the pole is relative to the real part of the poles.
(a) let α vary from 0.1 to 10 in a logarithmic fashion and gather the transient specifications to and compare them to those of the original system with no added pole. For what values of α does the added pole not have much impact on each of the transient specifications?

3. Repeat #1 and #2 above with $\zeta = 0.4$. Do your “rule of thumb” conclusions still hold here?