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ECE 484
HW # 8

10.1-1) A baseband channel has transfer function $H(f) = \frac{10^{-2}}{j2\pi f + 3000\pi}$

PSD $\Rightarrow$ $S_m(f) = 8\Pi(f/\alpha)$, $\alpha = 8000$ is transmitted with $H_c(f) = 10^{-3}/(j2\pi f + \alpha)$

and $S_n(f) = 10^{-8}$. If output SNR = 40 dB, find min bandwidth

$$SNR = 10\log\left(\frac{S_o}{N_o}\right) \Rightarrow 40 = 10\log\left(\frac{S_o}{N_o}\right) \Rightarrow \frac{S_o}{N_o} = 10^4$$

PSD $S_m(f) = 8\Pi\left(f/8000\right)$ $\Rightarrow$ bandwidth = 4000

but since $S_n(f) = 10^{-8}$ there are gaps

$$S_o = \int_{-\infty}^{\infty} S_m(f)\,|H_c(f)|^2\,df = \int_{-\infty}^{\infty} 8\Pi\left(\frac{f}{8000}\right)\left(\frac{10^{-4}}{4\pi^2 f^2 + 3000^2\pi^2}\right)df$$

$$S_o = \frac{8\times10^{-4}}{4\pi^2}\int_{-4000}^{4000}\left(\frac{1}{f^2 + \frac{3000^2\pi^2}{4\pi^2}}\right)df = \left(\frac{8\times10^{-4}}{4\pi^2}\right)\left(\frac{2}{3000}\tan^{-1}\left(\frac{2f}{3000}\right)\right)\Bigg|_{-4k}^{4k} = 1.87\times10^{-6}$$

$$N_o = \int_{-w}^{w} S_n(f)\,|H_c(f)|^2\,df = \frac{10^{12}}{4\pi^2}\int_{-w}^{w}\left(\frac{1}{f^2 + 1500^2}\right)df$$

$$\frac{S_o}{N_o} = 10^4 \Rightarrow \frac{1.87\times10^{-6}}{N_o} = 10^4 \Rightarrow \frac{1.87\times10^{-6}}{\frac{10^{-12}}{4\pi^2}\int_{-w}^{w}\left(\frac{1}{f^2+1500}\right)df} \Rightarrow \omega = 27.2\text{kHz}$$

$$\boxed{B = 2\omega = 54.4\text{ kHz}}$$

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10.1-2) a signal $m(t)$ with PSD $S_m(f) = \beta \Pi(f/\alpha)$, $\alpha = 8000$ is trans. with $H_c(f) = 10^{-3}/(j\pi 2f + \alpha)$. the channel PSD is $S_n(f) = 10^{-8}$

$$H_d(f) = \left(\frac{j\pi 2f + \alpha}{\alpha}\right)\Pi\left(\frac{2\pi f}{2\alpha}\right) \quad \text{and output SNR} = 55\,dB.$$

determine β and S_r/S_i

need SNR eq. $\Rightarrow \frac{S_r}{N_r} = 10^{5.5}$ and $S_r = \int_{-\infty}^{\infty} S_m(f)|H_c(f)|^2 |H_d(f)|^2 \, df$

$$S_r = \int_{-\infty}^{\infty} \beta \Pi\left(\frac{f}{8000}\right)\left(\frac{10^{-6}}{4\pi^2 f^2 + \alpha^2}\right)\left(\frac{4\pi^2 f^2 + \alpha^2}{\alpha^2}\right) df = \frac{\beta 10^{-6}}{\alpha^2}\int_{-\frac{\alpha}{2\pi}}^{\frac{\alpha}{2\pi}} df \Rightarrow S_r = \frac{\beta 10^{-6}}{\alpha\pi}$$

$S_n(f) = 10^{-8}$

$$N_r = \int_{-\infty}^{\infty} S_n(f)\left(\frac{10^{-6}}{\alpha^2}\right) df = \frac{10^{-8}\cdot 10^{-6}}{\alpha^2}\Bigg|_{-\frac{\alpha}{2\pi}}^{\frac{\alpha}{2\pi}} \Rightarrow \frac{10^{-14}}{\alpha\pi}$$

$$\frac{S_r}{N_r} = 10^{5.5} \Rightarrow \frac{\frac{\beta 10^{-6}}{\alpha\pi}}{\frac{10^{-14}}{\alpha\pi}} = 10^{5.5} = 10^{8}\beta = 10^{5.5} \Rightarrow \boxed{\beta = 3.162\times 10^{-3}}$$

$$S_r = \frac{\beta 10^{-6}}{\alpha\pi} \Rightarrow \boxed{S_r = 1.25\times 10^{-13}\ W}$$

$$S_i = \int_{-\infty}^{\infty} S_m(f)|H_c(f)|^2 df = \frac{3.16\times 10^{-3}}{4\pi^2}\int_{-\alpha}^{\alpha}\frac{1}{f^2 + \left(\frac{\alpha}{2\pi}\right)^2} df \Rightarrow \boxed{S_i = 1.13\times 10^{-7}\ W}$$

10.2-1) PSD of $S_n(f) = 10^{-12}$ and baseband signal $B = 5\,kHz$ and $SNR = 47\,dB$

a. What must be the signal power S_i at receiver input

$S_n(f) = \frac{N}{2} = 10^{-12} \Rightarrow N = 2\times10^{-12}$ and $SNR = 10\log\left(\frac{S_o}{N_o}\right) = 47\,dB$

$\frac{S_o}{N_o} = 50{,}118$ and $\frac{S_o}{N_o} = \frac{S_i}{NB} \Rightarrow S_i = NB\frac{S_o}{N_o} \Rightarrow \boxed{S_i = 5\times10^{-4}\,W}$

b. What is N_o?

$N_o = 2(10^{-12})5000 \Rightarrow \boxed{N_o = 10^{-8}}$

c. $S_i = \int_{-B}^{B} |H_c(f)|^2 \cdot S_r(f)\,df = |10^{-3}|^2 S_r \Rightarrow \boxed{S_r = \frac{S_i}{|10^{-3}|^2} = 501.2}$

10.2-4) A Gaussian baseband random process $m(t)$ is transmitted by AM. For 3σ loading. Find the output SNR as a function of γ and μ

$K^2 = \frac{m_p^2}{\overline{m^2}}$ and $\mu = \frac{m_p}{A}$ and $A = \frac{m_p}{\mu}$

$$\frac{S_o}{N_o} = \left(\frac{\overline{m^2}}{A^2+\overline{m^2}}\right)\gamma = \left(\frac{\mu^2}{(m_p^2/\overline{m^2})+\mu^2}\right)\gamma \Rightarrow \frac{S_o}{N_o} = \left(\frac{\mu^2}{K^2+\mu^2}\right)\gamma$$

$$K^2 = \frac{(3\sigma)^2}{\sigma^2} = 9 \Rightarrow \boxed{\frac{S_o}{N_o} = \left(\frac{\mu^2}{9+\mu^2}\right)\gamma}$$

10.3-1)

a. find S_i: $\frac{S_o}{N_o} = 27dB = 631 V/V \Rightarrow \frac{S_o}{N_o} = 3\beta^2\gamma\left(\frac{\overline{m^2}}{m_p^2}\right)$

$$\gamma = \frac{1}{3\beta^2}\left(\frac{S_o}{N_o}\right)\left(\frac{m_p^2}{\overline{m^2}}\right) = \left(\frac{1}{(3\cdot5^2)}\right)631\left(\frac{(3\sigma)^2}{\sigma^2}\right) = 75.72$$

$$N = 2S_n = 2\times10^{-10} \Rightarrow S_i = \gamma(NB) \Rightarrow \boxed{S_i = 227\,\mu W}$$

b. Find S_o:

$$\beta = \frac{\Delta\omega}{2\pi B} = \frac{k_f m_p}{2\pi B} \Rightarrow k_f\sigma = 1.57\times10^4$$

$$S_o = \alpha^2 k_f \overline{m^2} = \alpha^2 k_f^2\sigma^2 \Rightarrow \boxed{S_o = 2467.4\,kW}$$

c. Find N_o: $N_o = \frac{S_o}{\frac{S_o}{N_o}} = \frac{2467.4k}{631} \Rightarrow \boxed{N_o = 3.91\,kW}$

10.4-1)

a. find L:

$$\frac{S_o}{N_o} = 10^5 = 50\,dB \qquad \text{now find } \overline{m^2}$$

$$\overline{m^2} = \frac{1}{2m_p}\int_{-m_p}^{m_p} x^2\,dx = \frac{1}{2m_p}\frac{x^3}{3}\Big|_{-m_p}^{m_p} \Rightarrow \frac{\overline{m^2}}{m_p^2} = \frac{1}{3} \quad \text{and } \frac{S_o}{N_o} = 3L^2\frac{1}{3} \Rightarrow$$

$$L = \sqrt{\frac{10^5}{3(1/3)}} = 316.2 \quad \boxed{\text{so need } n = 9 \text{ for } L = 512}$$

b.

$$\frac{S_o}{N_o} = 3\cdot 512^2\left(\frac{1}{3}\right) = 262144 = \boxed{54.2\,dB = SNR}$$

C.

$$B_{pcm} = n2B = 9\cdot 2\cdot 4.5\,MHz \Rightarrow \boxed{B_{pcm} = 81\,MHz}$$

D.

$$56\,dB = 10^{5.6} \Rightarrow L = \sqrt{\frac{10^{5.6}}{3(1/3)}} = 631 \Rightarrow n = 10 \Rightarrow L = 1024$$

$$B_{pcm} = (10)(2)(4.5\,MHz) = 90\,MHz \Rightarrow \text{the \% increase is}$$

$$100\cdot\left(\frac{90-81}{81}\right)MHz = \boxed{11.1\%\text{ increase in } B_{pcm}}$$