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ECE 484
Homework 7

7.2-5) Use differential code with the pulse $p_3(t) = \begin{cases} \cos\left(\frac{\pi t}{T_b}\right) & |t| \le \frac{T_b}{2} \\ 0 & |t| > \frac{T_b}{2} \end{cases}$ to derive the PSD for a binary signal and determine the PSD $S_y(f)$

rewrite as rectangular pulse

$$p_3(t) = \cos\left(\frac{\pi t}{T_b}\right)\Pi\left(\frac{t}{T_b}\right) = 0.5\left[e^{\frac{j\pi t}{T_b}} + e^{\frac{j\pi t}{T_b}}\right]\Pi\left(\frac{t}{T_b}\right)$$

now find PSD

$$P(f) = 0.5T_b\left\{\operatorname{sinc}\left(\pi\left(f+\frac{1}{T_b}\right)T_b\right) + \operatorname{sinc}\left(\pi\left(f-\frac{1}{T_b}\right)T_b\right)\right\}$$

$$P(f) = 0.5T_b\left(\operatorname{sinc}(\pi+\pi f T_b) + \operatorname{sinc}(\pi-\pi f T_b)\right)$$

$$S_y(f) = \frac{|P(f)|^2}{T_b}\left[R_0 + 2\sum_{n=1}^{\infty} R_n \cos(2n\pi f T_b)\right]$$

now binary

$$R_0 = \lim_{N\to\infty}\frac{1}{N}\left[\frac{N}{2}(1)^2 + \frac{N}{2}(-1)^2\right] = 1$$

$$R_n = \lim_{N\to\infty}\frac{1}{N}\left[\frac{N}{4}(1)(1) + \frac{N}{4}(-1)(-1) + \frac{N}{4}(-1)(1) + \frac{N}{4}(1)(-1)\right] = 0 \Rightarrow$$

$$\boxed{S_y(f) = \frac{T_b}{4}\left[\operatorname{sinc}(\pi+\pi f T_b) + \operatorname{sinc}(\pi-\pi f T_b)\right]^{-2}}$$

7.3-1) A binary data stream needs to be transmitted at 5 Mb/s by means of binary signaling. To reduce ISI, a cosine roll off pulse of roll off factor $r = 0.25$ will be used. Determine the minimum required bandwidth for this transmission

$$B_r = \left(\frac{1+r}{2}\right) R_b \Rightarrow B_r = \left(\frac{1+0.25}{2}\right) 5M \Rightarrow \boxed{B_r = 3.125\, MHz}$$

7.3-2) Repeat prob. 6.2-9 if Nyquist criterion pulses with $r = 0.2$

$R_N = 2B = 2 \cdot 240 = 480\, Hz$ and $R_A = R_N\left(1 + \frac{20}{100}\right) = 576\, Hz$

for 9 bits

$C = nR_A \Rightarrow C = 5184$ b/s, but need to add 0.5% and transmit 5 of them

$C = 5\left[5184 + \left(\frac{0.5}{100}\right) 5184\right] = 26050$ b/s

and the bandwidth

$B_r = \frac{C}{2} \cong 13\, kHz$

and SNR

$SNR = \frac{3L^2}{\ln(1+\mu)^2} \Rightarrow L = \ln(1+\mu)\sqrt{\frac{SNR}{3}}$ and required SNR = 43 dB = 1.995×10^4

$L = \ln(1+100)\sqrt{\frac{1.995 \times 10^4}{3}} = 376.4 \Rightarrow L = 2^9 = 512$

R_b:

each has bandwidth of 240 ⇒ Nyquist rate = 480 samp/sec
and 20% above is 576 samp/s and each 9 bits ⇒ 5185 b/s
and 5 signals together ⇒ 25.925 kb/s and now add 0.5% ⇒ $R_b = 26.05$ kb/s
and $r = 0.2$

$$B_r = \left(\frac{1+0.2}{2}\right)(26.05k) \Rightarrow \boxed{B_r = 15.63\, kHz}$$

7.3-3) repeat problem 7.3-2 if M=4 pulse levels are transmitted such that each transmission of a pulse with a distinct level represents 2 bits. Generalize results when $M=2^m$ pulse levels are used in transmission

Know $R_b = 26.05$ kb/s $\Rightarrow R_p = \frac{26.05k}{2} = 13.025$

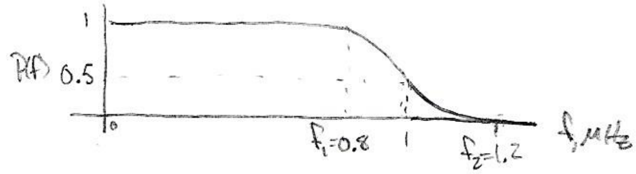
$B_r = \left(\frac{1+0.2}{2}\right)(13.025k) \Rightarrow \boxed{B_r = 7.82 \text{ kHz}}$

if $M = 2^m$

$R_p = \frac{26.05k}{m} = \frac{26.05k}{\log_2 M}$

$B_r = \left(\frac{1+0.2}{2}\right)\left(\frac{26.05k}{\log_2 M}\right) = \boxed{\frac{15.63}{\log_2 M} \text{ kHz} = B_r}$

7.3-6) The Fourier transform P(f) of the basic pulse p(t) used in a certain binary communication system is shown below



a. from the shape of P(f), explain at what pulse rate this pulse would satisfy Nyquist's first criterion.

- a pulse satisfying Nyquist also satisfies $|P(0.5R_b)| = 0.5|P(0)|$ and $|P(1MHz)| = 0.5|P(0)|$

So $\boxed{R_b = 2\ MHz}$

b. use the formula for the inverse Fourier transform of a basic pulse P(f)

$$p(t) = R_b \frac{\cos(\pi R_b t)}{1-4R_b^2t^2} \cdot \frac{\sin(\pi R_b t)}{\pi R_b t} = \frac{\sin(2\pi R_b t)}{\pi t(1-4R_b^2t^2)} \Rightarrow \boxed{p(t) = 10^6 \frac{\sin(4\pi t)}{2\pi t(1-16t^2)}}$$

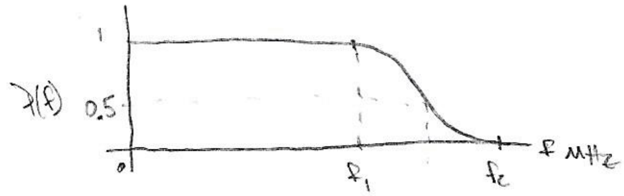
c. the bandwidth in excess of the minimum is $f_x = 0.2\ MHz$

$$r = \frac{f_x}{0.5R_b} = \frac{0.2}{0.5(2)} = 0.2 \Rightarrow \boxed{r = 0.2}$$

d. $p(t) = 10^6 \frac{\sin(4\pi t)}{2\pi t(1-16t^2)}$ and envelope of p(t) is proportional to $|t(1-16t^2)|^{-1}$

which decays at t^{-3} $\Rightarrow$ $\boxed{\text{pulse decays at } t^{-3}}$

7.3-7) Four level transmission at the pulse rate of 2Mb/s is to be transmitted by means of Nyquist first criteron pulses with P(f) shown below. f_1 & f_2 are adjustable. The channel available for transmission of this data has a bandwidth of 650kHz. Determine f_1 & f_2 and r



$R_b = 2M\,b/s$

and 4 level $\Rightarrow$ $\log_2 4 = 2$ bits

$$R_p = \frac{2M}{2} = 10^6 = 1MHz$$

$$B_r = \left(\frac{1+r}{2}\right)R_p \Rightarrow r = \frac{2B_r}{R_p} - 1 = \frac{2 \cdot 650k}{1M} - 1 \Rightarrow \boxed{r = 0.3}$$

now f

$$f_x = 0.5 r R_p = 150kHz \Rightarrow 0.5R_p = 500kHz$$

f_1, f_2 are f_x away

$$f_1 = 0.5R_p - f_x = 500 - 150 \Rightarrow \boxed{f_1 = 350kHz}$$

$$f_2 = 0.5R_p + f_x = 500 + 150 \Rightarrow \boxed{f_2 = 650kHz}$$