**5.1-2**

(a) $\omega_c = 2\pi \times 10^6$, $k_f = 2000\pi$, $k_p = \pi/2$.

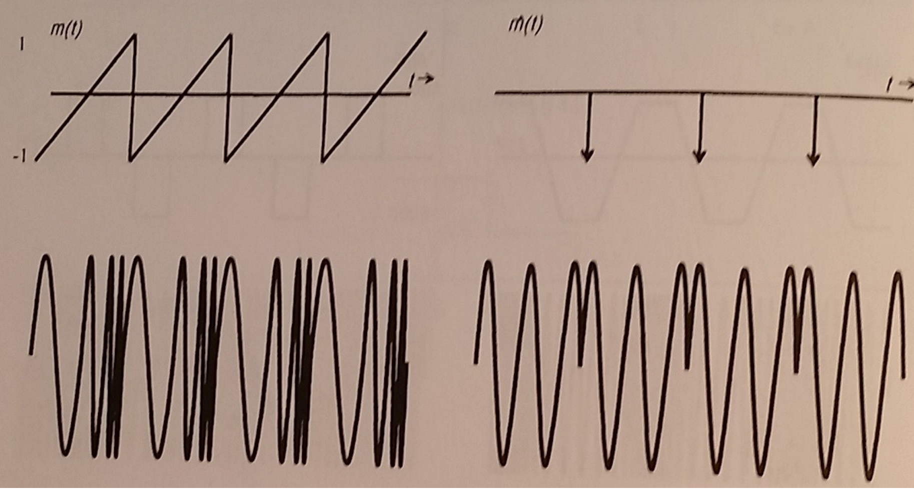
**FM:** The instantaneous frequency is $f_i = 10^6 + 1000m(t)$, $(f_i)_{\min} = 10^6 - 1000 = 999$ kHz, $(f_i)_{\max} = 10^6 + 1000 =$ 1001 kHz. As shown in Fig. S5.1-2 the instantaneous frequency of the FM wave increases linearly from 999 to 1001 kHz over $10^{-3}$ s, then switches back to 999 kHz and repeats.

**PM:** Since $m(t)$ has jump discontinuities, the direct approach will be used. When one cycle of the sawtooth is centered on the origin, $m(t) = 2000t$ over that cycle. Hence,

$$\begin{aligned}\varphi_{\text{PM}}(t) &= \cos\left[2\pi \times 10^6 t + \frac{\pi}{2} m(t)\right]\\ &= \cos\left[2\pi \times 10^6 t + \frac{\pi}{2} 2000t\right]\\ &= \cos\left[2\pi\left(10^6 + 500\right)t\right].\end{aligned}$$

As shown in Fig. S5.1-2, at the discontinuity there is a jump of $2k_p = \pi$, otherwise the carrier frequency is cons at $10^6 + 500$ Hz.

(b) This is equivalent to another PM signal with $f_c = 1000.5$ kHz and periodic rectangular message that swit from 1 to $-1$, as shown in the example at the beginning of the chapter. It is necessary to keep $k_p$ less than $\pi$ s those periodic signal jumps at those points of discontinuity $\Delta = 2$; otherwise a larger $k_p$ will give rise to ph ambiguity when $k_p\Delta > 2\pi$.



**5.1-4** We are given $\omega_c = 10000\pi$, and that over $|t| \leq 1$,

$$\varphi_{EM}(t) = 10 \cos 13,000\pi t$$

(a) If this were a PM signal with $k_p = 1000$, we would have

$$\begin{aligned}\varphi_{PM}(t) &= 10 \cos 13000\pi t = 10 \cos [\omega_c t + k_p m(t)] \\ &= 10 \cos [10000\pi t + 1000 m(t)]\end{aligned}$$

Clearly, $m(t) = 3\pi t$ over this interval.

(b) For an FM signal with $k_f = 1000$,

$$\varphi_{FM}(t) = A \cos \left[\omega_c t + k_f \int_0^t m(\alpha)\, d\alpha\right] = 10 \cos \left[10,000\pi t + 1000 \int_0^t m(\alpha) d\alpha\right].$$

Therefore,

$$\int_0^t m(\alpha) d\alpha = 3\pi t$$

and $m(t) = 3\pi$ over the interval.

**5.2-1** The signal bandwidth of $m(t)$ is to be approximated by its own fifth harmonic frequency, $B = 5/(0.25)$ Hz.

(a) $k_f = 20\pi$, $m_p = 3$, $\Delta f = k_f m_p/(2\pi) = 30$ Hz. Therefore, $B_{FM} = 2(\Delta f + B) = 2(30 + 20) = 100$ Hz.

(b) $k_p = \pi/2$, $\dot{m}_p = 3/0.05 = 60$, $\Delta f = k_p \dot{m}_p/(2\pi) = 15$ Hz. Therefore, $B_{PM} = 2(\Delta f + B) = 2(15 + 20) =$

**5.2-9**

(a) Since $J_n(\beta)$ are the exponential Fourier coefficients of $g(t) = e^{j\beta \sin \omega_m t}$. Thus, applying Parseval's Theorem,

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = \text{power of } g(t) = P_g$$

On the other hand, since $g(t)$ is periodic with period $T_m = 2\pi/\omega_m$ and $|g(t)|^2 = |e^{j\beta \sin \omega_m t}|^2 = 1$,

$$P_g = \frac{1}{T_m} \int_0^{T_m} |g(t)|^2 \, dt = 1$$

and we therefore have

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

(b)

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(\beta \sin x - nx) + j \sin(\beta \sin x - nx) dx.$$

Since $\sin(x)$ and $[\beta \sin(x) - nx]$ are both odd functions of $x$,

$$\int_{-\pi}^{\pi} \sin(\beta \sin x - nx)\, dx = 0.$$

Now, since $\cos(x)$ is even whereas $\beta \sin(x) - nx$ is odd function of $x$,

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(\beta \sin x - nx)\, dx = \frac{1}{\pi} \int_0^{\pi} \cos(\beta \sin x - nx)\, dx$$

Changing the variable of integration to $v = \pi - x$ gives

$$J_n(\beta) = \frac{1}{\pi} \int_{\pi}^{0} \cos(\beta \sin(\pi - v) - n\pi + nv)(-1)\, dx.$$

Now recall that $\sin(\pi - x) = \sin(x)$, $\cos(x - \pi) = -\cos(x)$, $\cos(x - n\pi) = (-1)^n \cos(x)$. Thus, we have

$$\begin{aligned} J_n(\beta) &= \frac{(-1)^n}{\pi} \int_{\pi}^{0} \cos(\beta \sin v + nv)(-1)\, dv \\ &= \frac{(-1)^n}{\pi} \int_{0}^{\pi} \cos(\beta \sin v + nv)\, dv \\ &= (-1)^n J_{-n}(\beta). \end{aligned}$$

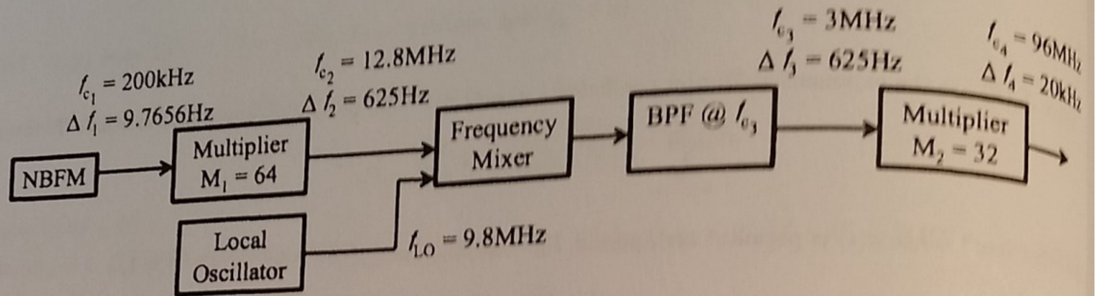



Fig. S5.3-1

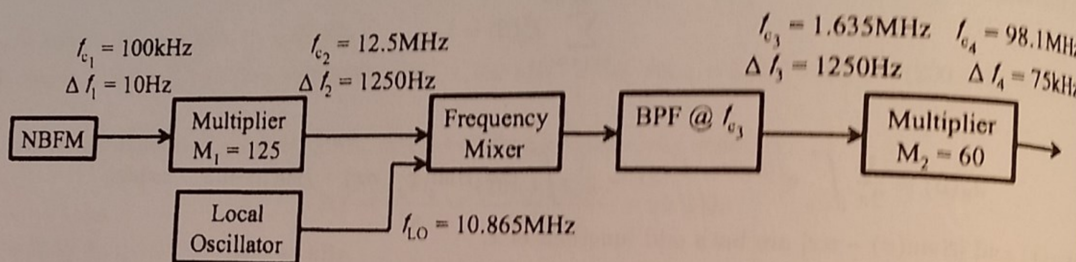



**Fig. S5.3-2**

**5.3-2**

(a) The design is shown in Fig. S5.3-2. In this case, the NBFM generator generates $f_{c_1} = 100$ kHz, and $\Delta f_1 =$ Hz. The final WBFM should have $f_{c_4} = 98.1$ MHz, and $\Delta f_4 = 75$ kHz. The total factor of frequency multiplicati needed is $M_1 \cdot M_2 = \frac{\Delta f_4}{\Delta f_1} = 7500$. With this, we can use $M_1 = 125$, $M_2 = 60$.

To find $f_{LO}$, there are three possible relationships: $f_{c_3} = f_{c_2} \pm f_{LO}$, $f_{LO} - f_{c_2}$. Each should be tested to determi the one that will require $10 \times 10^6 \leq f_{LO} \leq 11 \times 10^6$. First, we test $f_{c_3} = f_{c_2} - f_{LO}$. This case leads to 9 MHz $= f_{c_4} = 60 f_{c_3} = 60(f_{c_2} - f_{LO}) = 60(125 f_{c_1} - f_{LO}) = 7.5 \times 10^8 - 60 f_{LO}$. Thus, we have $f_{LO} = 10.8$ MHz, which is in the desired range. We won't test the other cases since this one works. Thus, the final design $M_1 = 125$, $M_2 = 60$, and $f_{LO} = 10.865$ MHz. This gives $f_{c_2} = 125 f_{c_1} = 12.5$ MHz, $\Delta f_2 = M1 \cdot \Delta f_1 = 1250$ H $f_{c_3} = f_{c_2} - f_{LO} = 12.5 - 10.865 = 1.635$ MHz, $\Delta f_3 = 1250$ Hz. The bandpass filter used will be centered at 1.6 MHz.

(b) Given the multiplication factors already used, the tunable range given by

$$60(f_{c_2} - f_{LO}) = 60(f_{c_2} - (10 \text{ MHz to } 11 \text{ MHz})) = 90 \text{ MHz to } 150 \text{ MHz}$$

**5.4-4** We can use small error analysis to find the Laplace transform of the phase error $\theta_e(t)$ as follows

$$\Theta_e(s) = \frac{s}{s + AKH(s)}\Theta_i(s)$$

For $\theta_i = kt^2$, $\Theta_i(s) = 2k/s^3$ and

$$\Theta_e(s) = \frac{2k}{s^2[s + AKH(s)]}$$

If $H(s) = 1$, the steady-state phase error is

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} s\Theta_e(s) = \lim_{s\to 0} \frac{2k}{s(s + AK)} = \infty$$

Hence, the incoming signal cannot be tracked. If $H(s) = \frac{s+a}{s}$, then

$$\Theta_e(s) = \frac{2k}{s^2[s + \frac{AK(s+a)}{s}]} = \frac{2k}{s[s^2 + AK(s+a)]}$$

and

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} s\Theta_e(s) = \lim_{s\to 0} \frac{2k}{s^2 + AK(s+a)} = \frac{2k}{AKa}$$

Hence, the incoming signal can be tracked within a constant phase $2k/AKa$ radians. Now if

$$H(s) = \frac{s^2 + as + b}{s^2}$$

then

$$\Theta_e(s) = \frac{2k}{s^2[s + \frac{AK(s^2 + as + b)}{s^2}]} = \frac{2k}{s^3 + AK(s^2 + as + b)}$$

and

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} s\Theta_e(s) = \lim_{s\to 0} \frac{2ks}{s^3 + AK(s^2 + as + b)} = 0$$

In this case, the incoming signal can be tracked with zero phase error.

**5.6-1** Given that the IF frequency is $f_{\text{IF}} = 455$ kHz and $f_c = 1530$ kHz, it is clear that an image station exists at a distance of $2f_{\text{IF}} = 910$ kHz. This means that at another carrier frequency of $f_c' = 1530 - 910 = 620$ kHz, $f_{\text{LO}}' = f_c' + f_{\text{IF}} = 1075$ kHz, and $f_c - f_{\text{LO}}' = f_{\text{IF}}$. Thus, at $f_c'$ the station at $f_c$ will be heard owing to a poor RF-stage bandpass filter.

**5.6-3**

**(a)** With $f_{\text{IF}} = 455$ kHz and a desired range of 9.4 to 9.9 MHz, $[f_{\text{LO}}]_{\min} = 9.4 + 0.455 = 9.855$ MHz and $[f_{\text{LO}}]_{\max} = 9.9 + 0.455 = 10.355$ MHz.

**(b)** Since the range of the 31-meter band is only 500 kHz, it is not possible to receive an image station and a desired station from that band. The image stations exist at a distance of $2f_{\text{IF}} = 910$ kHz, which means that for any given station within the desired range, the image station is outside the same band.