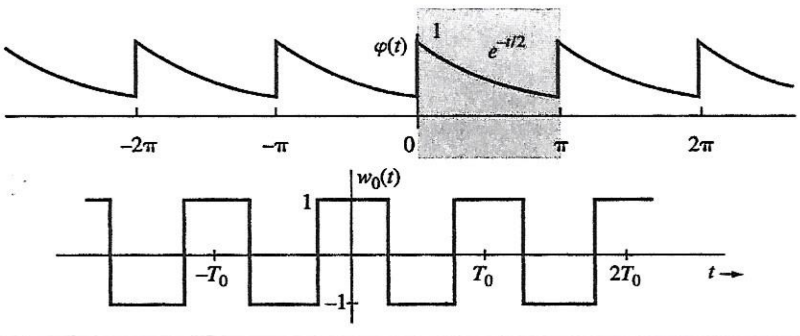
Josh Andrews
ECE 484 - HW 2

1

2.1-1 Find the average power of the signals in Fig. P2.1-1.

average power is the power over 1 period (periodic signal)

a)
b)



a) $P_g = \frac{1}{T_0}\int_0^{T_0} |g(t)|^2\, dt \Rightarrow P_g = \frac{1}{\pi}\int_0^{\pi} |e^{-t/2}|^2\, dt$

$P_g = \frac{1}{\pi}\int_0^{\pi} e^{-t}\, dt \Rightarrow P_g = \frac{-e^{-t}}{\pi}\Big|_{t=0}^{t=\pi} = \frac{-e^{-\pi}+1}{\pi}$

$$\boxed{P_g = \frac{1}{\pi}(1-e^{-\pi})}$$

b. $P_g = \frac{1}{T_0}\int_0^{T_0} |g(t)|^2\, dt \Rightarrow P_g = \frac{1}{T_0}\left[\int_0^{\frac{T_0}{4}} 1^2\, dt + \int_{\frac{T_0}{4}}^{\frac{3T_0}{4}} (-1)^2\, dt + \int_{\frac{3T_0}{4}}^{T_0} 1^2\, dt\right]$

$P_g = \frac{1}{T_0}\left[t\Big|_0^{\frac{T_0}{4}} + t\Big|_{\frac{T_0}{4}}^{\frac{3T_0}{4}} + t\Big|_{\frac{3T_0}{4}}^{T_0}\right] = \frac{1}{T_0}\left(\cancel{\frac{T_0}{4}} + \cancel{\frac{3T_0}{4}} - \cancel{\frac{T_0}{4}} + T_0 - \cancel{\frac{3T_0}{4}}\right)$

$$\boxed{P_g = 1}$$

2.1-7) Show that the power of a signal given by $g(t) = \sum_{k=m}^{n} D_k e^{j\omega_k t}$ where $\omega_i \neq \omega_k$ for all $i \neq k$ is parseval's theorem $P_g = \sum_{k=m}^{n} |D_k|^2$

Because the signal is complex and periodic ⇒

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t) g^*(t)\, dt$$ and plugging in $g(t)$

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left(\sum_{k=m}^{n} D_k e^{j\omega_k t}\right)\left(\sum_{i=m}^{n} D_k e^{j\omega_i t}\right)^* dt$$

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left(\sum_{k=m}^{n} D_k e^{j\omega_k t}\right)\left(\sum_{i=m}^{n} D_i^* e^{-j\omega_i t}\right) dt$$

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=m}^{n} \sum_{i=m}^{n} D_k D_i^* e^{j(\omega_k - \omega_i)t}\, dt$$

Now split up and solve

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=m}^{n} \sum_{i=m}^{n} D_k D_i^* e^{j(\omega_k - \omega_i)t}\, dt + \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=m}^{n} \sum_{k=m}^{n} D_k D_k^* e^{j(\cancel{\omega_k - \omega_k})t}\, dt$$

contains t so can integrate assuming finite value recieved

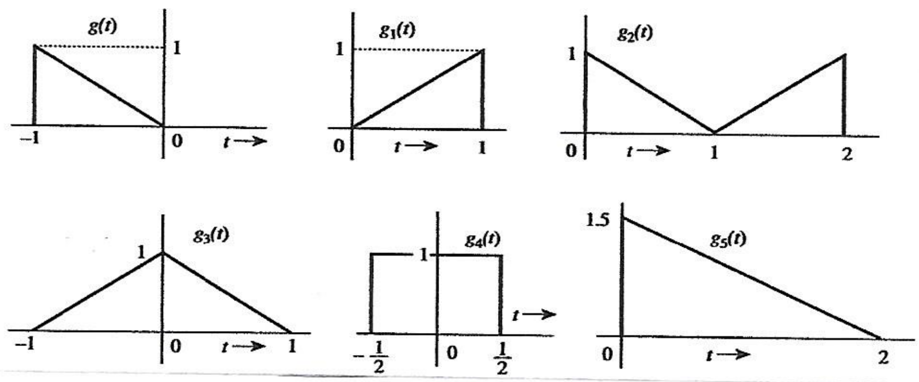
$\lim_{T\to\infty} \frac{1}{T} \cdot$ finite value ⇒ goes to ϕ

t disappears, factor

$\left[\sum_{k=m} |D_k|^2\right] \cancel{\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 1\, dt}$ $\frac{T}{T} = 1$

$$\boxed{P_g = \sum_{k=m}^{n} |D_k|^2}$$

2.3-1 In Fig. P2.3-1, the signal $g_1(t) = g(-t)$. Express signals $g_2(t)$, $g_3(t)$, $g_4(t)$, and $g_5(t)$ in terms of signals $g(t)$, $g_1(t)$, and their time-shifted, time-scaled, or time-inverted versions. For instance, $g_2(t) = g(t-T) + g_1(t-T)$ for some suitable value of T. Similarly, both $g_3(t)$ and $g_4(t)$ can be expressed as $g(t-T) + g(t+T)$ for some suitable value of T; and $g_5(t)$ can be expressed as $g(t)$ time-shifted, time-scaled, and then multiplied by a constant. (These operations may be performed in any order.)



- looking at g(t) and $g_1(t)$ ⇒ $\boxed{g(-t) = g_1(t)}$
- and $g_2(t)$ is comprised of g(t) and $g_1(t)$ ⇒ $\boxed{g_2(t) = g(t-1) + g_1(t-1)}$
- $g_3(t)$ is also comprised g(t) and $g_1(t)$ ⇒ $\boxed{g_3(t) = g(t-1) + g_1(t+1)}$
- $g_4(t)$ need to have signals shifted ⇒ $\boxed{g_4(t) = g(t-0.5) + g_1(t+0.5)}$
- and looking at $g_5(t)$ ⇒ $\boxed{g_5(t) = 1.5\, g(\frac{t}{2} - 1)}$

2.3-5 For an energy signal $g(t)$ with energy E_g, show that the energy of any one of the signals $-g(t)$, $g(-t)$, and $g(t-T)$ is E_g. Show also that the energy of $g(at)$ as well as $g(at-b)$ is E_g/a. This shows that neither time inversion nor time shifting affects signal energy. On the other hand, time compression of a signal by a factor a reduces the energy by the factor a. What is the effect on signal energy if the signal is (a) time-expanded by a factor a $(a > 1)$ and (b) multiplied by a constant a?

energy of $g(t)$ and $-g(t)$ is $E_g = \int_{-\infty}^{\infty} |g^2(t)|\,dt$

$E_{-g} = \int_{-\infty}^{\infty} |g^2(t')|\,dt'$ $(t=-t')$ so $\underline{E_g = E_{-g}}$

energy of $g(t-T) = \int_{-\infty}^{\infty} |g^2(t-T)|\,dt \Rightarrow E_{g(t-T)} = \int_{-\infty}^{\infty} |g^2(t')|\,dt'$ where $t' = t-T$

So $\underline{E_{g(t-T)} = E_g}$ (inversion and shifting = no change)

now with 'a'

$g(at) = \int_{-\infty}^{\infty} |g^2(at)|\,dt \Rightarrow E_{g(at)} = \int_{-\infty}^{\infty} |g^2(at)|\,dt$ $(t'=at) \Rightarrow$

$E_{g(at)} = \frac{1}{a}\int_{-\infty}^{\infty} |g^2(t')|\,dt'$ So $\underline{E_{g(at)} = \frac{1}{a}E_g}$

and shifting (a & b real)

$g(at-b) = \int_{-\infty}^{\infty} |g^2(at-b)|\,dt \Rightarrow E_{g(at-b)} = \int_{-\infty}^{\infty} |g^2(at-b)|\,dt$ $(t' = at-b)$, $dt' = a\,dt$ $\Rightarrow$

$E_{g(at-b)} = \int_{-\infty}^{\infty} |g^2(t')|\,\frac{dt'}{a} \Rightarrow \underline{E_{g(at-b)} = E_g/a}$

a. If time expanded by a $\Rightarrow E_{g(\frac{t}{a})} = \int_{-\infty}^{\infty} |g^2(\tfrac{t}{a})|\,dt$ $(t_1 = \frac{t}{a}) \Rightarrow E_{g(\frac{t}{a})} = \int_{-\infty}^{\infty} |g^2(t_1)|\,a\,dt_1 \Rightarrow$

$E_{g(\frac{t}{a})} = a\left[\int_{-\infty}^{\infty} |g^2(t_1)|\,dt_1\right] \Rightarrow \boxed{E_{g(\frac{t}{a})} = aE_g}$

b. If mult by a $\Rightarrow E_{ag(t)} = \int_{-\infty}^{\infty} |a^2 g^2(t)|\,dt$ $(t'=at) \Rightarrow E_{ag(at)} = a^2\int_{-\infty}^{\infty} |g^2(t')|\,dt'$

$\Rightarrow E_{ag(at)} = a^2 E_g \Rightarrow \boxed{E_{ag(t)} = a^2 E_g}$

2.5-1 Derive $|e|^2$ in alternate way with $e = (g - cx)$ and

$$|e|^2 = (g - cx)\cdot(g - cx) = |g|^2 + c^2|x|^2 - 2cg\cdot x$$

derivative with respect to c

$$(|e|^2)' = (|g|^2 + c^2|x|^2 - 2cg\cdot x)' = 2cx^2 - 2gx$$

and equate to ϕ $\Rightarrow$ $2cx^2 - 2gx = 0$ $\Rightarrow$ $2cx^2 = 2gx$

$$cx^2 = gx \Rightarrow c = \frac{gx}{x^2} \Rightarrow \boxed{c = \frac{\langle g, x\rangle}{\langle x, x\rangle}}$$

2.5-5 **2.5-5** Energies of the two energy signals $x(t)$ and $y(t)$ are E_x and E_y, respectively.

(a) If $x(t)$ and $y(t)$ are orthogonal, then show that the energy of the signal $x(t) + y(t)$ is identical to the energy of the signal $x(t) - y(t)$, and is given by $E_x + E_y$.

(b) If $x(t)$ and $y(t)$ are orthogonal, find the energies of signals $c_1x(t) + c_2y(t)$ and $c_1x(t) - c_2y(t)$.

(c) We define E_{xy}, the correlation of the two energy signals $x(t)$ and $y(t)$ as

$$E_{xy} = \int_{-\infty}^{\infty} x(t)y^*(t)\,dt$$

If $z(t) = x(t) \pm y(t)$, then show that

$$E_z = E_x + E_y \pm (E_{xy} + E_{yx})$$

a. IF orthogonal $\Rightarrow \int_{-\infty}^{\infty} x^*(t)y(t)\,dt = 0$ or $\int_{-\infty}^{\infty} x(t)y^*(t)\,dt = 0$

$x(t) + y(t) \rightarrow E_{(x+y)} = \int_{-\infty}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{\infty} |y(t)|^2\,dt + \cancel{\int_{-\infty}^{\infty} x^*(t)y(t)\,dt}^{0} + \cancel{\int_{-\infty}^{\infty} x(t)y^*(t)\,dt}^{0} \Rightarrow$

$E_{(x+y)} = \int_{-\infty}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{\infty} |y(t)|^2\,dt = \boxed{E_x + E_y}$ ✓

$x(t) - y(t) \rightarrow E_{(x-y)} = \int_{-\infty}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{\infty} |y(t)|^2\,dt - \cancel{\int_{-\infty}^{\infty} x^*(t)y(t)\,dt}^{0} - \cancel{\int_{-\infty}^{\infty} x(t)y^*(t)\,dt}^{0} \Rightarrow$

$E_{(x-y)} = \int_{-\infty}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{\infty} |y(t)|^2\,dt = \boxed{E_x + E_y}$ ✓

b. looking at part a,

for both $E_{(c_1x(t))} \pm E_{(c_2y(t))} = c_1^2\int_{-\infty}^{\infty} |x(t)|^2\,dt + c_2^2\int_{-\infty}^{\infty} |y(t)|^2\,dt \pm c_1c_2\cancel{\int_{-\infty}^{\infty} x^*(t)y(t)\,dt}^{0} \pm c_1c_2\cancel{\int_{-\infty}^{\infty} x(t)y^*(t)\,dt}^{0}$

$\Rightarrow \boxed{c_1x(t) \pm c_2y(t) = c_1^2 E_{c_1x(t)} + c_2^2 E_{c_2y(t)}}$

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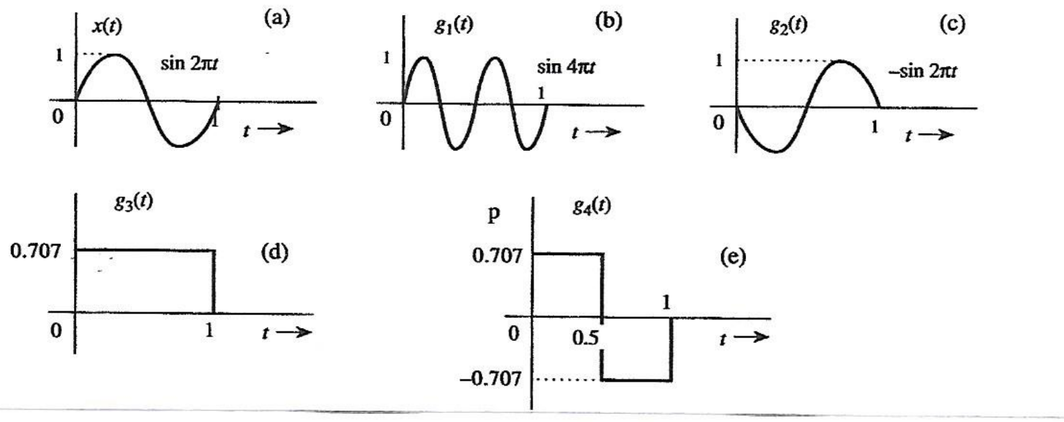
2.6-1 Find the correlation coefficient ρ between of signal $x(t)$ and each of the four pulses $g_1(t)$, $g_2(t)$, $g_3(t)$, and $g_4(t)$ shown in Fig. P2.6-1. To provide maximum margin against the noise along the transmission path, which pair of pulses would you select for a binary communication?



first need to find energy of each signal

$x(t) = \sin 2\pi t \Rightarrow E_x = \int_0^1 \sin^2(2\pi t)\,dt = \int_0^1 \frac{1-\cos(4\pi t)}{2}\,dt = \left[\frac{t}{2} - \frac{\sin 4\pi t}{2(4\pi)}\right]_{t=0}^{t=1} = 1/2$

$g_1(t) = \sin 4\pi t \Rightarrow E_{g_1} = \int_0^1 \sin^2 4\pi t\,dt = \int_0^1 \frac{1-\cos 8\pi t}{2}\,dt = \left[\frac{t}{2} - \frac{\sin 8\pi t}{2(8\pi)}\right]_{t=0}^{t=1} = 1/2$

$g_2(t) = -\sin 2\pi t \Rightarrow E_{g_2} = \int_0^1 \sin^2 2\pi t\,dt = 1/2$

$g_3(t) = \begin{cases} 0.707 & 0 \le t \le 1 \\ 0 & \text{else} \end{cases} \Rightarrow E_{g_3}(t) = \int_0^1 0.707^2\,dt = 1/2 \int_0^1 dt = 1/2$

$g_4(t) = \begin{cases} 0.707 & 0 \le t \le 0.5 \\ -0.707 & 0.5 \le t \le 1 \\ 0 & \text{else} \end{cases} \Rightarrow E_{g_4}(t) = 1/2\left(\int_0^{0.5} dt + \int_{0.5}^1 dt\right) = 1/2$

now find correlation coefficient

$\rho_{xg_1} = \frac{1}{\sqrt{E_x E_{g_1}}} \int_0^1 x(t) g_1(t)\,dt = 2\int_0^1 \sin(2\pi t)(2\sin(2\pi t)\cos(2\pi t))\,dt = 2\int_0^1 (1-\cos(4\pi t))\cos(2\pi t)\,dt$

$= 2\int_0^1 \cos(2\pi t)\,dt - \int_0^1 \cos(6\pi t)\,dt - \int_0^1 \cos(2\pi t)\,dt = 0 \Rightarrow \boxed{\rho_{xg_1} = 0}$

$\rho_{xg_2} = -2\int_0^1 \left(\frac{1-\cos(4\pi t)}{2}\right)dt = -2\left[\frac{t}{2} - \frac{\sin 4\pi t}{2(4\pi)}\right]_{t=0}^{t=1} = -1 \Rightarrow \boxed{\rho_{xg_2} = -1}$

$\rho_{xg_3} = 2(0.707)\int_0^1 \sin(2\pi t)\,dt = 2(0.707)\left[\frac{\cos(2\pi)}{2\pi} - \frac{\cos(0)}{2\pi}\right] = 0 \Rightarrow \boxed{\rho_{xg_3} = 0}$

$\rho_{xg_4} = 2(0.707)\left[\int_0^{0.5} \sin(2\pi t)\,dt - \int_{0.5}^1 \sin(2\pi t)\,dt\right] = 2(0.707)\left[-\frac{\cos 2\pi t}{2\pi}\Big|_0^{0.5} + \frac{\cos 2\pi t}{2\pi}\Big|_{0.5}^1\right] = 2(0.707)\left(\frac{4}{2\pi}\right) = \frac{2.828}{\pi}$

$\boxed{\rho_{xg_4} = \frac{2.828}{\pi}}$

best choice x(t) and $g_2(t)$ due to lowest correlation factor

2.7-2 a) For each of the signals in previous problem determine orthonormal basis functions of dimension 4

• for $x(t)$ $\Rightarrow$ $E_0 = \int_0^1 \sin^2(2\pi t)\,dt = \int_0^1 \left[\frac{1-\cos(4\pi t)}{2}\right]dt = \left[\frac{t}{2} - \frac{\sin 4\pi t}{4\pi}\right]\Big|_0^1 \Rightarrow E_0 = 1/2$

$$f_0(t) = \frac{e_0(t)}{\sqrt{E_0}} = \frac{x(t)}{\sqrt{1/2}} = \frac{\sin 2\pi t}{\frac{1}{\sqrt{2}}} \Rightarrow \boxed{f_0(t) = \sqrt{2}\sin 2\pi t}$$

• for $g_1(t)$ $\Rightarrow$ $E_1 = \int_0^1 \sin^2(4\pi t)\,dt = \left[\frac{t}{2} - \frac{\sin 8\pi t}{8\pi}\right]\Big|_0^1 = 1/2$

$$f_1(t) = \frac{g_1(t)}{\sqrt{1/2}} = \frac{\sin 4\pi t}{\frac{1}{\sqrt{2}}} \Rightarrow \boxed{f_1(t) = \sqrt{2}\sin 4\pi t}$$

• for $g_2(t)$ $\Rightarrow$ $e_2(t) = g_2(t) - [C_{20}f_0(t) + C_{21}f_1(t)]$

$$C_{20} = \int_{-\infty}^{\infty} g_2(t)f_0(t) = -\sqrt{2}\int_0^1 \sin^2 2\pi t\,dt = -\sqrt{2}\left[\frac{t}{2} - \frac{\sin 4\pi t}{4\pi}\right]\Big|_0^1 = -\frac{1}{\sqrt{2}}$$

$$C_{21} = \int_{-\infty}^{\infty} g_2(t)f_1(t) = -\sqrt{2}\left[\frac{\sin(2\pi t)}{4\pi} - \frac{\sin(6\pi t)}{12\pi}\right]\Big|_0^1 = 0$$

$$e_2(t) = -\sin(2\pi t) - \left[\frac{-1}{\sqrt{2}}(\sqrt{2}\sin 2\pi t) + 0\right] = 0 \quad \text{and} \therefore \Rightarrow \boxed{f_2(t) = 0}$$

- for $g_3(t)$ $\Rightarrow$ $e_3(t) = g_3(t) - [C_{30}f_0(t) + C_{31}f_1(t) + C_{32}f_2(t)]$

$$C_{30} = \int_0^1 \sin 2\pi t\,dt = 0$$

$$C_{31} = \int_0^1 \sin 4\pi t\,dt \doteq 0 \qquad \Rightarrow E_3 = 1/2 \quad \Rightarrow f_3(t) = \frac{0.707}{\sqrt{1/2}} \Rightarrow \boxed{f_3(t) = 1}$$

$$C_{32} = f_2(t) = 0$$

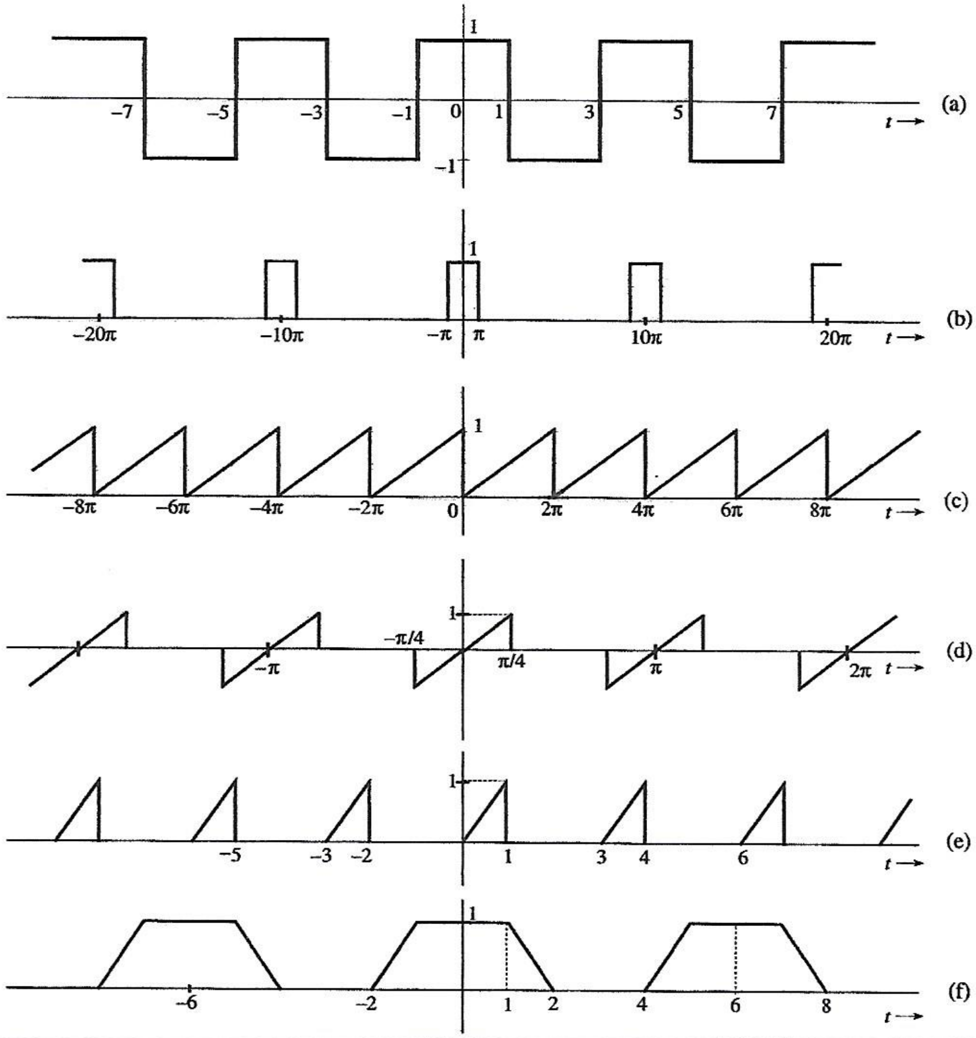
(previous problem)

b) $x(t) = \sin 2\pi t = \frac{\sqrt{2}\sin 2\pi t}{\sqrt{2}} = \frac{f_0(t)}{\sqrt{2}}$, $g_1(t) = \sin 4\pi t = \frac{\sqrt{2}\sin 4\pi t}{\sqrt{2}} = \frac{f_1(t)}{\sqrt{2}}$, $g_2(t) = -\frac{\sqrt{2}\sin 2\pi t}{\sqrt{2}} = \frac{-f_0(t)}{\sqrt{2}}$

$g_3(t) = 0.707 = \frac{1}{\sqrt{2}}f_3(t)$ $\qquad$ $g_4(t) = g_3(2t) - g_3(2t-1)$

$$\begin{bmatrix} x(t) \\ g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} f_0(t) \\ f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}$$

2.8-2



for each of the signals find the compact trigonometric Fourier series and sketch the amplitude and phase spectra. if sine/cosine term absent explain

a. $T_0 = 4 \Rightarrow \omega_0 = \pi/2$ and even sym. $\Rightarrow g(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos(n\omega_0 t) + b_n \sin n\omega_0 t$

$a_0 = \frac{1}{T_0}\int_{T_0} g(t)\,dt = \frac{1}{4}\int_{-2}^{2} g(t)\,dt = \frac{1}{2}\int_0^2 [g(t)]\,dt = \frac{1}{2}[\int_0^1 dt + \int_1^2 -dt \Rightarrow \underline{a_0 = 0}$

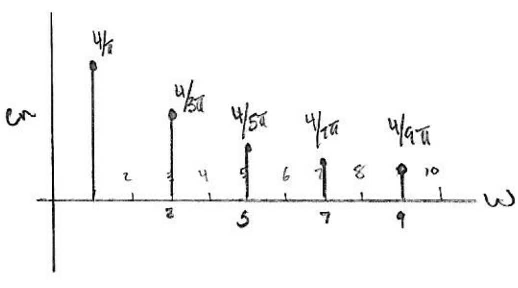
$a_n = \frac{1}{2}\int_0^2 [g(t)\cos\frac{n\pi}{2}t + g(t)\cos\frac{n\pi}{2}t]\,dt = \int_0^2 g(t)\cos\frac{n\pi}{2}t\,dt = \underline{\frac{4}{n\pi}[\sin\frac{n\pi}{2}]}$

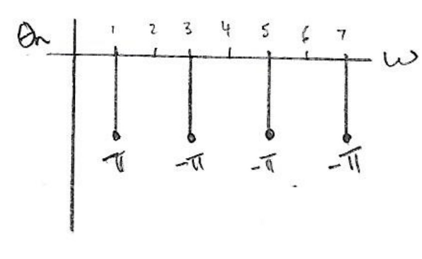
$b_n = \frac{1}{2}\int_0^2 (g(t)\sin\frac{n\pi}{2}t - g(t)\sin(\frac{n\pi}{2}t))\,dt = 0$

$g(t) = \sum_{n=1}^{\infty}\frac{4}{n\pi}[\sin(\frac{n\pi}{2})\cos(\frac{n\pi}{2}t)]$

$C_n = \frac{4}{n\pi}$ for n odd, 0 for even

$\theta_n = \begin{cases} -\pi & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$



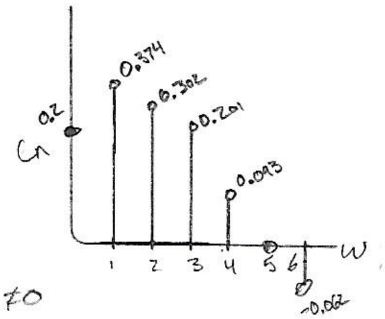


2.8-2 b. even sym $\Rightarrow b_n = 0$, $T_0 = 10\pi \Rightarrow \omega_0 = 1/5$

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t)$$

$$a_0 = \frac{1}{T_0}\int_{T_0} g(t)\,dt = \frac{1}{10\pi}\int_{-\pi}^{\pi} 1\,dt \quad a_0 = 1/5$$

$$a_n = \frac{2}{10\pi}\int_{-\pi}^{\pi} \cos\frac{nt}{5}\,dt = \frac{1}{5\pi}\cdot\frac{5}{n}\sin\frac{nt}{5}\Big|_{-\pi}^{\pi} \Rightarrow a_n = \frac{2}{n\pi}\sin\frac{n\pi}{5}$$



$$g(t) = \frac{1}{5} + \sum_{n=1}^{\infty} \frac{2\sin n\pi/5}{n\pi}\cos\frac{nt}{5}$$

so

$$C_n = \begin{cases} \frac{2\sin\frac{n\pi}{5}}{n\pi} & n \neq 0 \\ 1/5 & n = 0 \end{cases}$$

$$\theta_n = \phi$$