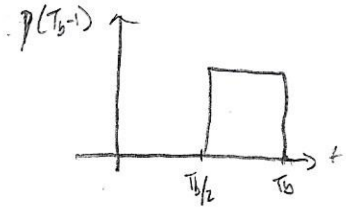
Josh Andrews
ECE 484
HW 9

11.2-1) a) Determine optimum architecture

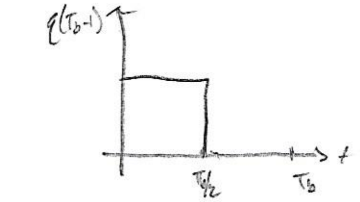
$q(t) = p_0(t) = u(t) - u(t - T_b/2)$

$p(t) = p_0(t - T_b/2) = u(t - T_b/2) - u(t - T_b)$

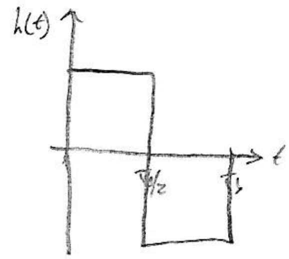
$$h(t) = p(T_b - t) - q(T_b - t) = u(T_b/2 - t) - u(-t) - [u(T_b - t) - u(T_b/2 - t)]$$



$u(T_b - t) - u(T_b/2 - t)$



$u(\frac{T_b}{2} - t) - u(-t)$



optimum

b. find optimum threshold

$$p_0(T_b) = \int_{-\infty}^{\infty} h(\tau) p(T_b - \tau)\, dt = \int_0^{T_b/2} [u(\tfrac{T_b}{2} - t) - u(-t)] d\tau - \int_{T_b/2}^{T_b} [u(\tfrac{T_b}{2} - t) - u(-t)] d\tau = T_b/2$$

$$q_0(T_b) = \int_0^{T_b/2} [u(T_b - t) - u(T_b/2 - t)] d\tau - \int_{T_b/2}^{T_b} [u(T_b - t) - u(T_b/2 - t)] d\tau = -T_b/2$$

$$P_e = P[0]\, Q\left(\frac{a_0 - q_0(T_b)}{\sigma_b}\right) + P[1]\, Q\left(\frac{p_0(T_b) - a_0}{\sigma_b}\right) = 0.4 \left(\frac{a_0 - q_0(T_b)}{\sigma_b}\right) + 0.6\, Q\left(\frac{p_0(T_b) - a_0}{\sigma_b}\right)$$

$$\Rightarrow e^{\left(\frac{p_0(T_b)^2 - a_0^2}{2\sigma_b^2} - \frac{a_0^2 - q_0^2(T_b)}{2\sigma_n^2}\right)} = 1.5 \quad \Rightarrow \quad 2 a_0 T_b = 0.81 \sigma_n^2 \Rightarrow \boxed{a_0 = -0.405 \cdot \frac{\sigma_b^2}{T_b}}$$

$$\boxed{P_c = 0.4 Q\left(0.5 \frac{T_b}{\sigma_b} + 0.405 \frac{\sigma_n}{T_b}\right) + 0.6 Q\left(0.5 \frac{T_b}{\sigma_n} - 0.405 \frac{\sigma_n}{T}\right)}$$

c. optimum threshold $= 0 \Rightarrow$ $\boxed{P_e = Q\left(\frac{0.5 T_b}{\sigma_n}\right)}$

11.2-4) when 0 transmitted

$$C_0 = C_{01} P(\epsilon | m=0) = C_{01} Q\left(\frac{a_0 - a_0(T_b)}{\sigma_n}\right) = C_{01} Q\left(\frac{a_0+1}{\sigma_n}\right)$$

when 1

$$C_1 = C_{10} P(\epsilon | m=1) = C_{10} Q\left(\frac{1-a_0}{\sigma_n}\right)$$

average

$$C = P_m(0) C_0 + P_m(1) C_1 = \tfrac{1}{2}\left[C_{01} Q\left(\frac{1+a_0}{\sigma_n}\right) + C_{10} Q\left(\frac{1-a_0}{\sigma_n}\right)\right]$$

and d/da

$$\frac{dC}{da_0} = 0 = \frac{1}{a\sigma_n\sqrt{2\pi}}\left[C_{01} e^{-\left(\frac{(1+a_0)^2}{2\sigma_n^2}\right)} - C_{10} e^{-\left(\frac{(1-a_0)^2}{2\sigma_n^2}\right)}\right. = e^{\left(\frac{2a_0}{\sigma_n^2}\right)}$$

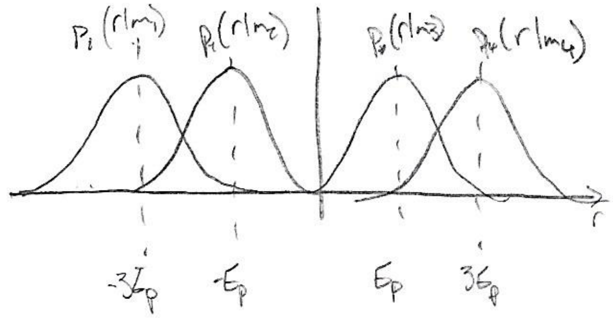
calc a_0

$$a_0 = \frac{\sigma_n^2}{2} \ln\left(\frac{C_{01}}{C_{10}}\right) = \frac{N}{4} \ln\left(\frac{C_{01}}{C_{10}}\right) \quad \text{as } \sigma_n^2 = \frac{N}{4}$$

11.2-6) $p_r(r|m_i) = \frac{1}{\sigma_n\sqrt{2\pi}} e^{\left(\frac{-[r-p_o^i(t_m)]^2}{2\sigma_n^2}\right)}$ where p_o^i are filter outputs at reciever

and $h(t) = p(t_m - t)$

$$p_o^i(t_m) = \int_{-\infty}^{\infty} p^i(t)\, h(t_m - t)\, dt = k_i \int_{-\infty}^{\infty} p^2(t)\,dt = k_i E_p \Rightarrow p_o^i(t_m) = \begin{cases} -3E_p & i=1 \\ -E_p & i=2 \\ +E_p & i=3 \\ +3E_p & i=4 \end{cases}$$



b) $E_s = \frac{1}{4}[E_p + E_p + 9E_p + 9E_p] = 5E_p \Rightarrow$ thresholds are $-2E_p, 0, 2E_p$

So $P(C|m_1) = P(n < E_p) = 1 - Q\left(\sqrt{\frac{2E_p}{N}}\right)$ and $P(C|m_2) = P(-E_p < n < E_p) = 1 - 2Q\left(\sqrt{\frac{2E_p}{N}}\right)$

$$P_e = 1 - \sum_{i=1}^{4} P(C|m_i)P(m_i) = 1 - \frac{1}{2}\left[1 - Q\left(\sqrt{\frac{2E_p}{N}}\right) + 1 - 2Q\left(\sqrt{\frac{2E_p}{N}}\right)\right]$$

$$\boxed{P_e = \frac{3}{2} Q\left(\sqrt{\frac{2E_B}{5N}}\right)}$$

11.3-1) a) derive optimal reciever and threshold

$$h(t) = p(T_b - t) - q(T_b - t) = \sqrt{2}\sin\left(\frac{\pi(T_b-t)}{T_b}\right)\cos\left[\omega_c + \left(\frac{\Delta\omega}{2}\right)\right](T_b - t) - \sqrt{2}\sin\left(\frac{\pi(t-T_b)}{T_b}\right)\cos\left[\omega_c - \left(\frac{\Delta\omega}{2}\right)\right](T_b - t)$$

$$\boxed{h(t) = 2\sqrt{2}\sin\left(\frac{\pi t}{T_b}\right)\sin\omega_c(t-T_b)\sin\left(\frac{\Delta\omega}{2}\right)(t-T_b)}$$

$$E_p = \int_0^{T_b} p^2(t)\,dt = \frac{1}{2}\int_0^{T_b} 1 - \cos\left(\frac{2\pi t}{T_b}\right) dt = T_b/2$$

and $\boxed{a_0 = \frac{1}{2}(E_q - E_p) = 0}$

b) find bit error probability

$$E_{pq} = \int_0^{T_b} p(t)q(t)\,dt = \frac{1}{2}\int_0^{T_b}\left[1 - \cos\left(\frac{2\pi t}{T_b}\right)\right]\cos(\Delta\omega t)\,dt$$

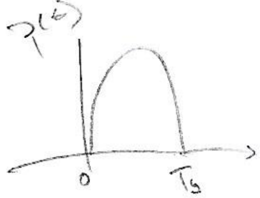
$$= \frac{1}{2}\left(\frac{\sin(\Delta\omega T_b)}{\Delta\omega}\right) - \frac{\Delta\omega T_b^2}{2}\left(\frac{\sin(\Delta\omega T_b)}{(\Delta\omega)^2 T_b^2 - 4\pi^2}\right) = \frac{T_b}{2}\operatorname{sinc}(\Delta\omega T_b)\left[\frac{1}{1 - \left(\frac{\Delta\omega}{2\pi}\right)^2 T_b^2}\right]$$

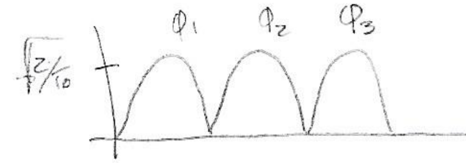
$$\boxed{P_b = Q\sqrt{\frac{T_b}{2N}\left(1 - \frac{\operatorname{sinc}(\Delta\omega T_b)}{1 - \left(\frac{\Delta\omega}{2\pi}\right)^2 T_b^2}\right)}}$$

c. Is it possible to find the optimum $\Delta\omega$ to minimize b.e.p.

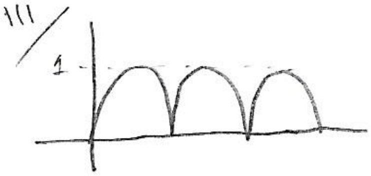
Yes as $f(\Delta\omega) = \dfrac{\operatorname{sinc}(\Delta\omega T_b)}{1 - \left(\frac{\Delta\omega}{2\pi}\right)^2 T_b^2}$ can be minimized

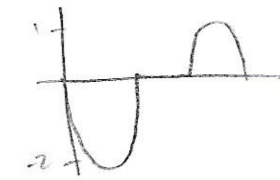
11.4-2)

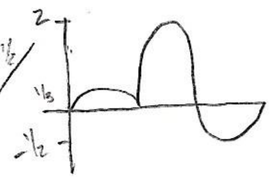
a) 

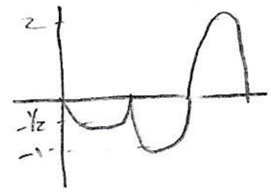
$\Rightarrow$ 

normalize $\sqrt{2/T_0}$ to 1

$\Rightarrow$ 111/ 

-2, 0, 1/ 

1/3, 2, -1/2/ 

-1/2, -1, 2/ 

b. $E_p = \int_0^{T} \frac{2}{T_0}\sin^2\left(\frac{\pi t}{T_0}\right)dt = \frac{1}{T_0}(T_0 - 0) = \infty$ and $E_y = \|y\|^2$

111/ $\sqrt{1^2+1^2+1^2}^{\,2} = \underline{3} = E_p$

-2, 0, 1/ $\sqrt{-2^2+0+1^2}^{\,2} = \underline{5} = E_p$

1/3, 2, -1/2/ $\sqrt{\frac{1}{3}^2+2^2+(-\frac{1}{2})^2} = \underline{4.36} = E_p$

-1/2, -1, 2 $\sqrt{(-\frac{1}{2})^2+(-1)^2+(2)^2}^{\,2} = \underline{5.25} = E_p$

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11.5-1) from previous question we know this signal is orthonormal

a) $\Rightarrow$ Since noise process is white $\Rightarrow$ $\underline{\varphi_k(t) = s_k(t)}$ for $k \in [-A, A]$

b) $y(t) = \sum_{i=1}^{5} y_i \varphi_i(t)$ and $y_i = s_{ki} + n_i$ where

$n_i = \int_{-\infty}^{\infty} n_w(t) \varphi_i(t) dt$ and $n_j n_k = N/2$ if $j = k$, 0 otherwise

$y_i = 1 + n_i$ when $i = k$ $\Rightarrow$ is Gaussian RV with mean 1 and variance $N/2$
$y_i = n_i$ when $i \neq k$ is Gaussian RV with mean 0 and variance $N/2$

c) $P_y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(\frac{-(y-m)^2}{2\sigma^2}\right)}$ $\Rightarrow$ the joint density is

$$(\pi N)^{-5/2} e^{\left(-\frac{1 - 2y_k + y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2}{N}\right)}$$