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ECE 484
Homework 6

6.2-2) Find Nyquist f_s & T_s for:

a. $\text{sinc}(2100\pi t) \overset{F}{\Longleftrightarrow} \frac{1}{2100}\Pi\left(\frac{f}{2100}\right)$ and $\Rightarrow B = 1050\,\text{Hz}$ so $f_s = 2B$

$$\boxed{f_s = 2.1\,\text{kHz} \qquad T_s = \frac{1}{2.1\,\text{kHz}} = 476\,\mu s}$$

b. $5\,\text{sinc}^2(200\pi t) \overset{F}{\Longleftrightarrow} \frac{5}{200}\Delta\left(\frac{f}{400}\right) \Rightarrow B = 200 \Rightarrow$

$$\boxed{f_s = 400\,\text{Hz} \qquad T_s = \frac{1}{400} = 2.5\,\text{ms}}$$

c. $\text{sinc}(2100\pi t) + \text{sinc}^2(200\pi t) \overset{F}{\Longleftrightarrow} \frac{1}{2100}\Pi\left(\frac{f}{2100}\right) + \frac{1}{200}\Delta\left(\frac{f}{400}\right)$

however because they are added, they will overlap and the total bandwidth will be the larger of the two.

In this case $B = 1050\,\text{Hz}$ $\Rightarrow$ as in part A $\boxed{f_s = 2.1\,\text{kHz},\ T_s = 476\,\mu s}$

d. $\text{sinc}(200\pi t)\,\text{sinc}(2100\pi t)$ so

$$\text{sinc}(200\pi t) \Longleftrightarrow \frac{1}{200}\Pi\left(\frac{f}{200}\right)$$
$$\text{sinc}(2100\pi t) \Longleftrightarrow \frac{1}{2100}\Pi\left(\frac{f}{2100}\right)$$

and because it is a product, it is the same as convolution

have to add the bandwidths together

$B = (1050 + 100)\,\text{Hz} \Rightarrow \boxed{f_s = 2.3\,\text{kHz} \text{ and } T_s = 435\,\mu s}$

6.2-4) A low-pass signal $g(t)$ sampled at rate of $f_s > 2B$ need reconstruction. The sampling interval is $T_s = 1/f_s$

a. If the reconstruction pulse used is: $p(t) = \Pi\left(\frac{t}{T_s} - \frac{1}{2}\right)$, specify $E(f)$ to recover $g(t)$

$$p(t) = \Pi\left[\frac{1}{T_s}\left(t - \frac{T_s}{2}\right)\right] \iff P(f) = T_s \operatorname{sinc}(\pi f T_s) e^{-j\pi f T_s}$$

and

$$E(f) \begin{cases} T_s/P(f) & |f| < B \\ \text{flexible} & \text{for } B < |f| < \left(\frac{1}{T_s} - B\right) \\ 0 & |f| > \left(\frac{1}{T_s} - B\right) \end{cases}$$

substitute in $P(f)$

$$E(f) = \frac{T_s}{T_s \operatorname{sinc}(\pi f T_s) e^{-j\pi f T_s}} \quad \text{for } |f| < B$$

$$\boxed{E(f) = \frac{e^{j\pi f T_s}}{\operatorname{sinc}(\pi f T_s)} \quad \text{for } |f| < B, \quad T_s = 1/f_s}$$

b. $p(t) = \Pi\left(\frac{t}{T_s/2} - \frac{1}{2}\right) \Rightarrow p(t) = \Pi\left(\frac{2}{T_s}\left(t - \frac{T_s}{4}\right)\right) \iff P(f) = \frac{T_s}{2} \operatorname{sinc}\left(\frac{\pi f T_s}{2}\right) e^{-j\pi f (T_s/2)}$

$$E(f) = \frac{T_s}{P(f)} \text{ for } |f| < B \Rightarrow E(f) = \frac{T_s}{\frac{T_s}{2} \operatorname{sinc}\left(\frac{\pi f T_s}{2}\right) e^{-j\pi f T_s/2}} \quad \text{for } |f| < B$$

$$\boxed{E(f) = \frac{2e^{j\pi f T_s/2}}{\operatorname{sinc}\left(\frac{\pi f T_s}{2}\right)} \quad |f| < B, \quad T_s = 1/f_s}$$

c. $p(t) = \sin\left(\frac{2\pi t}{T_s}\right)\left[u(t) - u\left(t - \frac{T_s}{2}\right)\right] = \frac{j}{2}\left(e^{-j2\pi t/T_s} - e^{j2\pi t/T_s}\right) \Pi\left[\frac{2}{T_s}\left(t - \frac{T_s}{4}\right)\right]$

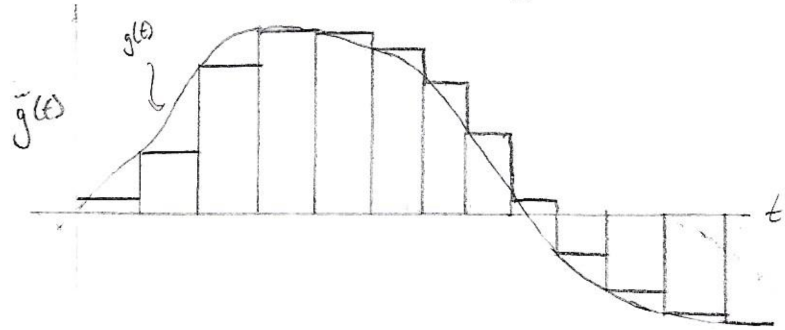
$$P(f) = \frac{jT_s}{4}\left[\operatorname{sinc}\left(\frac{\pi f T_s + \pi}{2}\right) e^{-j\pi(f+4)T_s/2} - \operatorname{sinc}\left(\frac{\pi f T_s - \pi}{2}\right) e^{-j\pi(f-4)T_s/2}\right]$$ and plug into $E(f)$

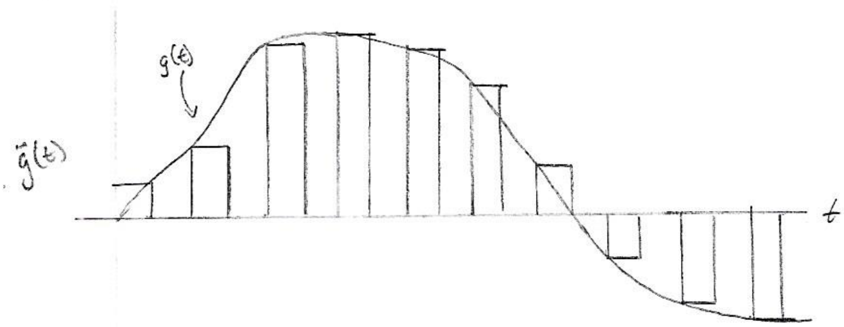
$$\boxed{E(f) = \frac{-1}{4j\left[\operatorname{sinc}\left(\frac{\pi f T_s + \pi}{2}\right) e^{-j\pi(f+4)T_s/2} - \operatorname{sinc}\left(\frac{\pi f T_s - \pi}{2}\right) e^{-j\pi(f-4)T_s/2}\right]} \quad \text{for } |f| < B, \quad T_s = 1/f_s}$$

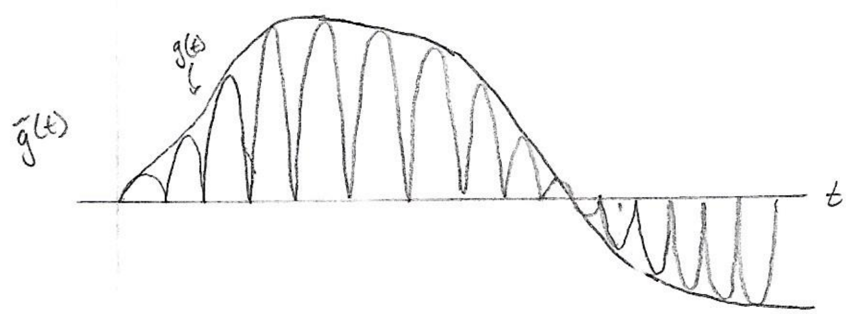
d. $p(t) = \Delta\left(\frac{t}{T_s/2} - \frac{1}{2}\right) = \Delta\left[\frac{2}{T_s}\left(t - \frac{T_s}{4}\right)\right] \overset{F}{\iff} P(f) = \frac{T_s}{4} \operatorname{sinc}^2\left(\frac{\pi f T_s}{4}\right) e^{-j\pi f T_s/2}$

$$\boxed{E(f) = \frac{4e^{j\pi f T_s/2}}{\operatorname{sinc}^2\left(\frac{\pi f T_s}{4}\right)} \quad |f| < B, \quad T_s = 1/f_s}$$

6.2-4) e. sketch the recovered signal for each part

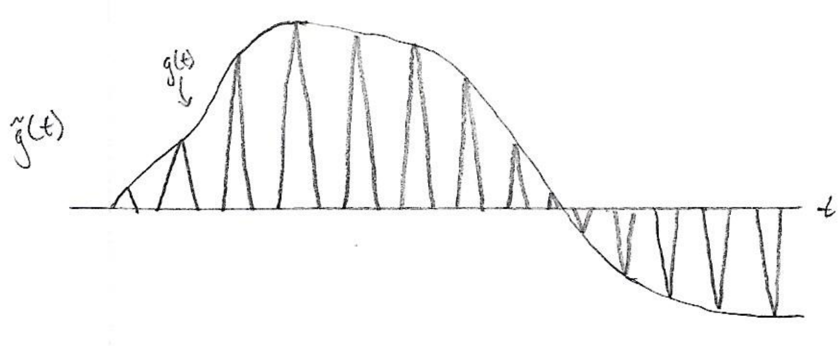
a) 

b) 

c) 

all graphs have equal T_s for each pulse drawn

C is not drawn very well

d) 

6.1-10) In nonideal sampler, the averaging filter impulse response is shown to be $q(t) = \Delta\left(\frac{t}{T_s/4}\right)$, design reconstruction to recover original analog signal

first need DC term of Fourier

$$Q_0 = \frac{1}{T_s}\int_{-T_s/8}^{T_s/8} \Delta\left(\frac{t}{T_s/4}\right) dt = \frac{2}{T_s}\int_0^{T_s/8} \frac{t}{T_s/8}\, dt = \frac{2}{T_s}\cdot\frac{(T_s/8)^2}{2(T_s/8)} = \frac{1}{8}$$

and rest

$$Q_n = \frac{1}{T_s}\int_{-T_s/8}^{T_s/8} \Delta\left(\frac{t}{T_s/4}\right) e^{-jn\omega_s t}\, dt = \frac{2}{T_s}\int_0^{T_s/8} \frac{t}{T_s/8} \cos\left(\frac{2n\pi t}{T_s}\right) dt$$

$$Q_n = \frac{n\pi \sin(n\pi + 4) + 4\cos(n\pi/4) - 4}{(n\pi)^2}$$

For the averaging filter

$$H_a(f) = \text{sinc}\left[\pi f T_s/4\right] \Rightarrow \boxed{F_0(f) = \frac{1}{T_s}\sum_n Q_n \,\text{sinc}\left[\pi(f+nf_s)\frac{T_s}{4}\right]}$$

and the reconstruction pulse is

$$p(t) = \Pi\left[\frac{t - 0.5T_s}{T_s}\right] \Leftrightarrow P(f) = T_s \,\text{sinc}(\pi f T_s)\, e^{-j\pi f T_s}$$

$$E(f) = \frac{T_s}{P(f)F_0(f)} \quad \text{for } |f| < B$$

Find the Equalizer freq resp.

$$E(f) = \frac{T_s}{\left(T_s \,\text{sinc}(\pi f T_s) e^{-j\pi f T_s}\right)\left(\frac{1}{T_s}\sum_n Q_n \,\text{sinc}\left[\pi(f+nf_s)\frac{T_s}{4}\right]\right)}$$

$$\boxed{E(f) = \frac{T_s\, e^{j\pi f T_s}}{\text{sinc}(\pi f T_s)\sum Q_n \,\text{sinc}\left[\pi(f+nf_s)\frac{T_s}{4}\right]}}$$

6.2-2) A TV signal (video and audio) has a bandwidth of 4.5 MHz. The signal is sampled, quantized, and binary coded to obtain a PCM signal

a. Determine the sampling rate if 20% above Nyquist rate

$B = 4.5\ MHz \Rightarrow$ Nyquist rate $= 2 \cdot 4.5 = 9\ MHz$

now 20% sampling

Sample rate $= \left(9 + \frac{20}{100} \cdot 9\right) MHz \Rightarrow \boxed{\text{Sample rate} = 10.8\ MHz}$

b. If samples quantized into 1024 levels, determine # of binary pulses to encode

$L = 1024$ and $2^n = L$

$2^n = 2^{10} \Rightarrow \boxed{\#\text{ of binary pulses} = 10}$

c. Determine binary pulse rate and minimum bandwidth to transmit

pulse rate = sample rate $\cdot n$ $\Rightarrow$ pulse rate $= 10.8\ MHz \cdot 10$

$\boxed{\text{binary pulse rate} = 108\ Mb/s}$

and

$B_{min} = \frac{\text{sample rate} \cdot n}{2} = \frac{108\ MHz}{2} \Rightarrow \boxed{B_{min} = 54\ MHz}$

6.2-3) 128 stereo stations in one data stream, for each station the bandwidth 15,000 Hz are sampled, quantized, and binary coded into PCM. must use time multiplexing

a) if maximum acceptable quantization error is 0.25%, find minimum # of bits

Quantization step is Δv ⇒ max error $= \frac{\Delta v}{2}$, need to split into L levels

$\Delta v = \frac{2m_p}{L}$ and error at most 0.25% of m_p

$0.25\% = \frac{m_p}{L}$ ⇒ $L = 400$ ⇒ need to round up to 512

$2^n = 512$ ⇒ $\boxed{n = 9}$

b) find minimum bit rate

$f_m = 15\text{ kHz}$ ⇒ $f_s = 2f_m = 30\text{ kHz}$ and 20% higher is 36 kHz

need 2 seperate channels for L and R and 9 bits each for 128 stations

Bit rate $= (36000)(2)(9)(128)$ ⇒ $\boxed{\text{Bitrate} = 82.944\text{ Mb/s}}$

c) if 5% more bits added, determine minimum bandwidth

total rate $= 82.944\text{M}\left(\frac{105}{100}\right)$ ⇒ total rate $= 87.09\text{ Mb/s}$

$B_{min} = \frac{\cancel{\text{total}}}{2}$ ⇒ $\boxed{B_{min} = 43.54\text{ MHz}}$

6.2-4) $m(t)$ is normalized $\pm 1V$ peak and $P_{avg} = 20mW$. If SQNR $\geq 43dB$, find # of bits and SNR

$$SNQR_{dB} = 10\log\left(3L^2\left(\frac{m^2(t)}{m_p^2}\right)\right) = 10\log\left(3L^2\left(\frac{20mW}{1V}\right)\right) = 48$$

$$10^{4.3} = 0.06L^2 \Rightarrow L = 522 \Rightarrow \text{round to } L = 1024 \Rightarrow \boxed{n = 10}$$

$$SQNR_{dB} = 10\log\left(3(1024)^2\left(\frac{20m}{1}\right)\right) \Rightarrow \boxed{SQNR_{dB} = 47.98\ dB}$$