Josh Andrews
ECE 484
Homework 5

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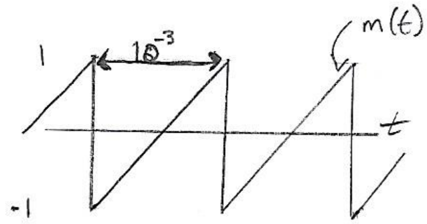
5.1-2) given



a) Sketch $\varphi_{FM}(t)$ and $\varphi_{PM}(t)$ for $m(t)$ if $\omega_c = 2\pi\times 10^6$, $K_f = 2000\pi$, $K_p = \pi/2$

first need $m(t)$, use sawtooth equation

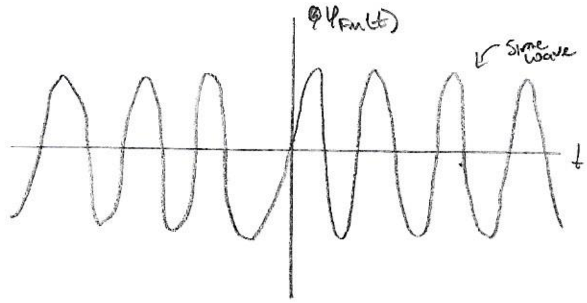
$$m(t) = \frac{2A}{T}t - A = \frac{2(1)}{10^{-3}}t - 1 \Rightarrow \underline{m(t) = 2000t - 1}$$

and now

$$\varphi_{FM}(t) = A\cos\Big[\underbrace{\omega_c t + K_f \int_{-\infty}^{t} m(t)\,dt}_{\Theta}\Big] \quad \text{and} \quad \omega_i = \frac{d\Theta}{dt} \Rightarrow \omega_i = \omega_c + K_f m(t)$$

$\omega_i = 2\pi\times 10^6 + 2000\pi(2000t-1)$ and changing to f $\Rightarrow f_i = 10^6 + 10^3 m(t)$

$f_{i_{min}} = 0.999$ MHz $\qquad f_{i_{max}} = 1.001$ MHz



← carrier frequency switches between the two every half cycle of $m(t)$

hard to notice difference

and for $\varphi_{PM}(t)$

$$\varphi_{PM}(t) = A\cos\Big[\underbrace{\omega_c t + K_p m(t)}_{\Theta}\Big] \quad \text{and} \quad \omega_i = \frac{d\Theta}{dt} \Rightarrow f_i = 10^6 + \frac{1}{4}\frac{dm(t)}{dt}$$

$\frac{dm(t)}{dt} = \pm 2000 \Rightarrow f_{i_{min}} = 0.9995$ MHz $\qquad f_{i_{max}} = 1.0005$ MHz

due to the discontinuities, the PM signal jumps π radians every $\frac{1}{1000}$ sec

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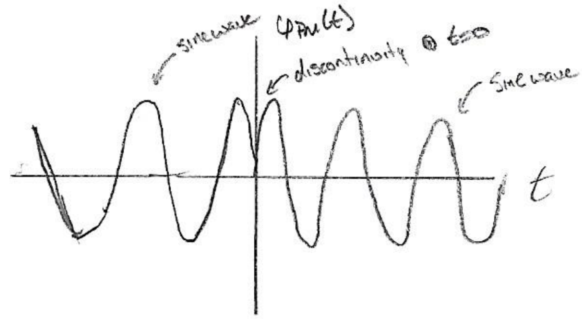
5.1-2)



b) $\varphi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$ and $\omega_i = \omega_c + k_p \dot{m}(t)$

and saw earlier that $\dot{m}(t) = 2000$, so plugging in $\Rightarrow$

$\omega_i = 2.001\pi \times 10^6$

The m(t) signal is increasing the PM wave's frequency
the discontinuities lead to the jumps in angle
and because $k_p(m_p + m_p) = \pi \Rightarrow$ they are always π radians

5.1-4) over $|t| \le 1$, $\varphi_{EM}(t) = 10\cos 13{,}000\pi t$ and $\omega_c = 10{,}000\pi$

a. assuming PM signal with $k_p = 1000$, find m(t) over $|t| \le 1$

$\varphi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$ and plug in values

$\varphi_{PM}(t) = 10\cos[10{,}000\pi t + 1000\, m(t)]$ and $\varphi_{EM}(t) = 10\cos 13{,}000\pi t = 10\cos[10000\pi + 1000(\underbrace{3\pi t}_{m(t)})]$

$\boxed{m(t) = 3\pi t}$

b. assuming FM signal with $k_f = 1000$, determine m(t)

$\varphi_{FM}(t) = A\cos[\underbrace{\omega_c t}_{\text{This stays the same}} + k_f \int_{-\infty}^{t} m(t)\,dt] \Rightarrow \int_{-\infty}^{t} m(t)\,dt = 3\pi t \rightarrow m(t) = 3\pi$

$\boxed{m(t) = 3\pi}$

5.2-1) a) From the figure, $m_p = 3V$ and the B is approximately the value of the 5^{th} harmonic frequency

$$B = 5\left(\frac{1}{T}\right) = 5\left(\frac{1}{0.2-(-0.05)}\right) \Rightarrow B = 20\,Hz$$

and now use Carson

$$B_{FM} = 2(\Delta f + B) = 2\left[\frac{1}{2\pi}k_f m_p + B\right] = 2\left[\frac{1}{2\pi}\cdot 20\pi \cdot 3 + 20\right] \Rightarrow \boxed{B_{FM} = 100\,Hz}$$

b) $$B_{PM} = 2(\Delta f + B) = 2\left[\frac{1}{2\pi}k_p \dot{m}_p + B\right] \quad \text{and} \quad \dot{m}_p = \dot{m}(t)_{max}$$

$$\dot{m}(t)_{max} = 60\,t \Rightarrow \dot{m}_p = 60$$

$$B_{PM} = 2\left(\frac{1}{2\pi}\cdot\frac{\pi}{2}\cdot 20 + 20\right) \Rightarrow \boxed{B_{PM} = 70\,Hz}$$

5.2-9) a. Applying Parseval's theorem, show that $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

by infinite series $\sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega t} = e^{j\beta \sin \omega t}$

$$\sum_{n=-\infty}^{\infty} |J_n(\beta)|^2 = \frac{1}{2\pi/\omega}\int_{-\pi/\omega}^{\pi/\omega} |e^{j\beta\sin\omega t}|^2\,dt = \frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega} dt = 1 \Rightarrow \text{real, so}$$

$$|J_n(\beta)|^2 = J_n^2(\beta) \Rightarrow \boxed{\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1} \checkmark$$

b. $$J_n\beta = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin x - nx)}\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos(\beta\sin x - nx) + j\sin(\beta\sin x - nx)\,dx$$

$$\operatorname{Im}[J_n(\beta)] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{\sin(\beta\sin x - nx)\,dx}_{0} \Rightarrow \operatorname{Im}[J_n(\beta)] = 0 \quad \text{so} \quad J_n(\beta) = \operatorname{Re}[J_n(\beta)]$$

$$\operatorname{Re}[J_n(\beta)] = \frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(\beta\sin x - nx)\,dx = \frac{1}{\pi}\int_{0}^{\pi}\cos(\beta\sin x - nx)\,dx$$

$$J_{-n}(\beta) = \frac{1}{\pi}\int_0^{\pi}\cos(\beta\sin x + nx) = (-1)^n \underbrace{\frac{1}{\pi}\int_0^{\pi}\cos(\beta\sin x - nx)\,dx}_{J_n(\beta)} \Rightarrow$$

$$\boxed{J_{-n}(\beta) = (-1)^n J_n(\beta)}$$

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5.3-2) a) Design block diagram of Armstrong FM modulator to generate carrier with $f_c = 98.1$ MHz and $\Delta f = 75$ kHz

Starting with the generator, need to get the Δf from 10 Hz to 75kHz

$$\frac{75000}{10} = 7500$$

and with the multipliers, factor out $\Rightarrow 7500 = \underset{\uparrow\ 2\text{ doubler}}{2^2} \cdot \underset{\uparrow\ 1\text{ tripler}}{3} \cdot \underset{\uparrow\ 4\text{ quintupler}}{5^4}$

but will need to shift frequency too

$X = \frac{98100000}{100{,}000} = 981 \Rightarrow \frac{981}{7500}$ ← ratio to lower carrier frequency

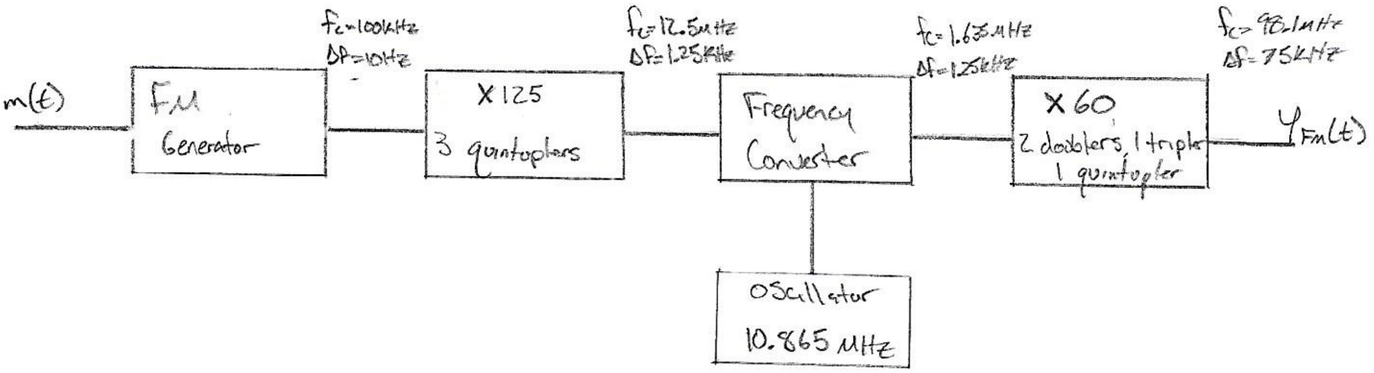
$f_{signal} - f_{oscillator} = \frac{981}{7500} f_{carrier} \Rightarrow f_s = 1.15 \underset{\uparrow\ 10\text{ to }11\text{ MHz}}{f_o} \Rightarrow$ range from 11.5 to 12.6 MHz

and in order to reach a frequency in that range need to look at where to put oscillator with respected to doublers, ...

So what multipliers satisfy that $(\underset{\uparrow\ \text{generator frequency}}{100{,}000} \cdot \overset{\text{multipliers}}{x}) =$ something in that range 11.5 - 12.6 MHz 3 quintuplers = 12.5

and finally need f_o

$f_o = \frac{f_s}{1.15} \Rightarrow f_o = 10.865$



5.3-2) b. Determine the tunable range of the carrier frequency

It is used to lower carrier frequency and oscillator $\Rightarrow$ 10 - 11 MHz $\Rightarrow$ lowest carrier when oscillator 11 MHz

$f_{c_{min}} = 60(12.5-11)\text{ MHz} \Rightarrow f_{c_{min}} = 90 \text{ MHz}$

and highest carrier with lowest oscillator freq.

$f_{c_{max}} = 60(12.5-10)\text{ MHz} \Rightarrow f_{c_{max}} = 150 \text{ MHz} \Rightarrow$

$\boxed{\text{The tunable range is } 90 \text{ to } 150 \text{ MHz}}$

5.4-4) From small error analysis, $\Theta_e(s) = \dfrac{s}{s + AKH(s)}\Theta_i(s)$ where $\Theta_i(t) = kt^2 \iff \Theta_i(s) = \dfrac{2k}{s^3}$

$$\Theta_e(s) = \frac{2k}{s^2(s + AKH(s))}$$

and to evaluate error $\lim_{t\to\infty}\Theta_e(t) = \lim_{s\to 0} s\,\Theta_e(s) = \lim_{s\to 0}\dfrac{2k}{s(s+AKH(s))}$

for $H(s)=1$

$$\lim_{s\to 0}\frac{2k}{s(s+Ak)} = \infty \Rightarrow \boxed{\lim_{t\to\infty}\Theta_e(t) = \infty \quad \text{Cannot be tracked}}$$

for $H(s) = \frac{s+a}{s}$

$$\lim_{s\to 0}\frac{2k}{s(s+Ak(\frac{s+a}{s}))} = \lim_{s\to 0}\frac{2k}{s^2+Ak(s+a)} = \frac{2k}{Aka} \Rightarrow \boxed{\lim_{t\to\infty}\Theta_e(t) = \frac{2k}{Aka} \quad \text{Can be tracked with constant phase error}}$$

for $H(s) = \frac{s^2+as+b}{s}$

$$\lim_{s\to 0}\frac{2k}{s(s+Ak(\frac{s^2+as+b}{s}))} = \lim_{s\to 0}\frac{2ks}{s^3+Ak(s^2+as+b)} = 0 \Rightarrow \boxed{\lim_{t\to\infty}\Theta_e(t) = 0 \quad \text{Can be tracked with 0 phase error}}$$

5.6-1) $f_c = 1530$ kHz with $f_{IF} = 455$ kHz so

$f_{LO} = f_c + f_{IF} = (1530 + 455)$ kHz $= 1985$ kHz

but another carrier with

$f_c' = (1985 + 455)$ kHz $= 2440$ kHz is also heard because the difference is also 455 kHz between new f_c' and f_{lo}

This will cause the station to be heard at both 1530 kHz and 2440 kHz

5.6-3) a) $f_{IF} = 455$ kHz and f_c can range from 9.4 to 9.9 MHz ⇒

$f_{LO} = f_c + f_{IF}$ ⇒ $f_{LO_{low}} = 9.4$ MHz $+ 455$ kHz $= 9.855$ MHz

⇒ $f_{LO_{high}} = 9.9$ MHz $+ 455$ kHz $= 10.355$ MHz

frequency of the oscillator for this receiver is
9.855 MHz to 10.355 MHz

b) In order to receive image station, needs to be seperated by $2f_{IF} = 910$ kHz

$f_{image} = (9.4\text{MHz} + 910\text{kHz})$ to $(9.9\text{MHz} + 910\text{kHz})$

$= 10.31$ MHz to 10.81 MHz

↑

This is out of the 9.4 - 9.9 MHz range and
this receiver cannot receive the image station