Week 3 – Sections 3.1-3.3

3.1 Cournot's oligopoly game

- Players: The firms.
  Actions: Each firm's set of actions is the set of its possible outputs (nonnegative numbers).
  Preferences: Each firm's preferences are represented by its profit
  $$\pi_i(q_1, \ldots, q_n) = q_i P(q_1 + \cdots + q_n) - C_i(q_i)$$
  where cost to firm i of producing q_i units of the good is $C_i(q_i)$, market price is $P(Q)$; P is called the "inverse demand function", firm i's revenue is $q_i P(q_1 + \cdots + q_n)$

- Duopoly Example:
  $C_i(q_i) = cq_i$
  $P(Q) = \alpha - Q$ if $Q \leq \alpha$; 0 if $Q > \alpha$ (Assume $c < \alpha$)
  $\pi_1(q_1, 0) = q_1(\alpha - c - q_1)$
  $(q^*_1, q^*_2) = ((\alpha - c)/3, (\alpha - c)/3)$ - Nash Eqbm.

- If, in the duopoly, the firms acted collusively, they could increase profits with decreased outputs.

- Generalizing using common property: Payoff of firm i may be written as $f_i(q_i, q_1 + \cdots + q_n)$, where the function f_i is decreasing in its second argument (given the value of its first argument, q_i).

3.2 Bertrand's Model of Oligopoly:

- Each firm chooses a price, and produces enough output to meet the demand it faces, given the prices chosen by all the firms. A single good is produced by n firms; each firm can produce q_i units of the good at a cost of $C_i(q_i)$.
  "Demand Function" D: if the good is available at the price p then the total amount demanded is D(p).

- Game:
  Players: The firms.
  Actions: Each firm's set of actions is the set of possible prices (nonnegative numbers).
  Preferences: Firm i's preferences are represented by its profit, equal to $p_i D(p_i)/m - C_i(D(p_i)/m)$ if firm i is one of m firms setting the lowest price (m = 1 if firm i's price p_i is lower than every other price), and equal to zero if some firm's price is lower than p_i.

- In a duopoly example, where each firm charges the price c, Nash Eqbm: $(p^*_1, p^*_2) = (c, c)$

- In Cournot's game a firm changes its behaviour if it can increase its profit by changing its output, on the assumption that the other firms' outputs will remain the same and the price will adjust to clear the market. In Bertrand's game a firm changes its behaviour if it can increase its profit by changing its price, on the assumption that the other firms' prices will remain the same and their outputs will adjust to clear the market.

3.3 Hotelling's model of electoral competition

- The median favourite position: the position m with the property that exactly half of the voters' favourite positions are at most m, and half of the voters' favourite positions are at least m.

- For two candidates with positions (x_1, x_2), the Nash Eqbm: (m, m)

(Exercises solved on paper, will upload later)