Week 2 – Sections 2.6-2.9

- A Nash equilibrium is an action profile a^* with the property that no player i can do better by choosing an action different from a_i^* , given that every other player j adheres to a_j^* .

- If, whenever the game is played, the action profile is the same Nash equilibrium a^*, then no player has a reason to choose any action different from her component of a^*; there is no pressure on the action profile to change. Expressed differently, a Nash equilibrium embodies a stable "social norm": if everyone else adheres to it, no individual wishes to deviate from it.

- (a'_i , a_{-i}) denotes the action profile in which every player j except i chooses her action a_j as specified by a, whereas player i chooses a'_i. If $a'_i = ai$ then of course $(a'_i , a_{-i}) = (a_i, a_{-i}) = a$.

- For an action profile a^* to be a Nash equilibrium: no player i has any action a_i for which she prefers (a_i , a_{-i}) to a^*. Equivalently, for every player i and every action a_i of player i, the action profile a^* is at least as good for player i as the action profile (a_i , a_{-i}).

- Number of Nash equilibria for a game can be 0, 1 or more.

- In Prisoner's Dilemma, it is optimal for a player to choose Fink regardless of the action she expects her opponent to choose. In most of the games we study, a player's Nash equilibrium action does not satisfy this condition: the action is optimal if the other players choose their Nash equilibrium actions, but some other action is optimal if the other players choose non-equilibrium actions

- Ex 25.1: 2, 2 0, 3 3, 0 1, 1
  $2+2\alpha$, $2\alpha+2$ 3α, 3 3, 3α $1+\alpha$, $1+\alpha$
  $2+2\alpha > 3\alpha \Rightarrow \alpha < 2$, $2+2\alpha < 3 \Rightarrow \alpha < ½$ - Prisoner's Dilemma
  For $\alpha >= ½$, N.Eq = (Q,Q)

- Ex 25.2: a. D,U > D,D > U,X

	Sit	Stand
Sit	(1,1)	(2,0)
Stand	(0,2)	(0,0)

  Prisoner's Dilemma? N.Eq = Sit, Sit
  b. U,D > D,D > U,U > D,U

	Sit	Stand
Sit	(2,2)	(0,3)
Stand	(3,0)	(1,1)

  Prisoner's Dilemma? Yes N.Eq = Stand, Stand

- For BoS: (B,B), (S,S) are Nash Equilibria.
- An equilibrium is immune to any unilateral deviation; coordinated deviations by groups of players are not contemplated. However, the existence of two equilibria raises the possibility that one equilibrium might more likely be the outcome of the game than the other.

- Ex 28.1: a. N.Eq.: (Stag,…, Stag), and (Hare,…, Hare)
  b. t Stags, n-t Hares : N.Eq. where m<= t <=k

- The theory of Nash equilibrium is neutral about the equilibrium that will occur in a game with many equilibria.

- Ex 31.1: $a_i = 0$, k+ contributions > $a_i = 1$, k+ contributions > $a_i = 0$, k- contributions > $a_i = 1$, k- contributions

k contributions = N.Eq.
k+ contributions = not N. Eq.
k > contributions > 0 = not N.Eq.
0 contributions = N.Eq

- Ex 32.1: Keynesian Beauty Contest

- Some games have equilibria in which a player is indifferent between her equilibrium action and some other action, given the other players' actions. We say that such a Nash equilibrium is not a strict equilibrium.

- Precisely, an action profile a^* is a strict Nash equilibrium if for every player i we have $u_i(a^*) > u_i(a_i, a^*_{-i})$ for every action $a_i = a^*_i$ of player i.

- Every member of the set $B_i(a_{-i})$ is a best response of player i to a_{-i}: if each of the other players adheres to a_{-i} then player i can do no better than choose a member of $B_i(a_{-i})$.

- The action profile a^* is a Nash equilibrium of a strategic game with ordinal preferences if and only if every player's action is a best response to the other players' actions:

$$a^*_i \text{ is in } B_i(a_{-i}) \text{ for every player i}$$

- If each player i has a single best response to each list a_{-i} of the other players' actions, denoted by $B_i(a_{-i})$ by $b_i(a_{-i})$ (that is, $B_i(a_{-i}) = \{b_i(a_{-i})\}$)

$$a^*_i = b_i(a_{-i}) \text{ for every player i}$$

  a collection of n equations in the n unknowns a^*_i , where n is the number of players in the game.

- The definition of a Nash equilibrium in terms of best response functions suggests a method for finding Nash equilibria:
  • find the best response function of each player
  • find the action profiles that satisfy a^*_i is in $B_i(a_{-i})$ for every player i

- Strict domination: In a strategic game with ordinal preferences, player i's action a''_i strictly dominates her action a'_i if $u_i(a''_i , a_{-i}) > u_i(a'_i , a_{-i})$ for every list a_{-i} of the other players' actions, where u_i is the payoff function.

- If an action strictly dominates the action a_i, we say that a_i is strictly dominated. Since a player's Nash equilibrium action is a best response to the other players' Nash equilibrium actions, a strictly dominated action is not used in any Nash equilibrium.

- In a strategic game with ordinal preferences, player i's a''_i weakly dominates her action a'_i if

$$u_i(a''_i , a_{-i}) \geq u_i(a'_i , a_{-i}) \text{ for every list } a_{-i} \text{ of the other players' actions}$$

  and

$$u_i(a''_i , a_{-i}) > u_i(a'_i , a_{-i}) \text{ for some list } a_{-i} \text{ of the other players' actions}$$