Stamatics - Game Theory - Assignment 1

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Question 1

a) Having no NE
Matching Pennies

P1/P2	Head	Tail
Head	(1*,-1)	(-1, 1*)
Tail	(-1, 1*)	(1*, -1)

b) unique NE which is not a WDSE
Prisoner's Dilemma

P1/P2	Quiet	Fink
Quiet	(2, 2)	(0, 3*)
Fink	(3*, 0)	(1*, 1*)

Nash Equillibrium (Fink, Fink) is an SDSE.

c) a unique WDSE which is not an SDSE

P1/P2	L	C	R
T	(0, 0)	(1, 0)	(1, 1)
M	(1, 1)	(1, 1)	(3, 0)
B	(1, 1)	(2, 1)	(2, 2)

d) a unique DSE and another profile which is only an NE.
Stag Hunt

P1/P2	Stag	Hare
Stag	(2*, 2*)	(0, 1)
Hare	(1, 0)	(1*, 1*)

Question 2

a) (C, C) is an SDSE
$-X > -2 \implies X < 2$

b) (C, C) is a WDSE but not an SDSE
$-X \geq -2$ but (C, C) is not an SDSE $\implies X = 2$

c) (C, C) is an NE but not a WDSE
For (C, C) to be an NE, $-X \leq -2$. Since it is not a WDSE, $-X > -2 \implies X < 2$.

d) (C, C) is not even an NE
$-X < -2 \implies X > 2$

Question 3

S1/S2	y=3	y=4
x=1	(2, -2)	(3, -3)
x=2	(1, -1)	(2, -2)

Clearly, no Nash Equillibrium exists.

Question 4

P1/P2	A	B	C	D
A	5,2	2,6*	1,4	0,4
B	0,0	3*,2*	2*,1	1,1
C	7,0	2,2	1,5*	5*,1
D	9*,5	1,3	0,2	4,8*

Nash Equillinrium is (B, B)

Question 5

The highest possible average that would occur if everyone guessed 100 is 66+2/3. Therefore, choosing a number that lies above $66\frac{2}{3}$ is strictly dominated for every player. These guesses can thus be eliminated. Once these strategies are eliminated for every player, $66\frac{2}{3}$ becomes the new highest possible average (that is, if everyone chooses $66\frac{2}{3}$).

Similarly, any guess above $44\frac{4}{9}$ is weakly dominated for every player since no player will guess above $66\frac{2}{3}$, and $\frac{2}{3}$ of $66\frac{2}{3}$ is $44\frac{2}{3}$. This process will continue as this logic is continually applied. With each step, the highest possible logical answer keeps getting smaller.
We will be left with the case when every player chooses 1 or when every player chooses 0. That is, the action profiles (0, 0,..., 0) and (1, 1,..., 1) are the Nash Equillibria.

Another way to find NE is to look at action profiles of the form $(k, k, ..., k)$. If one player chooses $k-1$ instead of k, $\frac{2}{3}^{rd}$ s of the average decreases to $\left(\frac{2k}{3} - \frac{2}{30}\right)$ which is closer to $k-1$ than it is to k (for all k $\geq$ 2).

Referred to this article

Question 6

The strategic game for this situation is modeled by:
Players: The n people.
Actions: Each person's set of actions is Contribute, Don't contribute.
Preferences: as given in the question.
When g players contribute)
i) If $g \geq k$, best response for player i is to not contribute.
ii) If $g = k-1$, best response for player i is to contribute.

iii) If $g < k - 1$, best response for player i is to not contribute.

So, Nash Equlibria will be given by all those action profiles in which k people contribute. The case where no one contributes is also a Nash Equillibrium.

Question 7

Let us create a best response mapping:

Player's Action	Best Responses
0	10
1	9, 10
2	8, 9, 10
3	7, 8, 9, 10
4	6, 7, 8, 9, 10
5	5, 6, 7, 8, 9, 10
6	5, 6
7	6
8	7
9	8
10	9

Using symmetry, we can conclude that there are 4 NE for this game: (5, 5), (5, 6), (6, 5), and (6, 6).

Question 8

$$\pi(p_1, p_2) = \begin{cases} (p_i - c_i)(\alpha - p_i);\ \text{if } p_i < p_j \\ \frac{1}{2}(p_i - c_i)(\alpha - p_i);\ \text{if } p_i = p_j \\ 0;\ \text{if } p_i > p_j \end{cases}$$

1) If $p_2 < c_1$, Firm 1's profit is negative if $p_1 \leq p_2$, otherwise 0. Best responses to $p_2 : B_1(p_2) = p_1 : p_1 > p_2$

2) If $p_2 = c_1$, Firm 1's profit is negative if $p_1 < p_2$, otherwise 0. Best responses to $p_2 : B_1(p_2) = p_1 : p_1 \geq p_2$

3) If $p_2 > c_1$, Firm 1's profit is positive (and increasing) for $c_1 < p_1 < p_2$. Otherwise it is either negative or 0. Best responses to $p_2 : B_1(p_2) = \phi$ [Since distribution is continuous.]

Question 9

By symmetry, we can claim that the NE will be of the type (t, t), assuming $t \leq K$.

If $t < \dfrac{K}{2}$, payoff/ profit for both bidders will be $\dfrac{K}{2} - t$. A bidder can then bid $t + \delta$ and her profit will increase to $K - t - \delta$ (for $\delta < \dfrac{K}{2}$)

If $t = \dfrac{K}{2}$, her profit is 0. The bidder can then bid $t + \delta$ and her profit will increase to $K - t - \delta$.

If $t > \frac{K}{2}$, her profit is negative. The bidder can then bid 0 and her payoff will increase.

No Nash Equilibrium exists in such a case (unless edge case).
Let us look at an edge case, with K = 2. The NE for this can be easily found as (0, 0).