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Week-4/5- Chapter 7

→ $T = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$

$S_1, S_2, \ldots S_n$ are the strategy sets
$u_i : S_1 \times S_2 \times S_3 \ldots \times S_n \rightarrow \mathbb{R}$ for $i \in \{1, 2, \ldots n\}$
are utility fns.

→ Player i with S_i as the set of pure strategies, a mixed strategy σ_i of player i is a probability distribution over S_i.

$\sigma_i : S_i \rightarrow [0, 1]$ is a mapping that assigns to each pure strategy $s_i \in S_i$, a probability $\sigma_i(s_i)$ such that

$$\sum_{s_i \in S_i} \sigma_i(s_i) = 1$$

→ Degenerate mixed strategy $e(s_i)$ or s_i has $\sigma_i(s_i) = 1$ and $\sigma_i(s_j) = 0$ for all $j \neq i$.

→ The set of all mixed strategies of player i, called the mixed extension of S_i, is given by $\Delta(S_i)$ where

$$S_i = \{s_{i1}, s_{i2}, \ldots s_{im}\}$$

&

$$\Delta(S_i) = \left\{ (\sigma_{i1}, \ldots \sigma_{im}) \in \mathbb{R}^m : \sigma_{ij} \geq 0 \text{ for } j = 1, \ldots, m \text{ and } \sum_{j=1}^{m} \sigma_{ij} = 1 \right\}$$

→ Mixed extension of Γ:

$$\Gamma_{ME} = \langle N, (\Delta(S_i)), (U_i) \rangle$$

where $U_i : \Delta(S_1) \times \Delta(S_2) \ldots \times \Delta(S_n) \rightarrow \mathbb{R}$

→ $$\sigma(s_1, \ldots s_n) = \prod_{i \in N} \sigma_i(s_i)$$

↓

Probability of a pure strategy profile $(s_1, \ldots s_n)$

[$\sigma_1, \ldots \sigma_n$ are mutually independent]

→ convex comb^n of $y_1, y_2, \dots, y_n$ to be the sum

$\sum \lambda_i y_i$ where $\sum \lambda_i = 1$ $(\lambda_i \in [0,1])$

$$\mu_i(\sigma_i, \sigma_{-i}) = \sum_{s_i \in S_i} \sigma_i(s_i)\, \mu_i(s_i, \sigma_{-i})$$

where:

$$\mu_i(s_i, \sigma_{-i}) = \sum_{s_{-i} \in S_{-i}} \left(\prod_{j \neq i} \sigma_j(s_j) \right) \mu_i(s_i, s_{-i})$$

→ $\sigma \in \times_{i \in N} \Delta(S_i)$

$$\max_{\sigma_i \in \Delta(S_i)} \mu_i(\sigma_i, \sigma_{-i}) = \max_{s_i \in S_i}$$

→ support of a mixed strategy σ_i

$$\delta(\sigma_i) = \{ s_i \in S_i : \sigma_i(s_i) > 0 \}$$

→ Payoff for player i corresponding to any pure strategy having +ve probability is the same & is no less than the payoff corresponding to any pure strategy having zero probability (whenever all other players are playing their Nash Eqbm mixed strategy