Stamatics - Game Theory - Assignment 2 - upto MSNE

Janhvi Rochwani (200467)

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1. Let us say player 1 plays (Quiet, Fink) with probabilities $(p, 1-p)$ and player 2 does the same with probabilities $(q, 1-q)$. For player 1 to have a mixed strategy Nash Equilibrium, she must be indifferent between playing Q and F given players 2's probability distribution. That is,

$$u_1(Q, (q, 1-q)) = u_1(F, (q, 1-q))$$
$$2 + 0 \cdot (1-q) = 3 + 1 \cdot (1-q)$$
$$1 = 0$$

Hence, there is no such probability distribution, or, there is no MSNE for Prisoners' Dilemma.

2. A) Similar to Q1,

$$u_1(Head, (q, 1-q)) = u_1(Tail, (q, 1-q))$$
$$(-1) \cdot q + 2 \cdot (1-q) = 2 \cdot q + (-1) \cdot (1-q)$$
$$q = \frac{1}{2}$$

And

$$u_2((p, 1-p), Head) = u_2((p, 1-p), Tail)$$
$$1 \cdot p + (-1) \cdot (1-p) = (-1) \cdot p + 1 \cdot (1-p)$$
$$p = \frac{1}{2}$$

MSNE: $\left(\frac{1}{2}, \frac{1}{2}\right) X \left(\frac{1}{2}, \frac{1}{2}\right)$

B)

$$u_1(L) = u_1(R)$$
$$0 = 2 \cdot q \implies q = 0$$
$$1 \cdot p + 2 \cdot (1-p) = 2 \cdot p + 1 \cdot (1-p)$$
$$p = \frac{1}{2}$$

MSNE: $\left(\frac{1}{2}, \frac{1}{2}\right) X \, (0, 1)$

C) "MSNE" comes out to be (-1, -1, 3) X (-1, -1, 3). These probabilities are not valid, and hence, there is no MSNE.

3. The game can be seen as:

P1/P2	Bach	Stravinski
Bach	(a, b)	(0, 0)
Stravinski	(0, 0)	(b, a)

$$u_1\left(Bach, \left(\frac{2}{5}, \frac{3}{5}\right)\right) = u_1\left(Stravinski, \left(\frac{2}{5}, \frac{3}{5}\right)\right)$$

4. Let us consider a MSNE where number of players is N and probability distribution of (Reporting, Not Reporting) or (R, X) is $\vec{P_i} = (p_i, 1-p_i)$ for the i^{th} player.
   We can say that player 1 should be indifferent between reporting and not reporting given this probability distribution.

   Taking N=2, we can write:

$$p_2(V-C) + (1-p_2)(V-C) = p_2 V + (1-p_2) \cdot 0$$

   On solving, $p_1 = p_2 = 1 - \dfrac{C}{V}$

For N = 3:

$$u_1(R, \vec{p_2}, \vec{p_3}) = V - C$$

and

$$u_1(X, \vec{p_2}, \vec{p_3}) = V - V(1 - p_2)(1 - p_3)$$

In this case, $p_1 = p_2 = p_3 = 1 - \sqrt{\frac{C}{V}}$

For general case, we can write:

$$u_1(R, \vec{P_2}, \vec{P_3}, ..., \vec{P_N}) = u_1(X, \vec{P_2}, \vec{P_3}, ..., \vec{P_N})$$

$$(V - C) = V - (1 - p_2)(1 - p_3)...(1 - p_N) \cdot V$$

or,

$$\prod_{i=2}^{n}(1 - p_i) = \frac{C}{V}$$

By symmetry, for any N and i, p_i comes out to be $1 - \left(\frac{C}{V}\right)^{\dfrac{1}{N-1}}$.
The probability that no one reports the crime will be:

$$P(\text{all X's}) = \prod_{i=1}^{N}(1 - p_i) = \left(\frac{C}{V}\right)^{\dfrac{N}{N-1}}$$

Since $0 < \frac{C}{V} < 1$, **the probability that no one reports the crime decreases with increase in N**