3 Time-varying volatility models – GARCH and stochastic volatility

Time-varying volatility models have been popular since the early 1990s in empirical research in finance, following an influential paper ‘Generalized Autoregressive Conditional Heteroskedasticity’ by Bollerslev (1986). Models of this type are well known as generalised autoregressive conditional heteroscedasticity (GARCH) in the time series econometrics literature. Time-varying volatility has been observed and documented in as early as 1982 (Engle 1982) and was initially concerned with an economic phenomenon – time-varying and autoregressive variance of inflation. Nevertheless, it was data availability and strong empirical research interest in finance, motivated by exploring any kind of market inefficiency, that encouraged the application and facilitated the development of these models and their variations. For instance, the GARCH in mean model is related to asset pricing with time-varying risk instead of constant risk in traditional models such as the CAPM. An exponential GARCH (EGARCH) model addresses asymmetry in volatility patterns which are well observed in corporate finance and financial markets and can sometimes be attributed to leverage effects. GARCH with *t*-distribution reflects fat tails found in many types of financial time series data where the assumption of conditional normality is violated. Finally, multivariate GARCH models are helpful tools for investigating volatility transmissions and patterns between two or more financial markets.

Although GARCH family models have time-varying variance, the variance is not stochastic. Therefore, GARCH is not exactly the ARMA equivalent in the second moment. Stochastic volatility, as discussed in Section 3.3 is not only time varying, but also stochastic, and is probably the closest equivalent to an AR or ARMA process in the second moment.

3.1 ARCH and GARCH and their variations

3.1.1 ARCH and GARCH models

A stochastic process is called autoregressive conditional heteroscedasticity (ARCH) if its time-varying conditional variance is heteroscedastic with autoregression

$$y_t = \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2) \tag{1a}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 \tag{1b}$$

Equation (1a) is the mean equation where regressors can generally be added to the right-hand side along side $_t$. Equation (1b) is the variance equation, which is an ARCH(q) process where autoregression in its squared residuals has an order of q, or has q lags.

A stochastic process is called GARCH if its time-varying conditional variance is heteroscedastic with both autoregression and moving average.

$$y_t = \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_p^2 \tag{2a}$$

$$= \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{2b}$$

Equation (2) is a GARCH (p, q) process where autoregression in its squared residuals has an order of q, and the moving average component has an order of *p*.

One of the advantages of GARCH over ARCH is parsimonious, that is, fewer lags are required to capture the property of time-varying variance in GARCH. In empirical applications a GARCH (1, 1) model is widely adopted. While in ARCH, for example, a lag length of five for daily data may still not be long enough. We demonstrate this with a GARCH (1, 1) model. Extending the variance process backwards yields:

$$
\begin{aligned}
\sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\
&= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \left(\alpha_0 + \alpha_1 \varepsilon_{t-2}^2 + \beta_1 \sigma_{t-2}^2 \right) \\
&\vdots \\
&= \frac{\alpha_0}{1-\beta_1} + \alpha_1 \sum_{n=1}^{\infty} \beta_1^{n-1} \varepsilon_{t-n}^2
\end{aligned} \tag{3}
$$

Indeed, only the first few terms would have noteworthy influence since $\beta^n_{1\,n\rightarrow\infty}$ 0. This shows how a higher order ARCH specification can be approximated by a GARCH (1, 1) process.

Similar to ARMA models, there are conditions for stationarity to be met. As the name of the model suggests, the variances specified above are conditional. The unconditional variance of GARCH would be of interest with respect to the property of the model. Applying the expectations operator to both sides of equation (2b), we have:

$$
E(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i E(\varepsilon_{t-1}^2) + \sum_{j=1}^{p} \beta_j E(\sigma_{t-j}^2)
$$

Noting $E(\sigma_t^2) = E(\varepsilon_{t-i}^2) = E(\sigma_{t-j}^2)$ is the unconditional variance of the residual, which is solved as:

$$
\sigma^2 = E(\sigma_t^2) = \frac{\alpha_0}{1 - \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j}
$$

It is clear that for the process to possess a finite variance, the following condition must be met:

$$
\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \tag{4}
$$

In commonly used GARCH (1, 1) models, the condition is simply $_1$ + $_1$ < 1. Many financial time series have persistent volatility, that is, the sum of $_i$ and $_j$ is close to being unity. A unity sum of $_i$ and $_j$ leads to the so-called integrated GARCH (IGARCH) as the process is not covariance stationary. Nevertheless, this does not pose as serious a problem as it appears. According to Nelson (1990), Bougerol and Picard (1992) and Lumsdaine (1991), even if a GARCH (IGARCH) model is not covariance stationary, it is strictly stationary or ergodic, and the standard asymptotically based inference procedures are generally valid. See Chapter 1 of this book for various definitions of stationarity and ergodicity.

3.1.2 Variations of the ARCH/GARCH model

Variations are necessary to adapt the standard GARCH model to the needs arising from examining the time series properties of specific issues in finance and economics. Here we present the model relating the return on a security to its time-varying volatility or risk – ARCH-in-Mean (ARCH-M), and the models of asymmetry – EGARCH and threshold GARCH (TGARCH).

The ARCH-M model

When the conditional variance enters the mean equation for an ARCH process, the ARCH-M model is derived:

$$y_t = \lambda_1 x_1 + \cdots + \lambda_m x_m + \varphi \sigma_t^2 + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2) \tag{5a}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 \tag{5b}$$

where x_k, $k = 1, \ldots, m$ are exogenous variables which could include lagged y_t. In the sense of asset pricing, if y_t is the return on an asset of a firm, then x_k, $k = 1, \ldots, m$ would generally include the return on the market and possibly other explanatory variables such as the price earnings ratio and the size. The parameter captures the sensitivity of the return to the time-varying volatility, or in other words, links the return to a *time-varying risk premium*. The ARCH-M model is generalised from the standard ARCH by Engle *et al*. (1987) and can be further generalised so that the conditional variance is GARCH instead of ARCH, and that the conditional standard deviation, instead of the conditional variance, enters the mean equation.

The EGARCH model

This model captures asymmetric responses of the time-varying variance to shocks and, at the same time, ensures that the variance is always positive. Itwas developed by Nelson (1991) with the following specification:

$$\ln(\sigma_t^2) = \alpha_0 + \beta \ln(\sigma_{t-1}^2) + \alpha \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{6}$$

where is the asymmetric response parameter or leverage parameter. The sign of is expected to be positive in most empirical cases so that a negative shock increases future volatility or uncertainty while a positive shock eases the effect on future uncertainty. This is in contrast to the standard GARCH model where shocks of the same magnitude, positive or negative, have the same effect on future volatility. In macroeconomic analysis, financial markets and corporate finance, a negative shock usually implies bad news, leading to a more uncertain future. Consequently, for example, shareholderswould require a higher expected return to compensate for bearing increased risk in their investment. A statistical asymmetry is, under various circumstances, also a reflection of the real world asymmetry, arising from the nature, process or organisation of economic and business activity, for example, the change in financial leverage is asymmetric to shocks to the share price of a firm.

Equation (6) is, exactly speaking, an EGARCH (1, 1) model. Higher order EGARCH models can be specified in a similar way, for example, EGARCH (p, q) is as follows:

$$\ln(\sigma_t^2) = \alpha_0 + \sum_{j=1}^{p} \beta_j \ln(\alpha_{t-j}^2) + \sum_{i=1}^{q} \left\{ \alpha_i \left(\left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \sqrt{\frac{2}{\pi}} \right) - \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right\} \tag{7}$$

The threshold GARCH model

It is also known as the GJR model, named after Glosten, Jagannathan and Runkle (1993). Despite the advantages EGARCH appears to enjoy, the empirical estimation of the model is technically difficult as it involves highly non-linear algorithms. In contrast, the GJR model is much simpler than, though not as elegant as, EGARCH. A general GJR model is specified as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \left\{ \alpha_i \varepsilon_{t-i}^2 + \delta_i \varepsilon_{t-i}^2 \right\} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{8}$$

where $_{i} = 0$ if $_{t-i} > 0$. So, $_{i}$ catches asymmetry in the response of volatility to shocks in a way that imposes a prior belief that for a positive shock and a negative shock of the same magnitude, future volatility is always higher, or at least the same, when the sign of the shock is negative. This may make sense under many circumstances but may not be universally valid. An alternative to the GJR specification is:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \left\{ \alpha_i^+ \varepsilon_{t-i}^2 + \alpha_i^- \varepsilon_{t-i}^2 \right\} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{9}$$

where $_{i}^{+} = 0$ if $_{t-i} < 0$, and α_i^- if $_{t-i} > 0$. In such a case, whether a positive shock or a negative shock of the same magnitude will have a larger effect on volatility will be subject to empirical examination.

3.2 Multivariate GARCH

We restrict our analysis to bivariate models as a multivariate GARCH with more than two variables would be extremely difficult to estimate technically and convey meaningful messages theoretically. A bivariate GARCH model expressed in matrices takes the form:

$$\mathbf{y}_t = \boldsymbol{\varepsilon}_t \tag{10a}$$

$$\boldsymbol{\varepsilon}_t | \Omega_{t-1} \sim N(0, \mathbf{H}_t) \tag{10b}$$

where vectors $\mathbf{y}_t = [y_{1,t}\ y_{2,t}]$ and $_{t} = [_{1,t}\ _{2,t}]$ and

$$\mathbf{H}_t = \begin{bmatrix} h_{11t} & h_{12t} \\ h_{21t} & h_{22t} \end{bmatrix}$$

is the covariance matrix which can be designed in a number of ways. Commonly used specifications of the covariance include constant correlation, VECH (full parameterisation), and BEKK (positive definite parameterisation) named after Baba, Engle, Kraft and Kroner (1990). We now introduce them in turn.

3.2.1 Constant correlation

A constant correlation means that:

$$\frac{h_{12t}}{\sqrt{h_{11t} h_{22t}}} = \rho$$

is constant over time or it is not a function of time. Therefore, h_{12t} is determined as:

$$h_{12t} = \rho \sqrt{h_{11t} h_{22t}} \tag{11}$$

An obvious advantage of the constant correlation specification is simplicity. Nonetheless, it can only establish a link between the two uncertainties, failing to tell the directions of volatility spillovers between the two sources of uncertainty.

3.2.2 Full parameterisation

The full parameterisation, or VECH, converts the covariance matrix to a vector of variance and covariance. As $_{ij} = _{ij}$, the dimension of the vector converted from an m x m matrix is $m(m + 1)/2$. Thus, in a bivariate GARCH process, the dimension of the variance/covariance vector is three. With a trivariate GARCH, the dimension of the vector is six, that is, there are six equations to describe the time-varying variance/covariance. Therefore, it is unlikely to be feasible when more than two variables are involved in a system. The VECH specification is presented as:

$$\operatorname{vech}(\mathbf{H}_t) = \operatorname{vech}(\mathrm{A}_0) + \sum_{i=1}^{q} \mathrm{A}_i \operatorname{vech}(\varepsilon_{t-i}\varepsilon'_{t-i}) + \sum_{j=1}^{p} \mathbf{B}_j \operatorname{vech}(\mathbf{H}_{t-j}) \tag{12}$$

where $\mathbf{H}_t$, $\mathbf{A}_0$, $\mathbf{A}_i$, $\mathbf{B}_j$ and $_{t\ t}$ are matrices in their conventional form, and vech (·) means the procedure of conversion of a matrix into a vector, as described above. For $p = q = 1$, equation (12) can be written explicitly as:

$$\begin{aligned}\mathbf{H}_t = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} &= \begin{bmatrix} \alpha_{11,0} \\ \alpha_{12,0} \\ \alpha_{22,0} \end{bmatrix} + \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} & \alpha_{13,1} \\ \alpha_{21,1} & \alpha_{22,1} & \alpha_{23,1} \\ \alpha_{31,1} & \alpha_{32,1} & \alpha_{33,1} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon^2_{2,t-1} \end{bmatrix} \\ &\quad + \begin{bmatrix} \beta_{11,1} & \beta_{12,1} & \beta_{13,1} \\ \beta_{21,1} & \beta_{22,1} & \beta_{23,1} \\ \beta_{31,1} & \beta_{32,1} & \beta_{33,1} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}\end{aligned} \tag{13}$$

So, the simplest multivariate model has 21 parameters to estimate.

3.2.3 Positive definite parameterisation

It is also known as BEKK, suggested by Baba, Engle, Kraft and Kroner (1990). In fact, it is the most natural way to deal with multivariate matrix operations. The BEKK specification takes the following form:

$$\mathbf{H}_t = \mathrm{A}'_0\mathrm{A}_0 + \mathrm{A}'_i\varepsilon_{t-i}\varepsilon'_{t-i}\mathrm{A}_\mathbf{i} + \mathbf{B}'_j\mathbf{H}_{t-j}\mathbf{B}_j \tag{14}$$

where $\mathbf{A}_0$ is a symmetric ($N \times N$) parameter matrix, and $\mathbf{A}_i$ and $\mathbf{B}_j$ are unrestricted ($N \times N$) parameter matrices. The important feature of this specification is that it builds in sufficient generality, allowing the conditional variances and covariances of the time series to influence each other, and at the same time, does not require the estimation of a large number of parameters. For $p = q = 1$ in a bivariate GARCH process, equation (14) has only 11 parameters compared with 21 parameters in the VECH representation. Even more importantly, the BEKK process guarantees that the covariance matrices are positive definite under very weak conditions; and it can be shown that given certain non-linear restrictions on $\mathbf{A}_i$ and $\mathbf{B}_j$, equation (14) and the VECH representation are equivalent (Engle and Kroner 1995). In the bivariate system with $p = q = 1$, equation (14) becomes:

$$\begin{aligned}\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} &= \begin{bmatrix} \alpha_{11,0} & \alpha_{12,0} \\ \alpha_{21,0} & \alpha_{22,0} \end{bmatrix} \\ &\quad + \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon^2_{2,t-1} \end{bmatrix} \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{bmatrix} \\ &\quad + \begin{bmatrix} \beta_{11,1} & \beta_{12,1} \\ \beta_{21,1} & \beta_{22,1} \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} \beta_{11,1} & \beta_{12,1} \\ \beta_{21,1} & \beta_{22,1} \end{bmatrix}\end{aligned} \tag{15}$$

We can examine the sources of uncertainty and, moreover, assess the effect of signs of shocks with equation (15). Writing the variances and covariance explicitly:

$$\begin{aligned}h_{11,t} &= \alpha_{11,0} + (\alpha^2_{11,1}\varepsilon^2_{1,t-1} + 2\alpha_{11,1}\alpha_{21,1}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \alpha^2_{21,1}\varepsilon^2_{2,t-1}) \\ &\quad + (\beta^2_{11,1}h_{11,t-1} + 2\beta_{11,1}\beta_{21,1}h_{12,t-1} + \beta^2_{21,1}h_{22,t-1})\end{aligned} \tag{16a}$$

$$
\begin{aligned}
h_{12,t} = h_{21,t} = \alpha_{12,0} + [\alpha_{11,1}\alpha_{12,1}\varepsilon_{1,t-1}^{2} + (\alpha_{12,1}\alpha_{21,1} \\
+ \alpha_{11,1}\alpha_{22,1})\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \alpha_{21,1}\alpha_{22,1}\varepsilon_{2,t-1}^{2}] \\
+ [\beta_{11,1}\beta_{21,1}h_{11,t-1} + (\beta_{12,1}\beta_{21,1} + \beta_{11,1}\beta_{22,1})h_{12,t-1} \\
+ \beta_{21,1}\beta_{22,1}h_{22,t-1}]
\end{aligned} \tag{16b}
$$

$$
\begin{aligned}
h_{22,t} = \alpha_{22,0} + (\alpha_{12,1}^{2}\varepsilon_{1,t-1}^{2} + 2\alpha_{12,1}\alpha_{22,1}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \alpha_{22,1}^{2}\varepsilon_{2,t-1}^{2}) \\
+ (\beta_{12,1}^{2}h_{11,t-1} + 2\beta_{12,1}\beta_{22,1}h_{12,t-1} + \beta_{22,1}^{2}h_{22,t-1})
\end{aligned} \tag{16c}
$$

Looking at the diagonal elements in the above matrix, that is, $h_{11,t}$ and $h_{22,t}$, we can assess the impact of the shock in one series on the uncertainty or volatility of the other, and the impact could be asymmetric or only be one way effective. In particular, one might also be interested in assessing the effect of the signs of shocks in the two series. To this end the diagonal elements representing the previous shocks can be rearranged as follows:

$$
\begin{aligned}
\alpha_{11,1}^{2}\varepsilon_{1,t-1}^{2} + 2\alpha_{11,1}\alpha_{21,1}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \alpha_{21,1}^{2}\varepsilon_{2,t-1}^{2} \\
= (\alpha_{11,1}\varepsilon_{1,t-1} + \alpha_{21,1}\varepsilon_{2,t-1})^{2}
\end{aligned} \tag{17a}
$$

$$
\begin{aligned}
\alpha_{12,1}^{2}\varepsilon_{1,t-1}^{2} + 2\alpha_{12,1}\alpha_{22,1}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \alpha_{22,1}^{2}\varepsilon_{2,t-1}^{2} \\
= (\alpha_{12,1}\varepsilon_{1,t-1} + \alpha_{22,1}\varepsilon_{2,t-1})^{2}
\end{aligned} \tag{17b}
$$

It is clear that $_{11,1}$ and $_{22,1}$ represent the effect of the shock on the future uncertainty of the same time series and $_{21,1}$ and $_{12,1}$ represent the cross effect, that is, the effect of the shock of the second series on the future uncertainty of the first series, and vice versa. The interesting point is that, if $_{11,1}$ and $_{21,1}$ have different signs, then the shocks with different signs in the two time series tend to increase the future uncertainty in the first time series. Similarly, if $_{12,1}$ and $_{22,1}$ have different signs, the future uncertainty of the second time series might increase if the two shocks have different signs. It seems that this model specification is appropriately fitted to investigate volatility spillovers between two financial markets.

The positive definite specification of the covariance extends the univariate GARCH model naturally, for example, a BEKK–GARCH (1, 1) model can reduce to a GARCH (1, 1) when the dimension of the covariance matrix becomes one. Therefore, it is of interest to make inquiry into the conditions for covariance stationarity in the general matrix form. For this purpose, we need to vectorise the BEKK representation, that is, to arrange the elements of each of the matrices into a vector. Due to the special and elegant design of the BEKK covariance, the vectorisation can be neatly and orderly derived, using one of the properties of vectorisation, that is, vech(ABC) = [C A]vech(B), where is the Kroneker product. In this case, the innovation matrix

$$
\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}' = \begin{bmatrix} \varepsilon_{1,t-1}^{2} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon_{2}^{2} \end{bmatrix}
$$

and the covariance matrix

$$
\mathbf{H}_{t-1} = \begin{bmatrix} h_{1,t-1} & h_{1,t-1}^{1/2}h_{2,t-1}^{1/2} \\ h_{2,t-1}^{1/2}h_{1,t-1}^{1/2} & h_{2,t-1} \end{bmatrix}
$$

are represented by *B*; and the fact that the parameter matrices **A** and **A** and **B** and **B** have already been transposed to each other further simplifies the transformation. For more details on these operations refer to Judge *et al*. (1988) and Engle and Kroner (1995). The vectorised $\mathbf{H}_t$ is derived as:

$$\text{vech}(\mathbf{H}_t) = (A_0 \otimes \mathbf{A}_0)'\text{vech}(\mathbf{I}) + (\mathbf{A}_i \otimes \mathbf{A}_i)'\text{vech}(\boldsymbol{\varepsilon}_t - 1\boldsymbol{\varepsilon}'_{t-1})$$
$$+ (\mathbf{B}_j \otimes \mathbf{B}_j)'\text{vech}(\mathbf{H}_{t-1}) \tag{18}$$

the unconditional covariance is:

$$E(\mathbf{H}_t) = [\mathbf{I} - (\mathbf{A}_i \otimes \mathbf{A}_i)' - (\mathbf{B}_j \otimes \mathbf{B}_j)']^{-1}\text{vech}(\mathbf{A}'_0 \otimes \mathbf{A}_0) \tag{19}$$

and the conditions for covariance stationarity is:

$$\text{mod}\lfloor(\mathbf{A}_\mathbf{I} \otimes \mathbf{A}_i)' + (\mathbf{B}_j \otimes \mathbf{B}_j)'\rfloor < 1 \tag{20}$$

That is, for $_t$ to be covariance stationary, all the eigenvalues of ($\mathbf{A}_i$ $\mathbf{A}_i$) + ($\mathbf{B}_j$ $\mathbf{B}_j$) are required to be less than one in modules. There are altogether four eigenvalues for a bivariate GARCH process as the Kroneker product of two (2×2) matrices produces a (4×4) matrix. These eigenvalues would be complex numbers in general. When the dimension of the covariance is one, equation (20) reduces to equation (4) for the univariate case.[1]

3.3 Stochastic volatility

ARCH/GARCH processes are not really stochastic, rather they are deterministic and the conditional variance possesses no unknown innovations at the time. ARCH and GARCH are not exactly the second moment equivalent to AR and ARMA processes in the mean. Stochastic volatility, as favoured by Harvey *et al*. (1994), Ruiz (1994), Andersen and Lund (1997) and others, is probably the closest equivalent to anARorARMAprocess in describing the dynamics of variance/covariance. Let us look at a simple case:

$$\begin{aligned} y_t &= \sigma_t \varepsilon_t \\ \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\ h_t &= \ln \sigma_t^2 \sim \text{ARMA}(q, p) \end{aligned} \tag{21}$$

The logarithm of the variance in a stochastic volatility model, $h_t = \ln \sigma_t^2$, behaves exactly as a stochastic process in the mean, such as random walks or AR orARMA processes. For example, if h_t is modelled as an AR(1) process, then:

$$h_t = \alpha + \rho h_{t-1} + v_t, \quad v_t \sim N(0, \sigma_v^2) \tag{22}$$

Alternatively when h_t is modelled as an ARMA(1, 1) process:

$$h_t = \alpha + \rho h_{t-1} + v_t + \theta v_{t-1}, \quad v_t \sim N(0, \sigma_v^2) \tag{23}$$

When the stochastic part of volatility, v_t , does not exist (i.e. $\sigma_v^2 = 0$), equation (22) does not reduce to ARCH (1) but to GARCH (1,0). So the difference in modelling variance is substantial between GARCH and stochastic volatility approaches. To estimate stochastic volatility models, expressing equation (21) as:

$$g_t = h_t + \kappa_t \tag{24}$$

where $g_t = \ln(y_t^2)$ and $\kappa_t = \ln(\varepsilon_t^2)$. We can see that h_t becomes part, or a component, of the (transformed) time series, in contrast to traditional statistical models where the variance expresses the distribution of variables in a different way. As the time series now has more than one component, neither is readily observable, so the components are often referred to as unobserved components. These components together form the whole system and individually describe the state of the system from certain perspectives, so they can be referred to as state variables as well. Such a specification poses problems as well as advantages: decomposition into components can be arbitrary and estimation can be complicated and sometimes difficult. Nevertheless, the state variables and their dynamic evolution and interaction may reveal the fundamental characteristics and dynamics of the system or the original time series more effectively, or provide more insights into the working of the system. Models of this type are usually estimated in the state space, often accompanied by the use of Kalman filters. See Chapter 8 for details of the state space representation and the Kalman filter.

3.4 Examples and cases

When the time comes to implement an empirical study, the problem may never be exactly the same as illustrated in the text. This is hopefully what a researcher expects to encounter rather than attempting to avoid if s/he imagines new discoveries in her/his study or would like to differentiate her/his study from others. This section provides such examples.

Example 1

This is an example incorporating macroeconomic variables into the conditional variance equation for stock returns by Hasan and Francis (1998), entitled ‘Macroeconomic factors and the asymmetric predictability of conditional variances’. The paper includes the default premium, dividend yield and the term premium as state variables in the conditional variance equation, though its main purpose is to investigate the predictability of the volatilities of large v. small firms. The paper shows that volatility surprises of small (large) firms are important in predicting the conditional variance of large (small) firms, and this predictive ability is still present when the equation of conditional variance includes the above-mentioned state variables.

The paper uses monthly returns of all NYSE and AMEX common stocks with available year-end market value information gathered from the Center for Research in Security Prices (CRSP) monthly master tape from 1926 to 1988. All stocks in the sample are equally divided into twenty size-based portfolios, S1 (smallest) to S20 (largest), according to the market value of equity at the end of the prior year. Monthly excess returns on each of the portfolios are obtained by averaging returns across all the stocks included in the portfolio. Their specification is as follows:

$$R_{i,t} = \alpha_{i,t} + \beta_i R_{i,t-1} + \mu_{i,1}\mathrm{JAN}_t + \gamma_i R_{j,t-1} + e_{i,t} \tag{25a}$$

$$h_{i,t} = \delta_{i,0} + \alpha_i e_{i,t-1}^2 + \theta_i h_{i,t-1} + \delta_{i1}\mathrm{JAN}_t + \varphi_j e_{j,t-1}^2 + \sum \omega_k Z_{k,t-1} \tag{25b}$$

The mean equation follows an AR(1) process. JAN_t is the dummy which is equal to one when in January and zero otherwise. $Z_{k,t}$ (k = 1, 2, 3) are the state variables of default premium (DEF), dividend yield (DYLD) and the term premium (TERM) respectively. These state variables are those used by Fama and French (1989) and Chen (1991). The effect of return and volatility spillovers across portfolios is through the inclusion of lagged returns on portfolio j in the mean equation for portfolio i, and the inclusion of lagged squared errors for portfolio j in the conditional variance equation of portfolio i. Squared errors for portfolio j are obtained through estimating a basic GARCH model whose conditional variance is a standard GARCH (1, 1) plus the January dummy. Therefore, the model is univariate rather than bivariate in nature.

The paper then estimates the model for two portfolios, the small stock portfolio and the large stock portfolio. Volatility spillovers across these two portfolios are examined. The major findings are that while return spillovers are from the small stock portfolio to the large stock portfolio only, volatility spillovers are bidirectional, though the effect of the small stock portfolio on the large stock portfolio is greater than vice versa. Only the main results are presented in Tables 3.1 and 3.2. Model (1) does not include the state variables, Models (2)–(4) include one of the state variables each, and Model (5) incorporates all the state variables. As there is not much difference in the mean equation results, only the results from Model (5) are provided.

Example 2

This is an example of the bivariate GARCH model applied to the foreign exchange market byWang andWang (2001). In this study, the daily spot and forward foreign exchange rates of the British pound, German mark, French franc and Canadian dollar against the US dollar are used. All of the data sets start from 2 January 1976 and end on 31 December 1990; so there are 3,758 observations in each series. These long period high-frequency time series data enable us to observe a very evident GARCH phenomenon in a bivariate system. The system of equations for the spot exchange rate, S_t , and the forward exchange rate, F_t , is specified as an extended VAR, which incorporates a forward premium into a simple VAR. In addition, the covariance of the extended VAR is time-varying which allows for and mimics volatility spillovers or transmission between the spot and forward foreign exchange markets. The model is given as follows:

Table 3.1 Small stock portfolio

Table 3.2 Large stock portfolio

$$\Delta s_t = c_1 + \gamma_1(f_{t-1} - s_{t-1}) + \sum_{i=1}^{m} \alpha_{1i} \Delta s_{t-i} + \sum_{i=1}^{m} \beta_{1i} \Delta f_{t-i} + \varepsilon_{1t}$$

$$\Delta f_t = c_2 + \gamma_2(f_{t-1} - s_{t-1}) + \sum_{i=1}^{m} \alpha_{2i} \Delta s_{t-i} + \sum_{i=1}^{m} \beta_{2i} \Delta f_{t-i} + \varepsilon_{2t}$$

$$\varepsilon_t | \Omega_{t-1} \sim N(0, H_t) \qquad (26)$$

where $s_t = \ln(S_t)$, $f_t = \ln(F_t)$, $s_t = s_{t-1}$, $ft = f_t - f_{t-1}$, and H_t is the time-varying covariance matrix with the BEKK specification.

The inclusion of the forward premium is not merely to set up an ECM model – it keeps information in levels while still meeting the requirements for stationarity. Although there are arguments about the property of the forward premium, its inclusion makes the system informationally and economically complete by reserving information in levels (original variables) and reflecting expectations in the market.

The bivariate GARCH effects are, in general, strong in both the spot and forward markets, though there exists a clear asymmetry in the volatility spillover patterns. That is, there are, to a lesser extent, volatility spillovers from the spot market to the forward market, compared with the other way round. Table 3.3 presents the results based mainly on the second moment.

In addition, the parameter for the forward premium is also reported, as it would validate the cointegration between the spot and forward exchange rates and the need to incorporate the forward premium. Consider the British pound first. a_{12} and a_{21} are both significant at the 1 per cent level, but the magnitude of the former is about half the size of the latter, implying that the effect of the shock in the forward market on the spot market volatility is bigger than that on the forward market induced by the shock in the spot market. Turning to the effects of the previous uncertainty, while b_{21} is significant, b_{12} is not significant at all, so the volatility spillovers are one-directional from the forward to the spot. Notice, b_{22} is also insignificant, which means there is only ARCH in the forward exchange rate. Further scrutiny on the signs of a_{12} and a_{22} suggests that the future volatility in the forward market would be higher if the two shocks have different signs. In the case of the German mark, the asymmetry is more apparent, where a_{12} is not significant at all but a_{21} is significant at the 1 per cent level. As such, the shock in the forward market would affect the future volatility in the spot market, but the shock in the spot market has no influence on the future volatility in the forward market. In addition, a_{11} and a_{21} have different signs, so the shock with opposite signs in these two markets would be inclined to increase the future volatility in the spot market. As far as the previous variance is concerned, b_{12} and a_{21} are both significant, but the size of the former is much smaller than that of the latter, so the asymmetry exists in this respect too. Again, b_{22} is not significant; the forward rate would only have the ARCH effect if it were not considered in a bivariate system. The strongest asymmetry occurs in the exchange rates of the Canadian dollar. The volatility spillovers are absolutely one-directional from the forward rate to the spot rate. That is, a_{12} and b_{12} are not significant at any conventional levels, whereas a_{21} and b_{21} are both significant at the 1 per cent level. Similar to the British pound, in the case of French franc, the influence of the previous variance is clearly one directional from the forward to the spot rate measured by b_{12} and b_{21}. Although the GARCH effect is strong in the forward rate as well as in the spot rate, b_{22} is close to being twice as big as b_{11}. Regarding the previous shocks, the influence is also more from forward to spot; both a_{12} and a_{21} are significant but a_{21} is much bigger than a_{12}. Therefore, the four currencies have similar asymmetric volatility spillover patterns. Another interesting point in the example of French franc is that the premium is not significant in either the spot or forward equations when the covariance matrix is assumed constant. The premium is significant in the forward equation when estimated in a multivariate GARCH framework. This suggests that the rejection/acceptance of a cointegration relationship is, to certain extent, subject to the assumption on the properties of the covariance.

In Table 3.4, all four eigenvalues for each currency are reported. Their positioning on the complex plane is displayed in Figure 3.1. It can been seen that the biggest of the eigenvalues for each currency is around 0.96 in modules, so the timevarying volatility is highly persistent. In the French franc case, the biggest module of eigenvalue is just above unity, suggesting that the unconditional covariance does not exist. There are two explanations to provide for this. First, according to Nelson (1990), Bougerol and Picard (1992) and Lumsdaine (1991), even if a GARCH (IGARCH) model is not covariance stationary, it is strictly stationary or ergodic, and the standard asymptotically-based inference procedures are generally valid. Second, the derivation of eigenvalues is based on the assumption that the spot variance and forward variance are equal in size. Nevertheless, the forward variance is smaller than the spot variance in the French franc case. Taking this into account, all of the modules of eigenvalue for the French franc become less than one and covariance stationarity exists. The analysis of the eigenvalues of the Kroneker product of the covariance matrices reveals that time-varying volatility is also highly persistent in a bivariate setting for foreign exchange rate data. In addition, though the BEKK specification has proved to be a helpful analytical technique for investigating volatility transmissions, especially the impact of the signs of the shocks in different markets, in empirical research, covariance stationarity is not so easy to satisfy and is not always guaranteed.

Table 3.3 Volatility spillovers between spot and forward FX rates

Table 3.4 Verifying covariance stationarity: the eigenvalues. Unconditional covariance: $E(\sigma_t^2) = [I - (A^* \otimes A^*)' - (B^* \otimes B^*)']^{-1}\mathrm{vec}(C_0^{*\prime} C_0^*)$

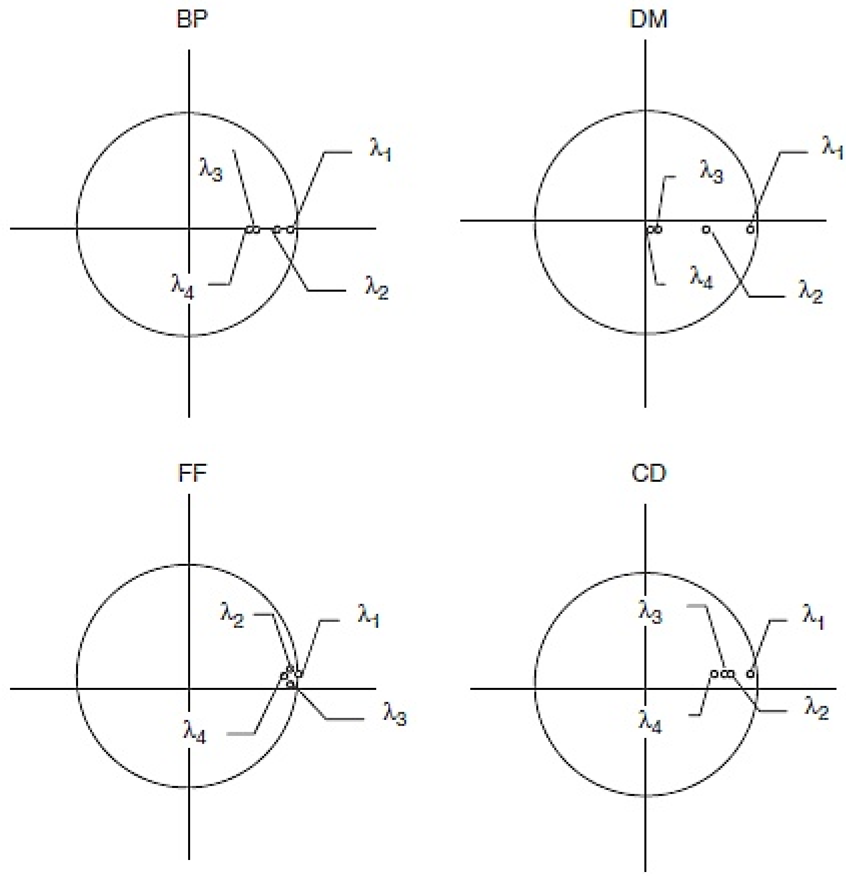

Figure 3.1 Eigenvalues on the complex plane. The horizontal axis is for the real part, and the vertical axis is for the imaginary part of the eigenvalue. The reference circle is the unit circle.

3.5 Empirical literature

While time-varying volatility has found applications in almost all time series modelling in economics and finance, it attracts most attention in the areas of financial markets and investment where a vast empirical literature has been generated, which has in turn brought about new forms and variations of this family of models. Time-varying volatility has become the norm in financial time series modelling, popularly accepted and applied by academics and professionals alike since the 1990s. Moreover, analysis of interactions between two or more variables in the first moment, such as in VAR and ECM, is extended through the use of time-varying volatility models to the second moment, to examine such important issues as volatility spillovers or transmissions between different markets.

One of the most extensively researched topics is time-varying volatility universally found in stock market indices. Although findings vary from one market to another, a pattern of time-varying volatility, which is also highly persistent, is common to most of them. Nevertheless, many of studies attempt to exploit new features and add variations in model specifications to meet the specific need of empirical investigations. To examine the characteristics of market opening news, Gallo and Pacini (1998) apply the GARCH model to evaluate the impact of news on the estimated coefficients of the model. They find that the differences between the opening price of one day and the closing price of the day before have different characteristics and have the effect of modifying the direct impact of daily innovations on volatility which reduces the estimated overall persistence of innovations. It is also claimed that the inclusion of this news variable significantly improves out-of-sample forecasting, compared with the simple GARCH model's performance. Brooks *et al*. (2000) adopt the power ARCH (PARCH) model proposed by Ding *et al*. (1993) to stock market returns in 10 countries and a world index. As PARCH removes the restriction implicitly imposed by ARCH/GARCH, that is, the power transformation is achieved by taking squaring operations of the residual, it can possess richer volatility patterns such as asymmetry and leverage effects. They find that the PARCH model is applicable to these return indices and that the optimal power transformation is remarkably similar across countries. Longin (1997) employs the analytical framework of Kyle (1985) where there are three types of traders: informed traders, liquidity traders and market makers. In such a setting, the paper models information as an asymmetric GARCH process so that large shocks are less persistent in volatility than small shocks. This, it is claimed, allows one to derive implications for trading volume and market liquidity. The study by Koutmos (1992) is one of the typical empirical applications of GARCH in finance in early times – risk-return trade-off in a time-varying volatility context and asymmetry of the conditional variance in response to innovations. The EGARCH in Mean (EGARCH-M) model is chosen for obvious reasons, as above, and the findings support the presence of these well-observed phenomena in 10 stock market return indices. Newly added to this literature is evidence from so-called emerging markets and the developing world. Investigating the behaviour of the Egyptian stock market in the context of pricing efficiency and the return–volatility relationship, Mecagni and Sourial (1999) employ a GARCH-M model to estimate four daily indices. Their results suggest that there is a tendency of volatility clustering in returns, and a positive but asymmetric link between risk and returns which is statistically significant during market downturns. They claim that the asymmetry in the risk–return relationship is due to the introduction of circuit breakers. Husain (1998) examines the Ramadhan effect in the Pakistani stock market using GARCH models. Ramadhan, the season of the holy month of fasting, is expected to have effects on stock market behaviour one way or another. The study finds that the market is indeed tranquil as the conditional variance declines in that month, but the season does not appear to have any impact on mean returns. Applying TGARCH models to two Eastern European markets, Shields (1997) reports findings contrary to those in the West that there is no asymmetry in the conditional variance in response to positive and negative shocks in these Eastern European markets.

International stock market linkages have attracted increasing attention in the process of so-called globalisation in a time when there are no major wars. Seeking excess returns through international diversification is one of the strategies employed by large multinational financial institutions in an ever intensifying competitive financial environment, while national markets, considered individually, appear to have been exploited to their full so that any non-trivial profitable opportunities do not remain in the context of semi-strong market efficiency. In particular, US investors have gradually given up the stand of regarding foreign markets as alien lands and changed their risk perspectives – international diversification benefits are more than off setting perceived additional risks. In the meantime, international asset pricing theory has been developed largely with a stratified approach which regards the international financial market as segmented as well as linked markets, adding additional dimensions to the original capital asset pricing model which is, ironically, universal, or in other words, global. Under such circumstances, it is not strange that applications of multivariate GARCH models have mushroomed during this period.

Investigating one of the typical features in emerging financial markets, Fong and Cheng (2000) test the information based hypothesis that the rate of information absorption in the conditional variance is faster for foreign shares (open to foreigners and locals) than for local shares (open to locals only) using a bivariate GARCH (1, 1) model for nine dual-listed stocks over the period 1991–1996. Their evidence indicates that the rate of information absorption is consistent with what was proposed by Longin (1977) that the rate of information absorption varies inversely with the number of informed traders. They claim that removing foreign ownership restrictions is likely to improve both market efficiency and liquidity. International risk transmission or volatility spillovers between two or more financial markets is by far the most intensively researched area. In this fashion, Kim and Rui (1999) examine the dynamic relationship between the US, Japan and UK daily stock market return volatility and trading volumes using bivariate GARCH models. They find extensive and reciprocal volatility spillovers in these markets. The results from return spillovers, or Granger causality in the mean equations, seem to confirm all reciprocal relationships but exclude London's influence on the New York Stock Exchange. Tay and Zhu (2000) also find such dynamic relationships in returns and volatilities in Pacific-Rim stock markets. Chou *et al*. (1999) test the hypothesis that the short-term volatility and price changes spillover from developed markets to emerging markets using the US and Taiwan data. They find substantial volatility spillover effect from the US stock market to the Taiwan stock market, especially for the model using close-to-open returns. There is also, it is claimed, evidence supporting the existence of spillovers in price changes. In contrast to the majority of the findings, Niarchos *et al*. (1999) show that there are no spillovers in means and conditional variances between the US and Greek stock markets and suggest that the US market does not have a strong influence on the Greek stock market. Many similar studies have emerged in recent years, for example, Dunne (1999) and Darbar and Deb (1997), to mention a few.

Inflation uncertainty remains one of the main application areas of GARCH modelling, following the first paper of this type on the topic by Engle (1982). In a recent study, Grier and Perry (1998), without much surprise, provide empirical evidence that inflation raises inflation uncertainty, as measured by the conditional variance of the inflation rate, for all G7 countries in the period from 1948 to 1993. Their results on the causal relationship from inflation uncertainty to inflation are mixed. In three countries, increased inflation uncertainty lowers inflation; while in two countries increased inflation uncertainty raises inflation. These findings have been extended to cover the developing world as well. Applying a similar testing procedure, Nas and Perry (2000) find evidence supporting the claim that inflation raises inflation uncertainty in Turkey over the full sample period of 1960–1998 and in the three sub-samples. They again show mixed results for the effect of inflation uncertainty on inflation, and claim that this is due to institutional and political factors in the monetary policy-making process in Turkey between 1960 and 1998. Wang *et al*. (1999) examine the causal relationships between inflation, inflation uncertainty as measured with the conditional variance of the aggregate inflation rate and relative price variability in sectoral price indices. They find that, although inflation does Granger cause inflation uncertainty, relative price variability is more a source of inflation uncertainty than the inflation level itself. In contrast, Grier and Perry (1996) present different findings in respect of these relationships and appear to contradict the results of their other studies. Various studies on the topic include Brunner and Hess (1993), and Loy and Weaver (1998).

In examining foreign exchange markets, time-varying volatility models have been widely adopted to study various issues ranging from time-varying risk premia, volatility spillovers between the spot and forward exchange market, and hedging strategies, to the effect of monetary policy. Searching for an explanation for the departure from uncovered interest parity (UIP), Tai (1999) examines the validity of the risk premium hypothesis using a GARCH-M (1, 1) model. The empirical evidence supports the notion of time-varying risk premia in explaining the deviations from UIP. It also supports the idea that foreign exchange risk is not diversifiable and hence should be priced in both the foreign exchange market and the equity market. Hu’s (1997) approach is to examine the influence of macroeconomic variables on foreign exchange risk premia. The paper assumes that money and production follow a joint stochastic process with bivariate GARCH innovations based on Lucas’s asset pricing model and implies that the risk premium in the foreign exchange market is due to time-varying volatilities in macroeconomic variables. Testing the model for three currencies shows that the time-varying risk premium is able to explain the deviation of the forward foreign exchange rate from the future spot rate. It is claimed that the model partially supports the efficient market hypothesis after accounting for time-varying risk premia. Investigating the effect of central bank intervention, Dominguez (1993) adopts GARCH models to test whether the conditional variance of exchange rates has been influenced by the intervention. The results indicate that intervention need not be publicly known for it to influence the conditional variance of exchange rate changes. Publicly known Fed intervention generally decreases exchange rate volatility, while secret intervention operations by both the Fed and the Bundesbank generally increase the volatility. Kim and Tsurumi (2000),Wang andWang (1999), Hassapis (1995), Bollerslev and Melvin (1994), Copeland and Wang (1993), Mundaca (1991), Bollerslev (1990) and many other studies are also in this important area.

As mentioned earlier, time-varying volatility has become the norm in financial time series modelling, popularly accepted and applied by academics and professionals alike since the 1990s. Therefore, it does not appear to be feasible to completely list the application areas and individual cases. Among other things not covered by the brief survey in this section, there are applications in option modelling, dynamic hedging, the term structure of interest rates and interest rate-related financial instruments.

Questions and problems

1. Describe ARCH and GARCH in comparison with AR and ARMA in the mean process.
2. Discuss many variations of GARCH and their relevance to financial modelling.
3. What is the stochastic volatility model? Discuss the similarities and differences between a GARCH-type model and a stochastic volatility model.
4. Compare different specifications of multivariate GARCH models and comment on their advantages and disadvantages.

5. Collect data from Datastream to test for GARCH phenomena, using the following time series:
    1. foreign exchange rates of selected industrialised nations and developing economies *vis-à-vis* the US$, taking the log or log difference transformation if necessary prior to the test;
    2. CPI of the UK, US, Japan, China, Russia and Brazil, taking any necessary transformation prior to the test;
    3. total return series of IBM, Microsoft, Sage, Motorola, Intel, Vodafone, and Telefonica, taking any necessary transformation prior to the test.

    What do you find of their characteristics?
6. Collect data from Datastream and apply various multivariate GARCH models to the following time series:
    1. the spot and forward foreign exchange rates of selected industrialised nations and developing economies *vis-à-vis* the US$, taking the log or log difference transformation if necessary prior to the test;
    2. the stock market return indices of the US (e.g. S&P 500) and the UK (e.g. FTSE 100);
    3. the stock market return indices of Japan and Hong Kong.

    What do you find of their links in the second moment?
7. Discuss and comment on the new developments in modelling time-varying volatilities.

Note

1 Equation (14) becomes $h_{11,t} = \alpha_{11,0}^2 + \sum_{i=1}^{q} \alpha_{11,i}^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_{11,j}^2$, so $\alpha_{11,0}^2, \alpha_{11,i}^2$ and $\beta_{11,j}^2$ are equivalent to $_0$, $_i$ and $_j$ respectively, in equation (4).

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