BSSE 22053

Probability Distributions and Applications

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References

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Chapter 1 – Summary on Basic Concepts

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1.1 Introduction to Types of Data

- In the study of statistics, we are concerned basically with the presentation and interpretation of *chance* or *randomness* of outcomes that occur in a planned study or scientific investigation.
- Hence, the statistician is often dealing with either *numerical data*, representing counts or measurements, or *categorical data*, which can be classified according to some criterion.
- We shall refer to any recording of information, whether it be numerical or categorical, as an *observation*.

For example, each month from January through April during the last year constitute a set of numerical data as observations.

Similarly, the categorical data $N, D, N, N,$ and D, representing the items found to be defective or nondefective when five items are inspected, are recorded as observations.

- Statisticians use the word *experiment* to describe any process that generates a set of data.
- A simple example of a statistical experiment is the tossing of a coin. In this experiment, there are only two possible outcomes, heads or tails.

  Another experiment might be the launching of a missile and observing of its velocity at specified times.

1.2 Sample Space

- Definition 1 – Sample Space

  The set of all possible outcomes of a statistical experiment is called the *sample space* and is represented by the symbol S.

- Each outcome in a sample space is called an *element* or a *member* of the sample space, or simply a *sample point*.
- If the sample space has a finite number of elements, we may list the members separated by commas and enclosed in braces.

  Thus, the sample space S, of possible outcomes when a coin is flipped, may be written $S = \{H, T\}$, where H and T correspond to heads and tails, respectively.

Example 1.1: Consider the experiment of tossing a die.

If we are interested in the number that shows on the top face, the sample space is $S_1 = \{1,2,3,4,5,6\}$.

If we are interested only in whether the number is even or odd, the sample space is simply $S_2 = \{even, odd\}$.

- Example 1.1 illustrates the fact that more than one sample space can be used to describe the outcomes of an experiment.

1.3 Events

- For any given experiment, we may be interested in the occurrence of certain *events* rather than in the occurrence of a specific element in the sample space.

- Definition 2 – Event

  An *event* is a subset of a sample space S.

  The class of all events associated with a given experiment is defined to be the *event space*.

- Note: The above Definition 2 does not precisely define what an event is.

  An event will always be a subset of the sample space, but for sufficiently large sample spaces not all subsets will be events.

  Thus the class of all subsets of the sample space will not necessarily correspond to the event space.

If the sample space consists of only a finite number of points, then the corresponding event space will be the class of all subsets of the sample space.

A one-point subset of a sample space will always be an event and will be called an *elementary event*.

Example 1.2

Consider the experiment of tossing a die and let A be the event that the outcome is divisible by 3.

This event will occur if the outcome is an element of the subset $A = \{3,6\}$ of the sample space $S_1 = \{1,2,3,4,5,6\}$.

Example 1.3

Suppose that three items are selected at random from a manufacturing process.

Each item is inspected and classified defective, D, or nondefective, N.

The sample space is

$$S = \{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}.$$

The event B that the number of defectives in this experiment is greater than 1. This will occur if the outcome is an element of the subset

$$B = \{DDN, DND, NDD, DDD\}.$$

Example 1.4

The experiment is to record the number of traffic deaths in the city of Colombo next year.

Any nonnegative integer is a conceivable outcome of this experiment; so the sample space $S = \{0,1,2,\cdots\}$.

The set $A = \{\text{fewer than } 500 \text{ deaths}\} = \{0,1,\cdots,499\}$ is an event.

$A_i = \{\text{exactly } i \text{ deaths}\}, i = 1,2,\cdots$, is an elementary event.

There is an infinite number of points in the sample space, and each point is itself an (elementary) event; so there is an infinite number of events.

Each subset of S is an event.

Example 1.5

Select a light bulb, and record the time in hours that it burns before burning out.

Any nonnegative number is a conceivable outcome of this experiment; so the sample space $S = \{x | x \geq 0\}$.

For this sample space not all subsets of S are events; however, any subset that can be exhibited will be an event.

For example, let

$$A = \{\text{bulb burns for at least k hours but burns out before } m\}$$
$$= \{x | k \leq x \leq m\},$$

then A is an event for any $0 \leq k \leq m$.

Example 1.6

Given the sample space $S = \{t|t \geq 0\}$, where t is the life in years of a certain electronic component, then the event A that the component fails before the end of the fifth year is the subset $A = \{t|0 \leq t \leq 5\}$.

- Since an event is a set, we can combine events to form new events using the various set operations:

  (i) $A \cup B$ is the event that occurs iff A occurs *or* B occurs (or both).

  (ii) $A \cap B$ is the event that occurs iff A occurs *and* B occurs.

  (iii) A', the complement of A, also written $\bar{A}$, is the event that occurs iff A does *not* occur.

- Two events A and B are called *mutually exclusive* if they are disjoint, that is, if $A \cap B = \emptyset$.

  In other words, the two events A and B are mutually exclusive iff they cannot occur simultaneously.

  Three or more events are mutually exclusive if every pair of them are mutually exclusive.

1.4 Probability of an Event

- Throughout the remainder of this chapter, we consider only those experiments for which the sample space contains a finite number of elements.
- The likelihood of the occurrence of an event resulting from such a statistical experiment is evaluated by means of a set of real numbers, called *weights* or *probabilities*, ranging from 0 to 1.
- To every point in the sample space we assign a probability such that the sum of all probabilities is 1.
- To find the probability of an event A, we sum all the probabilities assigned to the sample points in A.

  This sum is called the *probability* of A and is denoted by $P(A)$.

- Definition 3 – Probability of an Event

  The *probability* of an event A is the sum of the weights of all sample points in A. Therefore,

  $$0 \leq P(A) \leq 1, P(\emptyset) = 0 \text{ and } P(S) = 1.$$

  Furthermore, if $A_1, A_2, A_3, \cdots$ is a sequence of mutually exclusive events, then

  $$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) \cdots.$$

- If the sample space for an experiment contains N elements, all of which are equally likely to occur, we assign a probability equal to $\frac{1}{N}$ to each of the N points.

  Such a sample space is called an *equiprobable space*.

  The probability of any event A containing n of these N sample points is then the ratio of the number of elements in A to the number of elements in S.

- If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is $P(A) = \frac{n}{N}$.