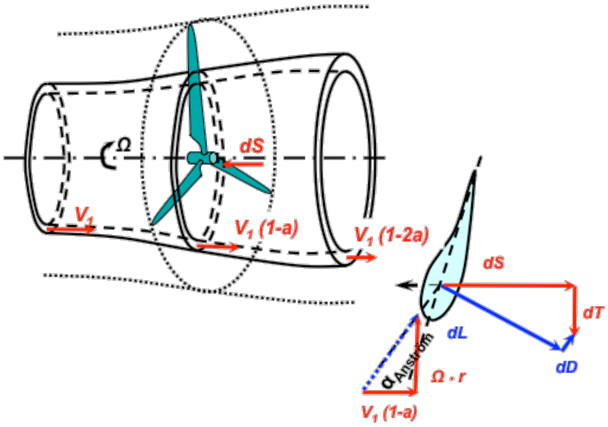
Design of wind turbine systems
SS 2016

Lecture 4:
Blade Element Momentum Theory (BEM)



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ForWind – Wind Energy Systems

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I. Blade Element Momentum theory (BEM)
- Background
- Algorithm (basics)

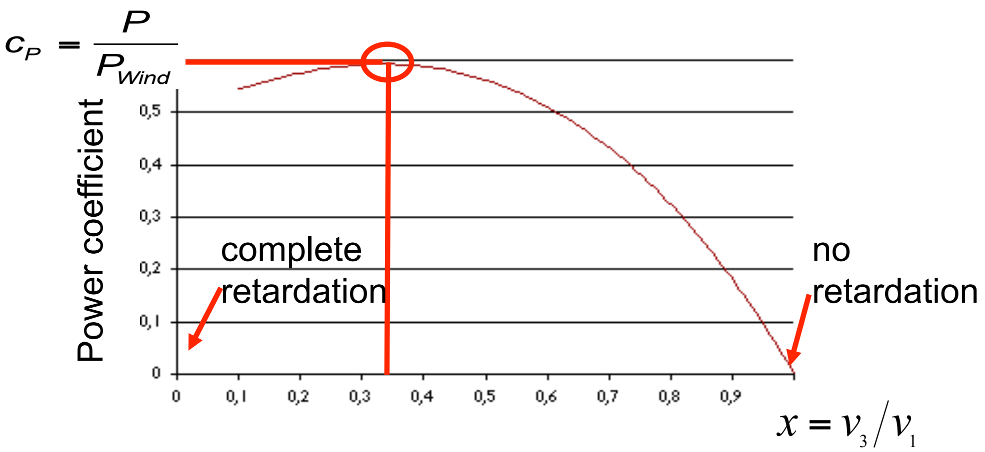
II. Correction to the basic BEM theory
- Wake rotation
- Free stream separation
- Finite number of blades

Notice

References:

- Scriptum on BEM:
  Grant Ingram, Wind Turbine Blade Analysis using the Blade Element Momentum Method. Version 1.0, 2005, Durham University,
  http://www.dur.ac.uk/g.l.ingram/download/wind_turbine_design.pdf
- R. Gasch, „Wind Power Plants“, 3rd edition, Springer 2012
- T. Burton, et al., Wind Energy Handbook, 2nd edition, Wiley 2012
- E. Hau, Wind Turbines, 2nd edition, Springer 2006
- S. Streiner, Beitrag zur numerischen Simulation der Aerodynamik und Aeroelastik großer Windkraftanlagen mit horizontaler Achse, Dissertation, Universität Stuttgart, 2011.

Maximum power coefficient acc. to Betz (1926)



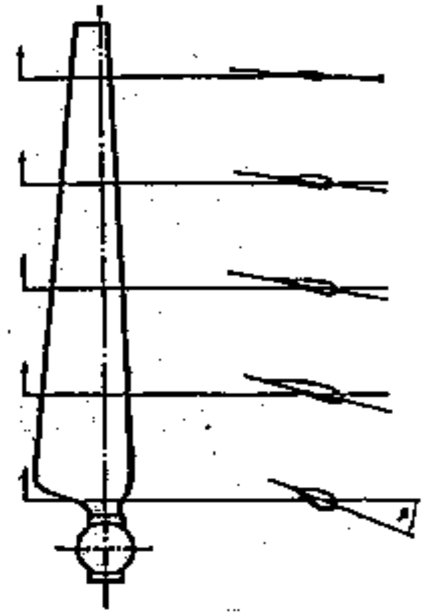
Maximum power coefficient acc. to Betz is $c_{P,Betz} = \frac{P}{P_{Wind}} = 0{,}593$,

if wind speed V_1 is reduced to $V_3 = V_1/3$ far behind the rotor
In the most favourable case, assuming power extraction without any losses, *only 59% of wind power can be used*.

Optimum rotor design according to Betz

Input:
design tip speed ratio λ_D
design value of lift coefficient c_L
design value of inflow angle α
(blade number N)

Output:
blade chord t(r) or *t/R(r/R)*
blade twist α_{twist} (r/R)



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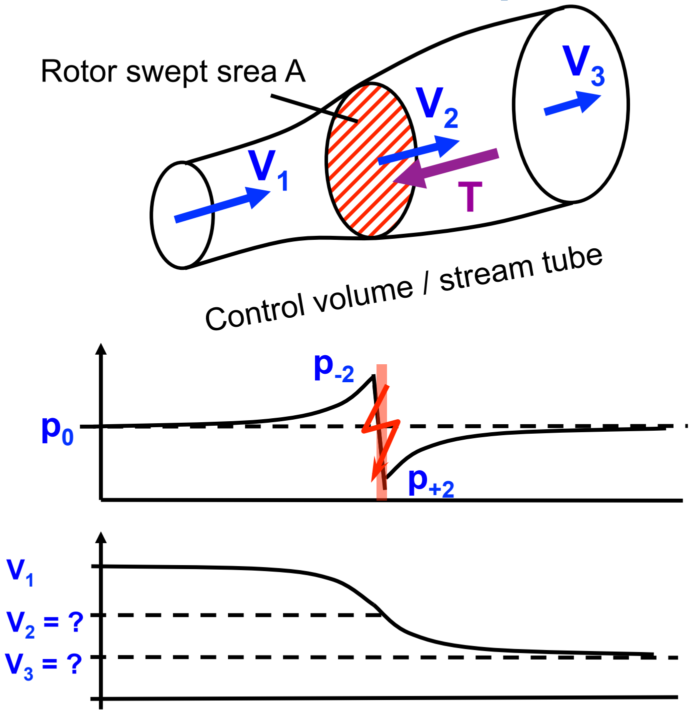
Blade element momentum theorry

Actuator disk model: speed retardation and pressure



Rotor modelled as actuator disk

induced velocity

$$V_i = V_1 - V_2 = a\,V_1$$

a = axial induction factor

(non-dimensional)

$$\boxed{V_2 = V_1\,(1 - a)}$$

Rotor thrust T is equal to

- pressure difference acting on actuator disk

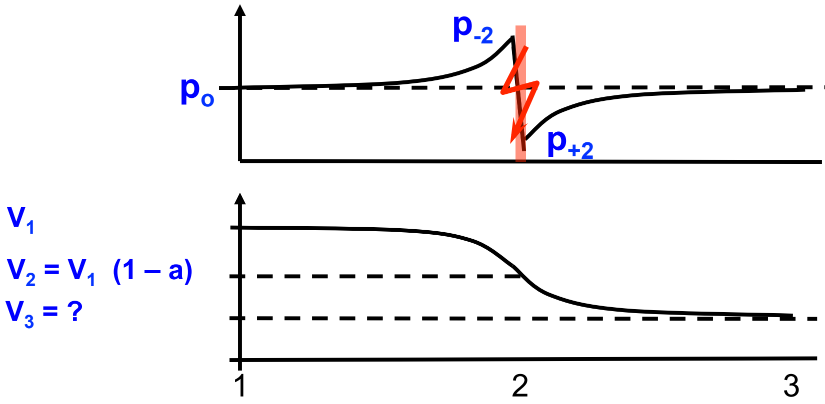
$$T = (p_{-2} - p_{+2})\,A$$

- momentum difference between inlet and outlet of stream tube

$$T = \dot{m}\,(V_1 - V_3)$$

$$\dot{m} = \rho\,A\,V_2$$

Actuator disk model: equations



Rotor thrust T equal to

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- Momentum difference between inlet and outlet of stream tube

$$T = \dot{m}\,(V_1 - V_3)$$

$$\dot{m} = \rho\,A\,V_2$$

Specific potential energy conservation at stations

- 1-2 : $p_0 + \frac{1}{2}\rho V_1^2 = p_{-2} + \frac{1}{2}\rho V_2^2$
- 2-3 : $p_0 + \frac{1}{2}\rho V_3^2 = p_{+2} + \frac{1}{2}\rho V_2^2$

$$\frac{1}{2}\,\rho\,(V_1^2 - V_3^2) = (p_{-2} - p_{+2}) = \frac{T}{A}$$

$$\frac{1}{2}\,\rho\,(V_1^2 - V_{3_3}^2)\,A = \dot{m}\,(V_1 - V_3)$$

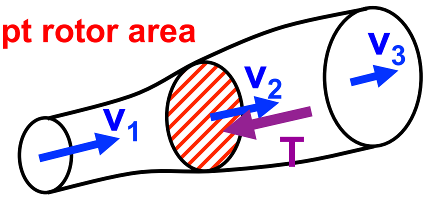
$$V_2 = V_1\,(1 - a) = \frac{1}{2}\,(V_1 + V_3)$$

$$\boxed{V_3 = V_1\,(1 - 2a)}$$

Explanation of the magnitude of the rotor thrust

$$\text{thrust coefficient} = \frac{\text{thrust}}{\text{dynamic pressure} \cdot \text{swept rotor area}}$$

thrust = thrust coefficient · dynamic pressure · swept rotor area



Optimum thrust coefficient = 8/9 ≈ 0,9

Drag coefficient of a solid disc ≈ 1,17

⇒ Thrust force has similar magnitude than the drag force on a solid plate covering the entire swept area !

Example: wind turbine ø 80 m at rated wind speed 11 m/s

⇒ Swept area 5.000 m²

⇒ Thrust approx. 330 kN, mass flow 45 t/s

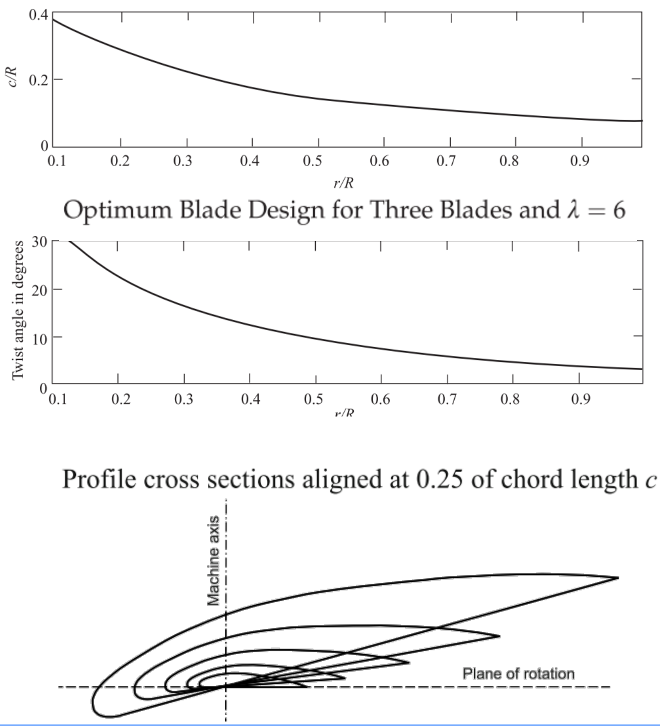
[Fig. REpower]

What's the limit of the actuator disk model?

- The induction at the rotor is due to the aerodynamical forces on the blade
- Rotors with well designed blades approach Betz/ Schmitz limit at the design point

… how does the induction vary outside the design range?

The actuator disk model needs help from airfoil theory!



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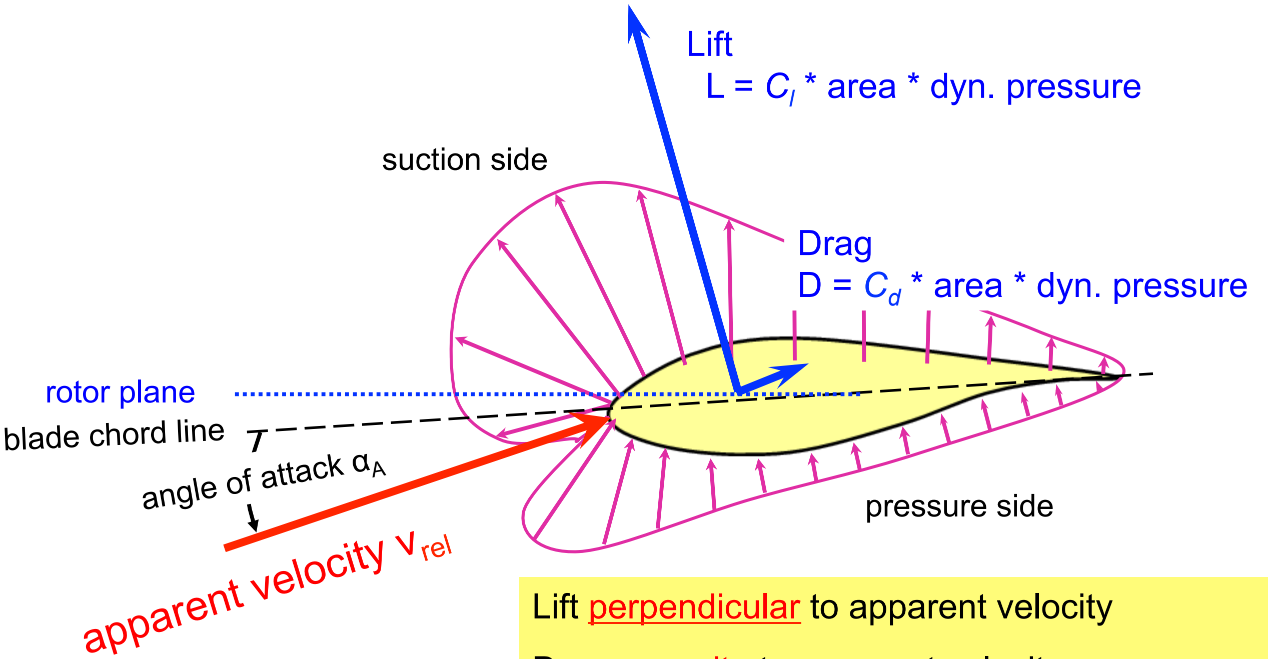
Distribution of pressure at an airfoil

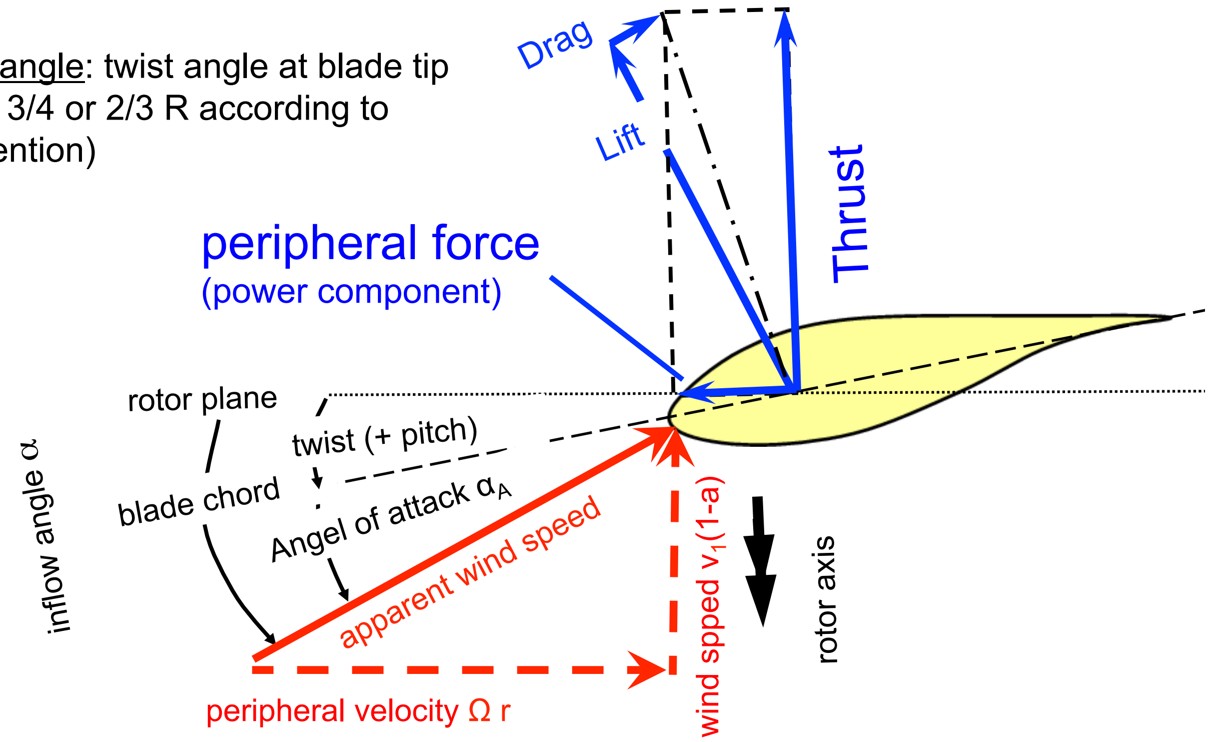


Lift perpendicular to apparent velocity

Drag opposite to apparent velocity

Forces at a blade section

pitch angle: twist angle at blade tip (or at 3/4 or 2/3 R according to convention)

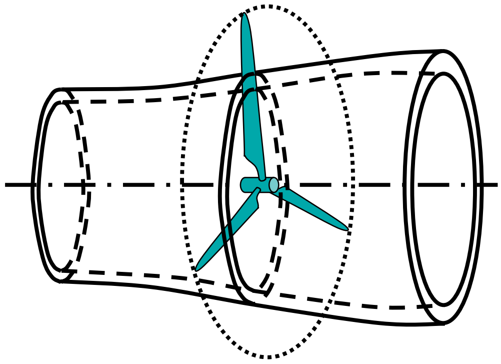


Blade Element Momentum theory (BEM) (1/4)

Basic idea: subdivision of the rotor in annular rings
balance of forces in axial (and tangential) directions

Forces from global momentum balance (dependent on induced velocity)	=	*Forces at the local blade element* (dependent on induced velocity)

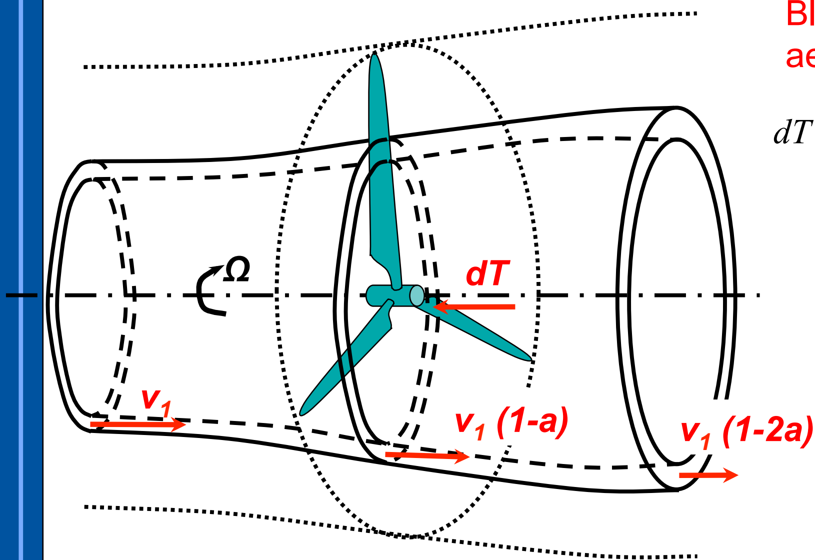
=> iterative determination of induced velocities



Assumptions:

1. *stream tube theory* and segmentation into isolated rings (no radial interaction)
2. *no radial flow along the blades* (problems for stall and at blade tip)
3. *no tangential changes within the ring* (yet empirical correction for finite number of blades)

Blade Element Momentum theory (2/4)

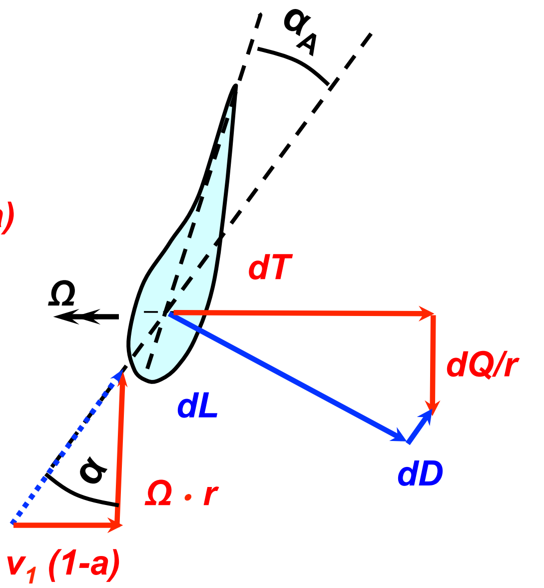


Momentum theory:
thrust over a ring

$$dT = \dot{m}\,(V_1 - V_3) = \rho\; 2\pi\; r\; dr\; v_1^2\; 4a\;(1-a)$$

Blade element theory:
aerodynamic thrust

$$dT = N\; c\; dr\; \frac{\rho}{2}\; v_{rel}^2\left(c_L \cos\alpha + c_D \sin\alpha\right)$$



Blade Element Momentum method (3/4)

Thrust by thrust coefficient from momentum balance
(on annulus area)

=

Thrust on local blade element
(at all N blade elements width dr)

inflow angle α

$$c_L = c_L(\alpha)\,;\, c_D = c_D(\alpha) \qquad \tan(\alpha) = \frac{v_1(1-a)}{\Omega \cdot r}$$

$$4a(1-a)\cdot\frac{\rho}{2}v_1^2 \cdot 2\pi\; r\; dr = Nc\frac{\rho}{2}v_{rel}^2\left(c_L \cos\alpha + c_D \sin\alpha\right)dr$$

inflow velocity v_{rel}

$$v_{rel}^2 = \left[v_1(1-a)\right]^2 + \left[\Omega \cdot r\right]^2$$

Induction factor a determined iteratively!

Blade Element Momentum method (4/4)

start induction
$a_0 = 0$

inflow angle α
$$\tan(\alpha) = \frac{v_1(1-a_i)}{\Omega \cdot r}$$

angle of attack α_A
$$\alpha_A = \alpha - (\alpha_{Twist} + \alpha_{Pitch})$$

Thrust coefficient from
$c_L(\alpha_A), c_D(\alpha_A)$ curves
$$C_{Thrust} = C_L \cos\alpha + C_D \sin\alpha$$

Evaluation of new induction factor a_{i+1}
$$a_{i+1} = \left(\frac{4\sin^2\alpha}{\frac{N\,c}{2\pi\,r}\,C_{Thrust}} + 1\right)^{-1}$$

Convergence test
$|a_{i+1} - a_i| > tollerance$
$\Rightarrow a_i = a_{i+1}$
$|a_{i+1} - a_i| \le tollerance$
$\Rightarrow a := a_{i+1}$

- Next element
- Next inflow condition

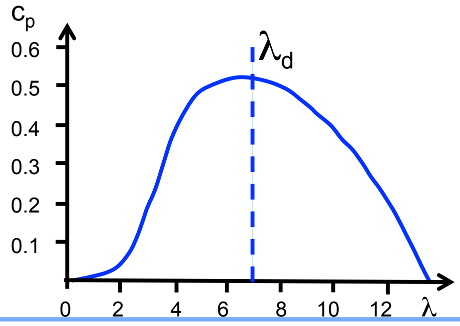
Blade Element Momentum method (4/4)

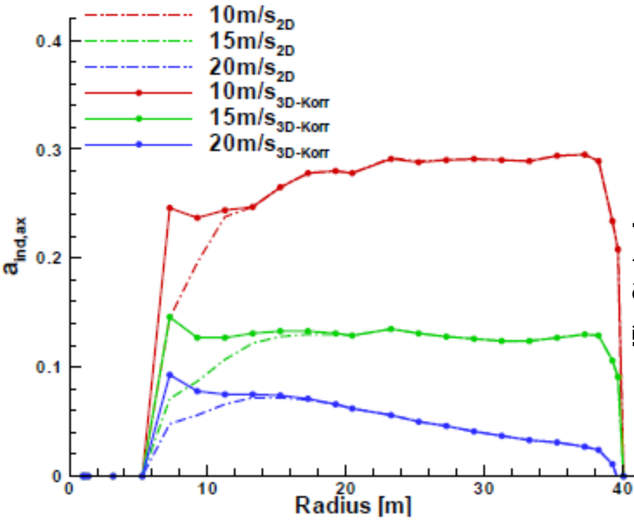
Output :

- Induction factor at each blade section

Utilization:

- evaluation of performance curves (e.g. c_p-λ, power curve)
- Evaluation of loads on turbines elements (blades, gear-box, …)
- Input data for wake models





[Fig. Streiner]

Extension of the basic BEM theory:
Wake rotation (1/3)

Basic momentum theory:

- Momentum balance in the control volume
  $\Rightarrow$ thrust force at the rotor

… what is missing?

- Balance of angular momentum in the control volume

[Fig.: www.windpower.org]

The torque Q is applied on the rotor by the flow	⇔	The rotor applies the opposite torque -Q to the flow

Actio = Reactio

Extension of the basic BEM theory:
Wake rotation (1/3)

Schmitz diagram

[Fig.: www.windpower.org]

Basic momentum theory:

- Momentum balance in the control volume
  ⇒ thrust force at the rotor

… what is missing?

- Balance of angular momentum in the control volume

The torque Q is applied on the rotor by the flow	⇔	The rotor applies the opposite torque -Q to the flow

=> the lower tip speed ratio, the higher rotational losses

Extension of the basic BEM theory:
Wake rotation (1/3)

Basic momentum theory:

- Momentum balance in the control volume
  ⇒ thrust force at the rotor

… what is missing?

- Balance of angular momentum in the control volume

[Fig.: www.windpower.org]

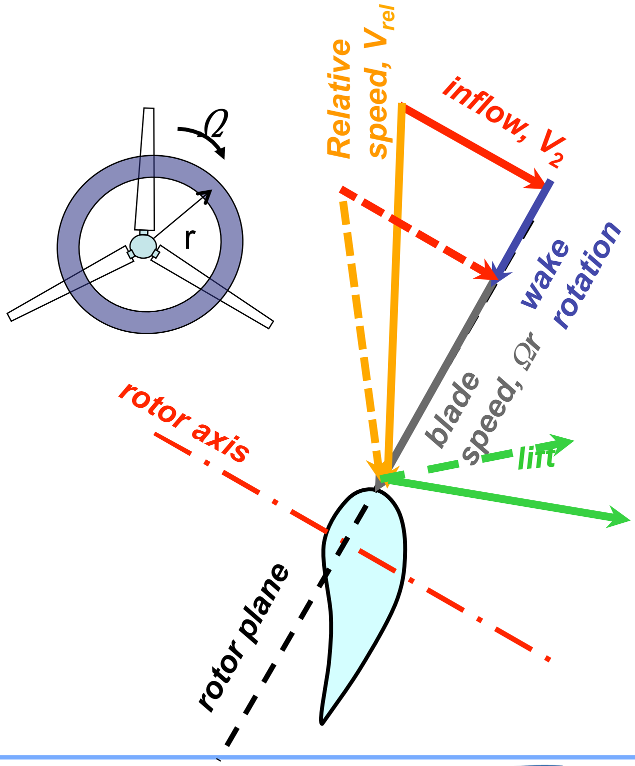
The torque Q is applied on the rotor by the flow	⇔	The rotor applies the opposite torque -Q to the flow

Relevant for $\lambda < 4$

Extension of the basic BEM theory:
Wake rotation (2/3)

Angular momentum theory:

- Wake and rotor rotate in opposite directions
- The angular momentum changes only across the rotor
- The tangential induction is introduced as $a' \Omega r$ where is the induction factor
- The flow rotation is assumed…
  - 0 before the rotor
  - $a' \Omega r$ at the blade center
  - $2 a' \Omega r$ in the wake



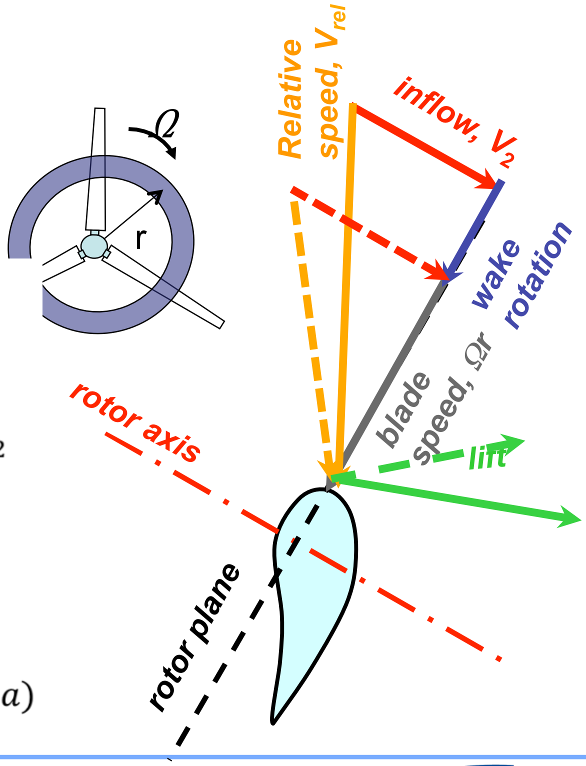
Extension of the basic BEM theory:
Wake rotation (3/3)

- Reduction of inflow angle
  => higher thrust, lower torque
- Increase of reltive speed
  => higher loads

$$\boldsymbol{tan(\alpha)} = \frac{(1-a)v_1}{\boldsymbol{(1+a')\Omega r}}$$

- $v_{rel}^2 = (1-a)^2 v_1^2 + [(1+a')\boldsymbol{\Omega r}]^2$

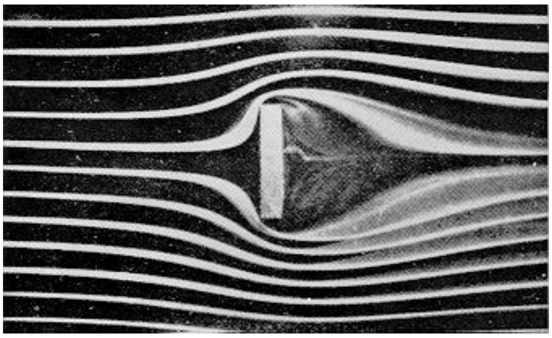
- a and a' are linked by
  $\boldsymbol{\lambda_r^2} a' = a(1-a)$
  $\Rightarrow c_{p_r} = 4a(1-a)^2 = 4\boldsymbol{\lambda_r^2} a'(1-a)$



Extension of the basic BEM theory:
Free-stream separation (1/2)

In some conditions a rotor approaches the behaviour of a solid disc

… what happens to a solid disc?



- Increase of static pressure on the front of the disc
- The flow spreads out radially on the disc
- Due lack of energy the flow separates and follows the main stream
- Behind the disc the is stagnation at low static pressure

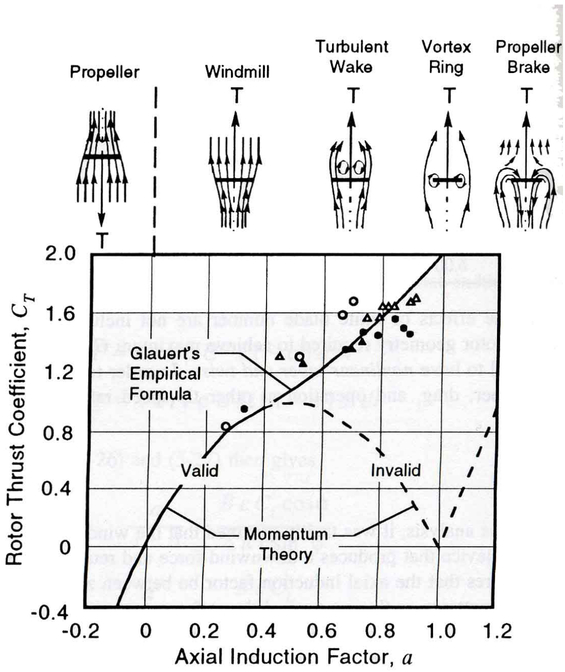
Extension of the basic BEM theory:
Free-stream separation (2/2)

The same happens to a rotor spinning at high tip speed ratio:

- The overall permeability of the rotor decreases
- Part of the flow spreads radially on the blade
- It separates at the blade tip
- It creates a low pressure region behind the rotor

⇒Higher thrust than expected is generated

⇒Correction of c_T Based on Glauert's experimental data

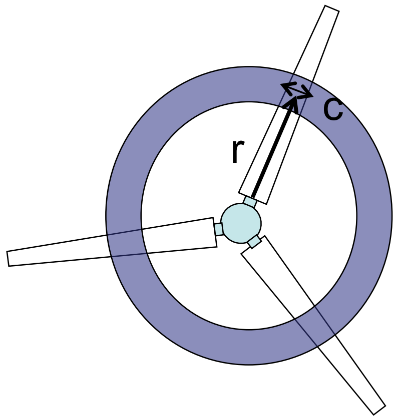


Extension of the basic BEM theory:
Finite number of blades (tip losses) (1/2)

- BEM:
  - induction factor uniform over the each rotor ring
- Real rotor:
  - the induction is at the each element of the N blades
  - the local solidity σr decrease towards the tip

$$\sigma_r = N\,c\,/\,(2\,\pi\,r\,)$$

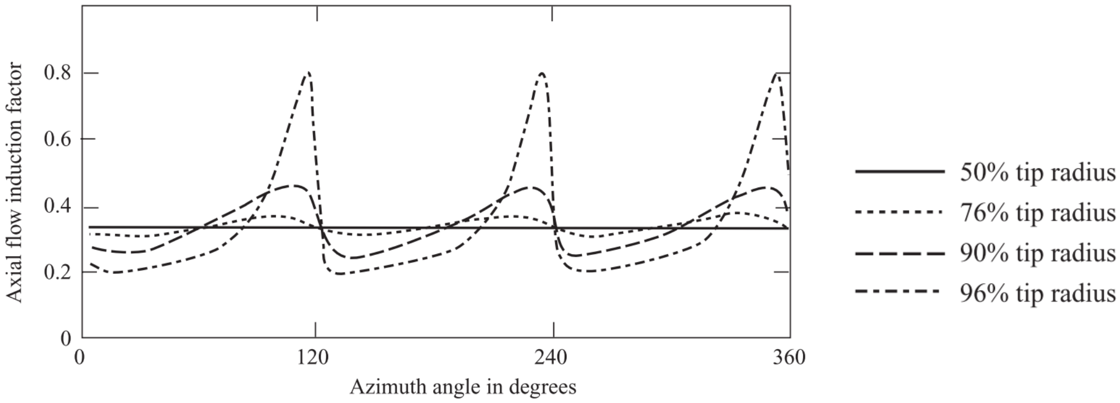
⇒ the relative surface spanned by the blade decrease radially



- The induction factor is not uniform over the ring
- Uniformity
  increases with *N*
  decreases with *r*

Extension of the basic BEM theory:
Finite number of blades (tip losses) (2/2)

Axial induction factor simulated for a 3 blade turbine with a vortex model



[Fig.: Burton]

Evident non-uniform induction over the azimuth at the tip

Prandtl proposed an approximation to be used with BEM based on theoretical considerations

Summary

I. Blade Element Momentum theory (BEM)
- Unknown induction in disk actuator model
- Betz/Schmitz limit ⇒ induction at design point
- Unknown relative speed in airfoil theory
- Disk actuator + Airfoil theory = BEM
  ⇒ induction on the blade

II. Correction to the basic BEM theory
- Actio-reatio ⇒ Wake rotation
- High tip-speed-ratio = low permeability
  ⇒ rotor ~ solid disk
  ⇒ higher thrust (Glauert conrrection)
- Induction at the blade only,
  not on rotor ring
  ⇒ higher induction at the blade position
  ⇒ effect increases towards the tip

Annex I: BEM algorithms

1. Initialize a and a‘ to 0
2. Evaluate the inflow angle α .... $\alpha = \arctan\left(\frac{V_1(1-a)}{\Omega r(1+a')}\right)$
3. Evaluate the angle of attack of the profile α_A .... $\alpha_A = \alpha - \alpha_{twist}$
4. Interpolate the $C_{L,D}$-α_A curves at the angle of attack α_A
5. Compute $C_{Thrust,Torque}$ .... $C_{Torque} = C_L \sin\alpha - C_D \cos\alpha \qquad C_{Thrust} = C_L \cos\alpha + C_D \sin\alpha$
6. Evaluate a and a‘ .... $a = \left(\frac{4\sin^2\alpha}{\sigma\, C_{Thrust}} + 1\right)^{-1} \quad a' = \left(\frac{4\sin\alpha\cos\alpha}{\sigma\, C_{Torque}} - 1\right)^{-1} \quad \sigma = \frac{N\, c}{2\pi\, r} \quad N: \text{number of blade}$
7. If the difference between the computed induction factors (a,a’) and their initialization values is out of tolerance reinitialize a and a‘ with the newly evaluated values and restart the process from point 2.
8. Evaluate the local torque and the power generated .... $dT = \frac{1}{2}\rho N \frac{V_1^2(1-a)^2}{\sin^2\alpha} c\, C_{Thrust} dr \qquad dQ = \frac{1}{2}\rho N \frac{V_1(1-a)\Omega r(1+a')}{\sin\alpha\cos\alpha} c\, C_{Torque} dr$

- Prandtl Correction .... $F_{Tip} = \frac{2}{\pi}\cos^{-1}\left[\exp\left\{\frac{-N/2(1-r/R)}{(r/R)\sin\alpha}\right\}\right] \qquad a = \left(\frac{4F_{Tip}\sin^2\alpha}{\sigma\, C_{Thrust}} + 1\right)^{-1} \qquad a' = \left(\frac{4F_{Tip}\sin\alpha\cos\alpha}{\sigma\, C_{Torque}} - 1\right)^{-1}$
- Spera Correction for $a>a_c$ .... $a = \frac{1}{2}\left(a_c^2 + K(1-2a_c) - \sqrt{(K(1-2a_c)+2)^2 + 4(Ka_c^2 - 1)}\right) \qquad K = \frac{4F_{Tip}\sin^2\alpha}{\sigma\, C_{Thrust}},\ a_c \approx 0.2$
- Blade geometry description