**Institut für Physik**

**AG Windenergiesysteme (WE-Sys) • Prof. Dr. Dipl.-Ing. Martin Kühn**

# Design of Wind Energy Systems – Summer Term 2016
# Tutorial 2: Blade Element Momentum Theory

Davide Trabucchi MSc, Juan José Trujillo MSc

| | | |
|---|---|---|
| Name: | | |
| Surname: | | |
| Student Nr. | | |
| Email: | | |
| Date of issue: 26.04.2016<br>Submission deadline: 03.05.2016 | | Group Nr. |

**Exercise 1**

During the second tutorial some iteration of the blade element momentum (BEM) algorithm has been run for the rotor in Table 1 at the undisturbed inflow wind speed $v_1$ of 10 m/s considering ten uniformly distributed annular rings. In this exercise we consider the same specification with a different design of the rotor (see Table 1 and Table 3). Mainly, the design parameters are chosen to increase the contribute of the lift without seeking for an optimal gliding factor.

Table 1 - Turbine design specification

| **Turbine specification** | |
|---|---|
| Rotor diameter [m] | 100 |
| Number of blade [-] | 3 |
| Design wind speed [m/s] | 10 |
| Design tip-speed-ratio [-] | 7 |
| Design lift coefficient* [-] | 1.3 |
| Maximal tip-speed [m/s] | 70 |
| | |
| Air density [kg/m$^3$] | 1.225 |

*Based on maximum lift coefficient

1.1 Implement the BEM algorithm and validate it against the results achieved in the class exercise (see Table 3). The following assumption should be considered:

- no wake rotation ($a'$=0)
- no tip-losses (finite number of blades)
- no drag is produced by the blade elements ($c_D(\alpha_A)$ =0)
- free-stream correction according to Spera (1994)

- linear approximation of the $c_L(\alpha_A)$ curve (see Table 3):

1.2 When the turbine operates in partial load (undisturbed inflow $v_1$ in the range from 4 to 10 m/s), the rotor and the blades work at the design condition. This means that the pitch and the rotational speed are regulated accordingly. Can you evaluate the overall thrust and arodynamical power coefficient given at an inflow $v_1$ of 8 m/s on the basis of the results provided in Exercise 1.1?

**Exercise 2**

The turbine described in Table 1 and Table 3 reaches its rated power when the wind speed is 10 m/s. In the full load range the design condition are abandoned in order to keep the tip speed below the critical values given by noise limitation and blade abrasion (70-90 m/s) as well as to reduce the structural loads.

2.1 Which operational parameters should be changed in order to comply with the mentioned constrains?

2.2 Evaluate with the BEM solver implemented in Exercise 1.1 which pitch angle is required in order to achieve the rated power at the inflow wind speed $v_1$ of 14 m/s. Remember that the pitch angle should be added to the design twist of each blade element.

2.3 Calculate for the conditions from point 2.2 the flap moment distribution along the blade and compare the results with those at 10 m/s given in Table 3.

Table 2 - Approximation of lifting coefficient

Note: use this table in case you cannot interpolate conveniently the lift coefficient curve given in the Excel-sheet from CIP-Tutorial.

| **Angle of attack range** | **$c_L$ approximation** |
|---|---|
| $-20° \le \alpha_A < -10°$ | $c_L = -0.98 + 0.056\ \alpha_A$ |
| $-10° \le \alpha_A < 0°$ | $c_L = 0.2 + 0.062\ \alpha_A$ |
| $0° \le \alpha_A < 15°$ | $c_L = 0.2 + 0.095\ \alpha_A$ |
| $15° \le \alpha_A < 22°$ | $c_L = 1.6$ |
| $\alpha_A \ge 22°$ | $c_L = 2 - 0.02\ \alpha_A$ |

Table 3 – Blade design and results from simplified BEM algorithm at design wind speed (10 m/s) for each blade element.

| Position [m] | Chord [m] | Twist [°] | Induction factor [-] | Flap moment [Nm] |
|---|---|---|---|---|
| 2.5 | 13.59 | 52.12 | 0.305 | 2462227 |
| 7.5 | 8.22 | 22.23 | 0.303 | 2279357 |
| 12.5 | 5.46 | 10.67 | 0.297 | 2011516 |
| 17.5 | 4.03 | 5.04 | 0.292 | 1686641 |
| 22.5 | 3.18 | 1.77 | 0.288 | 1331524 |
| 27.5 | 2.62 | -0.36 | 0.285 | 972091 |
| 32.5 | 2.22 | -1.85 | 0.283 | 633704 |
| 37.5 | 1.93 | -2.95 | 0.281 | 342127 |
| 42.5 | 1.71 | -3.79 | 0.28 | 122304 |
| 47.5 | 1.53 | -4.46 | 0.278 | 0 |

**General BEM algorithm including the wake rotation and the free-stream/turbulent wake effect**

| | |
|---|---|
| 1 Initialize a and a' to 0 | |
| 2 Evaluate the inflow angle $\alpha_A$ | $\alpha = \arctan\left(\dfrac{V_1(1-a)}{\Omega r(1+a')}\right)$ |
| 3 Evaluate the angle of attack of the profile | $\alpha_A = \alpha - (\alpha_{twist} + \alpha_{pitch})$ |
| 4 Interpolate the $C_{L,D}$-$\alpha_A$ curves at the angle of attack $a_A$ | |
| 5 Compute thrust and torque coefficient $C_T$ and $C_Q$ | $C_T = C_L \cos\alpha + C_D \sin\alpha$<br>$C_Q = C_L \sin\alpha - C_D \cos\alpha$ |
| 6 Evaluate a and a ' | $a' = \left(\dfrac{4\sin\alpha\cos\alpha}{\sigma\ C_Q} - 1\right)^{-1}$ ; $a = \left(\dfrac{4\sin^2\alpha}{\sigma\ C_T} + 1\right)^{-1}$<br>N: number of blades<br>$\sigma = {}^{N\ c}\!/_{2\pi\ r}$ |
| 7 If a > a c<br>⇒ Spera correction | $a = \dfrac{1}{2}\left(2 + K(1-2a_c) - \sqrt{(K(1-2a_c)+2)^2 + 4(Ka_c^2 - 1)}\right)$<br>where:<br>$K = \dfrac{4\sin^2\alpha}{\sigma\ C_T},\ \ a_c \approx 0.2$ |
| 8 If the difference between the computed induction factors (a, a') and their initialization values is out of tolerance reinitialize a and a ' with the newly evaluated values and restart the process from point 2. | |