Design of wind energy systems
SS 2016

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ForWind
Center for Wind Energy Research

Tutorial 2: Blade Element Momentum Theory

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ForWind – Wind Energy Systems

Outline

- Class exercise
  - Blade design
  - BEM (partial load)

- Home exercise
  - Distributed soon

Assignment

Data

- Consider the turbine defined in the table as an actuator disk
- Consider the disk evenly subdivided in 10 annular rings

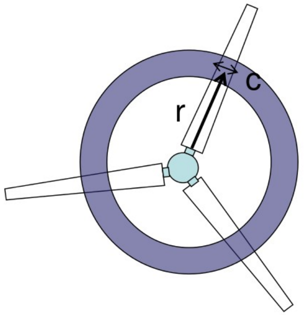
Task

- Design a blade following Schmitz and using a NACA 63-215 profile (see C_L in tables of CIP-Tutorial)
- Evaluate the axial induction factor of the designed blade by means of BEM for an inflow speed 10 m/s

Turbine specification

Rotor diameter [m]	100
Number of blade [-]	3
Electrical conversion efficiency [-]	0.95
Design wind speed [m/s]	10
Design tip-speed-ratio [-]	7
Design lift coefficient * [-]	0.65
Air density [kg/m3]	1.225

* Based on max. gliding factor

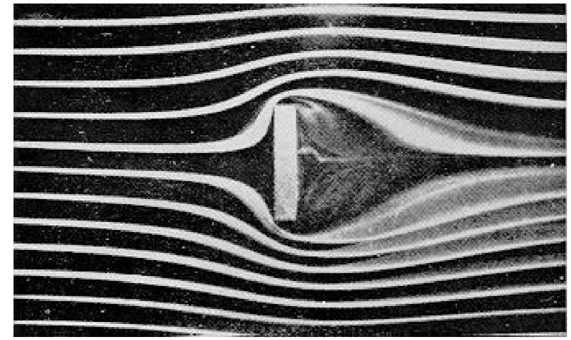


r = (rout+rin) / 2

BEM approximation/extension

During this tutorial the following assumptions are taken in order to implement the blade element momentum method

- No wake rotation
- No drag on blade element
- No tip-loss correction
- Wake separation, turbulent wake state (see next slide)



[Fig.: www.windpower.org]

Wake separation

- Turbulent wake state:

high tip-speed-ratio/induction-factor

↓

rotor ~ impermeable disk

↓

CThrust increases

- Spera (1994) empirical correction

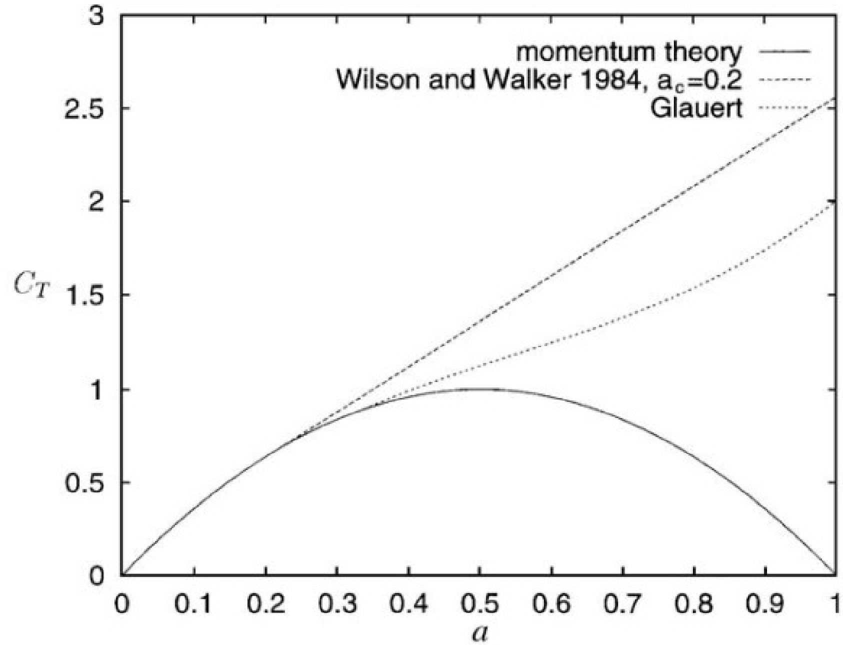


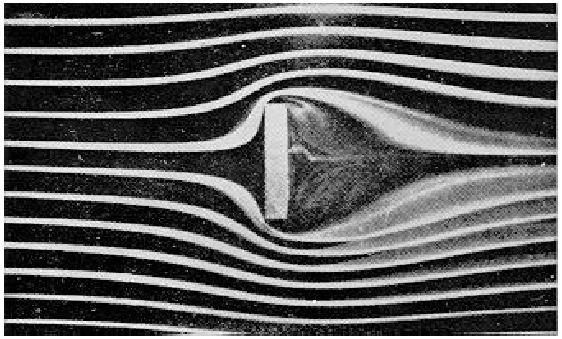
Fig. Hansen

$$a = \frac{1}{2}\left(2 + K(1-2a_c) - \sqrt{\left(K(1-2a_c)+2\right)^2 + 4(Ka_c^2 - 1)}\right)$$

$$K = \frac{4sin^2(\alpha)}{\sigma C_{Thrust}}$$

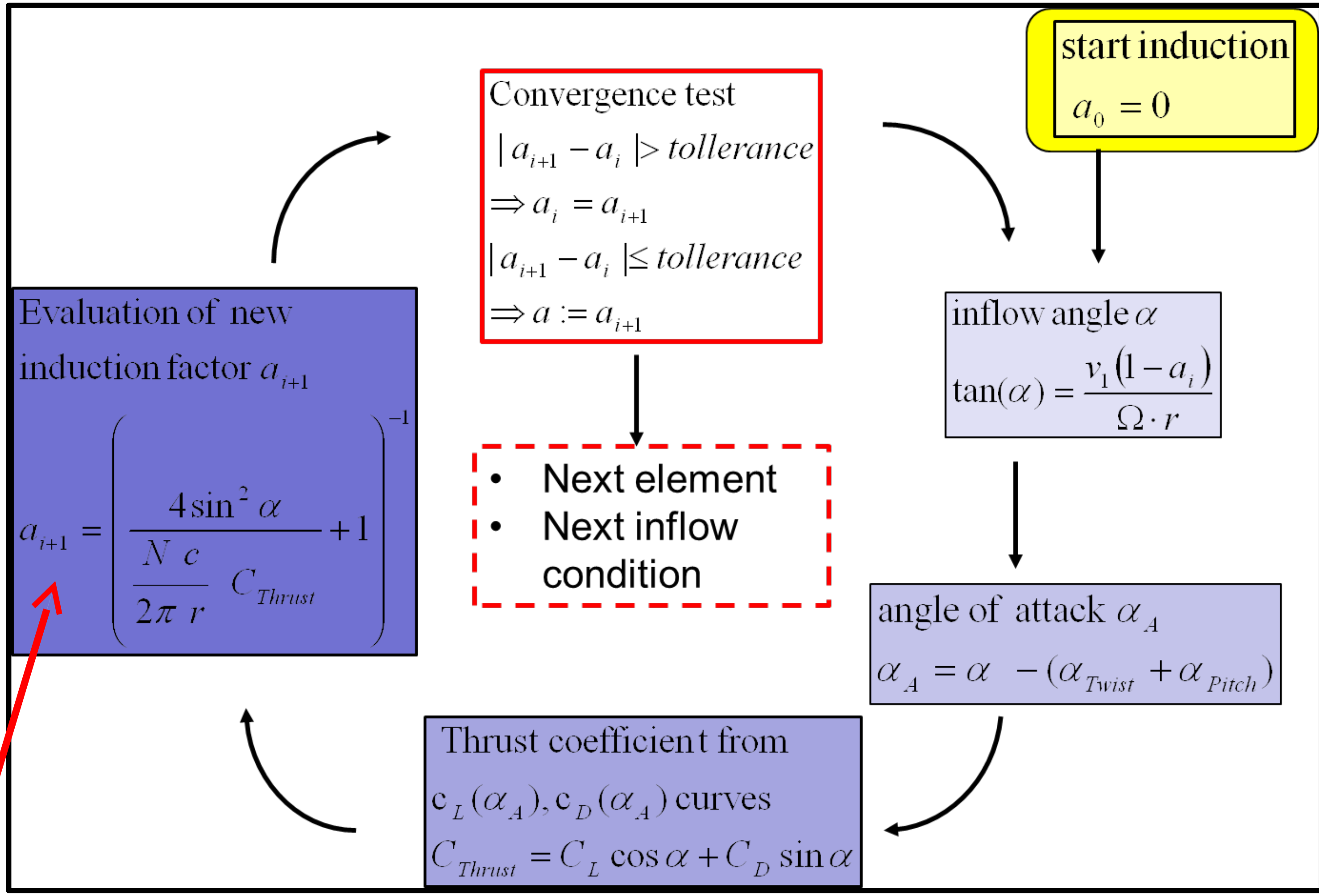
$$a_c \approx 0.2\,;\sigma = \frac{Nc}{2\pi r}$$

$$C_T = C_L\cos(\alpha) + C_D\,sin(\alpha)$$



BEM Algorithm

- CD=0
- if ai+1>ac
  ai+1 according
  to Spera



[from lecture 02, slide 13]

$$K = \frac{4\sin^2(\alpha)}{\sigma C_{Thrust}}$$

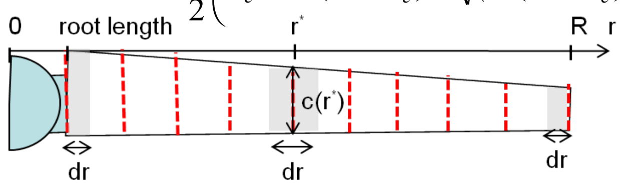
$$a_c \approx 0.2$$

$$\sigma = \frac{Nc}{2\pi r}$$

$$a = \frac{1}{2}\left(2 + K(1-2a_c) - \sqrt{(K(1-2a_c)+2)^2 + 4(Ka_c^2 - 1)}\right)$$

Annex I: BEM algorithms

1. Initialize a and a‘ to 0
2. Evaluate the inflow angle α .... $\alpha = \arctan\left(\dfrac{V_1(1-a)}{\Omega r(1+a')}\right)$
3. Evaluate the angle of attack of the profile αA .... $\alpha_A = \alpha - \alpha_{twist}$
4. Interpolate the CL,D-αA curves at the angle of attack αA
5. Compute CThrust,Torque .... $C_{Torque} = C_L \sin\alpha - C_D \cos\alpha \qquad C_{Thrust} = C_L \cos\alpha + C_D \sin\alpha$
6. Evaluate a and a‘ .... $a = \left(\dfrac{4\sin^2\alpha}{\sigma\, C_{Thrust}} + 1\right)^{-1} \qquad a' = \left(\dfrac{4\sin\alpha\cos\alpha}{\sigma\, C_{Torque}} - 1\right)^{-1} \qquad \sigma = \dfrac{N\,c}{2\pi\, r} \qquad N: \text{number of blade}$
7. If the difference between the computed induction factors (a,a’) and their initialization values is out of tolerance reinitialize a and a‘ with the newly evaluated values and restart the process from point 2.
8. Evaluate the local torque and the power generated .... $dT = \dfrac{1}{2}\rho N \dfrac{V_1^2(1-a)^2}{\sin^2\alpha} c\; C_{Thrust}\, dr \qquad dQ = \dfrac{1}{2}\rho N \dfrac{V_1(1-a)\Omega r(1+a')}{\sin\alpha\cos\alpha} c\; C_{Torque}\, dr$
9. Prandtl Correction .... $F_{Tip} = \dfrac{2}{\pi}\cos^{-1}\left[\exp\left\{\dfrac{-N/2(1-r/R)}{(r/R)\sin\alpha}\right\}\right] \qquad a = \left(\dfrac{4F_{Tip}\sin^2\alpha}{\sigma\, C_{Thrust}} + 1\right)^{-1} \qquad a' = \left(\dfrac{4F_{Tip}\sin\alpha\cos\alpha}{\sigma\, C_{Torque}} - 1\right)^{-1}$
10. Spera Correction for a>ac .... $a = \dfrac{1}{2}\left(a_c^2 + K(1-2a_c) - \sqrt{(K(1-2a_c)+2)^2 + 4(Ka_c^2 - 1)}\right) \qquad K = \dfrac{4F_{Tip}\sin^2\alpha}{\sigma\, C_{Thrust}},\; a_c \approx 0.2$
11. Blade geometry description



Aerodynamic profile (NACA 63-215)

- Use C_L table in Excel-sheet of CIP-Tutorial or use the approximations below to interpolate data for BEM
- C_L,C_D curve extended to α_A -180°,180°
- Linear approximation of $C_L(\alpha_A)$
  - $\alpha_A \geq -20°$, $C_L = -0.98 + 0.056\ \alpha_A$
  - $\alpha_A \geq -10°$, $C_L = 0.2 + 0.062\ \alpha_A$
  - $\alpha_A \geq 0°$, $C_L = 0.2 + 0.095\ \alpha_A$
  - $\alpha_A \geq 15°$, $C_L = 1.6$
  - $\alpha_A \geq 22°$, $C_L = 2 - 0.02\ \alpha_A$