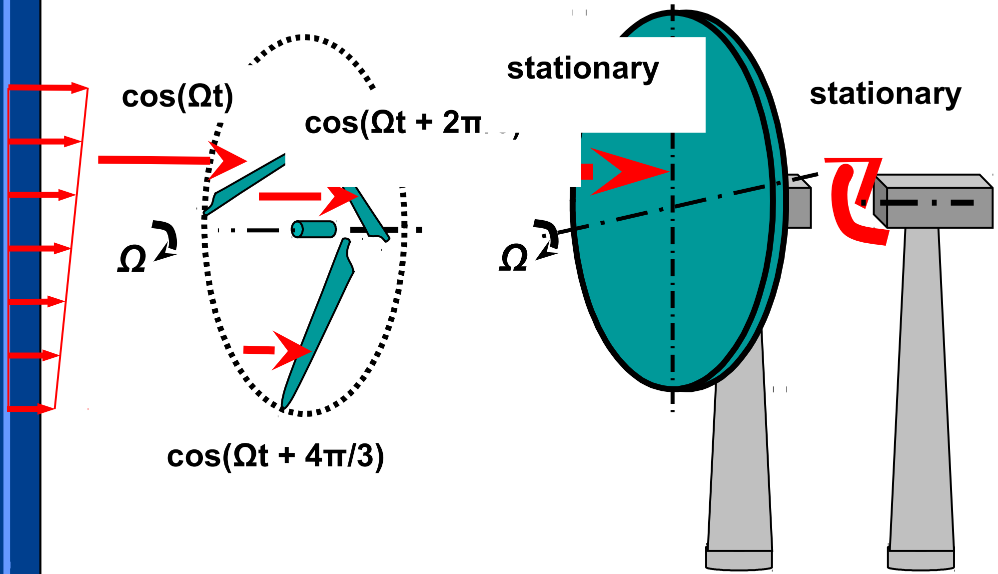
Design of Wind Energy Systems
SS 2016

Tutorial3: Dynamics and Campbell Diagram



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ForWind – Wind Energy Systems

Topics

1. Campbell diagram

2. Quantitative proof of transformation of blade bending moments from rotating to fixed coordinate system

1. Campbell diagram (resonance diagram)

- Useful diagram for detection of potential resonances between excitation frequency, i.e. rotor harmonics and eigenfrequencies as well as potential interactions between different eigenfrequencies
- Horizontal axis: rotor speed Ω
- Vertical axis: eigenfrequencies ω_i depending on rotor speed Ω
- excitation frequencies: straight rays starting at the origin related to $\omega = 1\Omega$, $\omega = 3\Omega$, $\omega = 6\Omega$, $\omega = 9\Omega$, etc. (3-bladed rotor)
- Potential resonances:

  intersection point of an excitation ray and an eigenfrequency in the operational range of the rotor speed
- potential interactions:

  coincidence of two eigenfrequencies
- used in the fields of rotor dynamics, dynamics of helicopter and wind turbines, etc.

Campbell-Diagramm of a typical 1.5 MW turbine

Please draw a Campbell diagram for the following parameters

- Rotor speed range Ω = 11 – 20 rpm
- Number of rotor blades 3
- 1st tower bending eigen frequency 0.35 Hz
- 1st flapwise blade bending frequency at stand-still 0.8 Hz, at 20 rpm 1.1 Hz
- 1st edgewise blade bending frequency at stand-still 1.95 Hz, at 20 rpm 2.05 Hz
- 2nd tower bending frequency 2.3 Hz
- 2nd flapwise blade bending frequency at stand-still 3.1 Hz, at 20 rpm 3.25 Hz
- 1st tower torsional eigen frequency 3.8 Hz

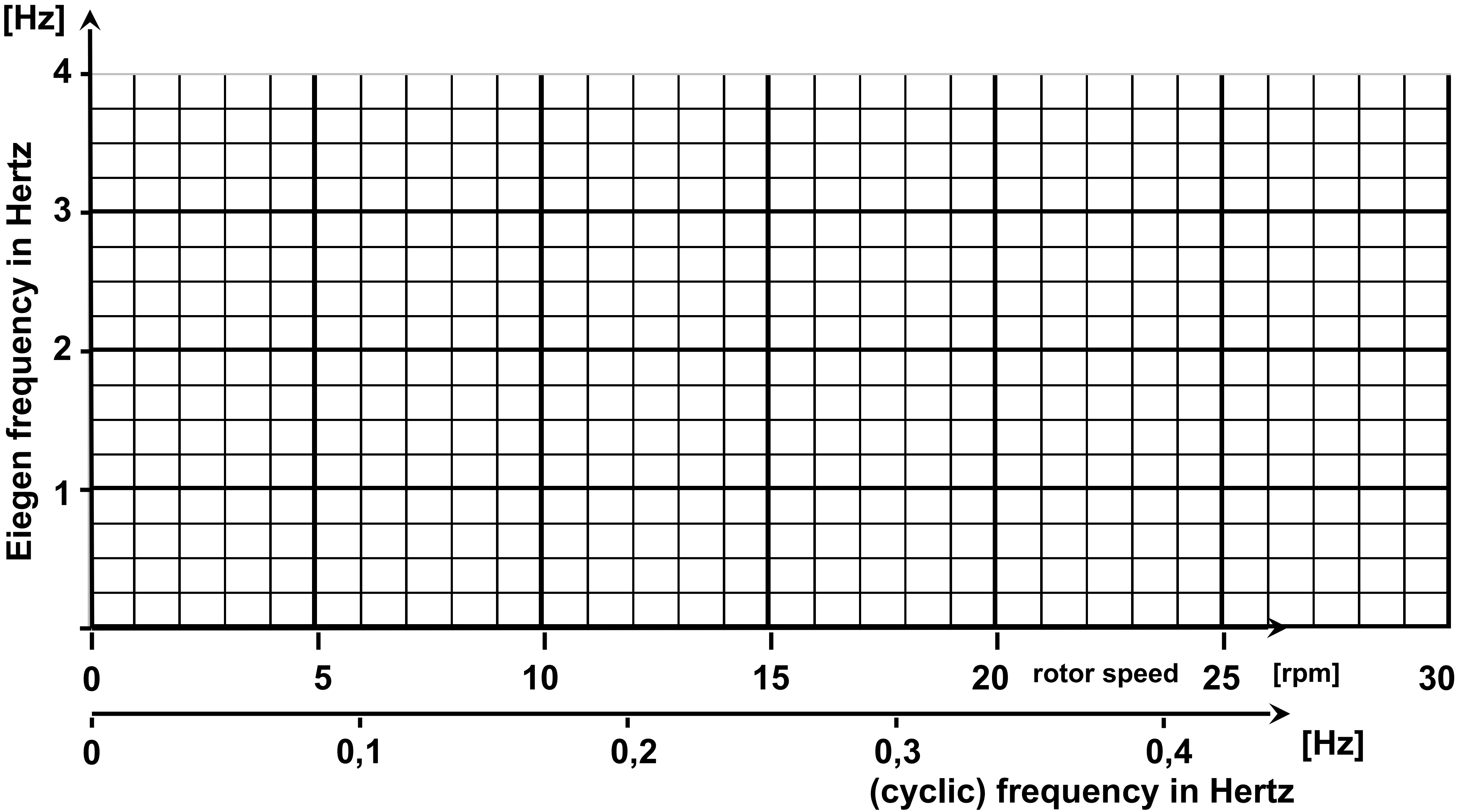
Hints:

- First draw the harmonic excitation rays 1Ω, 2Ω, 3Ω, etc.
  30 rpm at the rotor speed axis equals 0.5 Hz at the eigen frequency axis
  2•30 rpm = 1 Hz, 3•30 rpm = 1.5 Hz, etc.
- Next mark the rotor speed range at the rotor speed axis
- Now draw the graphs of the eigenfrequencies, i.e. more or less horizontal lines.

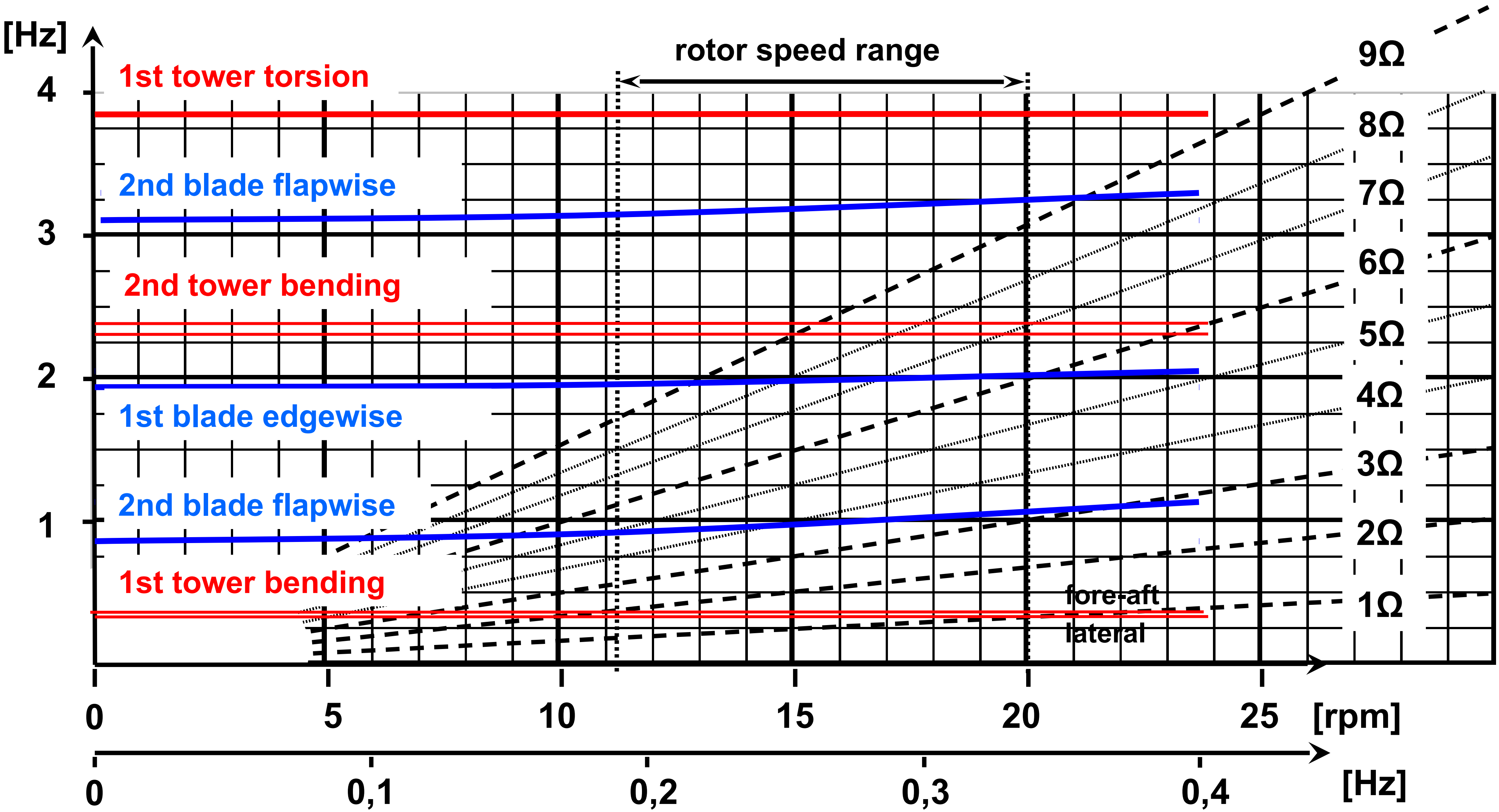
Questions:

- Where do possible resonances within the operational rotor speed range occur?
  I.e. identify intersections of an eigenfrequency curve and a harmonic excitation ray within the operational rotor speed range?
  Important blade excitation are 1Ω, 2Ω, 3Ω, etc.
  Important excitations of the tower-nacelle assembly include 1Ω (lateral), 3Ω, 6Ω, etc.

Campbell diagram of a typical 1.5 MW turbine



Campbell diagram of a typical 1.5 MW turbine (solution)



2. Quantitative proof of transformation of blade bending moments from rotating to fixed coordinate system

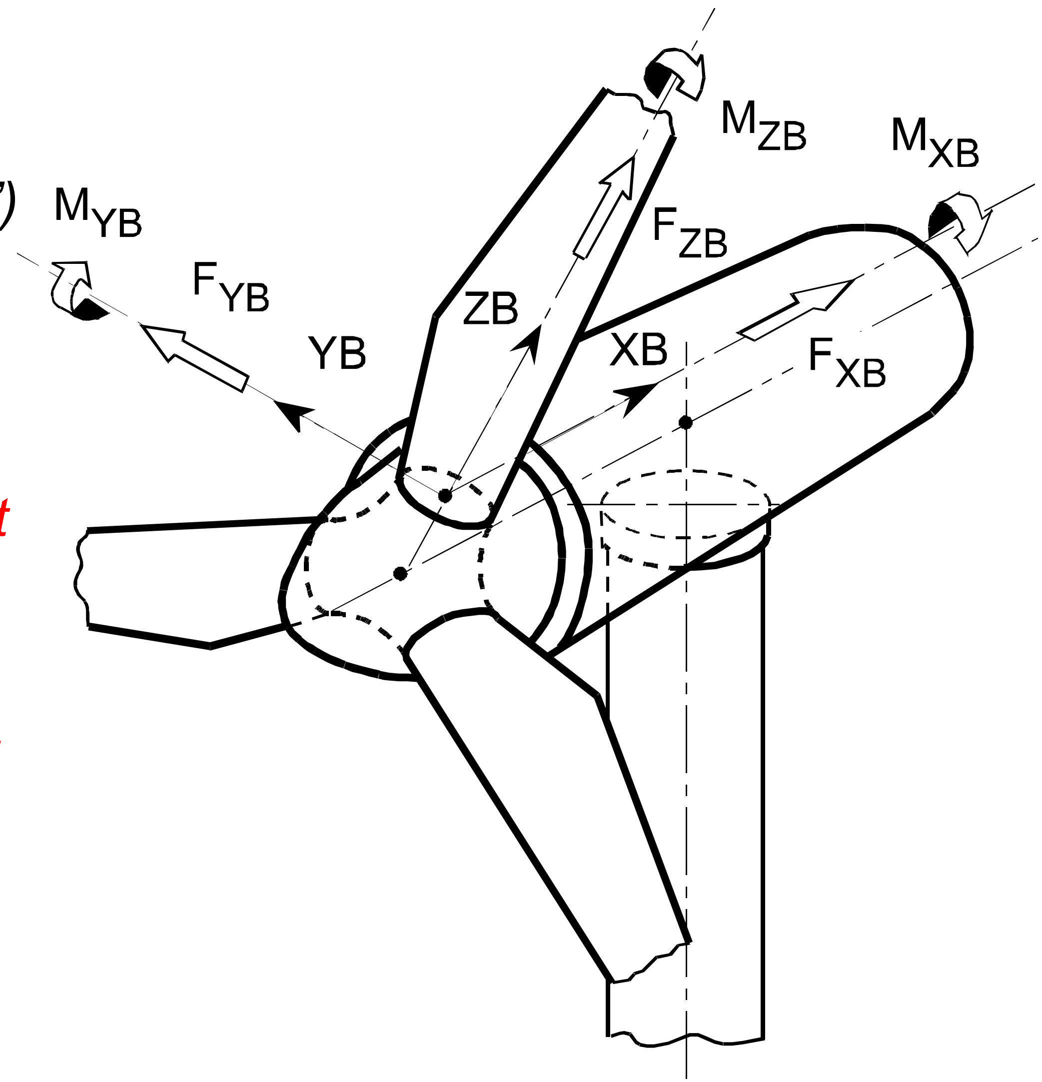
	Nacelle nodding moment	Nacelle yawing moment
Three bladed turbine	1.5 ΔM_o	Zero
Two bladed turbine	$\Delta M_0(1+\cos(2\Omega t))$	**$\Delta M_0 \sin(2\Omega t)$**

- Where $\Delta M_0 =$ variation of blade root moment due to shear

$$\psi = azimuth\ angle = \Omega t + (i-1)\cdot\frac{2\pi}{N_{blades}}$$

Blade Loads: Terminology & Coordinate Systems

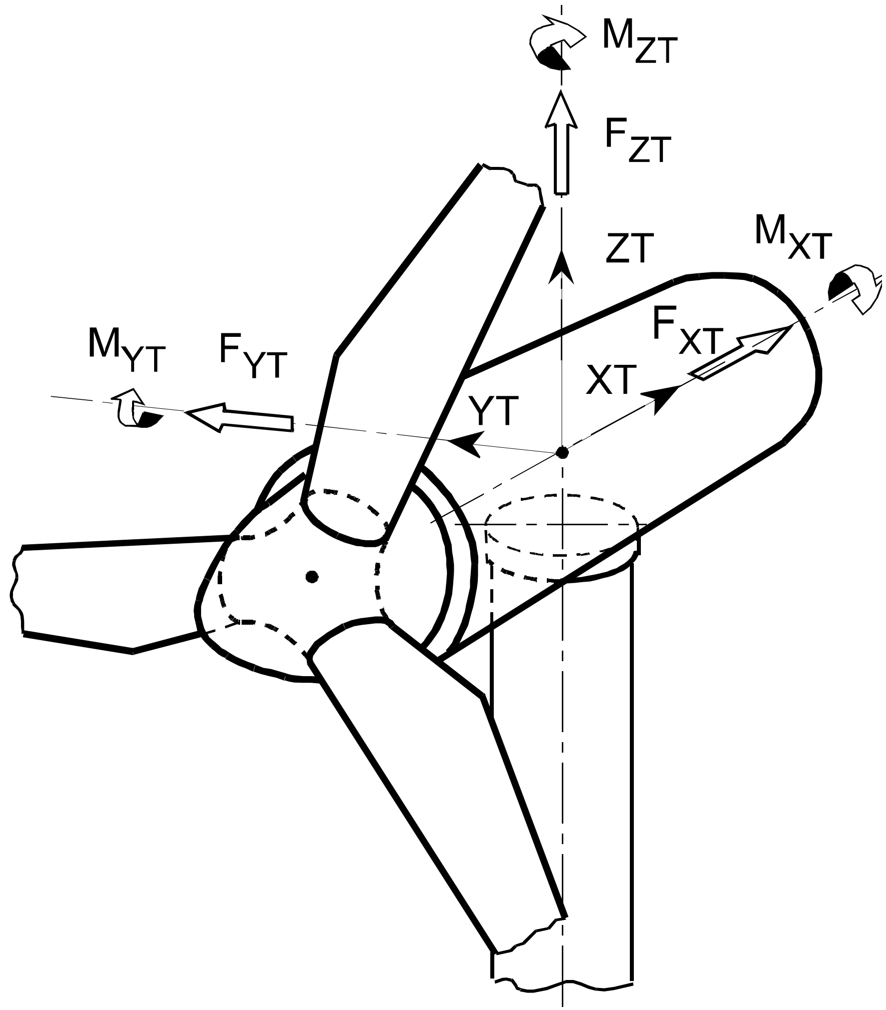
- *F_x: (horizontal) Thrust*
- *F_y: Tangential Force ("in-plane")*
- F_z: Axial Force
- *M_x: Edgewise Bending Moment („in-plane")*
- *My: Flapwise Bending Moment* („out-of-plane")
- M_z: Torsional Moment



[Fig. Garrad Hassan]

Nacelle Loads: Terminology & Coordinate Systems

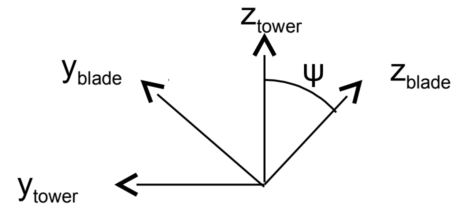
- *F_x: (horizontal) Thrust*
- *F_y: Shear Force*
- F_z: Vertical Force
- M_x: <u>Roll</u> Moment
- *M_y: <u>Tilt</u> Moment*
- M_z: <u>Yaw</u> Moment



[Fig. Garrad Hassan]

Quantitative proof of moment transformations

The transformation matrix is given by: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) & -\sin(\psi) \\ 0 & \sin(\psi) & \cos(\psi) \end{bmatrix}$



The blade out-of-plane bending moment is given by:

$$M_y^B = M_0 + \Delta M_0 \times \cos(\Omega t + \psi_0)$$

- Transfer the given out of plane bending moment into the tower coordinate system

  $$M_y^T = T \cdot M_y^B$$

- First, calculate the constant part of the blade out-of-plane bending moment, transferred to the tower coordinate system without wind shear (ΔM_0=0) for a three-bladed rotor
- Second, calculate the dynamic altering moment in dependency of the azimuth angle for a three bladed rotor
- Repeat the procedure for a two bladed turbine and compare the resulting equations.
- Calculate the dependency of the nacelle yawing moment of the azimuth angle for 2- and 3-bladed rotor

Trigonometric identities

	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π
	0^0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0	210^0	225^0	240^0	270^0	300^0	315^0	330^0	360^0
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\cot x$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$

Additionstheoreme

$$\begin{aligned}
\cos(x \pm y) &= \cos x \cos y \mp \sin x \sin y \\
\sin(x \pm y) &= \sin x \cos y \pm \cos x \sin y \\
\tan(x \pm y) &= \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}
\end{aligned}$$

doppelter Winkel

$$\begin{aligned}
\cos 2x &= \cos^2 x - \sin^2 x \\
&= 1 - 2\sin^2 x = 2\cos^2 x - 1 \\
\sin 2x &= 2 \sin x \cos x \\
\tan 2x &= \frac{2\tan x}{1-\tan^2 x} \\
\cot 2x &= \frac{\cot^2 x - 1}{2 \cot x}
\end{aligned}$$

halber Winkel

$$\begin{aligned}
\cos\frac{x}{2} &=^* \pm\sqrt{\frac{1}{2}(1+\cos x)} \\
\sin\frac{x}{2} &=^* \pm\sqrt{\frac{1}{2}(1-\cos x)} \\
\tan\frac{x}{2} &= \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} \\
&=^* \pm\sqrt{\frac{1-\cos x}{1+\cos x}} \\
\cot\frac{x}{2} &= \frac{1+\cos x}{\sin x} = \frac{\sin x}{1-\cos x} \\
&=^* \pm\sqrt{\frac{1+\cos x}{1-\cos x}}
\end{aligned}$$

Symmetrie

$\cos(-x)$	$= \cos x$	gerade Funktion
$\sin(-x)$	$= -\sin x$	ungerade Funktion
$\tan(-x)$	$= -\tan x$	ungerade Funktion
$\cot(-x)$	$= -\cot x$	ungerade Funktion

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

$$\begin{aligned}
\cos^2 x &= \frac{1}{2}(1+\cos 2x) & \sin x &=^* \frac{\tan x}{\pm\sqrt{1+\tan^2 x}} \\
\sin^2 x &= \frac{1}{2}(1-\cos 2x) & \cos x &=^* \frac{1}{\pm\sqrt{1+\tan^2 x}} \\
\cos x &= \sin\left(\frac{\pi}{2} \pm x\right) & \tan x &= \frac{\sin x}{\cos x} \\
\sin x &= \cos\left(\frac{\pi}{2} - x\right) & \cot x &= \frac{\cos x}{\sin x} = \frac{1}{\tan x}
\end{aligned}$$

$$\begin{aligned}
\sin x + \sin y &= 2 \sin\frac{x+y}{2} \cos\frac{x-y}{2} \\
\sin x - \sin y &= 2 \cos\frac{x+y}{2} \sin\frac{x-y}{2} \\
\sin x \cdot \sin y &= \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right) \\
\cos x + \cos y &= 2 \cos\frac{x+y}{2} \cos\frac{x-y}{2} \\
\cos x - \cos y &= -2 \sin\frac{x+y}{2} \sin\frac{x-y}{2} \\
\cos x \cdot \cos y &= \frac{1}{2}\left(\cos(x-y) + \cos(x+y)\right) \\
\sin x \cdot \cos y &= \frac{1}{2}\left(\sin(x-y) + \sin(x+y)\right)
\end{aligned}$$

[Fig. Wirth- Repetitorium der höheren Mathematik]