



Programming Languages

L8

CS4411-CS5212 : L8: Relational Concepts in Mozart OZ

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Think Relational



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Logic for Contextual Association

- Logic provide us with a universal structure of information governed by contextual associations. With such associations we can capture and represent rules and natural structures that are useful for building smarter, more human-like systems. [According to Joe's Theory of Everything \(JTE\)](#), relations between data items are the fabric of science. "For our course purposes, it is enough to say that relations among facts much or all of business, science, and knowledge can be described as links between facts.
- Facts represent relations not functions.**

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What is a Relation and how it differs from Function?

- A **"relation"** is just a relationship between sets of information. **Think of all the people in one of your classes, and think of their heights. The pairing of names and heights is a relation.**
- A **function** is a "well-behaved" relation. This means given a starting point, we know exactly where to go: given an x , we get only and **exactly one** y .

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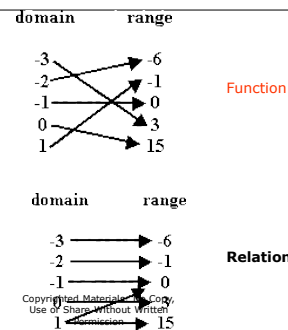
Relations have Multiple Behaviors?

- Let's return to our relation of your classmates and their heights, and let's suppose that the domain is the set of everybody's heights. Let's suppose that there's a pizza-delivery guy waiting in the hallway. And all the delivery guy knows is that the pizza is for the student in your classroom who is five-foot-five. Now let the guy in. Who does he go to? What if nobody is five-foot-five? What if there are **six people** in the room that are five-five?

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Function vs Relation in Math



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How to Think Relational?

- It is to represent **business rules** on data of the problem defined. Business rules specify conditions and relationships that must always be true, or must always be false.
- Business rules represent constraints (.i.e conditions or relations)

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Relational Programming = Constraints Satisfaction Problem (CSP)

- Constraint programming is a programming paradigm wherein relations between variables are stated in the form of constraints. Constraints differ from the common primitives of imperative programming languages in that they do not specify a step or sequence of steps to execute, but rather the properties of a solution to be found.
This makes constraint programming a form of declarative programming.

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Relational Programming

- ▶ A relation is also called a predicate **pred(a,b,c)**
- ▶ Tuple is an element in a relation
- ▶ Logic program is a specification of a relation (contrast to functional programming)
 - brother(sam, bill)**
 - brother(sam, bob)**
 - Brother is not a function, since it maps "sam" to two different range elements*
 - Brother is a relation*
- ▶ Relations are n-ary, not just binary
 - family(jane, sam, [ann, tim, sean])**
- ▶ Relations are multi-directional
 - It works backward, forward and hybrid

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Relation as Bidirectional Functions

- Functions are *directional* and computes output(s) from inputs.



- Relations are *bidirectional* and used to relate a tuple of parameters.



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Examples of Relations

- Parent-child relations e.g. **parent(X, Y)**
- Classification e.g. **male(X)** or **female(Y)**
- Operations e.g. **append(Xs, Ys, Zs)**
- Databases: **employee(Name, ...)** relational tables?
- Geometry problems: how are sides of rectangles related, e.g. **rect(X, Y, X, Y)**
- Each function is a special case of relation.

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Relational Programming

- ▶ Logic programs define relations and allow you to express patterns to extract various tuples from the relations
- ▶ Infinite relations cannot be defined by rote... need rules
 - (A, B) are related if B is $A * A$
 - (B, H, A) are related if A is $\frac{1}{2} B * H$
 or... gen all tuples like this **(B, H, B * H * 0.5)**
 Logic program uses Horn clauses for explicit definition (facts) and for rules

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The Relational Model in OZ

```

(S) ::= skip           empty statement
      | <S121n

```

Choice allows alternatives to be explored, while failure indicates no answer at that branch.

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Choice-Fail

- ▶ The relational computation model extends the declarative model with two new statements, **choice** and **fail**:
- ▶ The **choice** statement groups together a set of alternative statements. Executing a **choice** statement provisionally picks one of these alternatives. If the alternative is found to be wrong later on, then another one is picked.
- ▶ The **fail** statement indicates that the current alternative is wrong. A **fail** is executed implicitly when trying to bind two incompatible values,

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Relational Program uses **Backtracking** and generates a **Search Tree**

- ▶ A relational program is executed sequentially.
- ▶ The **choice** statements are executed in the order that they are encountered during execution.
- ▶ When a **choice** is first executed, its first alternative is picked. When a **fail** is executed, execution "backs up" to the most recent **choice** statement, which picks its next alternative. If there are none, then the next most recent **choice** picks another alternative, and so forth. Each **choice** statement picks alternatives in order from left to right.
- ▶ This execution strategy can be illustrated with a tree called the **search tree**. Each node in the search tree corresponds to a **choice** statement and each subtree corresponds to one of the alternatives.

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Execution of Relational Program

- ▶ A relational program is interesting because it can potentially execute in many different ways, depending on the choices it makes. We would like to control which choices are made and when they are made. For example, we would like to specify the search strategy: depth-first search, breadth-first search, or some other strategy. We would like to specify how many solutions are calculated: just one solution, all solutions right away, or new solutions on demand. Briefly, we would like the same relational program to be executed in many different ways.
- ▶ There are two ways to perform search in OZ:
 - 1. **Basic Search Engines**
 - 2. **General Search Engines**

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Basic Search Engines

- ▶ Using built-in Search.base functions:
 1. **Search.base.one**
 2. **Search.base.all**
- ▶ **base.one**
 - {Search.base.one + ScriptP?Xs}
 - returns a singleton list containing the first solution of the script + ScriptP (a unary procedure) obtained by **depth-first search**. If no solution exists, nil is returned.
- ▶ **base.all**
 - {Search.base.all + ScriptP?Xs}
 - returns the list of all solutions of the script + ScriptP (a unary procedure) obtained by depth-first search.

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Example

- ```

{Search.base.one proc {$ X}
 choice
 choice X=ape
 [] X=bear
 end
 [] X=cat
 end
end}

```
- ▶ returns the list [ape].

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## Example 2: From English to OZ KR

- Suppose you know the following:
- Every human who is intelligent and rich is happy. Happy people are kind. If you can count, you are intelligent. John is rich. Paul is intelligent. Lisa is rich, and she can count. Paul and John and Lisa are human.
- Write a program in Oz that is able to infer that Lisa is kind. Use the Oz Explorer to investigate the search tree.

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```

declare
% Every human who is intelligent and rich is happy.
proc (Happy X)
 (Human X)
 (Rich X)
 (Intelligent X)
end
% Happy people are kind
proc (Kind X)
 (Happy X)
end
% If you can count, you are intelligent
% John is intelligent
proc (Intelligent X)
 choice
 (CanCount X)
 [] X = john
 end
end
% Lisa is rich
proc (Rich X)
 X = lisa
end
% Lisa can count
proc (CanCount X)
 X = lisa
end
% Paul, John and Lisa are human
proc (Human X)
 choice
 X = paul
 [] X = john
 [] X = lisa
 end
end
(Browse (Search.base.all Kind))
(Explorer.all Find)

```

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Given the following  
relationships, define and  
test the following relations:

X is the father of Y  
X is the grandmother of Y  
X is the sister of Y  
X is the uncle of Y  
X is the cousin of Y

X is an ancestor of Y

```

proc (Parent X Y)
 choice
 X = katherine Y = bertrand
 [] X = katherine Y = frank
 [] X = katherine Y = rachel
 [] X = dora Y = kate
 [] X = dora Y = john
 [] X = anne Y = conrad
 [] X = amberley Y = bertrand
 [] X = amberley Y = frank
 [] X = amberley Y = rachel
 [] X = bertrand Y = kate
 [] X = bertrand Y = john
 [] X = bertrand Y = conrad
 end
end
proc (Female X)
 choice
 X = katherine
 [] X = rachel
 [] X = dora
 [] X = anne
 [] X = kate
 end
end
proc (Male X)
 choice
 X = amberley
 [] X = frank
 [] X = bertrand
 [] X = conrad
 [] X = john
 end
end

```

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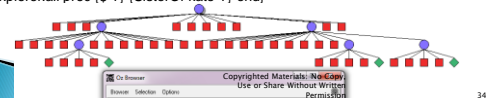
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```

proc (FatherOf X Y)
 (Parent X Y)
 (Male X)
end
proc (SisterOf X Y)
 Z in
 (Parent Z X)
 (Parent Z Y)
 (NotEq X Y)
 (Female X)
 end
end

```

% Who is Bertrand the father of?  
(Browse (Search.base.all proc (\$ Y) {FatherOf bertrand Y} end))  
(Explorer.all proc (\$ Y) {FatherOf bertrand Y} end)  
% Who is kate the sister of?  
(Browse (Search.base.all proc (\$ Y) {SisterOf kate Y} end))  
(Explorer.all proc (\$ Y) {SisterOf kate Y} end)



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```

declare
proc (S P0 P)
 P1 in
 (NP P0 P1)
 (VP P1 P)
 end
end
proc (NP P0 P)
 choice
 P0 = john|P
 [] P0 = mary|P
 end
end
proc (VP P0 P)
 choice
 P0 = whistles|P
 [] P0 = walks|P
 end
end

```

Given is the following fragment of English:

john walks  
john whistles  
mary walks  
mary whistles

captured by the following grammar

```

S --> NP VP
NP --> john | mary
VP --> walks | whistles

```

Write a program in Oz capable of recognizing the above sentence solve it.

(Explorer.all proc (\$ \_) {S [john walks] end})  
(Explorer.all proc (\$ \_) {S [john whistles] end})  
(Explorer.all proc (\$ \_) {S [mary walks] end})  
(Explorer.all proc (\$ \_) {S [mary whistles] end})

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Given is the following fragment of English:

john walks  
mary whistles  
mary likes john  
mary eats a banana  
every man likes mary  
...

captured by the following grammar

```

S --> NP VP
NP --> PN
NP --> DET N
VP --> IV
VP --> TV NP
PN --> john | mary
DET --> a | every
N --> banana | man
IV --> walks | whistles
TV --> likes | eats

```

**Exercise:** Write a program in Oz which is able to recognize them. Use the Oz Explorer to investigate the search tree.

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## General Search Engine: The Solve

- We provide encapsulated search by adding one function, **Solve**, to the computation model. This is an external search engine. Load them from your DZL.
- The call **(Solve F)** is given a zero-argument function **F** (or equivalently, a one-argument procedure) that returns a solution to a relational program. The call returns a lazy list of all solutions, ordered according to a depth-first search strategy. For example, the call:
  - **L=(Solve fun {\$} choice 1 [] 2 [] 3 end end)**
- returns the lazy list **[1 2 3]**. Because **Solve** is lazy, it only calculates the solutions that are needed. **Solve** is compositional, i.e., it can be nested: the function **F** can contain calls to **Solve**. Using **Solve** as a basic operation, we can define both **one-solution** and **all-solutions search**. To get one-solution search, we look at just the first element of the list and never look at the rest:
- The **Solve** function includes **SolveOne** and **SolveAll**.
- You need to **\insert 'Solve.oz'** at your program

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## Using the Solve: **SolveOne**, **SolveAll**

```

Buffers Files Tools Edit Search Mule Oz Help
\insert 'Solve.oz'

declare
fun (Digit)
 choice 0 [] 1 [] 2 [] 3 [] 4 [] 5 [] 6 [] 7 [] 8 [] 9 end
end
(Browse (SolveOne Digit))
(Browse (SolveAll Digit))

```

**Oz Browser**

Browser Selection Options

[0]

[0 1 2 3 4 5 6 7 8 9]

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## How about **SolveSome**? You can programm it.

Oz Programming Interface (emacs@SABAH-PC)

```

Buffers Files Tools Edit Search Mule Oz Help
\insert 'Solve.oz'

declare
fun (Digit)
 choice 0 [] 1 [] 2 [] 3 [] 4 [] 5 [] 6 [] 7 [] 8 [] 9 end
end
(Browse (List.take (Solve Digit) 5))

```

**Oz Browser**

Browser Selection Options

[0 1 2 3 4]

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## Using to Solve to satisfy more complicated constraints?

I want list of four numbers where  $N1 < N2 < N3 < N4$  where  $Ni$  is a Digit

```

Buffers Files Tools Edit Search Mule Oz Help
\insert 'Solve.oz'

declare
fun (DigitSeq)
 local L A B
 L = [(Digit) (Digit) (Digit) (Digit)]
 (L.1 < L.2.1) = true
 (L.2.1 < L.2.2.1) = true
 (L.2.2.1 < L.2.2.2.1) = true
 L
end
(Browse (SolveAll DigitSeq))
(Browse (Length (SolveAll DigitSeq)))

```

**Oz Browser**

Browser Selection Options

[0 1 2 3][0 1 2 4][0 1 2 5][0 1 2 6][0 1 2 7][0 1 2 8]  
 [0 1 2 9][0 1 3 4][0 1 3 5][0 1 3 6][0 1 3 7][0 1 3 8]  
 [0 1 3 9][0 1 4 5][0 1 4 6][0 1 4 7][0 1 4 8][0 1 4 9]  
 [0 1 5 6][0 1 5 7][0 1 5 8][0 1 5 9][0 1 6 7][0 1 6 8]  
 [0 1 6 9][0 1 7 8][0 1 7 9][0 1 8 9][0 2 3 4][0 2 3 5]  
 [0 2 3 6][0 2 3 7][0 2 3 8][0 2 3 9][0 2 4 5][0 2 4 6]  
 [0 2 4 7][0 2 4 8][0 2 4 9][0 2 5 6][0 2 5 7][0 2 5 8]  
 [0 2 5 9][0 2 6 7][0 2 6 8][0 2 6 9][0 2 7 8][0 2 7 9]  
 [0 2 8 9][0 3 4 5],...

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## Using the Explorer

Oz Programming Interface (emacs@SABAH-PC)

```

declare
fun (DigitSeq)
 local L A B
 L = [(Digit) (Digit) (Digit) (Digit)]
 (L.1 < L.2.1) = true
 (L.2.1 < L.2.2.1) = true
 (L.2.2.1 < L.2.2.2.1) = true
 L
end
(BrowseOne(DigitSeq))

```

**Oz Explorer**

Explorer Move Search Nodes Hide Options

tree diagram showing nodes and edges

**Oz Inspector**

Inspector Selection Options

[0 1 2 3]

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## Can I add constraints inside the Query? With Implicit Fail

```

\insert 'Solve.oz'
declare
proc {Vowel ?R}
 choice
 R=a
 [] R=e
 [] R=i
 [] R=o
 [] R=u
 end
end

```

% But can be used in a search with Solve

{Browse (List.take (Solve Vowel) 4)}

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## With Explicit Fail

```

declare
proc {Distinct ?Res}
 V1 V2
in
 {Vowel V1}
 {Vowel V2}
 if V1 == V2 then fail end
 Res=V1#V2
end
(Browse (List.take (Solve Distinct) 7))

```

Oz Browser

Browser Selection Options

[a#e a#i a#o a#u e#a e#i e#o]

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## How about Querying Family?

```

Buffers Files Tools Edit Search Mule Oz
proc {Dad ?Father ?Kid}
 choice
 Father=pop Kid=sarah
 []
 Father=pop Kid=john
 []
 Father=pop Kid=robert
 []
 Father=pop Kid=jill
 end
end
proc {Mom ?Mother ?Kid}
 (Dad ?Kid)
 Mother=mom
end
proc {Male ?Person}
 choice
 Person=pop
 []
 Person=robert
 []
 Person=john
 end
end
proc {Female ?Person}
 choice
 Person=mom
 []
 Person=sarah
 []
 Person=jill
 end
end

```

```

Browse (SolveAll proc ($ Who)
 (Dad pop Who)
end))
Browse (SolveAll proc ($ Who)
 (Mom mom Who)
end))
Browse (SolveAll proc ($ Who)
 (Male Who)
end))
Browse (SolveAll proc ($ Who)
 (Female Who)
end))

```

Browser Selection Options

[sarah john robert jill]  
[sarah john robert jill]  
[pop robert john]  
[mom sarah jill]

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## Querying Family: Another Example

```

\insert 'Solve.oz'
declare
proc {Son ?X ?Y}
 (Parent Y X) (Male X)
end
proc {Daughter ?X ?Y}
 (Parent Y X) (Female X)
end
proc {Parent ?Y ?X}
 choice
 (Dad Y X)
 []
 (Mom Y X)
 end
end
% What children are sons of what adults?
(Browse (SolveAll proc ($ Ans)
 Child#Adult=Ans
in
 (Son Child Adult)
end))

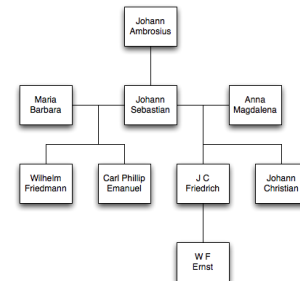
```

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## DIY Exercise: Family Tree

Represent and query this family tree in OZ. Represent at least three different family relationships and query each one forward and backward.



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## Relational Programming

- ▶ <http://www.idi.ntnu.no/emner/tdt4165/handouts/>
- ▶ The relational model of computation covered in the last lecture is an extension of the declarative model of computation with:
  - ▶ Δ non-deterministic **choice** statements,
  - ▶ Δ **failure** statements.
- ▶ This means an ability to describe and a process to infer or propagate solution(s) via (unification-matching-navigating through alternatives (choice/backtracking))

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## Concrete Representation of Relations in OZ?

- Relations can be represented using Functions or Procedures
  - Parameters in Relations can be either Input or Output
  - Relations works Backwards or Forward:
- Example:
- Append Relation may work as Concat or Split?

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## Is Append Fully Relational?

```

declare
proc {Append List1 List2 List3}
try List1 = nil List2 = List3
catch _ then Head Tail1 Tail3 in
 List1 = Head|Tail1
 List3 = Head|Tail3
 {Append Tail1 List2 Tail3} end end

{Browse {Append [1] [2 3] $}} % prints [1 2 3]
{Browse {Append [1] $ [1 2 3]}} % prints [2 3]

{Browse {Append $ [2 3] [1 2 3]}} % fails at nil = _|

```

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## Better version? Using or?

```

declare
proc {Append List1 List2 List3}
or List1 = nil
 List2 = List3
 [] Head Tail1 Tail3 in
 List1 = Head|Tail1
 List3 = Head|Tail3
 {Append Tail1 List2 Tail3} end end

{Browse {Append [1] [2 3] $}}
{Browse {Append [1] $ [1 2 3]}}
{Browse {Append $ [2 3] [1 2 3]}}

```

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## Fully Relational-Using Choice

```

declare
\insert 'solve.oz'
proc {Append List1 List2 List3}
choice
 List1 = nil
 List2 = List3
 [] Head Tail1 Tail3 in
 List1 = Head|Tail1
 List3 = Head|Tail3
 {Append Tail1 List2 Tail3} end end

{Browse {SolveAll
 fun ($)
 {Append $ [2 3] [1 2 3]} end}}

{Browse {SolveAll
 fun ($)
 List1 List2 in
 {Append List1 List2 [1 2 3]}
 solution(List1 List2) end}}

```

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More  
Concrete  
way to  
represent  
Append as  
a Relation

```

declare
proc {RAppend ?A ?B ?C}
choice
 A=nil B=C
 [] local As Cs X in
 A=X|As[]
 C=X|Cs
 {RAppend As B Cs}
 end
end

{Browse {SolveFirst
 proc ($) Z=S in {RAppend [3 4] [5 6] Z} end}}

{Browse {SolveFirst
 proc ($) X=S in {RAppend X [2 3] [1 2 3]} end}}

{Browse {SolveFirst
 proc ($) Y=S in {RAppend [3 4] Y [3 4 5 6]} end}}

```

Oz Browser

Browse Selection Options

[3 4 5 6]

[1]

[5 6]

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Relational  
Member

```

declare
proc {RMember ?X ?Ls}
local E Es in
 Ls=E|Es
 choice
 X=E
 [] {RMember X Es}
 end
end

proc {RMember2 ?X ?Ls}
choice
 Ls=X|_
 [] local Es in
 Ls=_|Es[]
 {RMember2 X Es}
 end
end

```

```

{Browse {SolveAll proc ($) {RMember 3 4 [3|2|nil]} V=ok end}}
{Browse {SolveAll proc ($) {RMember2 3 4 [3|2|nil]} end}}

```

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Relational  
Add to  
End  
of List

```

declare
proc {AddToEnd ?L ?X ?R}
choice
 L=nil R=X|nil
 [] local E Es Res in
 L=E|Es
 {AddToEnd Es X Res}
 R=E|Res
 end
end

fun {AddToEnd2 ?L ?X}
choice
 L=nil X|nil
 [] local E Es in
 L=E|Es
 E{AddToEnd2 Es X}
 end
end

{Browse {SolveFirst fun ($) {AddToEnd [a b c] d $} end}}
{Browse {SolveFirst fun ($) {AddToEnd [a b c] $ [a b c d]} end}}
{Browse {SolveFirst fun ($) {AddToEnd $ d [a b c d]} end}}
{Browse {SolveFirst fun ($) {AddToEnd2 [a b c] d $} end}}
{Browse {SolveFirst fun ($) {AddToEnd2 $ [a b c] d $} end}}
{Browse {SolveFirst fun ($) {AddToEnd2 $ $ [a b c d]} end}}

```

Oz Browser

Browse Selection Options

[a b c d]

d

[a b c]

[a b c d]

d

[a b c]

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## Relational Reverse

```

\insert 'SolveFirst.oz'
\insert 'SolveOne.oz'
\insert 'SolveAll.oz'
\insert 'Solve.oz'
\insert 'AddToEnd.oz'
declare
proc {Reverse L R}
 choice
 L=nil R=nil
 [] local E Es R2 in
 L=E|Es (Reverse Es R2) (AddToEnd R2 E R)
 end
end
end

(Browse (SolveAll proc ($ V) (Reverse [1 2 3 4] V) end))
(Browse (SolveAll proc ($ V) (Reverse [2 3 4] V) end))

```

Oz Browser

| Browser     | Selection | Options |
|-------------|-----------|---------|
| [[4 3 2 1]] |           |         |
| [[4 3 2]]   |           |         |

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## Relational Insert

```

Oz Programming Interface (emacs@SABAH-PC)
\insert 'Solve.oz'
\insert 'SolveAll.oz'

declare
proc {Insert X Impl OutL OutR}
 choice
 A | B = OutL in
 A = X
 B = Impl
 []
 [] B = Impl
 C | D = OutL in
 A = C
 (Insert X B D)
 end
 end
end

(Browse (SolveAll proc ($ S) (Insert y [a b c] S) end))
(Browse (SolveAll proc ($ S) (Insert S [a b c] [a b x c]) end))
(Browse (SolveAll proc ($ Y) S|T-Y in (Insert S T [a b x c]) end))

```

Oz Browser

| Browser                                 | Selection | Options |
|-----------------------------------------|-----------|---------|
| [y a b c] [a y b c] [a b y c] [a b c y] |           |         |
| [x]                                     |           |         |
| [a b x c] b[a x c]                      |           |         |

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## Relational Delete

```

\insert 'SolveAll.oz'
declare
proc {Delete X Impl OutL}
 choice
 local
 A | As = Impl in
 X = A
 As = OutL
 end
 [] local
 A | As = Impl
 B | Bs = OutL in
 B = A
 (Delete X As Bs)
 OutL = A | Bs
 end
 end
end

(Browse (SolveAll proc ($ S) (Delete 2 S [3 4 2]) end))
(Browse (SolveAll proc ($ S) (Delete 2 [3 4 2] S) end))
(Browse (SolveAll proc ($ S) (Delete 3 [3 5 4 2] S) end))
(Browse (SolveAll proc ($ S) (Delete 4 [3 5 4 2] S) end))
(Browse (SolveAll proc ($ S) (Delete 5 [3 5 4 2] S) end))

```

Oz Browser

| Browser                                   | Selection | Options |
|-------------------------------------------|-----------|---------|
| [[2 3 4 2] [3 2 4 2] [3 4 2 2] [3 4 2 2]] |           |         |
| [[3 4 2] [3 4 2]]                         |           |         |
| [5]                                       |           |         |
| nil                                       |           |         |
| [5]                                       |           |         |

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## How to Query a Relation?

Two ways:

- (1) using a function that uses solve
- (2) directly using solve

```

proc {Capitals ?State ?City}
 choice
 State=florida City=tallahassee
 State=iowa City=desMoines
 State=kansas City=topeka
 State=virginia City=richmond
 State=michigan City=lansing
 State=newYork City=albany
 State=california City=sacramento
 end
end

fun {QueryCapitals State City}
 Ans = (SolveOne
 proc ($ Ok)
 (Capitals State City)
 Ok=yes
 end)
 in if Ans \= nil then yes else no end
end

(Browse (QueryCapitals florida tallahassee))
(Browse (QueryCapitals newYork albany))
(Browse (QueryCapitals georgia atlanta))

% queries with unbound variables: What is the capital of iowa?
(Browse (SolveFirst proc ($ What) (Capitals iowa What) end))
% What state is albanys capital of?
(Browse (SolveFirst proc ($ What) (Capitals What albany) end))

```

Oz Browser

| Browser   | Selection | Options |
|-----------|-----------|---------|
| yes       |           |         |
| yes       |           |         |
| no        |           |         |
| desMoines |           |         |
| newYork   |           |         |

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## Example on Querying a relation in using direct solve: Agriculture Products

```

\insert 'SolveFirst.oz'
\insert 'SolveOne.oz'
\insert 'SolveAll.oz'
\insert 'Solve.oz'
\insert 'AddToEnd.oz'
declare
proc {Agriculture ?State ?Product}
 choice
 State=florida Product=oranges
 State=iowa Product=pork
 State=kansas Product=sunflowers
 State=virginia Product=ham
 State=michigan Product=cherries
 State=newYork Product=dairy
 State=california Product=wine
 end
end

% What is the capital and chief agricultural product of florida?
(Browse (SolveFirst proc ($ Ans)
 Town|Farmed = Ans
 in
 (Capitals florida Town)
 (Agriculture florida Farmed)
 end))

% What is the capital and chief agricultural product of iowa?
(Browse (SolveFirst proc ($ Ans)
 Town|Farmed=Ans
 in
 (Capitals iowa Town)
 (Agriculture iowa Farmed)
 end))

```

Oz Browser

| Browser             | Selection | Options |
|---------------------|-----------|---------|
| tallahassee#oranges |           |         |
| desMoines#pork      |           |         |

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## More Complex Relational Thinking: Coin Change?

- Given a set of coins and amount, Write a Relational OZ program to find out how many ways we can make the change of the amount using the coins given.
- Example (Computing change)
  - nominals: nickel (5 cents), dime (10 cents), quarter (25 cents)
    - amount: 25 cents
    - change:
      - 5 nickels
      - 3 nickels and 1 dime
      - 1 nickel and 2 dimes
      - 1 quarter

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## The Change Relation

```
fun {Change Amount Purse}
 choice Amount = 0 nil
 [] Amount > 0 = true
 Coin = {Purse}
 Rest = {Change Amount-Coin Purse} in
 Coin|Rest end end
```

The version of Change makes two choices:

- ▲ the amount is positive, and given a coin it is possible to change the rest; the coin is then returned together with the rest of the change;
- ▲ the amount is zero; the change is no coins.

If the amount is negative, the branch of the search simply fails (change impossible).

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## Testing the Coins Change

```
declare
 \insert 'solve.oz'
 declare
 Nickel = 5
 Dime = 10
 Quarter = 25
 Nominals = [Nickel Dime]
 fun {Pick List}
 Head|Tail = List in
 choice Head [] {Pick Tail} end end
 fun {Sum List}
 {FoldL List Number.'+' 0} end
 fun {Purse}
 {Pick Nominals} end
 declare
 \insert 'change-alternative.oz'
 {Browse
 fun ($)
 {Change Quarter Purse} end}}
```

**Notice:**  
the results list is  
redundant; some of its  
elements represent the  
same change, they just  
differ in the order of coin  
nominals.

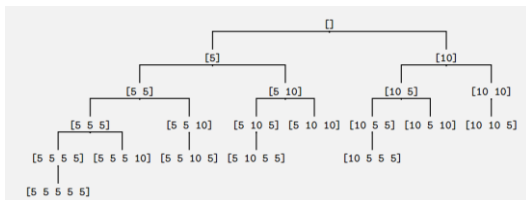
**How to eliminate this  
redundancy??????**

```
Oz Browser
Browser Selection Options
[[5 5 5 5 5] [5 5 5 10] [5 5 10 5] [5 10 5 5] [5 10 10]
[10 5 5 5] [10 5 10] [10 10 5]]
```

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## The Search Tree



The search is redundant, because after having chosen the second nominal, we can choose the first one again.

- ▲ If many more nominals were available, the search space would be exponentially larger

We can prevent the redundancy by demanding that a nominal that has been skipped over in a search branch can never be chosen again

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## Change without Redundancy

```
fun {Change Amount Nominals}
 choice Amount = 0 nil
 [] Amount > 0 = true
 Coin|Coins = Nominals in
 choice
 Rest = {Change Amount-Coin Nominals} in
 Coin|Rest
 [] {Change Amount Coins} end end end
```

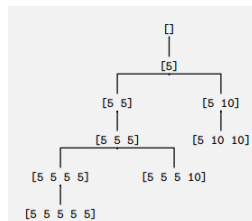
The search is no longer redundant, because after having chosen the second nominal, we cannot choose the first one any more.

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## Testing the Change without Redundancy

```
declare
 \insert 'solve.oz'
 declare
 Nickel = 5
 Dime = 10
 Quarter = 25
 Nominals = [Nickel Dime]
 fun {Pick List}
 Head|Tail = List in
 choice Head [] {Pick Tail} end end
 fun {Sum List}
 {FoldL List Number.'+' 0} end
 fun {Purse}
 {Pick Nominals} end
 declare
 \insert 'change-nonredundant.oz'
 {Browse
 fun ($)
 {Change Quarter Nominals} end}}
```



```
Oz Browser
Browser Selection Options
[[5 5 5 5 5] [5 5 5 10] [5 10 10]]
```

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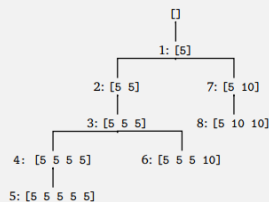
## Change with Different Searching Strategies

- The Solve we have used so far implements the depth-first search strategy.
- ▲ Whenever a node does not correspond to a final solutions, further choices are made, and the node's children are examined before the node's siblings.
- ▲ The relational model of computation in Oz uses depth-first search, but it's just because Solve is implemented this way.
- ▲ It is possible to reimplement Solve so that it uses, e.g., breadth-first
- search, or even allows the user to choose the strategy when Solve is called.

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## Example (Non-redundant change, depth-first search)

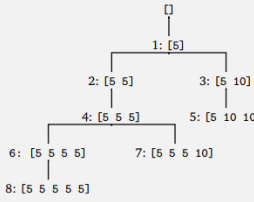


► Complete solutions are found at nodes 5, 6, and 8, in this order.

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## Example (Non-redundant change, breadth-first search)



► Complete solutions are found at nodes 5, 7, and 8, in this order.

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## Changing Solve.oz to include Strategies

```

fun (SolveAll WidthFirst Search)
 (Touchall (Solve WidthFirst Search)) end
fun (Touchall List)
 case List of
 all then skip
 () List then (Touchall List _) end
 List end
fun (Solve WidthFirst Search)
 fun (SolveSpace CSpace)
 case CSpace of
 all then nil
 () CSpace/CSpace then
 case CSpace
 of (SolveAll CSpace) then
 CSpace = (Space:clone CSpace) in
 (Space:clone CSpace) in
 (SolveSpace CSpace) end end
 else
 (SolveSpace (Space:ask CSpace) CSpace CSpace) end end end
 fun (SolveSpace SpaceState CSpace CSpace)
 use SpaceState
 of failed then
 (SolveSpace CSpace)
 () succeeded then
 (Space:merge CSpace) (SolveSpace CSpace)
 () alternativesN then
 (SolveSpace (HeadSpaceList (Choices CSpace N) CSpace)) end end
 fun (Choices CSpace N)
 (List:mapall
 (List:make N)
 fun (I I 1) (SolveSpace CSpace I) end) end
 fun (HeadSpaceList Choices CSpace)
 if WidthFirst == depth then (Append (Choices CSpace))
 else (Breadth:traverse(SpecificationError(WidthFirst)) nil end end in
 (SolveSpace (Space:clone CSpace)) end

```

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## Testing Change with Strategies

```

declare
 \insert 'solve.oz'
declare
 Nickel = 5
 Dime = 10
 Quarter = 25
 Nominals = [Nickel Dime]
 fun (Pick List)
 Head/Tail = List in
 choice Head () (Pick Tail) end end
 fun (Sum List)
 (FoldL List Number '+' 0) end
 fun (Purse)
 (Pick Nominals) end
 declare
 \insert 'solve-strategy.oz'
 (Browse
 (SolveAll depth
 fun ($) (Change Quarter Nominals) end))
 (Browse
 (SolveAll breadth
 fun ($) (Change Quarter Nominals) end))

```

Oz Browser

Browse Selection Options

```

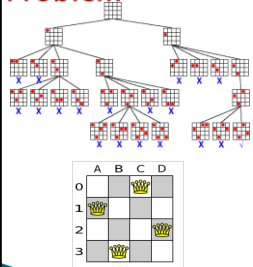
[[5 5 5 5 5] [5 5 5 10] [5 10 10]
[[5 10 10] [5 5 5 5 5] [5 5 5 10]]

```

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## Another Example: 4 Queens Problem



```

declare
 fun (Queens N)
 proc (PlaceQueens N ?Cs ?Us ?Ds)
 if N>0 then
 Ds2 Us2 in
 Us = | Us2
 Ds2 = | Ds
 (PlaceQueens N-1 Cs Us2 Ds2)
 (PlaceQueen N Cs Us Ds)
 else skip end
 end
 end
 proc (PlaceQueen N ?Cs ?Us ?Ds)
 choice
 Cs=N-
 Us=N-
 Ds=N-
 []
 _|Cs2=Cs
 _|Us2=Us
 _|Ds2=Ds in
 (PlaceQueen N Cs2 Us2 Ds2)
 end
 end
 end
 Qs=[MakeList N]
 in
 (PlaceQueens N Qs _ _)
 end
end
(Browse (Solve.solveOne fun ($) (Queens 4) end))
(Browse (Solve.solveOne fun ($) (Queens 5) end))
(Browse (Solve.solveOne fun ($) (Queens 6) end))
(Browse (Solve.solveOne fun ($) (Queens 7) end))
(Browse (Solve.solveOne fun ($) (Queens 8) end))

```

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## Solution

```

Oz Programming Interpreter
Buffers Files Tools Edit Search Mule Oz Help
\insert 'solve.oz'
declare
 fun (Queens N)
 proc (PlaceQueens N ?Cs ?Us ?Ds)
 if N>0 then
 Ds2 Us2 in
 Us = | Us2
 Ds2 = | Ds
 (PlaceQueens N-1 Cs Us2 Ds2)
 (PlaceQueen N Cs Us Ds)
 else skip end
 end
 end
 proc (PlaceQueen N ?Cs ?Us ?Ds)
 choice
 Cs=N-
 Us=N-
 Ds=N-
 []
 _|Cs2=Cs
 _|Us2=Us
 _|Ds2=Ds in
 (PlaceQueen N Cs2 Us2 Ds2)
 end
 end
 end
 end
end

```

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