III. Physikalisches Institut A

RWTH AACHEN UNIVERSITY

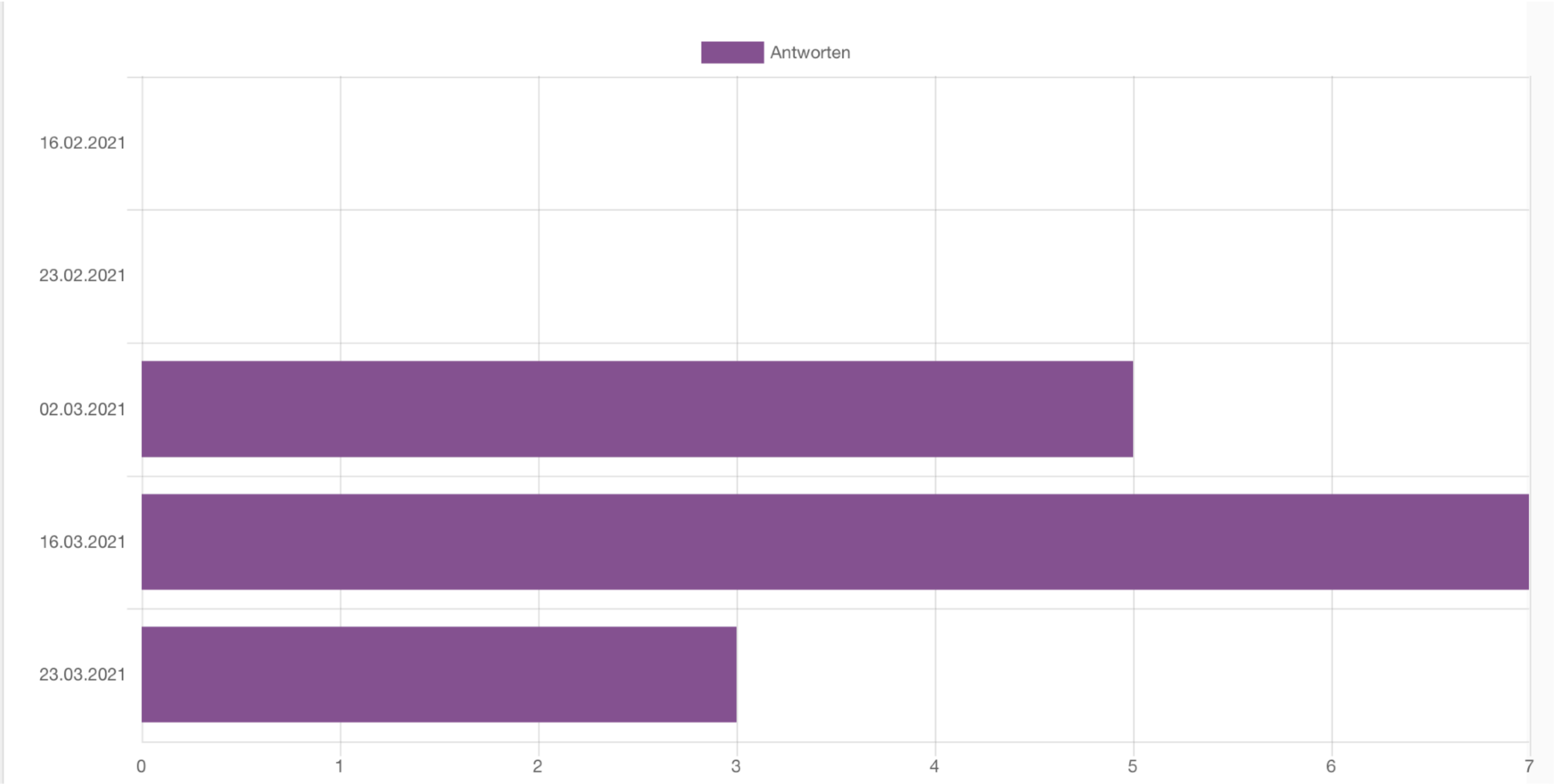
Experimental Techniques in Particle Physics (WS 2020/2021)

Interaction of particles with Matter (Part II)

17.11.2020

Prof. Alexander Schmidt

Date of the written exam



- **favourite choice: 16.03.2020**
- **is this a problem for anyone?**

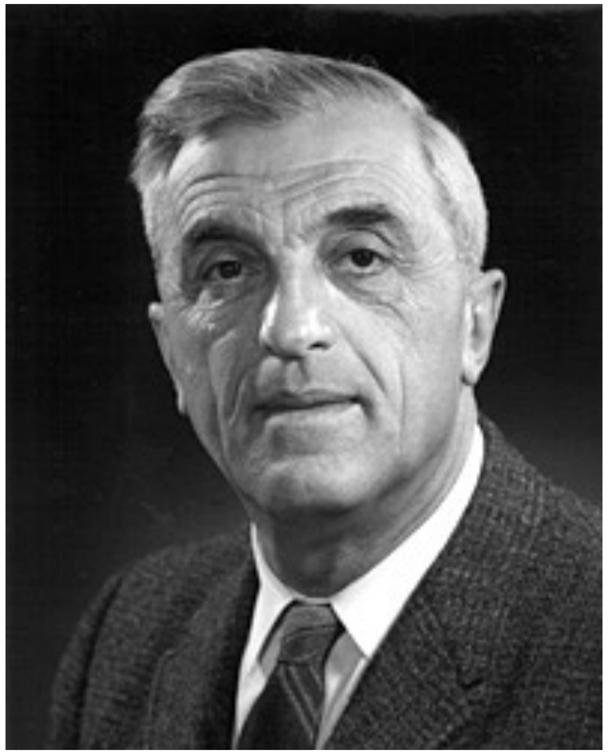
Repetition: Types of interaction

the following types of interactions between radiation and matter exist:

- ionisation and excitation caused by charged particles **(last week)**
- bremsstrahlung: emission of photons from accelerated charged particles **(today)**
- photon scattering and photon absorption **(today)**
- Cherenkov radiation **(later)**
- nuclear interaction of imminent hadrons with nuclei in material
- weak interaction (only detection method for neutrinos)

- usually **more than one** interaction process happens if the particle is not immediately absorbed

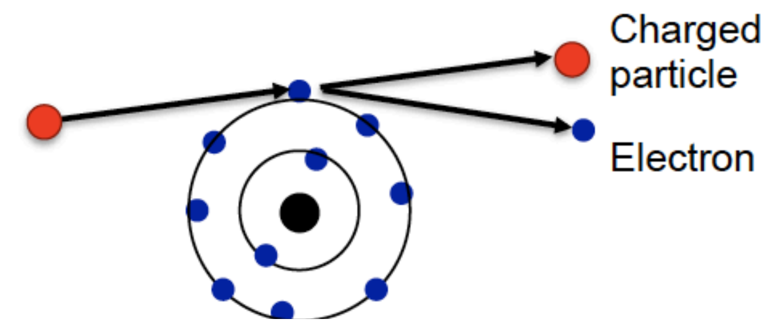
Repetition: Bethe Bloch



Felix Bloch (1905 - 1983)

Hans Bethe (1906 - 2005)

Repetition: Bethe Bloch

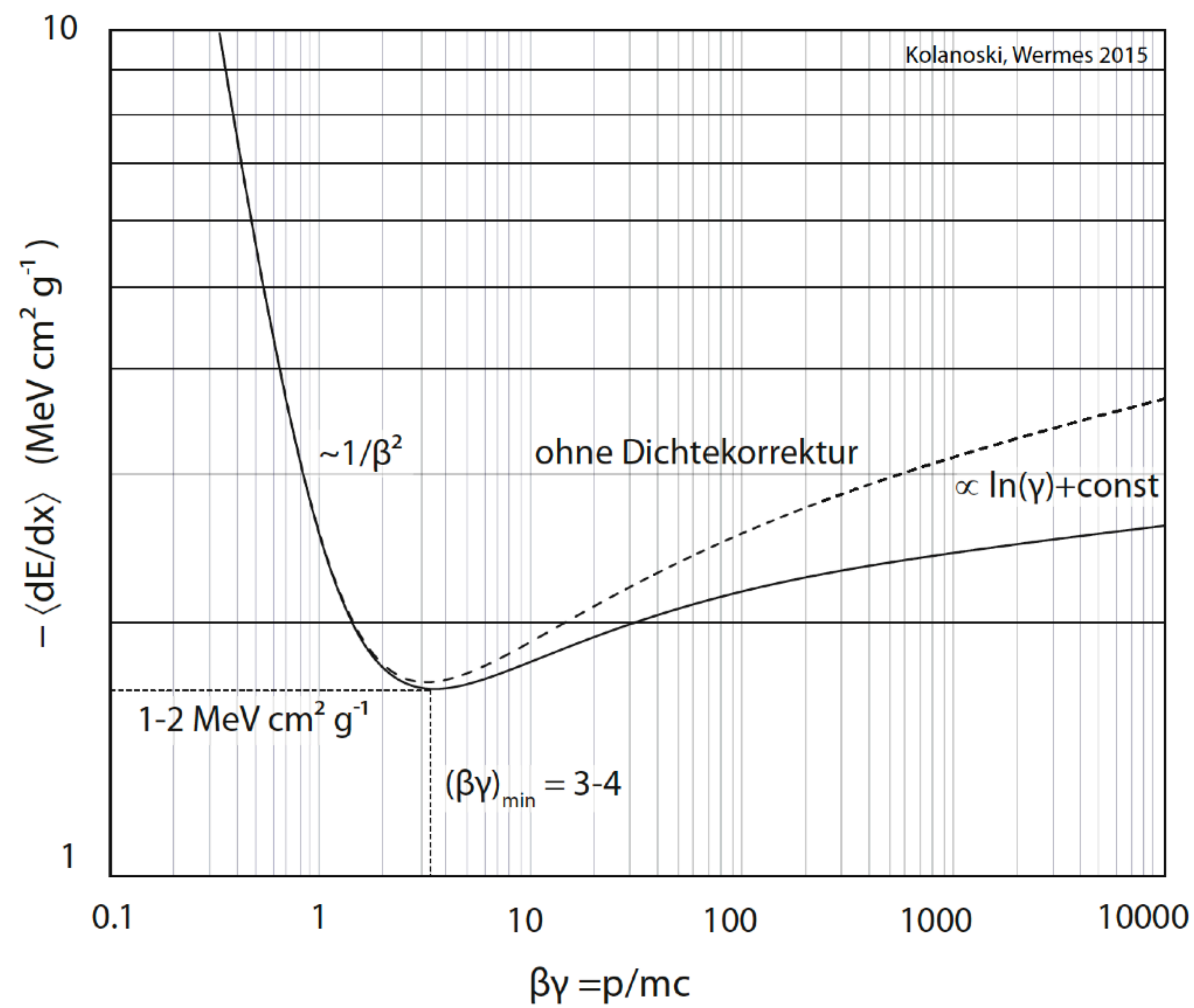


- energy loss through **ionisation**:

$$-\left\langle \frac{dE}{dx} \right\rangle = K \frac{Z}{A} \rho \frac{z^2}{\beta^2} \left[\frac{1}{2} \ln \frac{2\, m_e\, c^2\, \beta^2\, \gamma^2\, T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C(\beta\gamma,\, I)}{Z} \right]$$

with

- $K = 4\pi N_A r_e^2 m_e\, c^2 = 0.307 \text{ MeV } \text{ cm}^2/\text{mol}$
- *β, z* are charge and velocity of the projectile
- Z, A are atomic number and atomic mass of the medium
- *δ* is a density correction (necessary for high energies)
- *C/Z* is a “shell” correction (necessary for small *β*)

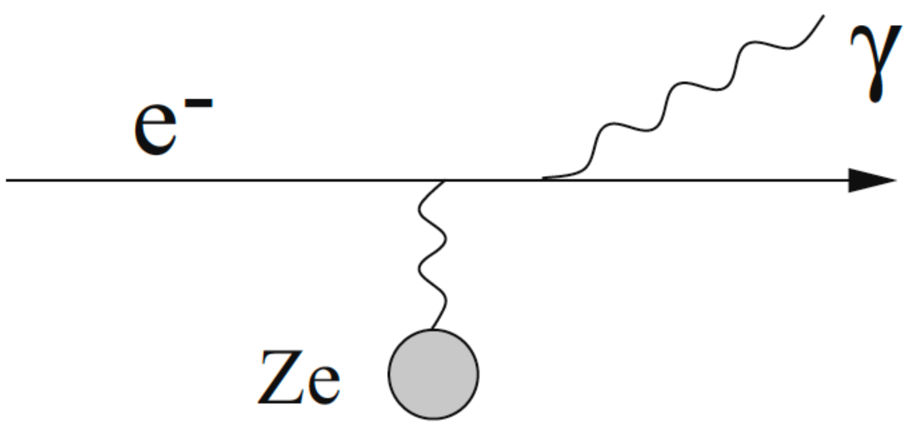


Bremsstrahlung

Bremsstrahlung

- at higher energies another energy loss mechanism starts to dominate: emission of photon radiation (**Bremsstrahlung**)

- particle is accelerated (slowed down) mostly in EM field of nucleus



- classical electrodynamics: emitted power of accelerated particle:

$$\frac{dW}{dt} = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 c^3}\,\ddot{\vec{x}}^{\,2}$$

- in E-field of the nucleus:

$$\ddot{\vec{x}}^{\,2} \propto \frac{z^2 Z^2 e^2}{m^2}$$

- inversely proportional to m^2 of incident particle (low mass particles radiate more)

- various corrections needed to go from classical electrodynamics to quantum mechanics

Bremsstrahlung

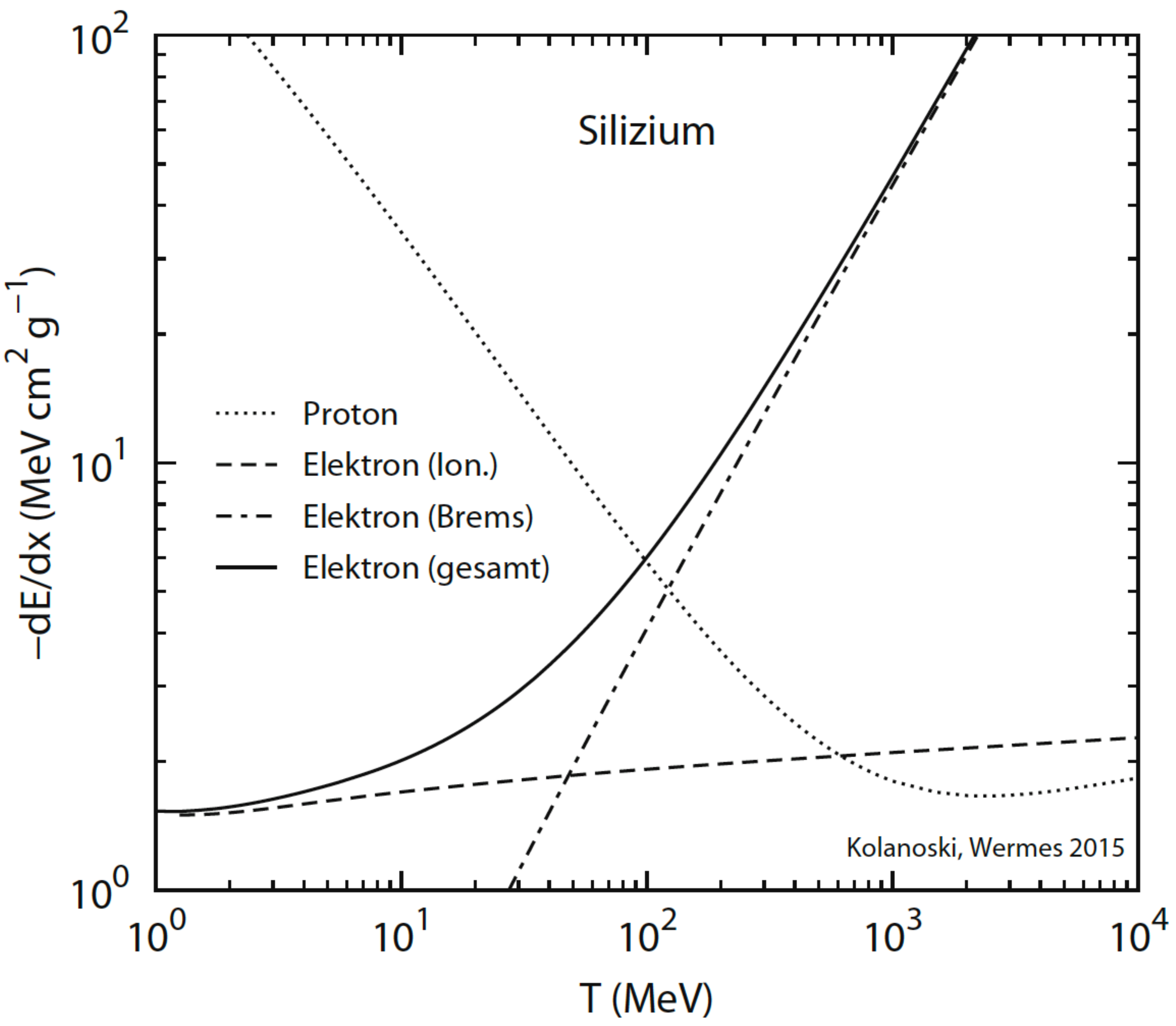
- various corrections are needed for going from classical electrodynamics to particle physics:
  - electron shell causes shielding of the charge of the nucleus (more shielding for smaller momenta)
  - Coulomb correction: modification of incoming plane wave approximation through atomic fields
  - dielectric suppression: the medium can be polarized, leading to cutoff of low-energy tails
  - scattering on shell-electrons (in addition to nucleus)
- the exact calculation of the spectrum is very elaborate (see literature)
- main result: $$\frac{d\sigma}{dE_\gamma} \propto \frac{Z^2}{m^2 E_\gamma}$$ (corrections need to be applied for the low- and high energy limits, but we ignore them here)
- simplified expression for high energy **particles** (relevant for us): $$\left(\frac{dE}{dx}\right)_{rad} = -\frac{E}{X_0}$$

  where the variable X_0 is defined as the "radiation length"
- for electrons: $$\rho X_0 = \frac{716.408\,\mathrm{g\,cm^{-2}}\,A}{Z(Z+1)\ln\frac{287}{\sqrt{Z}}}$$

Radiation length for electrons

Material	Z	Dichte	X_0	
		(g/cm^3)	(g/cm^2)	(cm)
Be	4	1.85	65.19	35.3
C (Graphit)	6	2.21	42.65	19.3
Al	13	2.70	24.01	8.9
Si	14	2.33	21.82	9.36
Fe	26	7.87	13.84	1.76
Cu	29	8.96	12.86	1.43
Ge	32	5.32	12.25	2.30
W	74	19.30	6.76	0.35
Pb	82	11.35	6.37	0.56
U	92	18.95	6.00	0.32
Szintillatoren:				
NaI	11, 53	3.66	9.49	2.59
CsI	55, 53	4.53	8.39	1.85
BaF_2	56, 9	4.89	9.91	2.03
$PbWO_4$	82, 74, 8	8.30	7.39	0.89
Polystyrol	1, 6	1.06	43.79	41.3
Gase (20° C, 1 atm):				
H_2	1	$0.0838 \cdot 10^{-3}$	61.28	731000
He	2	$0.1249 \cdot 10^{-3}$	82.76	662610
Luft	≈ 7.36	$1.205 \cdot 10^{-3}$	36.66	30423
Ar	18	$1.66 \cdot 10^{-3}$	19.55	11763
Xe	54	$5.48 \cdot 10^{-3}$	8.48	1547

Bremsstrahlung for electrons



- Ionisation dominates at lower energies, while Bremsstrahlung dominates at higher energies

Bremsstrahlung for other particles

- different processes dominate in different energy regimes depending on the particle type

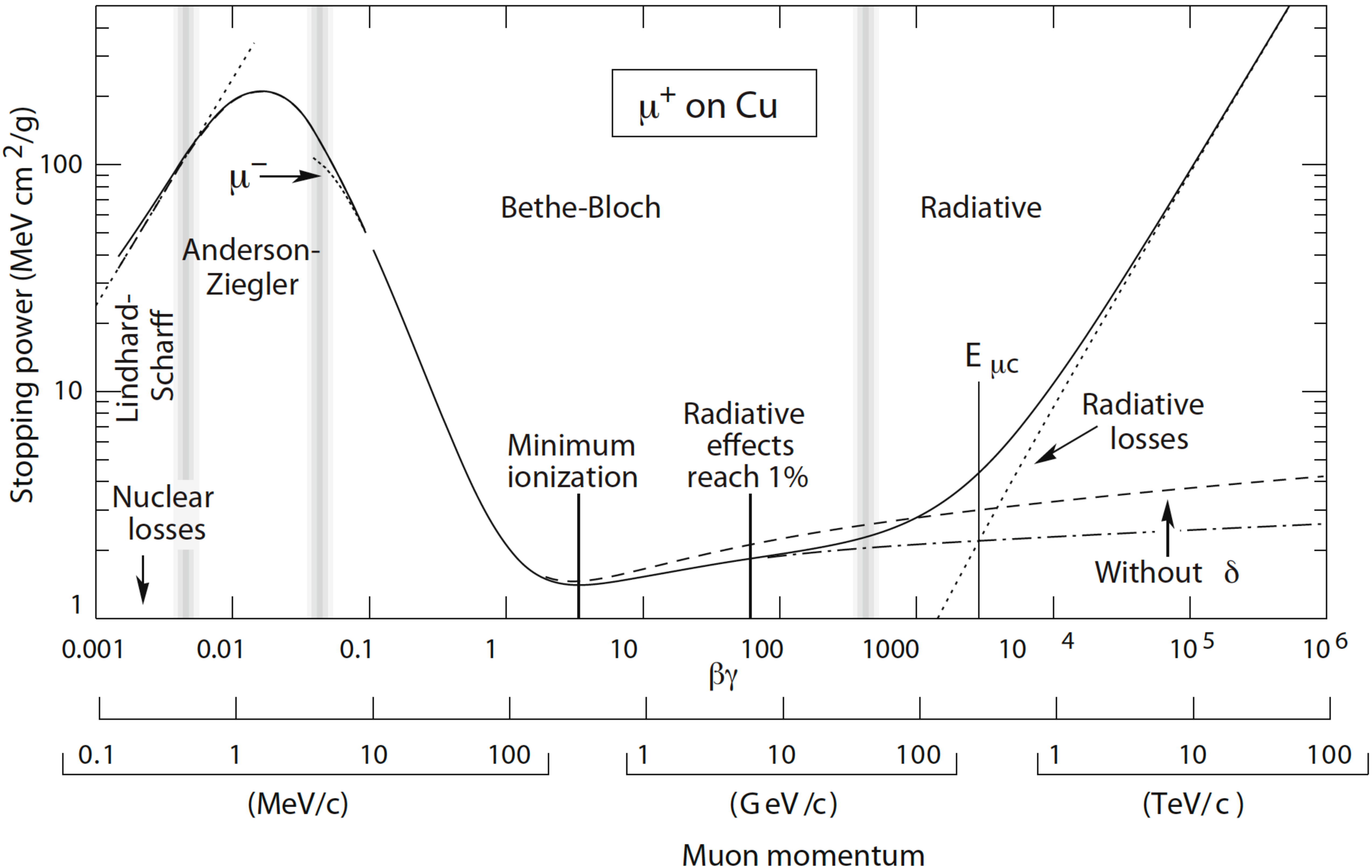
$$\begin{aligned} &\text{Ionisation:} && \propto \; Z \ln E/M \\ &\text{Bremsstrahlung:} && \propto \; Z^2 \, E/M^2 \end{aligned}$$

- the muon mass is 200 times larger than the electron mass
- muons radiate 40 000 times less than electrons

- muons at the LHC or from cosmic rays may reach energies so that Bremsstrahlung **is** relevant

- below 100 GeV, the Bremsstrahlung is negligible for muons

Variety of energy loss processes (of charged particles)



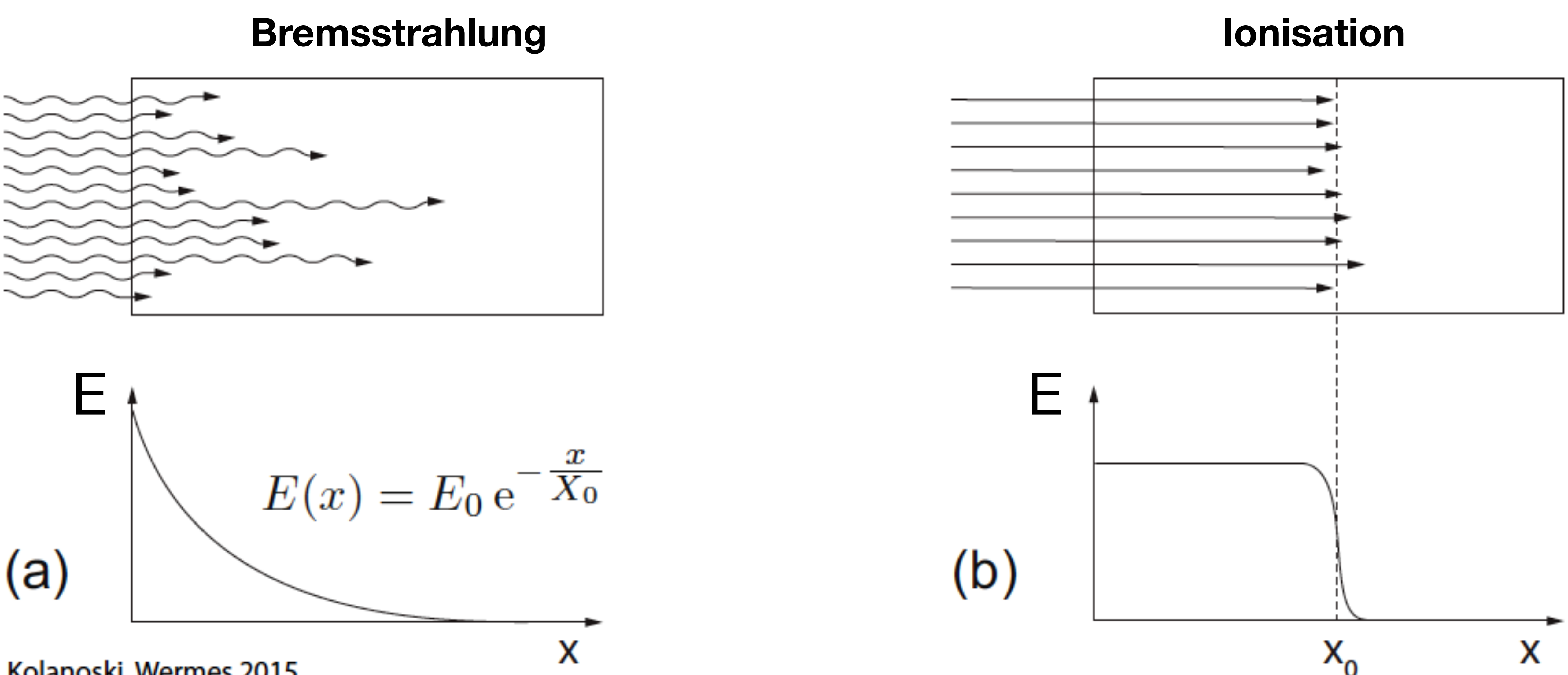
Bremsstrahlung

- integration of the energy loss $\left(\frac{dE}{dx}\right)_{rad} = -\frac{E}{X_0}$

  results in $E(x) = E_0\, \mathrm{e}^{-\frac{x}{X_0}}$

- in contrast to ionisation (remember the Bragg peak), the particle is not stopped abruptly

- it loses its energy in an exponential decrease



Kolanoski, Wermes 2015

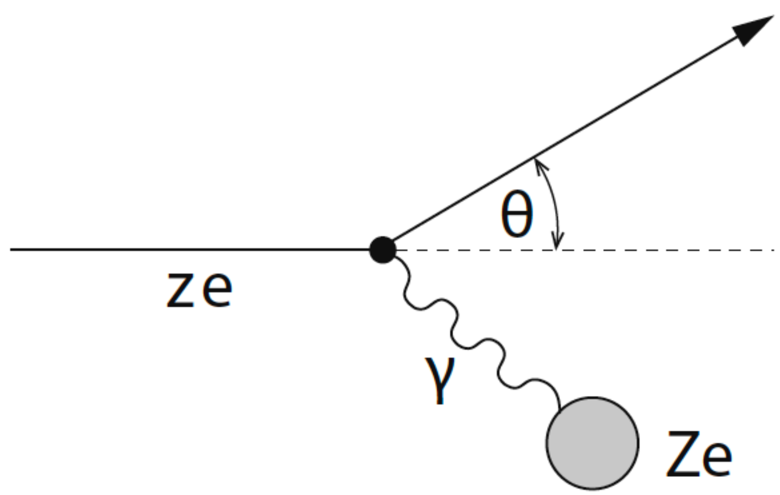
multiple scattering

Multiple scattering

multiple Coulomb scattering:

- several **elastic** scattering processes in the electric field of the nuclei while particle traverses material
- scattering particle is light compared to the nucleus
- a single scattering process is described by Rutherford scattering:

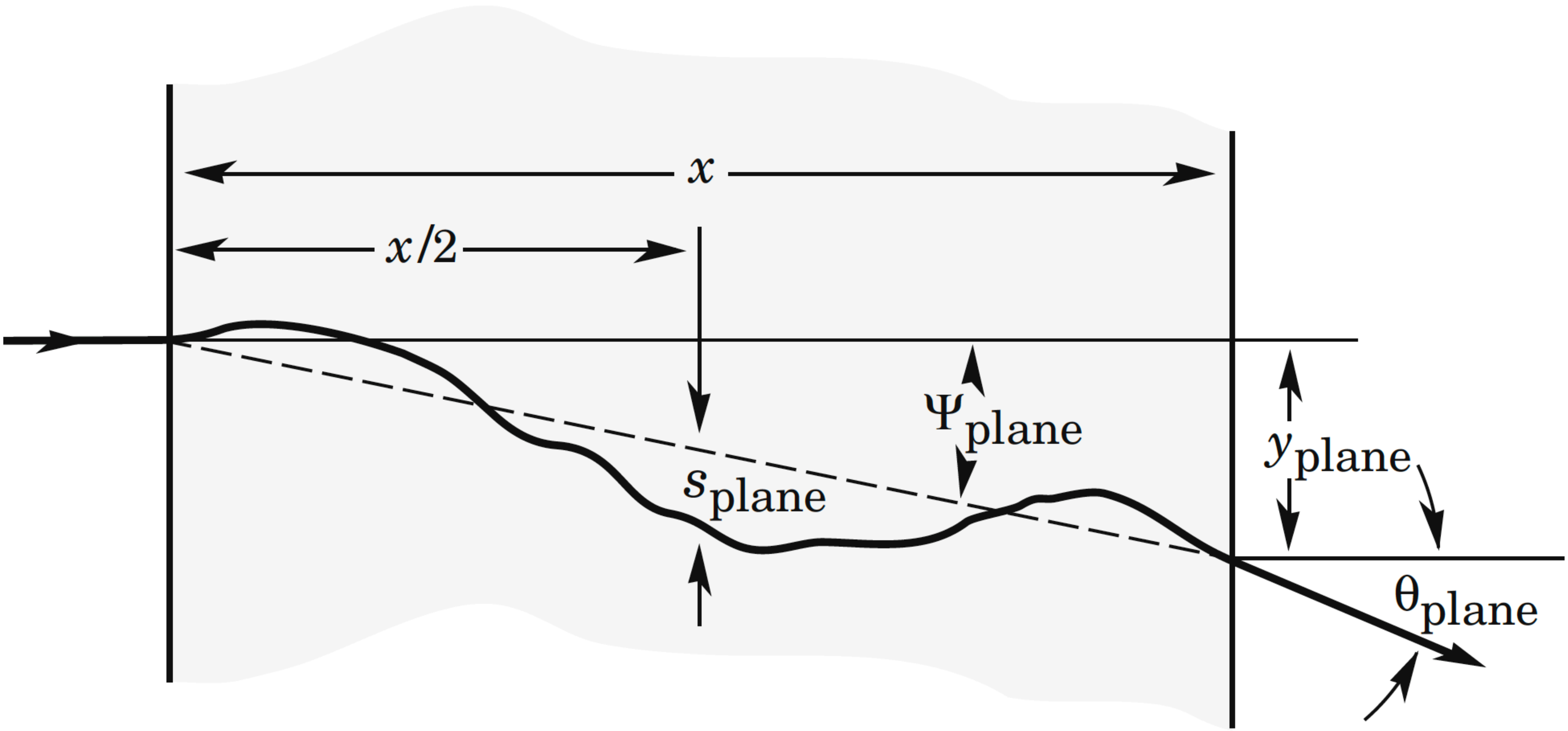
$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{Rutherford}} = z^2 Z^2 \alpha^2 \hbar^2 \frac{1}{\beta^2 p^2} \frac{1}{4\sin^4 \theta/2}$$



- the energy loss can be very small, especially if the scattered particle has small mass wrt the nucleus

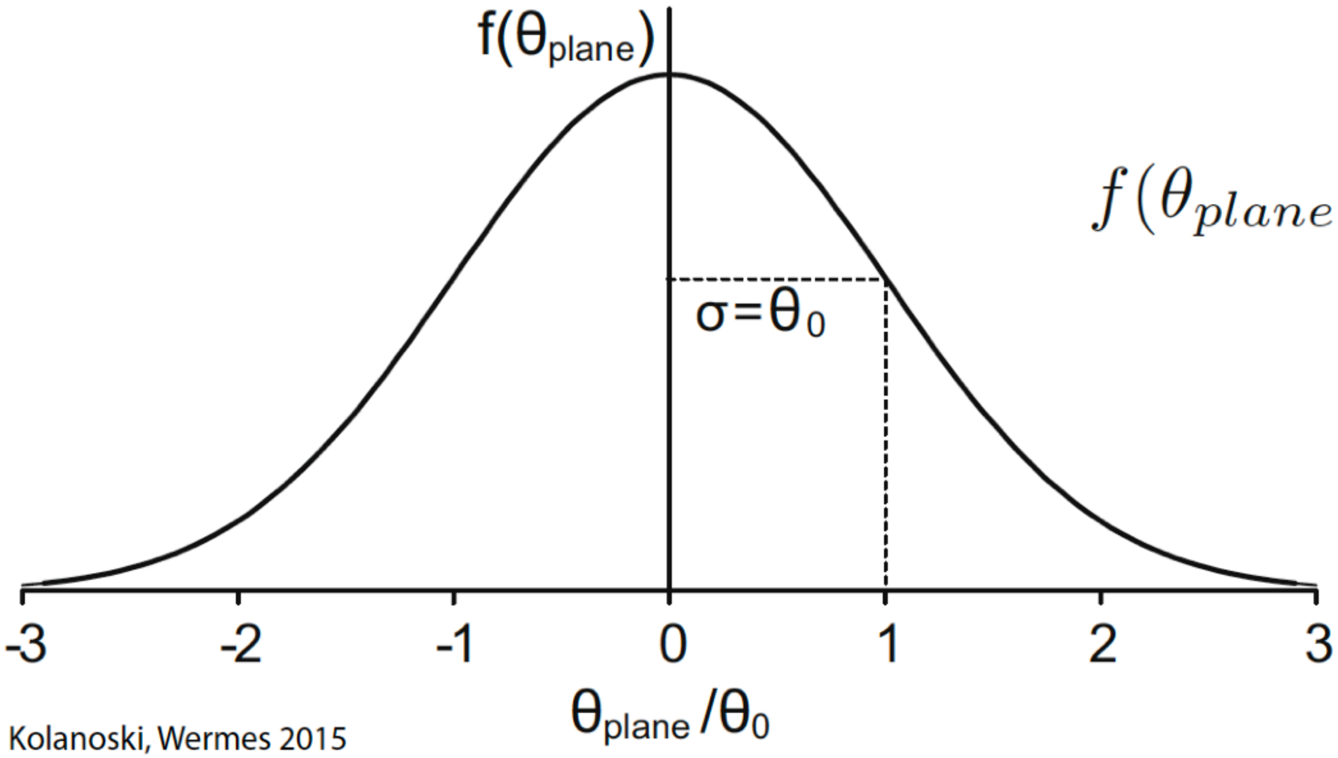
Multiple scattering

- projection into plane:



Multiple scattering

- Gauss approximation of the distribution of the scattering angle:



$$f(\theta_{plane})d\theta_{plane} = \frac{1}{\sqrt{2\pi}\,\theta_0}\exp\left(-\frac{\theta^2_{plane}}{2\theta_0^2}\right)d\theta_{plane}$$

- Rossi/Greisen approximation :

$$\theta_0 \approx \sqrt{\langle\theta^2_{plane}\rangle} \approx \frac{E_s}{\sqrt{2}\,p\,c\,\beta}\sqrt{\frac{x}{X_0}}$$

- with $E_s = m_e\,c^2\sqrt{\frac{4\pi}{\alpha}} = 21.2\,\mathrm{MeV}$

- Highland approximation:

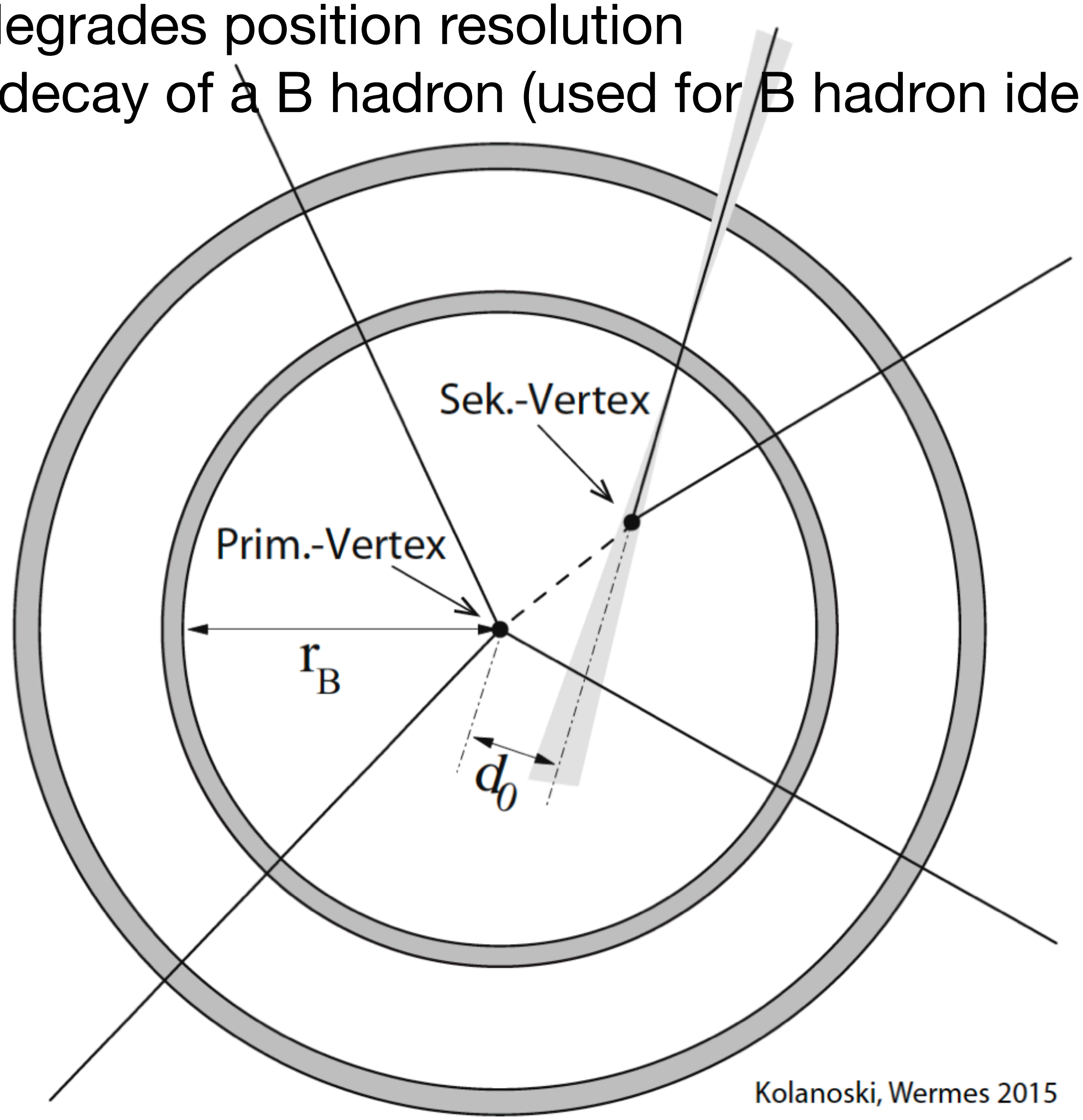
$$\theta_0 = \frac{13.6\,\mathrm{MeV/c}}{p\beta}z\sqrt{\frac{x}{X_0}}\left(1 + 0.038\ln\frac{x}{X_0}\right)$$

- for simulation, the material is treated as scattering slices to track the trajectory:

$$\langle\psi_{plane}\rangle = \frac{1}{\sqrt{3}}\theta_0\,, \qquad \langle y_{plane}\rangle = \frac{1}{\sqrt{3}}x\,\theta_0\,, \qquad \langle s_{plane}\rangle = \frac{1}{4\sqrt{3}}x\,\theta_0$$

Multiple scattering: application

- multiple scattering degrades position resolution
- example: displaced decay of a B hadron (used for B hadron identification):



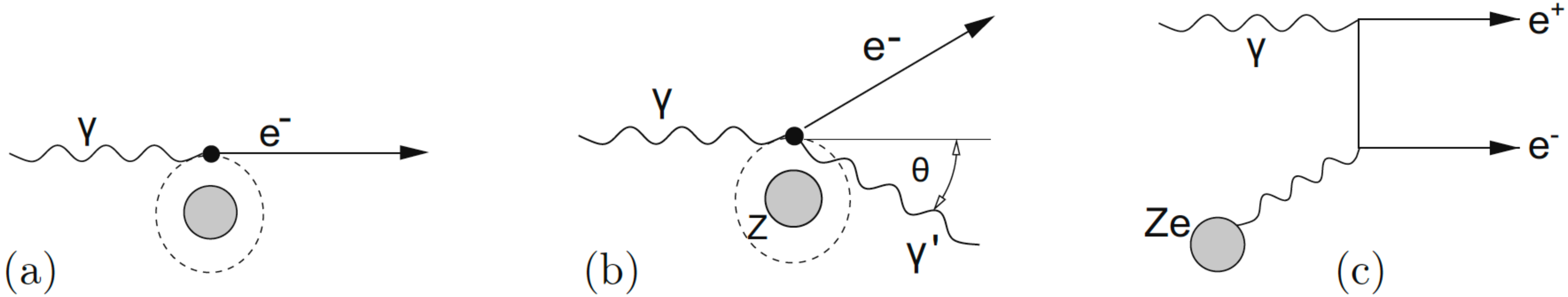
Error of the impact parameter d_0 of a 5 GeV pion for a beam pipe of thickness 1mm:

Material	X_0 [mm]	x/X_0	θ_0 [rad]	Δd_0 [µm]
Al	89	0.011	0.0003	15.0
Be	353	0.003	0.00015	7.5

photon interactions

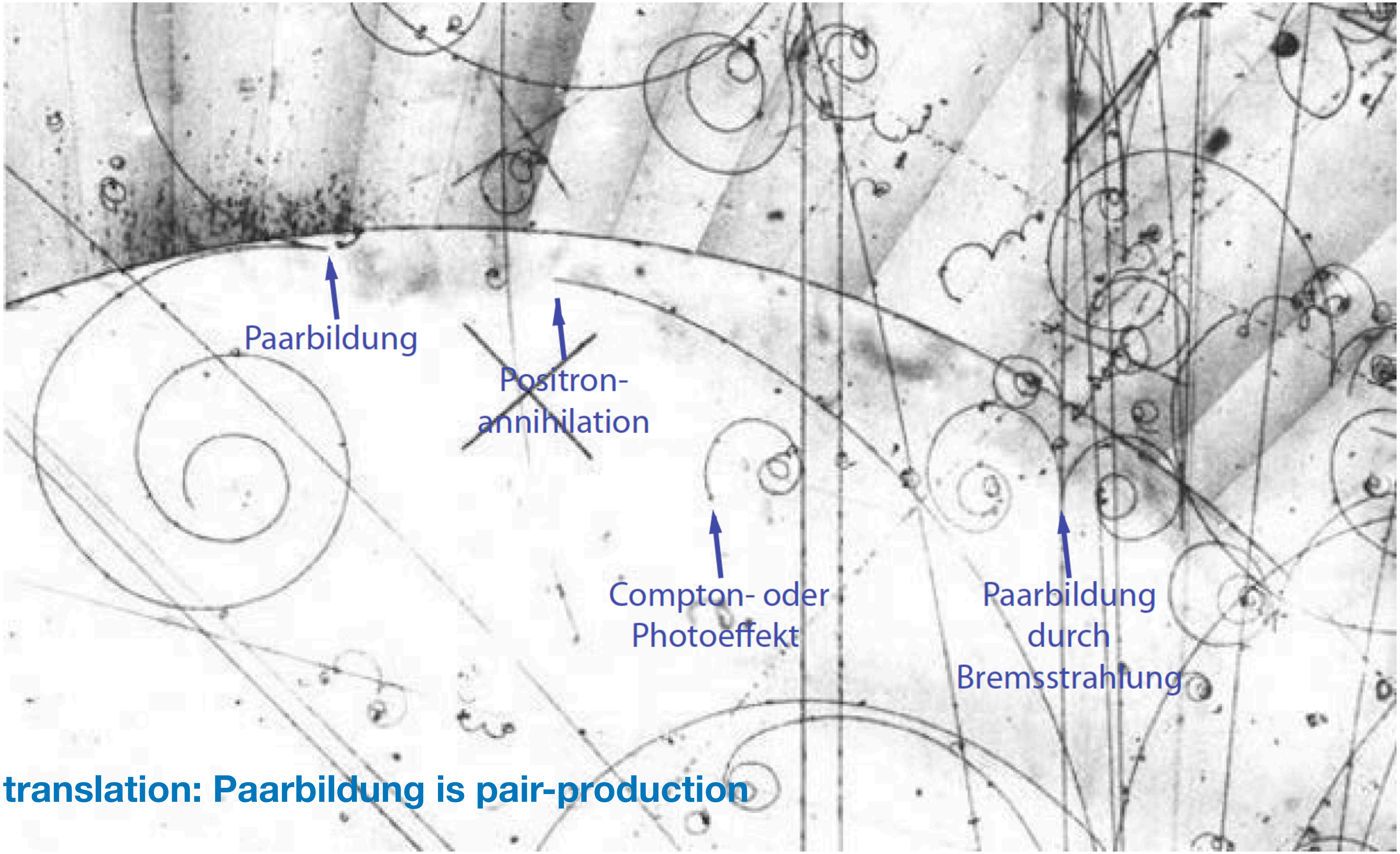
Photon interactions

- interaction of photons with matter
  - **photo effect**: a photon transmits all of its energy to the shell of an atom
  - **Compton effect**: a photon is scattered elastically at the shell of an atom
  - **pair-production**: a photon converts into e^+e^- pairs in the field of the atom



Kolanoski, Wermes 2015

- at lower energies (below ionisation threshold), there is also Thomson and Rayleigh scattering, which is coherent scattering at all shell electrons without energy deposit, these are therefore not relevant for particle detection



translation: Paarbildung is pair-production

- photo from a bubble chamber (15-foot bubble chamber, Fermilab)
- the chamber is filled with Helium-Neon mixture, charged particles leave visible tracks through ionisation (Bethe-Bloch)
- magnetic field causes clockwise curvature for negative particles
- the spirals are mainly electrons and positrons (the spirals change curvature because of energy loss through ionisation, this shows the direction of travel)
- photons themselves are not visible (they don't leave tracks in the material)

Photon interactions

- general: photon absorption rate is proportional to photon beam intensity

➡photon absorption or scattering follows an exponential law as function of depth

$$N(x) = N_0\, e^{-\mu x}$$

where *N* is the number of photons in a “beam”
and *μ* is the “absorption coefficient”

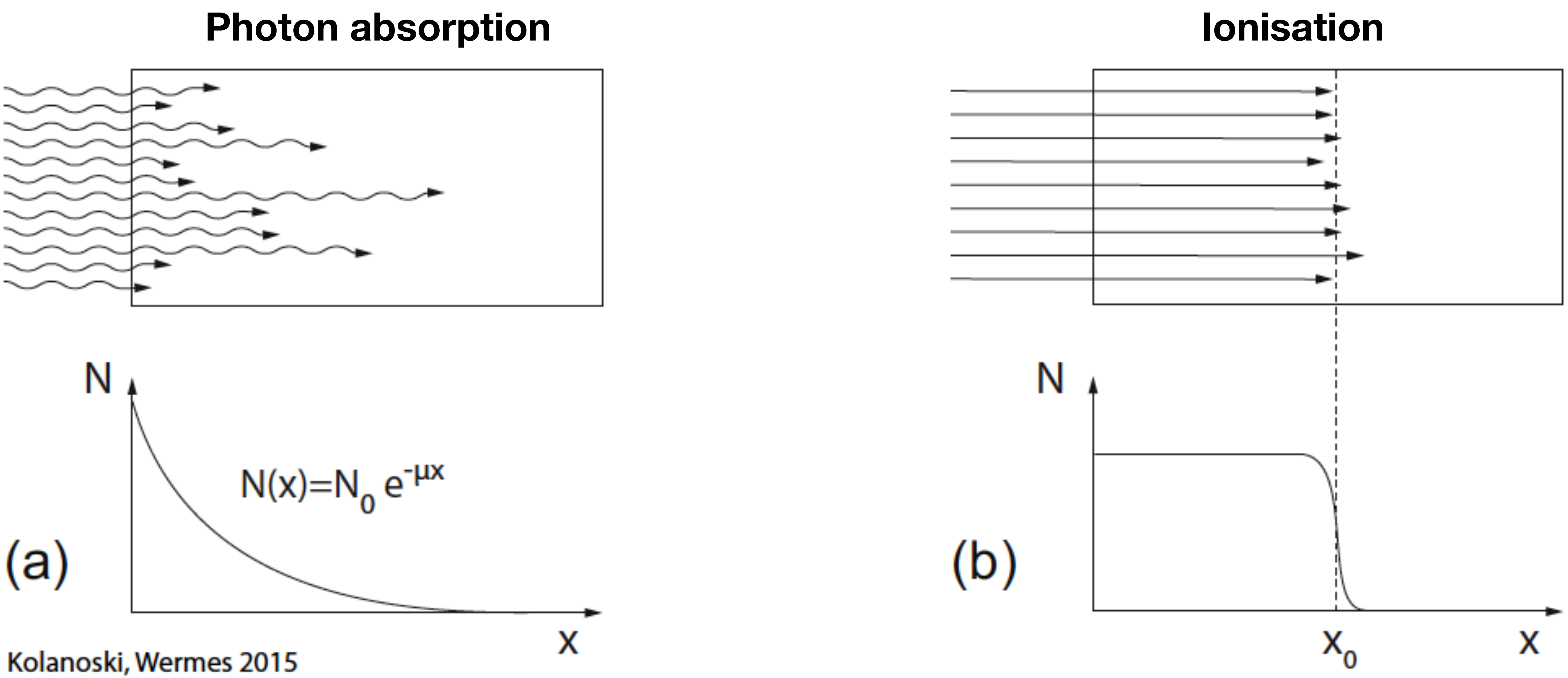
- note that *μ* depends on the energy through the cross section

$$\mu = \rho\, \frac{N_A}{A}\, \sigma = n\, \sigma$$

- this is very different from ionisation!

- the cross section depends on what process actually happens

Photon interactions



- attention: this is the absorption depth. In case of very high energy photons, secondary processes or even long cascades can occur (see later)

Photo Effect

- photo effect:

$$\gamma + \text{Atom} \longrightarrow (\text{Atom})^+ + e^-$$

  - kinetic energy of emitted electron: $T = E_\gamma - E_B$
  - calculation of the cross-section is super difficult (not discussed here)
  - can be measured and is tabulated

  - the resulting holes in the electron shell are filled by shell electrons from higher energy states

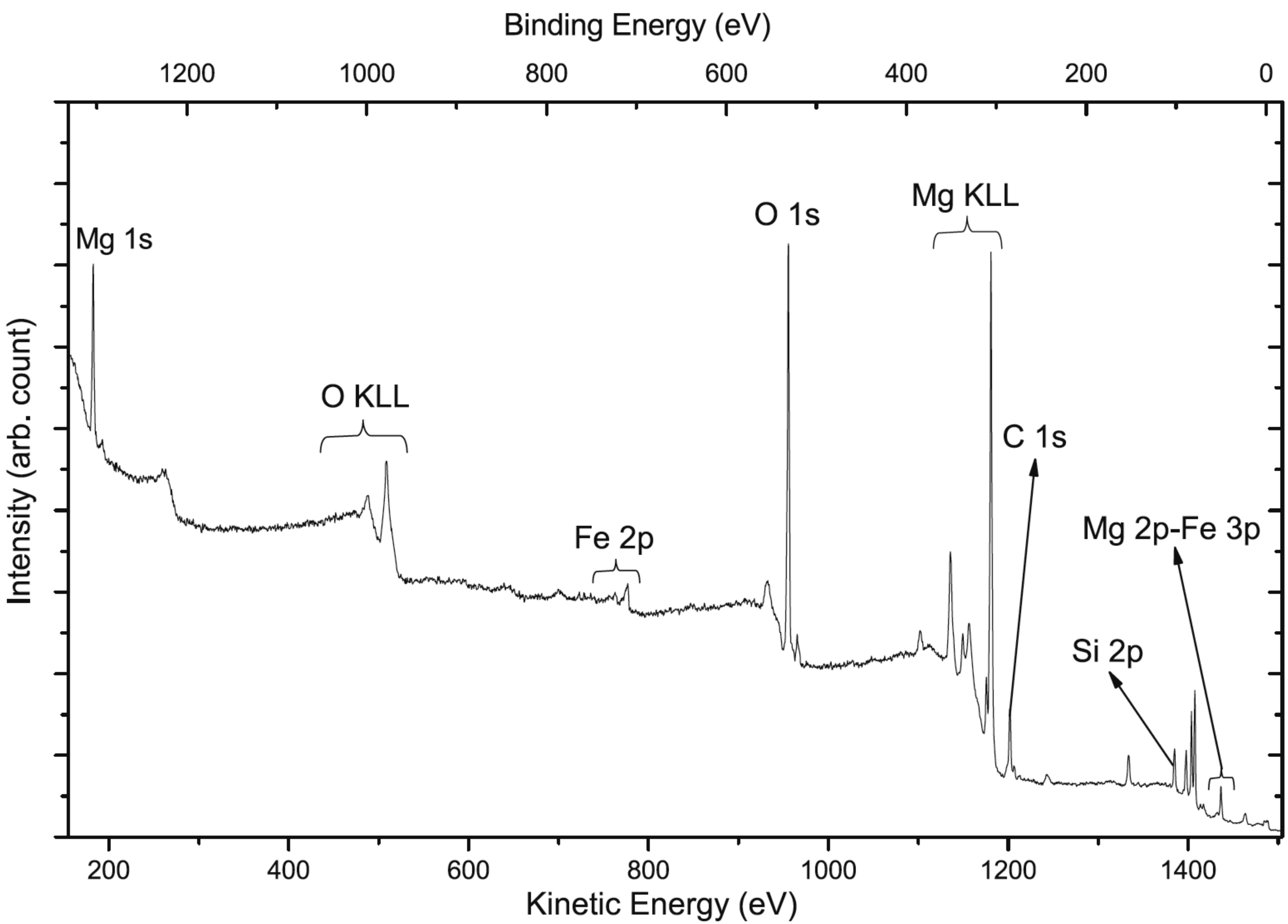
  - the energy released by these electron transitions is also emitted through photons and electrons (Auger electrons)

  - this is mostly discrete emission (see example next slide)

  - this is for example used for precise material characterisation of surfaces

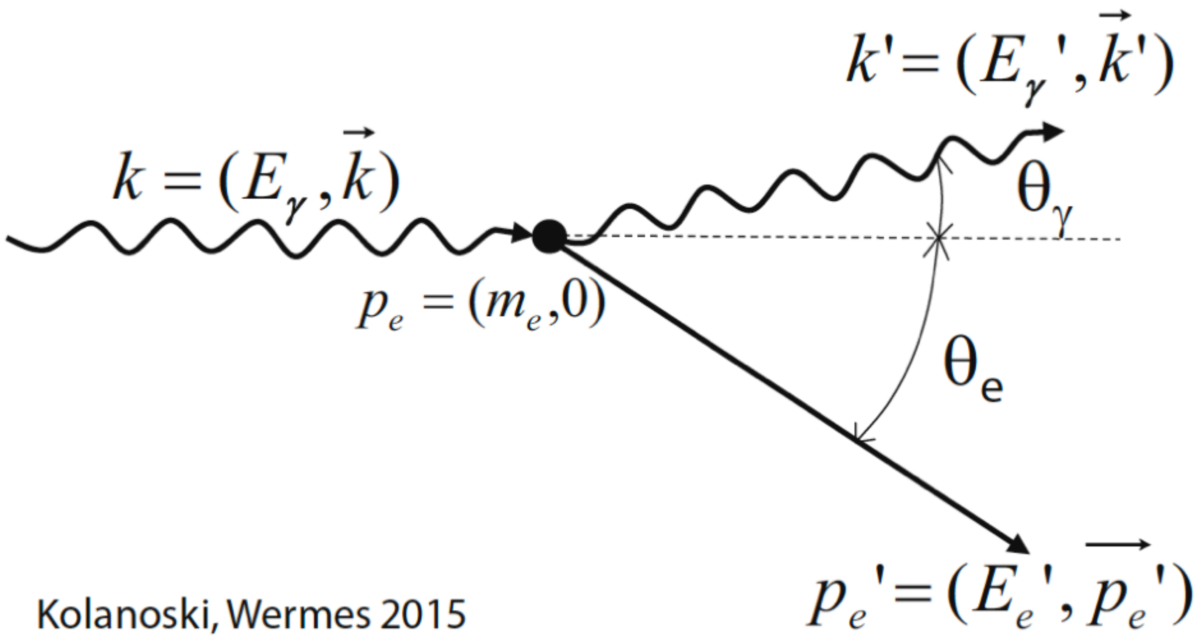
Photo Effect

- monochromatic (1486.6 eV) x-ray source incident on Olivino $(Mg, Fe)_2SiO_4$ surface



- the peaks correspond to the different elements and binding states

Compton Scattering



Kolanoski, Wermes 2015

- relativistic kinematics:

$$E'_\gamma = \frac{E_\gamma}{1+\epsilon\,(1-\cos\theta_\gamma)}$$

with $\epsilon = E_\gamma / m_e c^2$

- cross-section to be calculated with QED (Klein-Nishina formula):

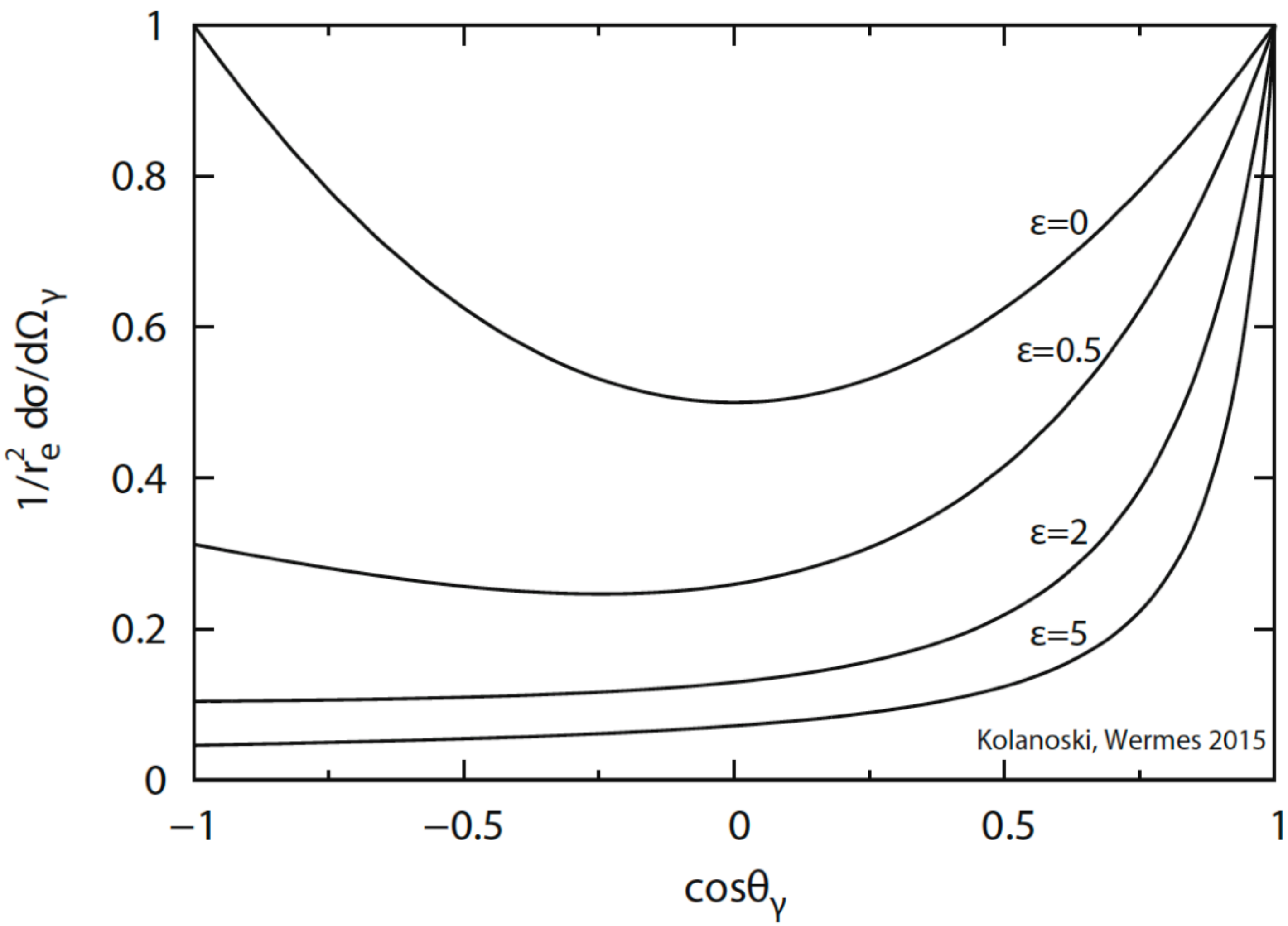
$$\frac{d\sigma_C}{d\Omega_\gamma} = \frac{r_e^2}{2\,[1+\epsilon(1-\cos\theta_\gamma)]^2}\left(1+\cos^2\theta_\gamma + \frac{\epsilon^2(1-\cos\theta_\gamma)^2}{1+\epsilon(1-\cos\theta_\gamma)}\right)$$

(for small energies identical to classical Thomson scattering, see next slide)

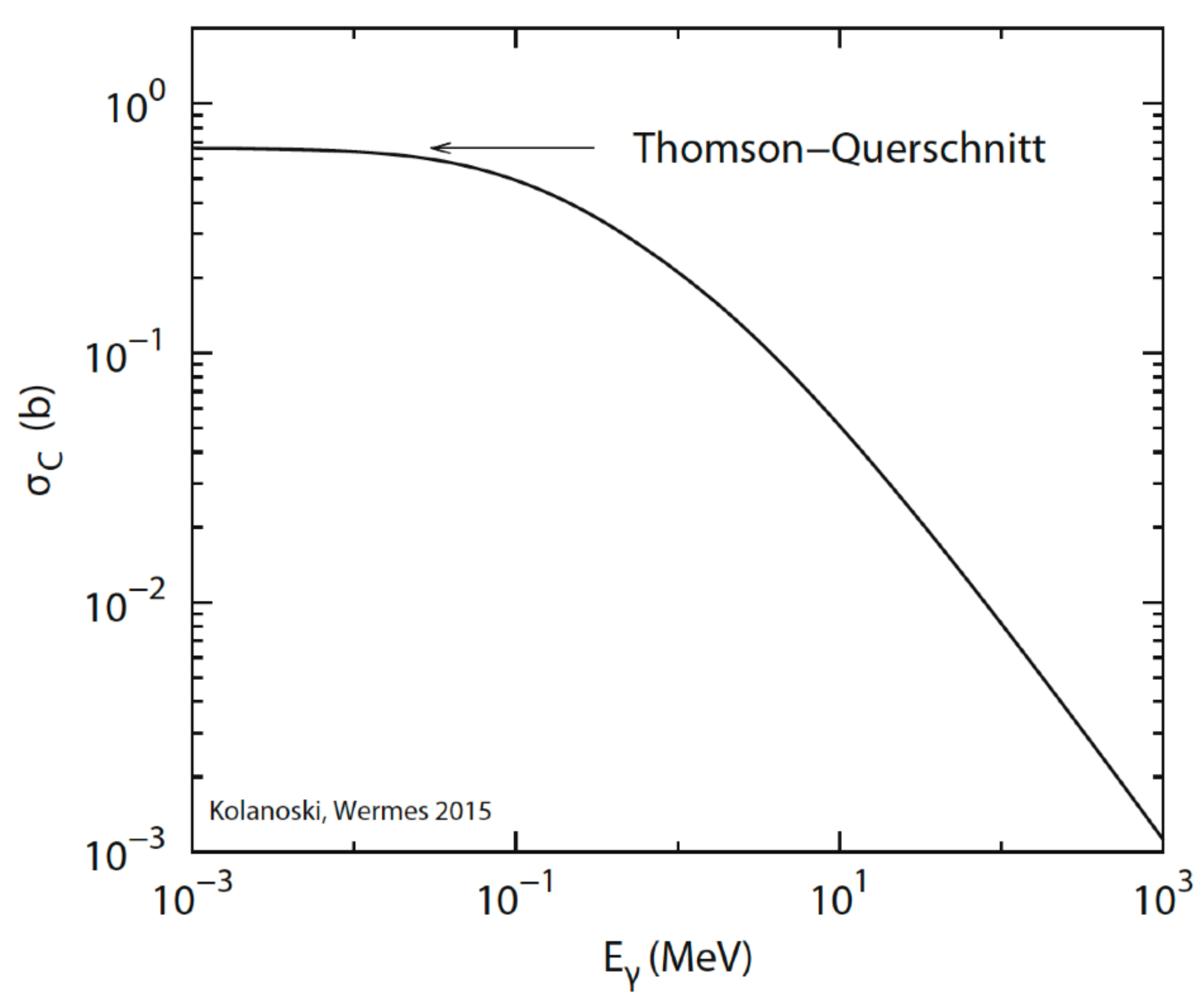
- valid for energies that are large compared to binding energy (quasi-free electrons)

Compton Scattering

- differential x-section:

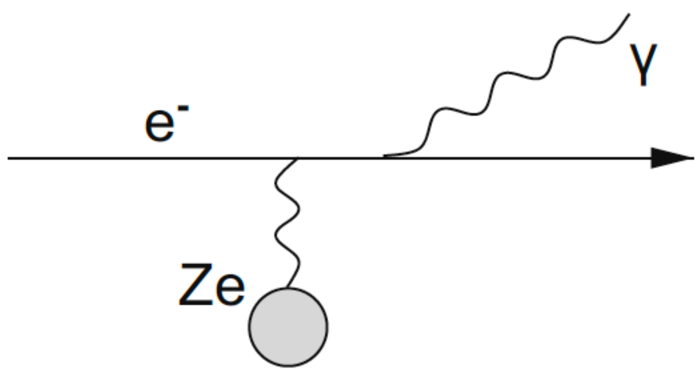


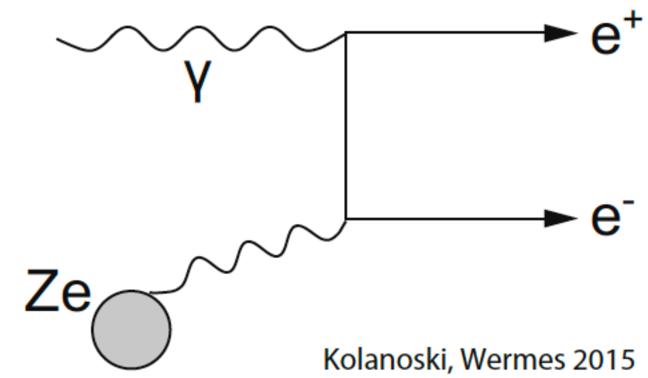
- total x-section:



- the electron recoil can be measured in detector (electric signal)

Pair production





- Bremsstrahlung

- pair-production

- pair-production is related to Bremsstrahlung (replace outgoing with incoming photon and incoming electron with outgoing positron)
- can be treated with similar mathematics as Bremsstrahlung (the processes are called “Bethe-Heitler” processes

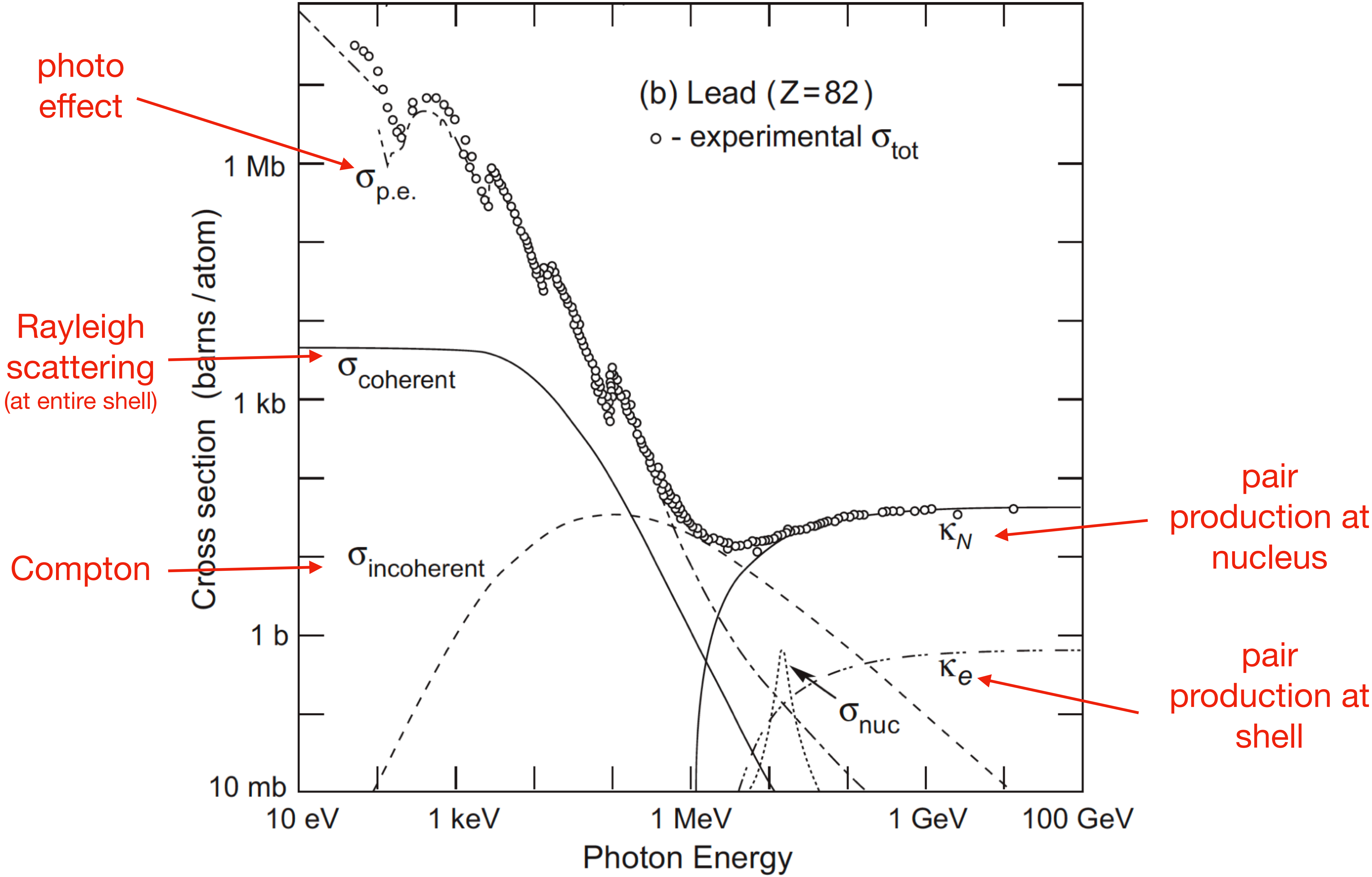
- the cross-section can be expressed in the high-energy approximation as

$$\sigma_{Paar} \approx \frac{7}{9} \frac{1}{X_0} \frac{A}{N_A \rho}$$

- it is independent of the energy !

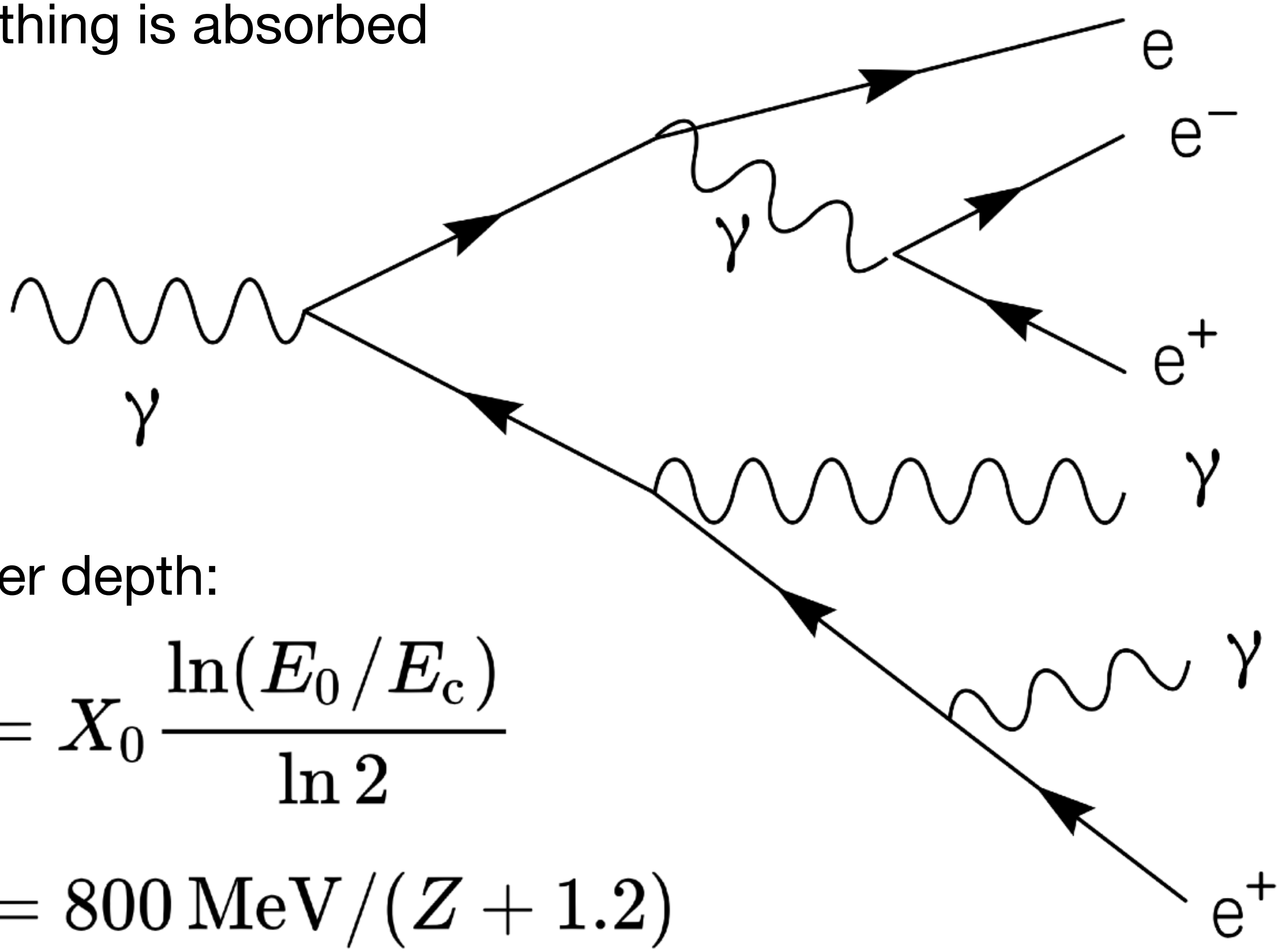
Photon interactions

- example: all processes for scattering of photons at lead



Photon interactions: showers

- high-energy photons can create long cascades of interactions until everything is absorbed



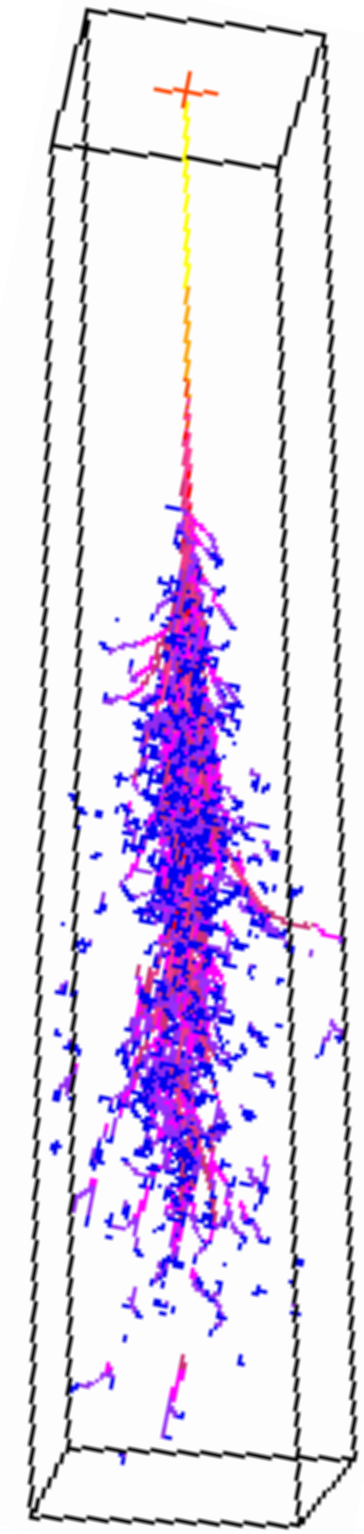
- shower depth:

$$X = X_0 \frac{\ln(E_0/E_c)}{\ln 2}$$

$$E_c = 800\,\mathrm{MeV}/(Z + 1.2)$$

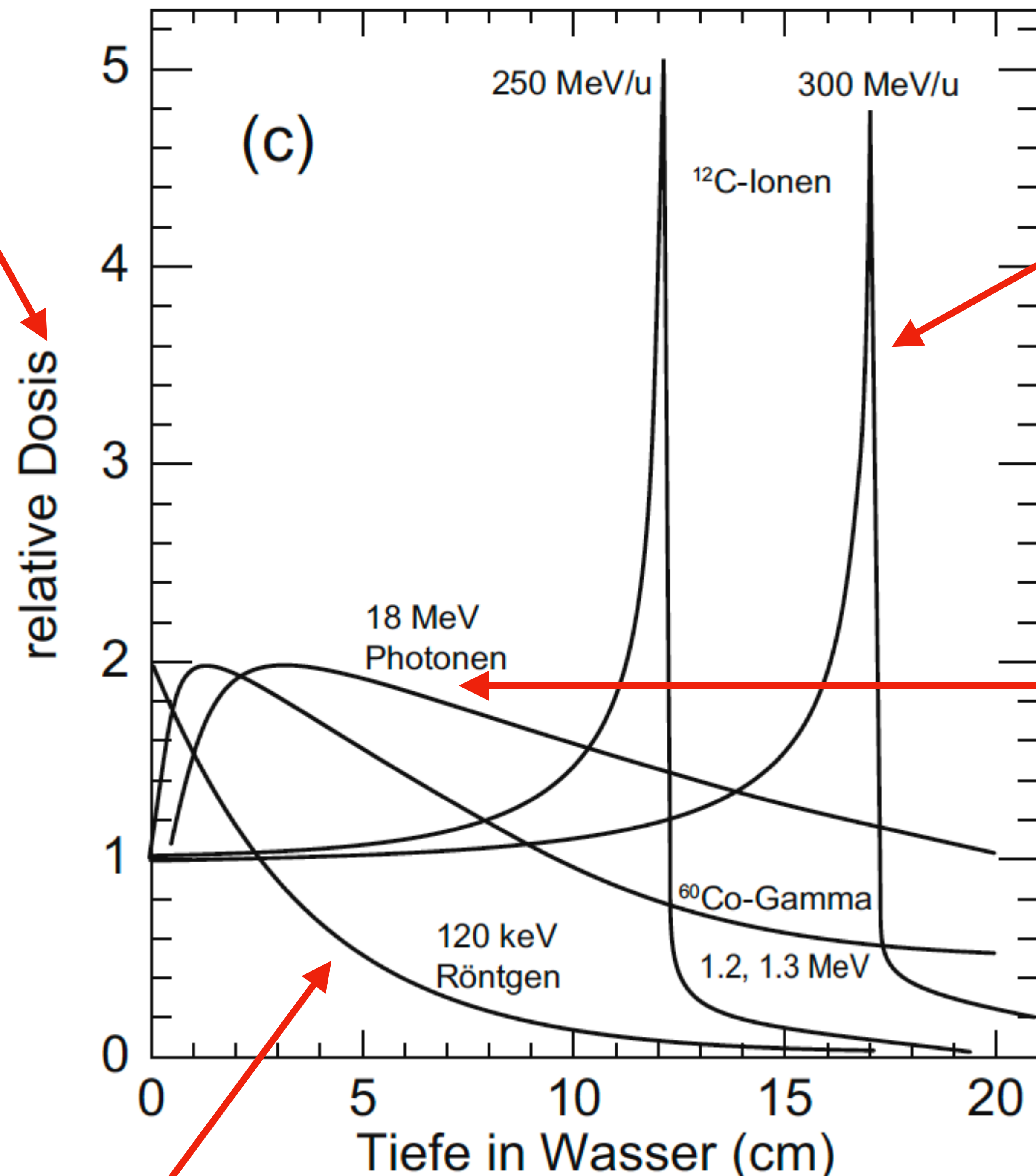
- transverse size of the shower is characterized by Moliere radius:

$$R_m = \frac{21\,\mathrm{MeV}}{E_C} X_0$$



Photon interactions: reminder from last lecture

axis denotes radiation dose, not interaction depth



charged particles, Bragg peak

high energy photons (they create EM showers)

the shower starts to build up until it reaches a maximum

low energy photons (exponential law)