III. Physikalisches Institut A

RWTH AACHEN UNIVERSITY

Experimental Techniques in Particle Physics (WS 2020/2021)

Interaction of particles with Matter (Part I)

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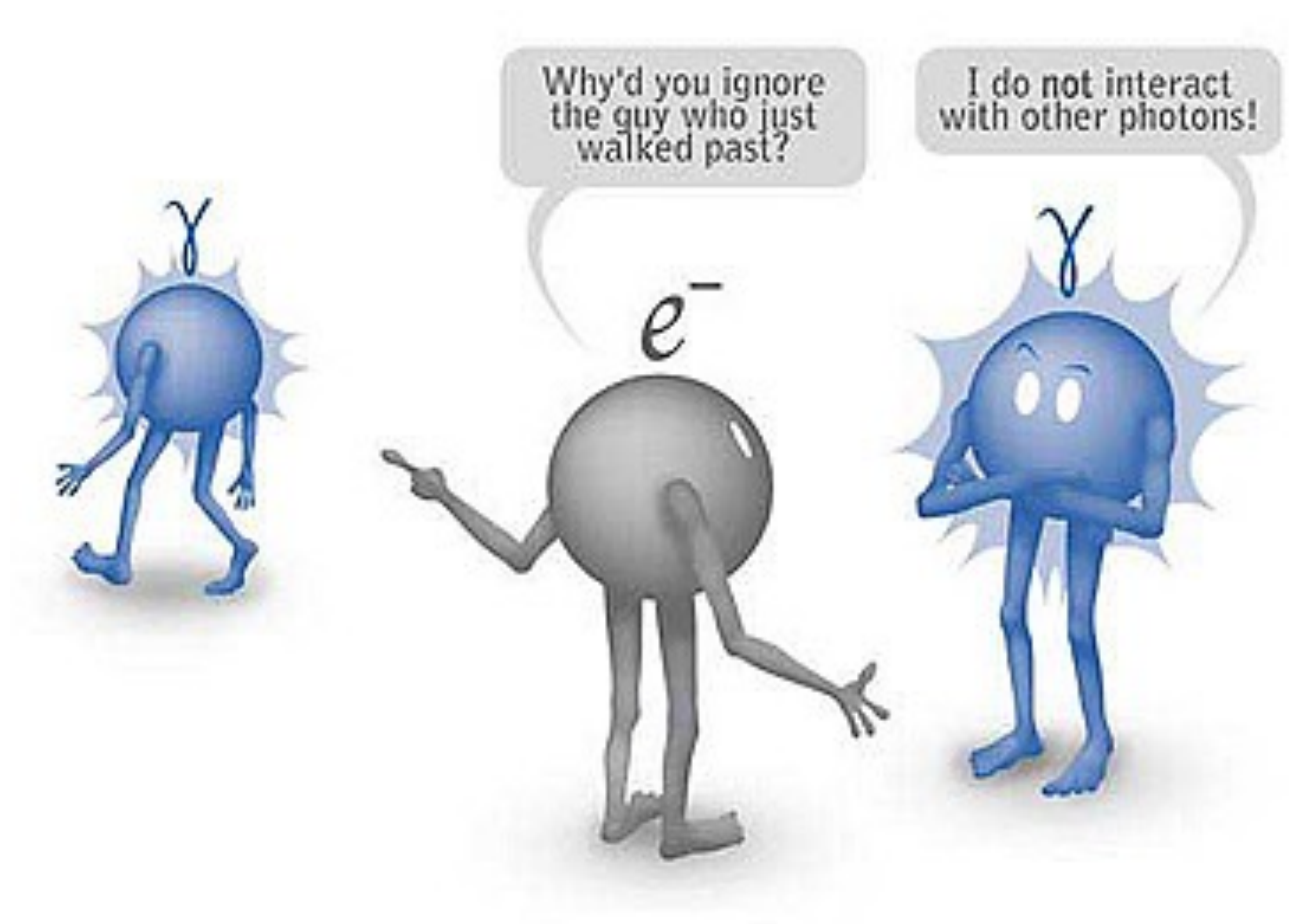
Introduction

- particles can only be detected through their interaction with “something”
- the interaction is registered by the detector, not the particle itself
- every effect of particles or radiation can be used as a working principle for a particle detector.

- one has to deduce the information about the particle from the interaction
- the detector consists of various types of material



Types of interactions relevant for particle detectors

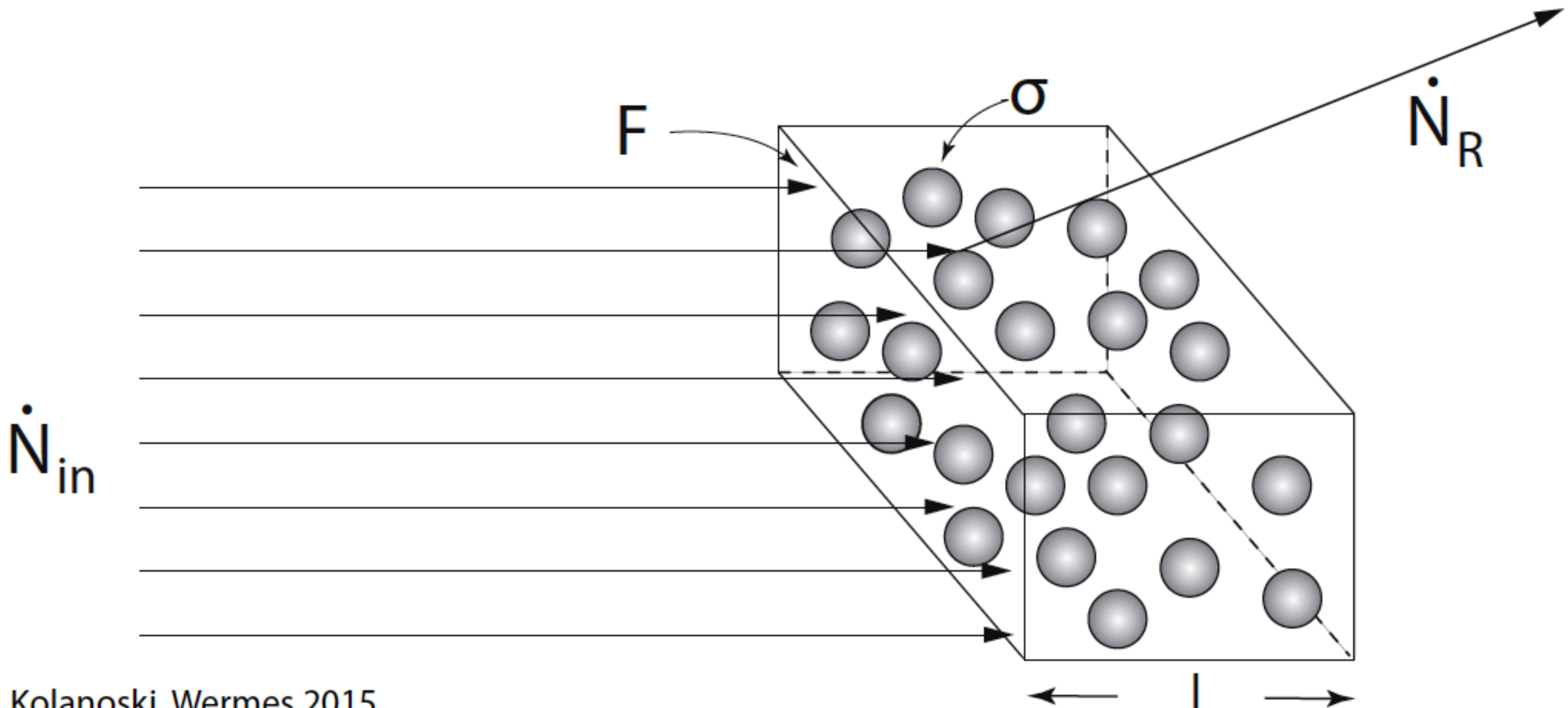
the following types of interactions between radiation and matter exist:

- ionisation and excitation of the material, caused by charged particles **(this lecture)**
- bremsstrahlung: emission of photons from accelerated charged particles **(next week)**
- photon scattering and photon absorption **(next week)**
- Cherenkov radiation **(next week)**
- nuclear interaction of imminent hadrons with nuclei in material
- weak interaction (the only detection method for neutrinos)

- usually **more than one** interaction process happens if the particle is not immediately absorbed

- In Axion detectors, the interaction of the Axion with the Electromagnetic field is used (this is a special case, which we can consider at the end of the semester in a dedicated lecture, if there is interest and time)

Cross section

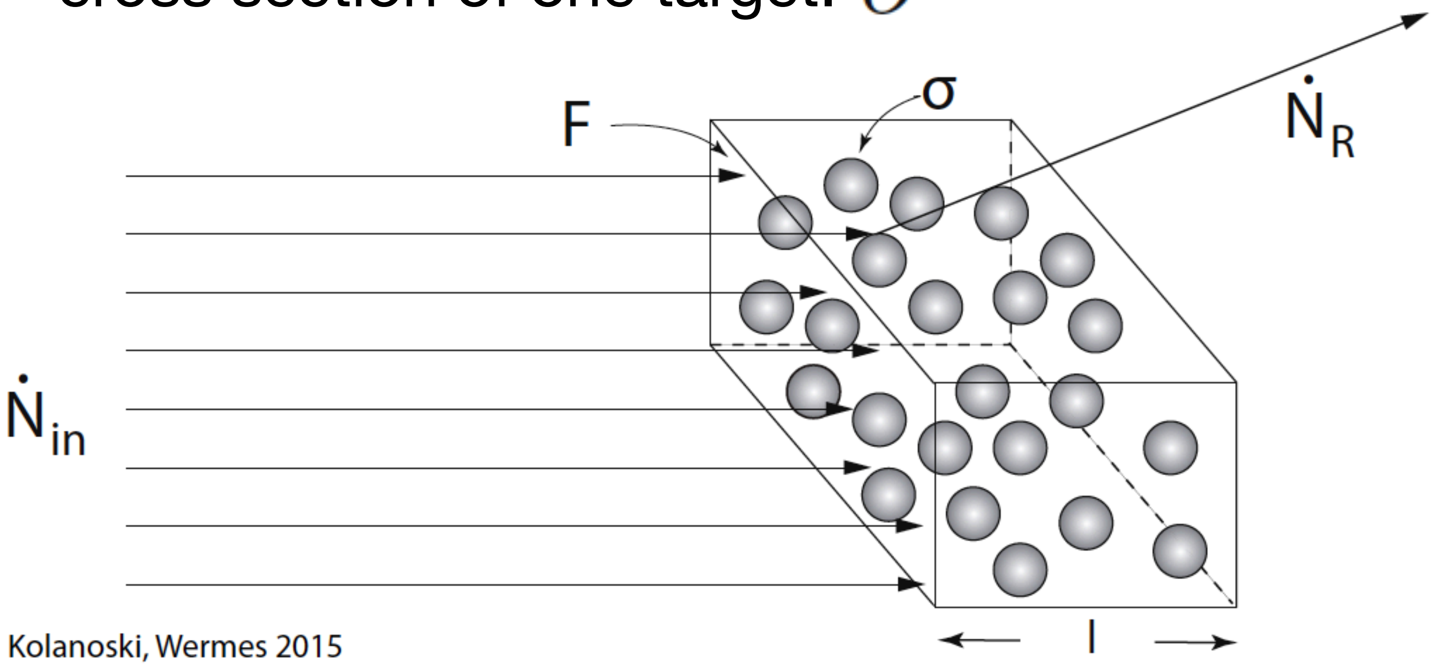


Kolanoski, Wermes 2015

Cross section

definition:

- cross section of one target: σ



Kolanoski, Wermes 2015

- area of full target section is *F*
- volume *V* contains
  $$N_T = \frac{\rho V}{M_T} N_A$$
  targets
- ρ is the target density
- M_T is the relative atomic mass
- $N_A = 6.022 \times 10^{23}/\mathrm{mol}$

- the number density is then $n = \dfrac{N_T}{V}$
- probability to hit a target $w = N_T \, \sigma / F$
- for thin targets: $w = \dfrac{\dot{N}_R}{\dot{N}_{in}}$
- it follows $w = \dfrac{\dot{N}_R}{\dot{N}_{in}} = \dfrac{N_T \, \sigma}{F} = n \, \sigma \, l$

$$\boxed{\sigma = \frac{\dot{N}_R}{\dot{N}_{in}} \frac{1}{n \, l}}$$

Bethe Bloch calculation

- now, when particles scatter, they don't just bounce off elastically. They lose energy and leave that energy in the material. This is the next topic.

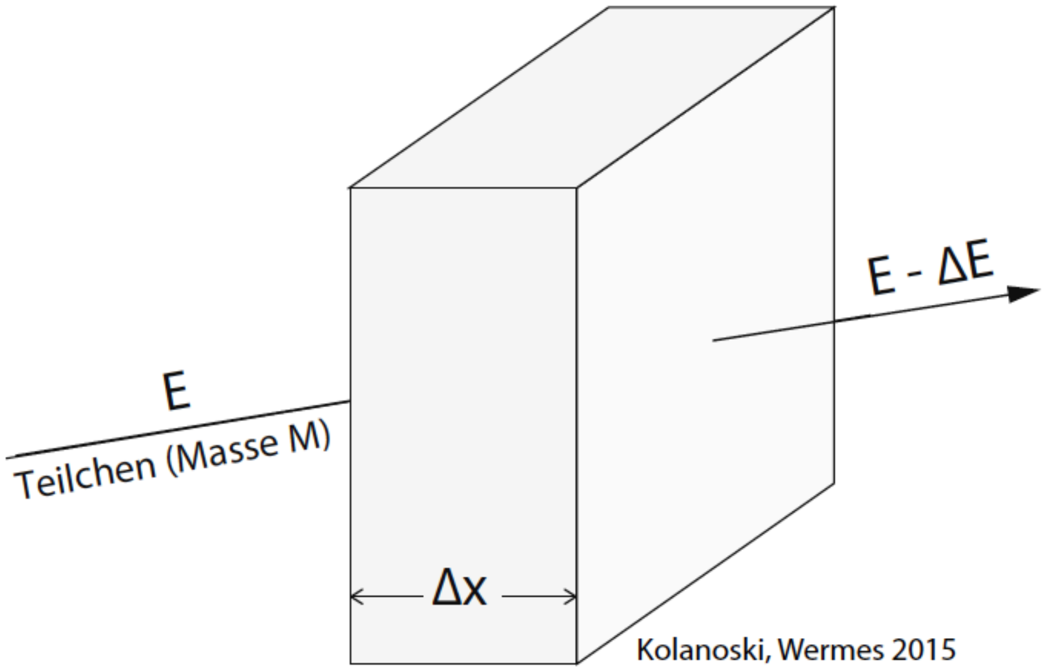
- **the derivation of Bethe-Bloch formula (energy loss through ionisation for charged particles)**
  can be found in many Textbooks, for example Kolanoski/Wermes "Teilchendetektoren", which is freely available for download.

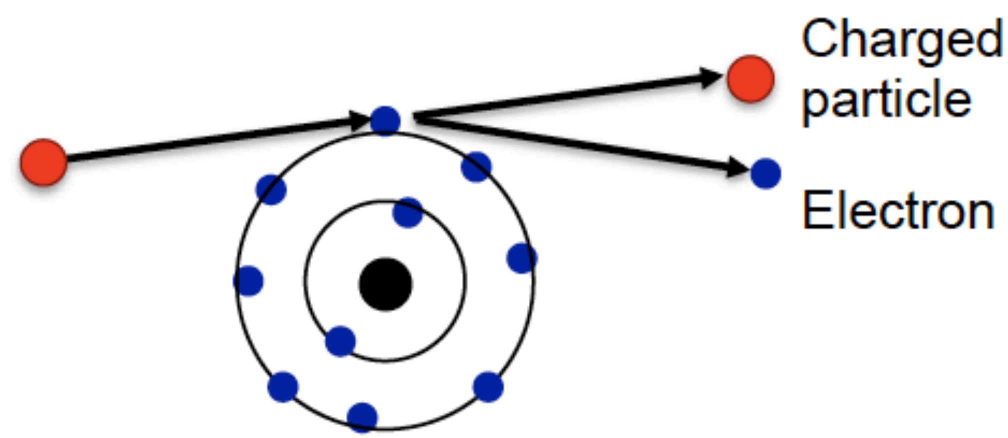
- the following gives a summary of the main steps and conclusions

Ansatz

energy loss through ionisation (scattering at electron in atomic shell):

- Bethe-Bloch formula describes the energy lost ΔE per path length Δx
- the energy ist lost through single processes occurring stochastically





- the mean energy loss *dE* per distance *dx* is

$$-\left\langle \frac{dE}{dx} \right\rangle = n \int_{T_{min}}^{T_{max}} T \, \frac{d\sigma_A}{dT}(M, \beta, T) \, dT$$

it depends on the properties of the medium *A*, the mass of the incident particle *M* and the velocity β.

$\frac{d\sigma_A}{dT}$ is the differential cross section for the loss of kinetic energy *T* in a collision

The difficult part is the calculation of $\frac{d\sigma_A}{dT}$

Bethe Bloch calculation

- the process can be described by scattering in Coulomb field of a single electron in the shell of an atom.
- the process is described by Rutherford scattering

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\, z^2\alpha^2\hbar^2 c^2}{\beta^2}\frac{1}{Q^4}$$

z is the charge of the projectile
M is the mass of the projectile
P, P' are the four vectors of the projectile (before, after scattering)
Q^2 is the squared momentum transfer (conservation of energy/momentum)
T is the kinetic energy of the electron after scattering

- with $Q^2 = -(P - P')^2 = -(p_e - p'_e)^2$ (relativistic kinematics)

$$= 2m_e c^2 T$$

- we obtain

$$\frac{d\sigma}{dT} = \frac{2\pi\, z^2\alpha^2\hbar^2}{\beta^2\, m_e}\frac{1}{T^2}$$

Bethe Bloch calculation

- extension of Rutherford scattering to Mott-scattering (including the Spin) brings an additional factor

$$\frac{d\sigma}{dT} = \frac{2\pi\, z^2 \alpha^2 \hbar^2}{\beta^2\, m_e\, T^2} \left(1 - \beta^2 \frac{T}{T_{max}}\right)$$

- insert into Bethe Bloch formula
  and calculate resulting integral with approximation $T_{min} \ll T_{max}$

$$-\left\langle \frac{dE}{dx} \right\rangle = n_e \int_{T_{min}}^{T_{max}} T\, \frac{2\pi\, z^2 \alpha^2 \hbar^2}{\beta^2\, m_e\, T^2} \left(1 - \beta^2 \frac{T}{T_{max}}\right) dT$$

$$= \frac{2\pi\, z^2 \alpha^2 \hbar^2}{\beta^2\, m_e}\, n_e \left(\ln \frac{T_{max}}{T_{min}} - \beta^2\right) .$$

[n_e is the electron density of the target (the target density)]

Bethe Bloch calculation

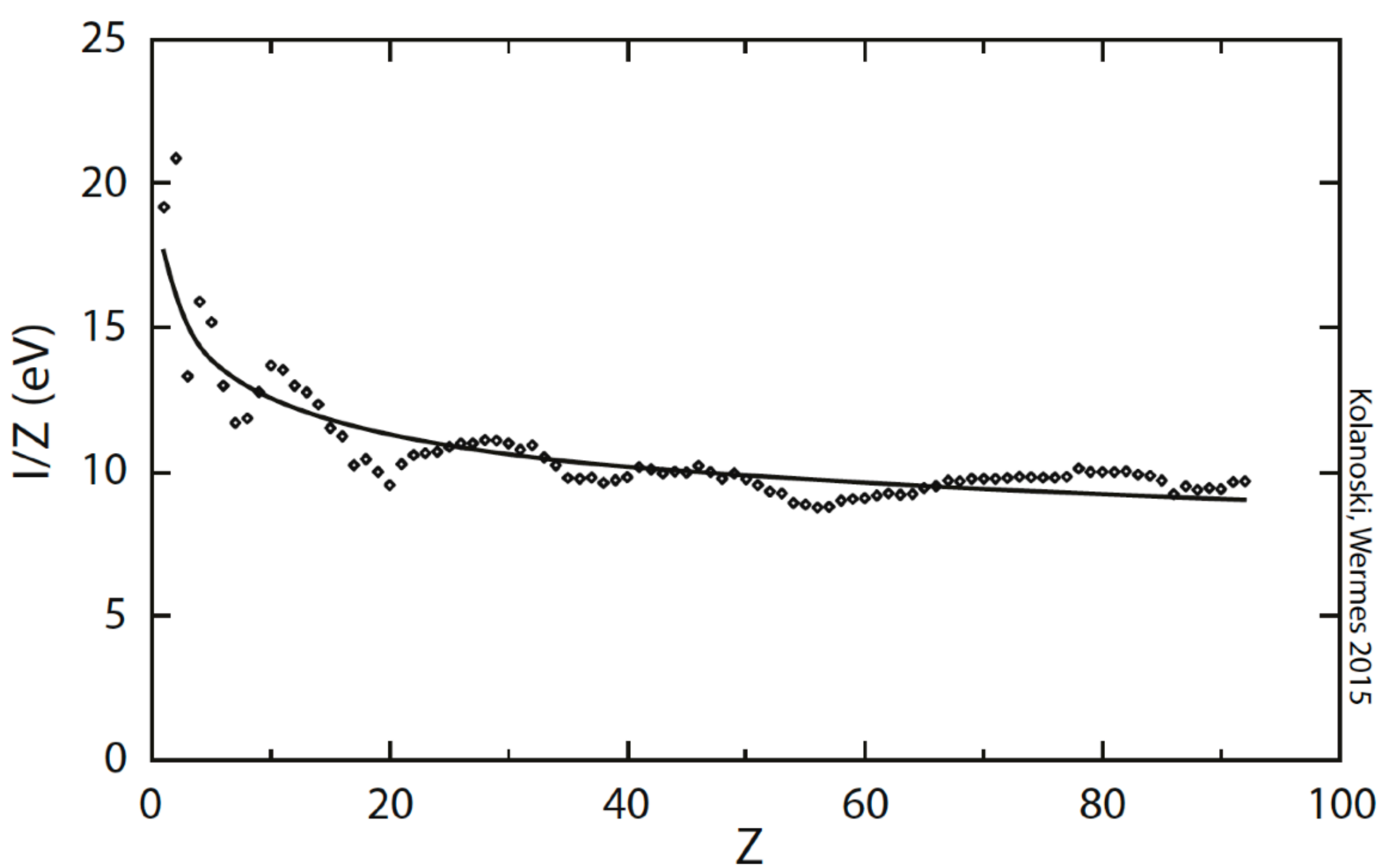
- T_{max} can be calculated from kinematics of elastic scattering (approximation with electron at rest compared to incoming particle)
- maximum energy transfer happens for central scatter, when all vectors *P*, *P'*, p'_e are parallel
- incoming particle: $E = \gamma M c^2, \qquad |\vec{P}| = \beta\,\gamma\,M c\,.$
- outgoing electron kinetic energy: $T = E'_e - m_e c^2$
- (some relativistic kinematics calculations) $T_{max} = \dfrac{2\,m_e\,c^2\,\beta^2\,\gamma^2}{1 + 2\,\gamma\,m_e/M + (m_e/M)^2}$
- result for special cases

$$T_{max} \approx \begin{cases} 2\,m_e\,c^2\,(\beta\gamma)^2 & \text{für} \quad \gamma m_e \ll M \\ \gamma\,M\,c^2 \;=\; E & \text{für} \quad \gamma \to \infty \qquad \text{(ultra-relativistic)} \\ m_e c^2(\gamma - 1) \;=\; E - m_e c^2 & \text{für} \quad M = m_e\,. \qquad \text{(incoming electron)} \end{cases}$$

- note that **all** kinetic energy is transferred in the ultra-relativistic case, and in the case M=m_e

Bethe Bloch calculation

- T_{min} is more more difficult to calculate, because
  - classical limit with $T_{min} = 0$ is not valid, because discrete states in QM
  - incoming particle may be slow or similar velocity as electron in atomic shell (approximiation with electron at rest not possible)
  - quantum mechanical interference effects
  - shielding effects of charge through atomic shell
- from literature $$T_{min} = \frac{I^2}{2\, m_e\, c^2\, \beta^2\, \gamma^2}$$
- where I is a mean excitation energy, which is usually not calculated but measured



Bethe Bloch calculation: result

- insert everything into Bethe Bloch formula:

$$-\left\langle \frac{dE}{dx} \right\rangle = K \, \frac{Z}{A} \, \rho \, \frac{z^2}{\beta^2} \left[\frac{1}{2} \ln \frac{2\, m_e\, c^2\, \beta^2\, \gamma^2\, T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C(\beta\gamma,\, I)}{Z} \right]$$

- with

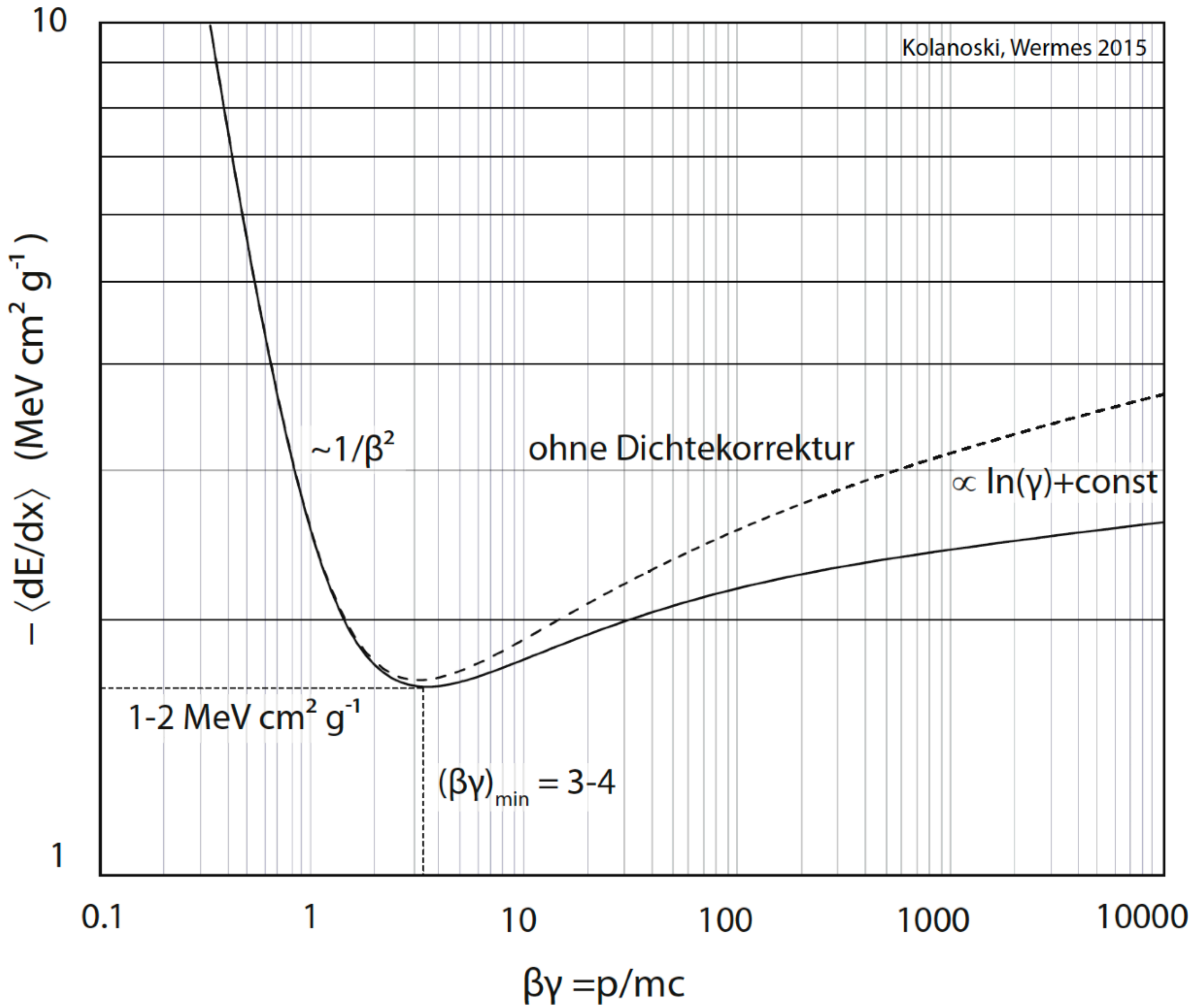
  $K = 4\pi N_A r_e^2 m_e \, c^2 = 0.307 \text{ MeV } \text{ cm}^2/\text{mol}$
  β, z are charge and velocity of the projectile
  Z, A are atomic number and atomic mass of the medium
  δ is a density correction (necessary for high energies)
  C/Z is a “shell” correction (necessary for small *β*)

- it is usually tabulated normalised to the density $\dfrac{dE}{\rho\, dx}$

  in units of $\dfrac{\text{MeV}}{\text{g cm}^{-2}}$

Bethe Bloch calculation: result

- energy dependence
- example: incoming pions on silicon



- at small energies the $1/\beta^2$ term dominates, while the ln(ɣ) term dominates at high energies
- there is a minimum (minimum ionisation)

Energy dependence

$$-\left\langle \frac{dE}{dx} \right\rangle = K \frac{Z}{A} \rho \boxed{\frac{z^2}{\beta^2} \left[\frac{1}{2} \ln \frac{2\, m_e\, c^2\, \beta^2\, \gamma^2\, T_{max}}{I^2}\right.} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C(\beta\gamma,\, I)}{Z} \left.\vphantom{\frac{1}{2}}\right]$$

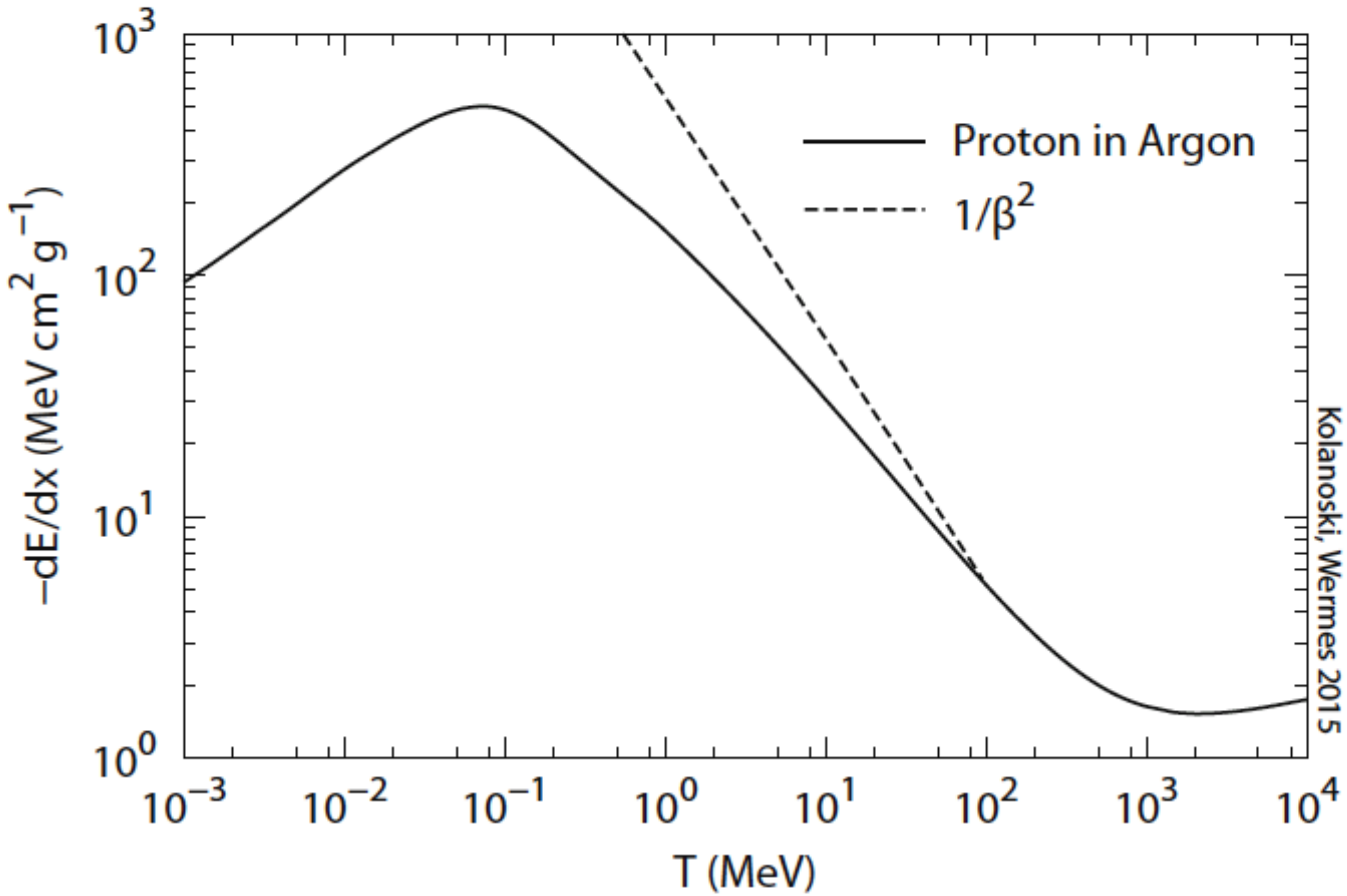
$$const. \frac{1}{\beta^2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} = const. \frac{1}{\beta^2} \left(\ln \frac{2m_e c^2 T_{max}}{I^2} + \ln \frac{\beta^2}{I^2} + \ln \frac{\gamma^2}{I^2} \right)$$

- at small energies: $\beta \rightarrow 0$ and $\gamma \rightarrow 1$, so the $1/\beta^2$ term dominates
- at high energies: $\beta \rightarrow 1$ and $\gamma \rightarrow$ infinity, so the $\ln(\gamma)$ term dominates

- the low-energy behaviour can be understood by considering the effective interaction “time” for the momentum transfer (the time the projectile spends in the field of the electron)
- for extremely low energies, the velocity of the electron becomes relevant, and shell corrections need to be applied

Energy dependence

- low-energy corrections for protons scattering in argon:



particle identification with Bethe-Bloch

Particle ID with dE/dx

- Bethe-Bloch has no explicit dependence on momentum or energy
- the momentum dependence is given through β and γ

- this means that particles with same momenta, but different masses have different β and γ

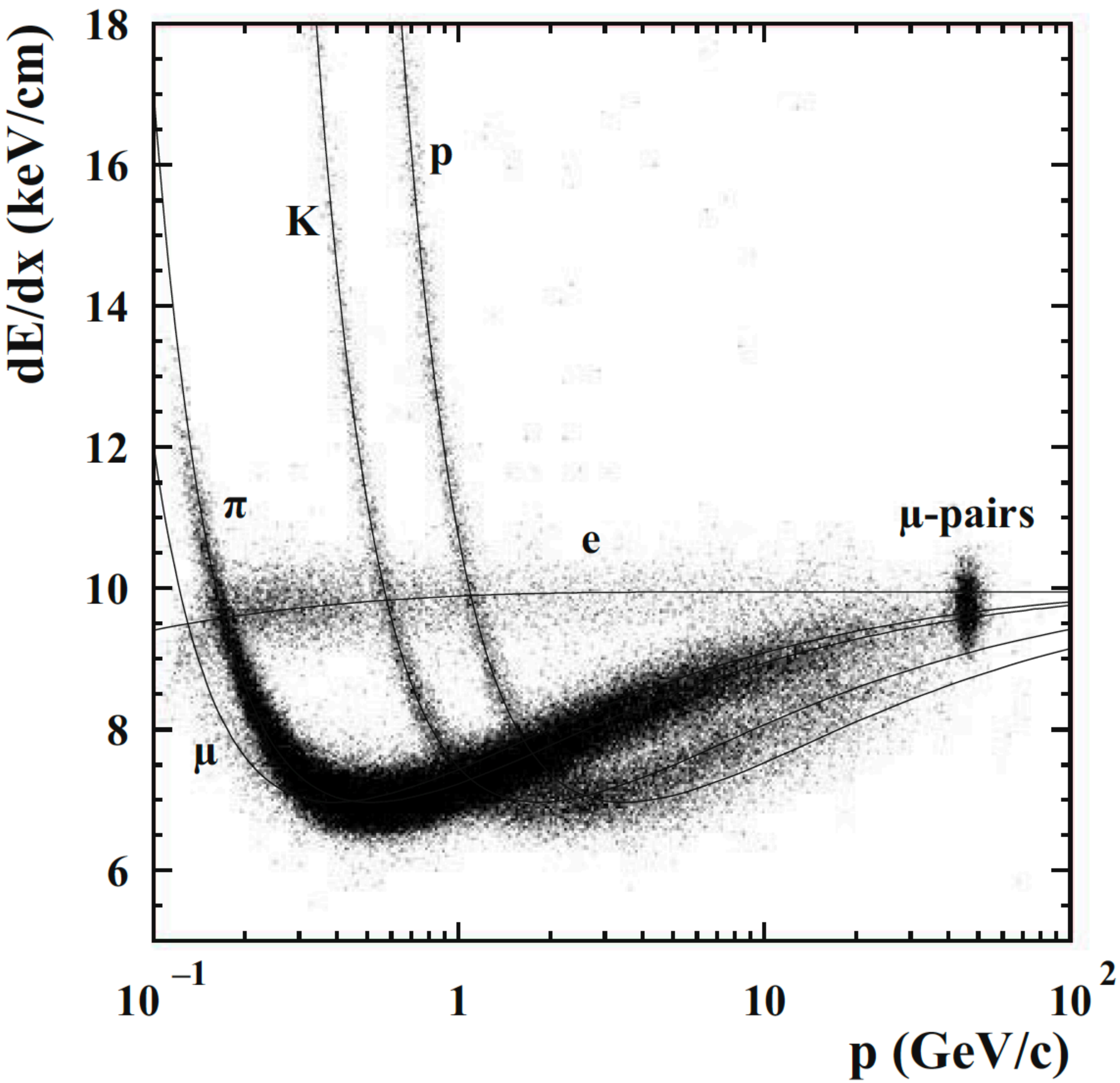
- this means that the dE/dx energy loss for particles with same momenta, but different masses is different as well

- in detectors, we often measure momenta (curvature in a magnetic field), so we don't know the particle type

→**particle types can be distinguished based on their dE/dx properties**
→**particle identification methods are based on this**

Particle ID with dE/dx

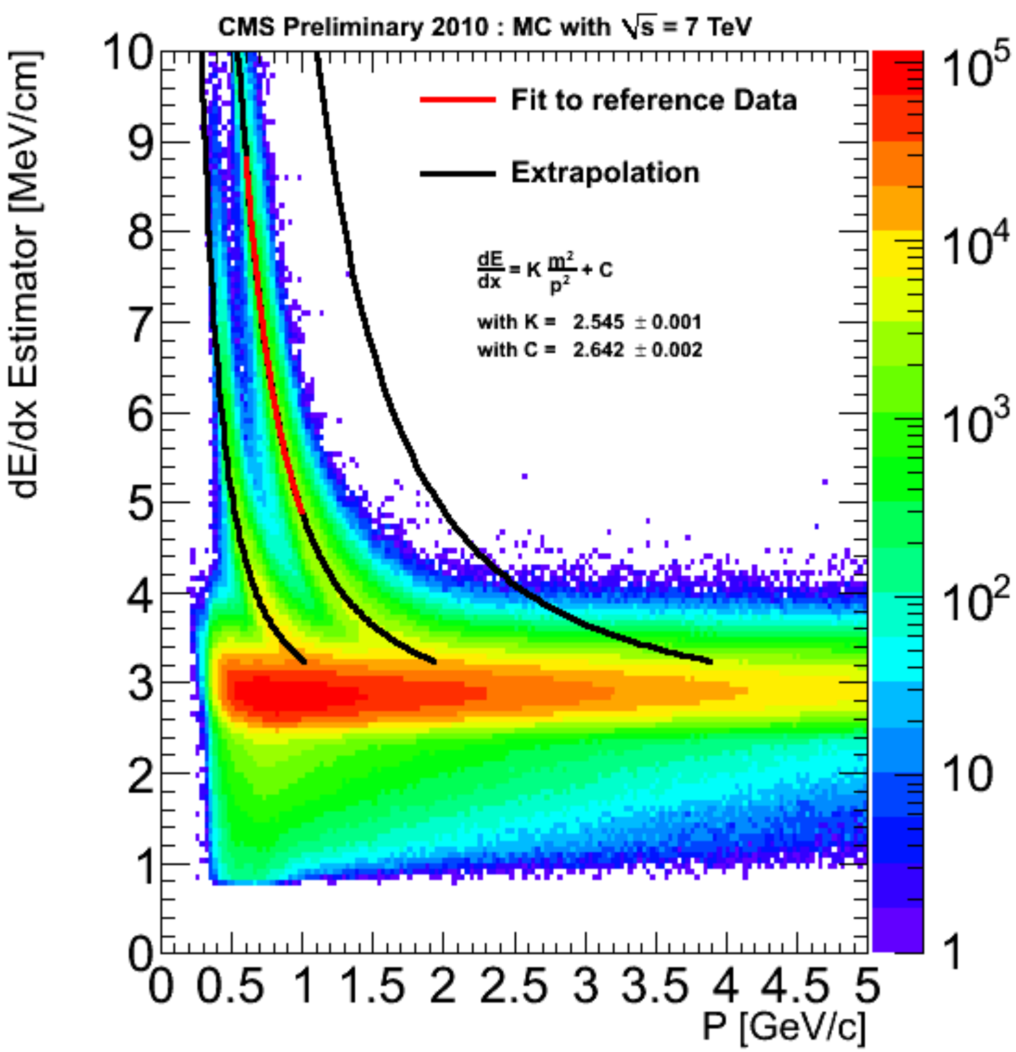
- application for particle identification (more details later)
- example: OPAL detector at LEP (drift chamber filled with argon)



Particle ID with dE/dx

- some big particle physics experiments have capabilities to measure dE/dx (and do particle identification this way) some have not

- for example: CMS experiment
    - does not have dE/dx measurement devices
    - can only measure the trajectories and total energy deposits (in showers) of charged particles
    - a priory a proton looks like a pion in the CMS tracking devices
    - (exception for very low momentum particles)



- for example: LHCb experiment
    - is able to do particle ID with Cherenkov radiation (see later)

Landau distribution

Landau Distribution

- the Bethe-Bloch formula describes the **average** energy loss per path length
- the actual energy loss is a stochastic process with fluctuations
- the energy is deposited through many (*N*) small deposits *δE*

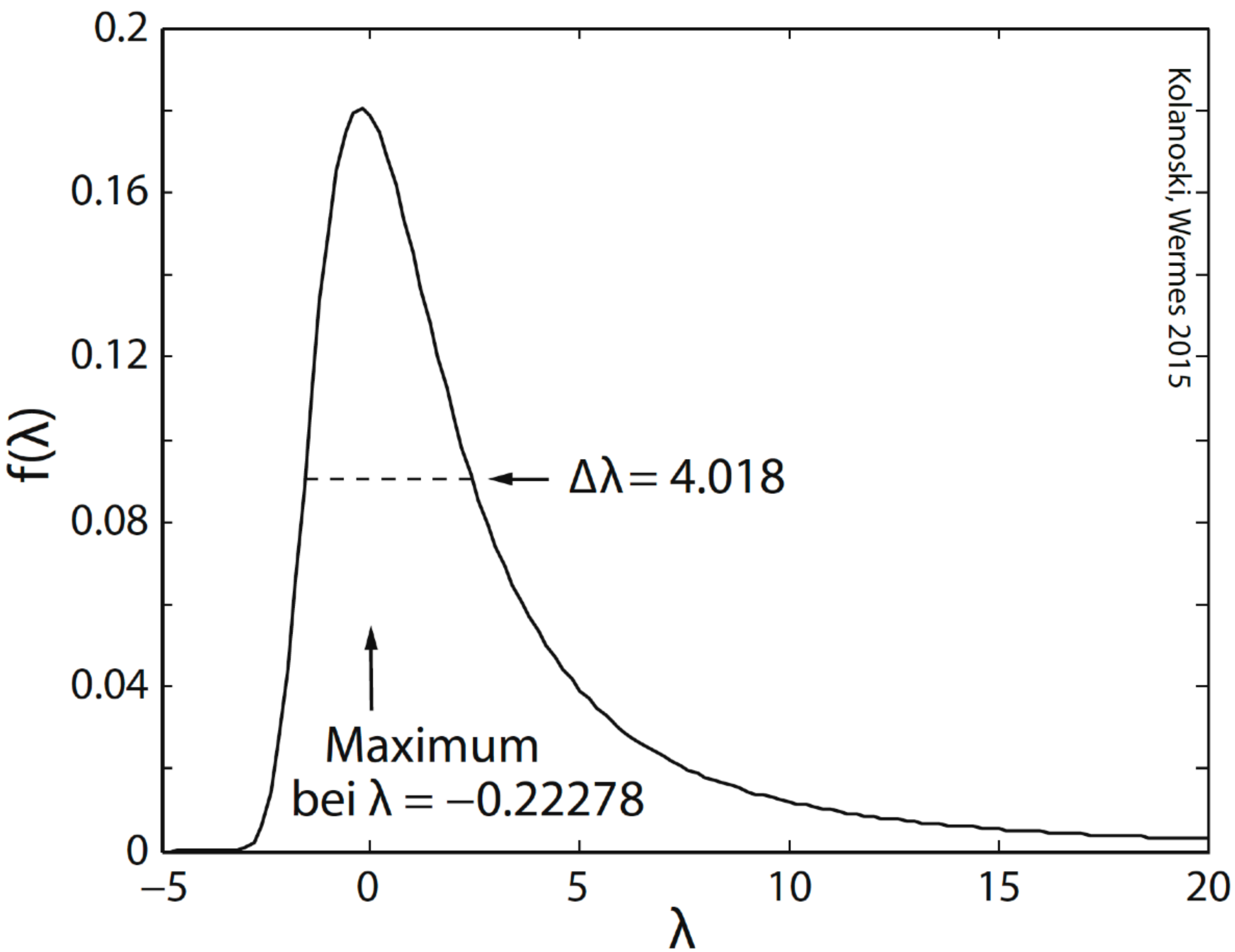
$$\Delta E = \sum_{n=1}^{N} \delta E_n$$

- both *N* and *δE* are subject to statistical fluctuations
- these fluctuations have impact on measurements in detectors:
  - momentum measurements can be degraded if particle loses energy before/after momentum measurement
  - particle identification gets more difficult for broader curves (see previous slide)
  - degradation of position resolution due to secondary ionisations (knock-on)

Landau Distribution

- we would expect a Gaussian distribution for *δE* due to the central limit theorem
- but in the relativistic case this is never true:
  - additional loss processes for higher energies (delta electrons)
  - the exact shape depends on mean value (Bethe-Bloch) and maximum possible value
  - there is a tail to higher energies
  - Landau derived the first analytic form of the distribution

$$f_L(\lambda) = \frac{1}{\pi}\int_0^\infty e^{-t\ln t-\lambda t}\sin(\pi t)\,dt$$



- the distribution is implemented in ROOT

Landau Distribution

- the distribution needs to be transformed to the actual energy loss, which depends on material:

$$\lambda = \lambda(\Delta E_w, \xi) = \frac{\Delta E - \Delta E_w}{\xi} - 0.22278$$

- where $$\xi = \frac{1}{2} K \frac{Z}{A} \rho \frac{z^2}{\beta^2} \Delta x$$

- and $$\Delta E_w = \xi \left(\ln \frac{2 m_e c^2 \beta^2 \gamma^2}{I} + \ln \frac{\xi}{I} + j - \beta^2 - \delta \right)$$

  is the maximum of the distribution

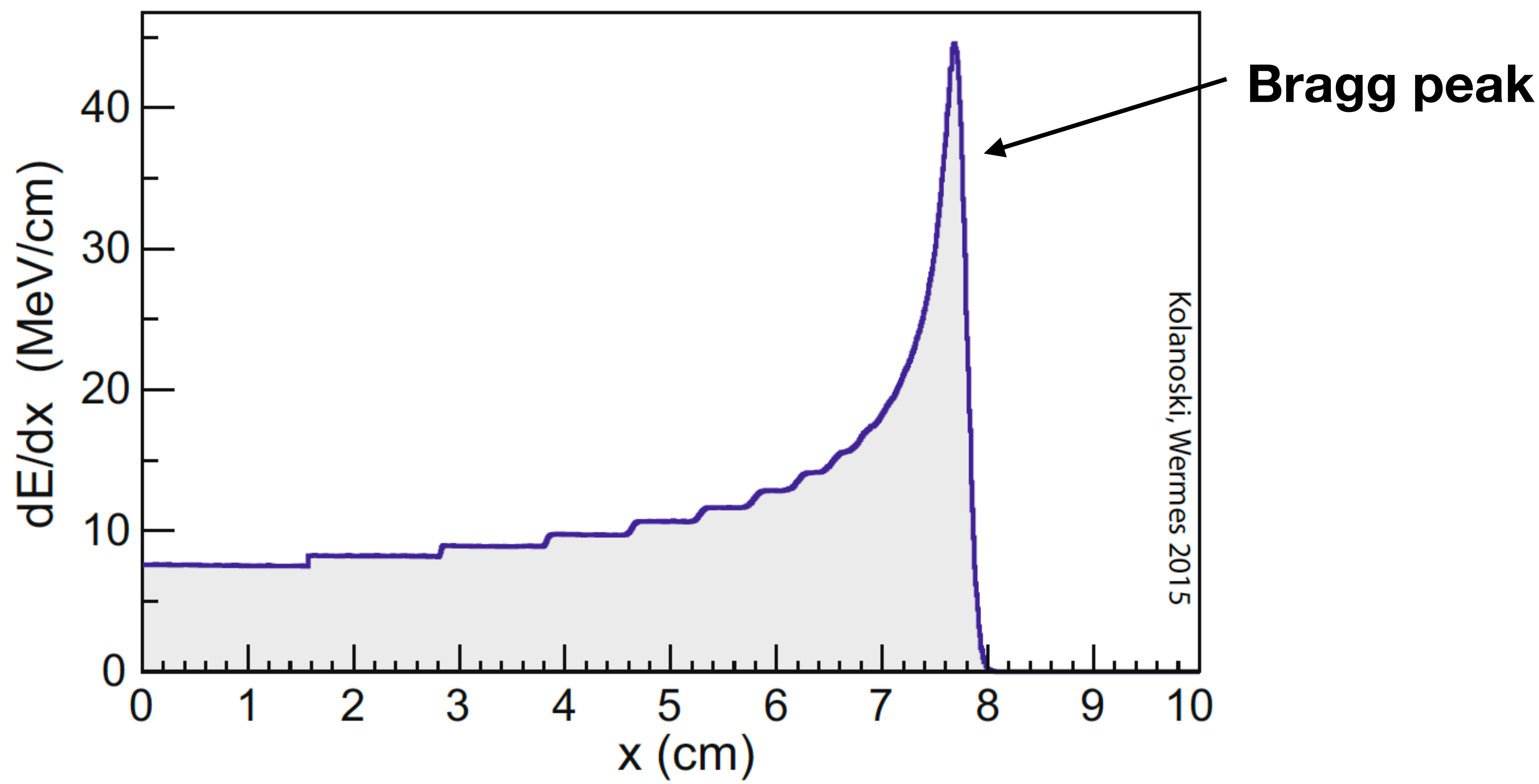
- the shape of the distribution depends on $$\kappa = \frac{\xi}{T_{max}}$$
- Gaussian for κ>1
- asymmetric for small κ values
- difficult to calculate analytically
- Geant 4 simulates this reliably for all κ value ranges

Bragg peak and radiation therapy

Bragg peak

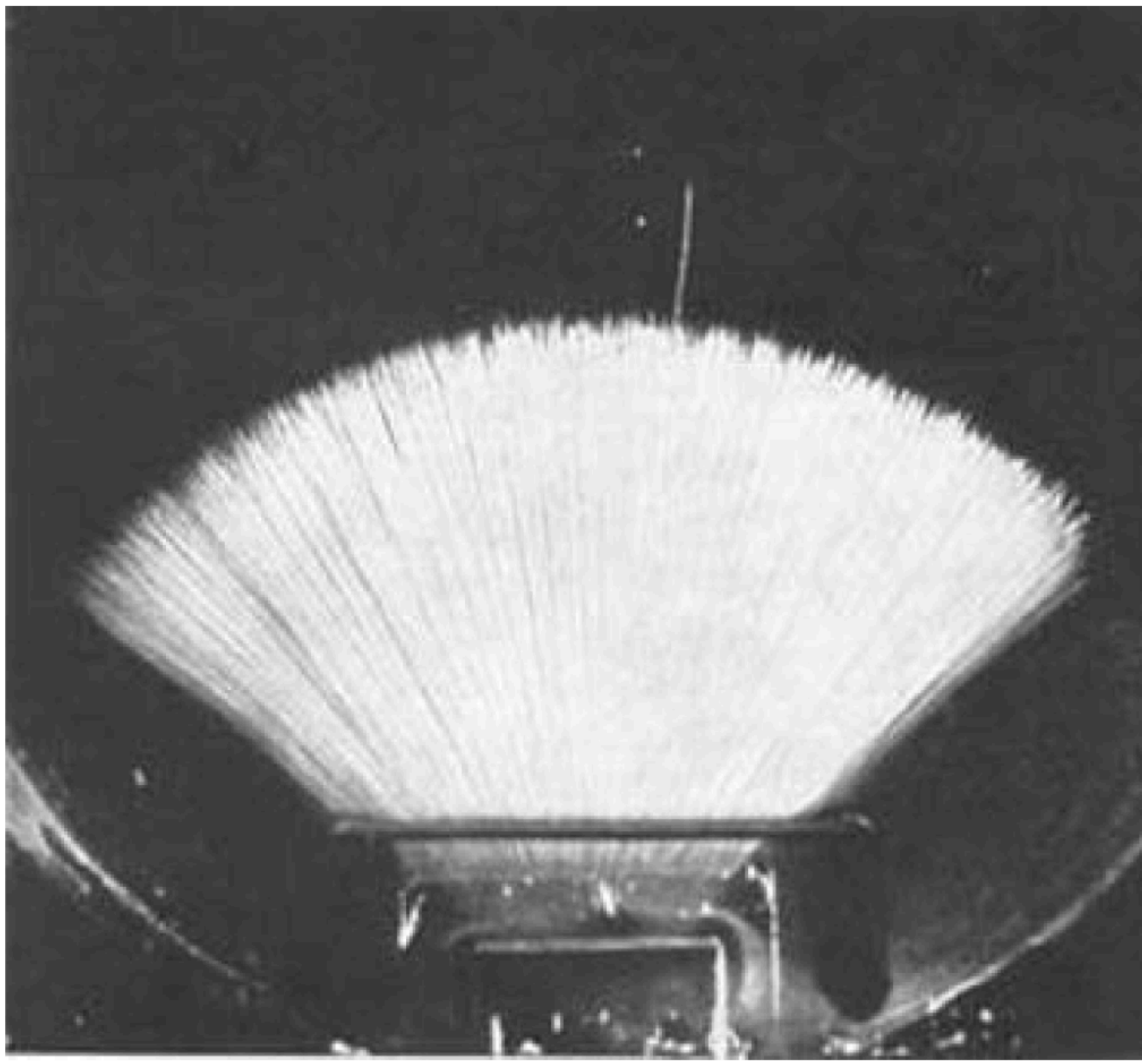
- dE/dx increases, when the particle slows down (due to the $1/\beta^2$ factor)
- while the particle loses energy in the medium, the *dE/dx* increases until it is stopped and absorbed
- at the end of the path most of the energy is deposited in the medium

- example: 100MeV protons in water (simulated with Geant 4)

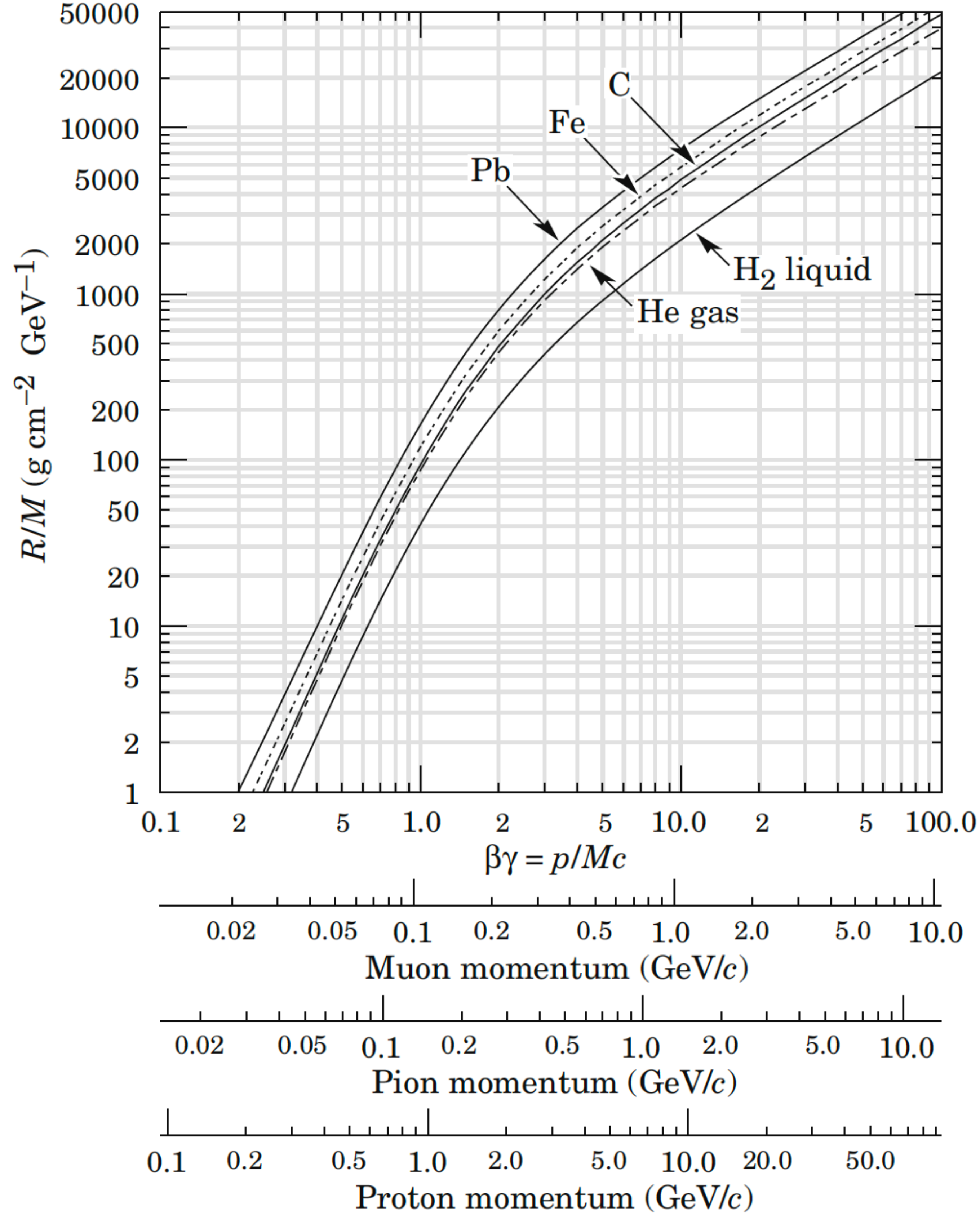


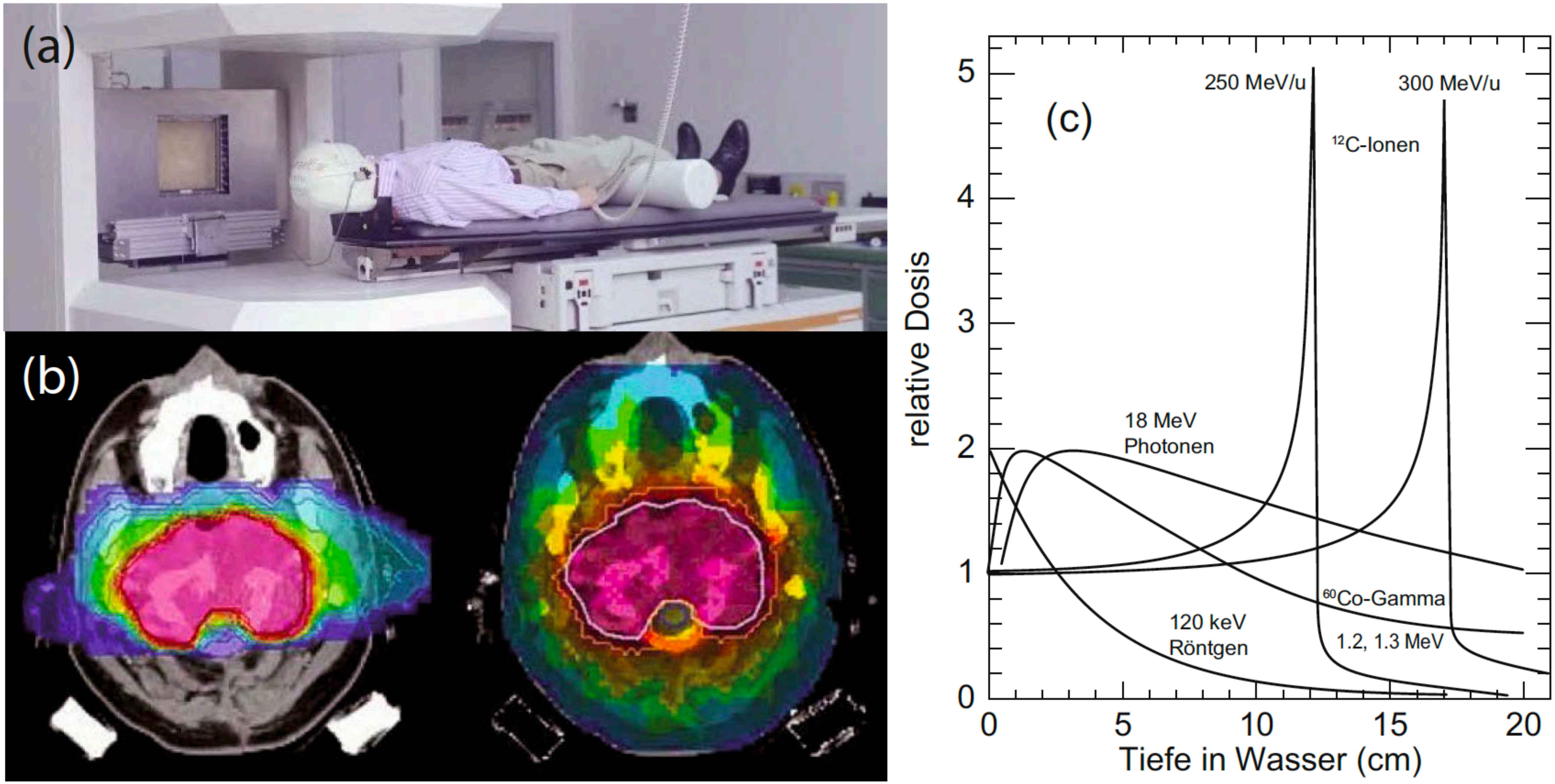
- most energy is deposited between 7 and 8 cm
- application: radiation therapy

- example: α-source in a cloud chamber



- depth as function of $\beta\gamma$




(a)
(b)
(c)
250 MeV/u
300 MeV/u
^{12}C-Ionen
18 MeV
Photonen
^{60}Co-Gamma
1.2, 1.3 MeV
120 keV
Röntgen
relative Dosis
Tiefe in Wasser (cm)
0
1
2
3
4
5
0
5
10
15
20

Date of the exam

- now that many other exams have been scheduled, we need to fix a date for the written exam:

- possibilities:

  16.02.2021
  23.02.2021
  02.03.2021
  16.03.2021
  23.03.2021

- **please vote for your favourite date(s) in the moodle poll, until 16.11.2020**

- **Please pick as many dates as possible**

- **We hope to find at least one date that fits for everyone.**

- **If more than one dates are possible, we will choose the earliest one**

- **It will be announced in the lecture next week and sent around as announcement**