

# ProbProg6

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## 1 Reasoning about Conditional Probabilities

### A

```
calls = 0;
minute = 0;
weekday = true; [5/7] weekday = false;
if (weekday = true) {
    while (minute < 60) {
        calls = calls+1 [1/6] skip;
        minute++;
    }
} else {
    while (minute < 60) {
        calls = calls+1 [1/20] skip;
        minute++;
    }
}
observe(calls==4);
```

### B

Weekday gets the predicate:  $wd$  to abbreviate this formula slightly.

$f = [wd]$

$f = (5/7 * [calls == 4|wd]) / ((5/7 * [calls == 4|wd] + 2/7 * [calls == 4|\neg wd]))$

Then:

$$[calls == 4|wd] = \frac{(1/6)^4 * (5/6)^{56} * \binom{60}{4}}{1 - ((1/6)^4 * (5/6)^{56} * \binom{60}{4})} = 1.404..%$$

$$[calls == 4|\neg wd] = \frac{(1/20)^4 * (19/20)^{56} * \binom{60}{4}}{1 - ((1/20)^4 * (19/20)^{56} * \binom{60}{4})} = 17.48..%$$

So that:

$$f = (5/7 * 1.404..% / (5/7 * 1.404..% + 17.48..%)) = 16.7228\%$$

## C

$$\begin{aligned} & cwp([wd = T], 1) \\ &= cwp(cwp[observer(calls = 4)]([wd = T], 1)) \\ &= cwp(cwp[wd = T[5/7]wd = F]([calls = 4] * ([wd = T], 1))) \\ &= cwp(5/7 * cwp[wd = T]([calls = 4]([wd = T], 1)) \\ &\quad + 2/7 * cwp[wd = F]([calls = 4]([wd = T], 1))) \end{aligned}$$

## 2 Basic Properties of expected runtime calculus

### A Counter Example

Take  $P = skip$ ,  $k = 5$ ,  $t = \lambda s \rightarrow 1$ .  
 $ert(skip, 5 * 1) == 5 * ert(skip, 1)$   
 $ert(skip, 5) == 5 * (1 + 1)$   
 $5 + 1 == 5 * 2$   
 $6 == 10$   
*false*

### B Counter Example

Take  $P = skip$ ,  $k = 1$ ,  $t = \lambda s \rightarrow 1$   
 $ert(skip, 1 + 1) == ert(skip, 1) + ert(skip, 1)$   
 $ert(skip, 2) == (1 + 1) + (1 + 1)$   
 $2 + 1 == 2 + 2$   
 $3 == 4$   
*false*

### C Counter Example

Take  $P = skip$ ,  $k = \frac{1}{1000}$ ,  $t = \lambda s \rightarrow 5$   
 $ert(skip, 5 * \frac{1}{1000}) \geq 5$   
 $ert(skip; 0, 005) \geq 5$   
 $1, 005 \geq 5$   
*false*