Induction Cases

④ "Sequential composition"

$$wp(P;Q, [F]) = [wp(P;Q, F)]$$
$$wp(P, wp(Q, [F])) = [wp(P, wp(Q, F))]$$
$$wp(P, [wp(Q, F)]) = [wp(P, wp(Q, F))]$$

$$[wp(P, wp(Q, F))] = [wp(P, wp(Q, F))]$$

(apply IH Q)

(apply IH P)

⑤ "if then else"

$$wp(if (G) P \ else \ Q, [F]) = [wp(if(G)P \ else \ Q, F)]$$
$$[G] \cdot wp(P, [F]) + [\neg G] \cdot wp(Q, [F])$$
$$= [(G \wedge wp(P, F)) \vee (\neg G \wedge wp(Q, F))]$$

$$[G] \ wp(P, [F]) + [\neg G] \ wp(Q, [F])$$
$$= [G] \cdot [wp(P, F)] + [\neg G] \cdot (wp(Q, F))]$$

$$[G] \cdot wp(P, [F]) + [\neg G] \cdot wp(Q, [F])$$
$$= [G] \cdot wp(P, [F]) + [\neg G] \cdot wp(Q, [F])$$

(apply IH twice for the right hand side)

⑥ "while" ~~$wp(P, [F]) = [wp(P, F)$~~

$$wp(while (G) P, [F]) = [wp(while (G) P, F)]$$

$$[\mu X. [G] \cdot wp(P, X) + [\neg G] \cdot [F]] = [\mu X ((G \wedge wp(P, X)) \vee (\neg G \wedge F))]$$

~~$[\mu X [G] \cdot wp(P, X) + [\neg G] [F]$~~

$$sup_{n \in \mathbb{N}} ([G] \cdot wp(P, X) + [\neg G] \cdot [F])^n (0) = [sup_{n \in \mathbb{N}} ((G \wedge wp(P, X)) \vee (\neg G \wedge F))^n (false)]$$

$$sup_{n \in \mathbb{N}} ([G] \cdot wp(P, X) + [\neg G] [F])^n (0) = sup_{n \in \mathbb{N}} [(G \wedge wp(P, X)) \vee (\neg G \wedge F)]^n (0)$$

$$sup_{n \in \mathbb{N}} ([G] \cdot wp(P, X) + [\neg G] \cdot [F])^n (0) = sup_{n \in \mathbb{N}} [[G] \cdot [wp(P, X)] + [\neg G] \cdot [F]]^n (0)$$

now apply IH, then the formulas are the same