ProbProg6

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1 Bayesian Networks

A

$Pr(R = 0|G = 0, S = 0)$
$= Pr(R = 0, G = 0, S = 0)/Pr(S = 0, G = 0)$
$= 0.8 * 0.6 * 1/((0.2 * 0.99 + 0.8 * 0.6) * (0.2 * 0.2 + 0.8 * 1)) = 0.8428$

B

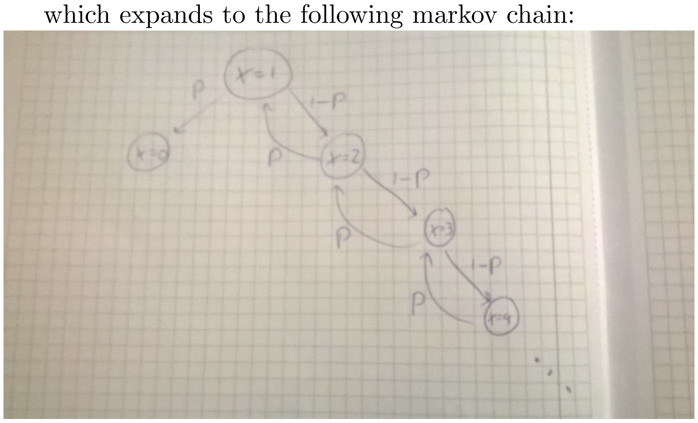
```
sR, sS, sG = R, S, G; flag = True;
while(flag) {
   R, S, G = sR, sS, sG; flag = False;
   R = 1 [0.2] R = 0;
   if (R==0) {
      S = 1 [0.4] S = 0;
   } else {
      S = 1 [0.01] S = 0;
   }
   if (R==0 && S==0) {
      G = 0;
   } else if (R==1 && S==0) {
      G = 1 [0.8] G = 0;
   } else if (R==0 && S==1) {
      G = 1 [0.9] G = 0;
   } else if (R==1 && S==1) {
      G = 1 [0.99] G = 0;
   }
   flag = (G+S==0);
}
```

2 Random walks

We've got the following program that models the situation:

```
x := 1;
while (x > 0) \{
     x— [p] x++
\}
```

which expands to the following markov chain:



If p is strictly larger than $1-p$, then over time eventually the execution will of the program will reach the left-most state, where x equals 0. This happens because the markov property, the memorylessness ensures that every step deeper into the chain, the probabilities remain p and $1-p$.

3 Computing the expected runtime

$P =$

```
1.      {b := true} [1/2] {b := false};
2.      if (b) {
3.              s := true;
4.      } else {
5.              {b := true} [1/2] {b := false};
6.              if (b) {
7.                      s := true;
8.              } else {
9.                      s := false;
10.             }
11.     }
```

$ert(P, 0) = ert(line1, ert(line2 - 11, 0))$
$ert(line2 - 11, 0) = 1 + [b] * ert(s := true, 0) + [\neg b] * ert(line5 - 10, 0)$
$ert(line5 - 10, 0) = ert(line5; line6 - 10, 0)$
$ert(line5 - 10, 0) = ert(\{b := true\}[1/2]\{b := false\}, ert(line6 - 10, 0))$

$ert(line6 - 10, 0) = 1 + [b] * ert(\{s := true\}, 0) + [\neg b] * ert(\{s := false\}, 0)$
$ert(P, 0) = 2 + 1 + 0.5 * 1 + 0.5 * (2 + 1 + 0.5 * 1 + 0.5 * 1)$
$ert(P, 0) = 5.5$

4 From pGCL programs to Bayesian networks

A

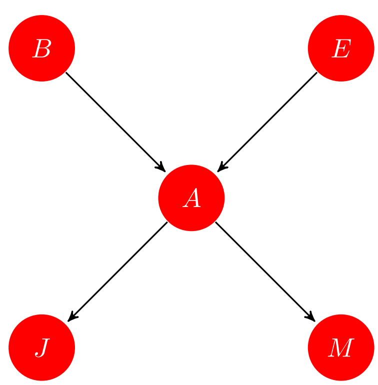
B=True	B=False
0.001	0.999

E=True	E=False
0.002	0.998

	A=True	A=False
B=True,E=True	0.95	0.05
B=True,E=False	0.94	0.06
B=False,E=True	0.29	0.71
B=False,E=False	0.01	0.99

	J=True	J=False
A=True	0.90	0.10
A=False	0.05	0.95

	M=True	M=False
A=True	0.70	0.30
A=False	0.01	0.99



B

$$
\begin{aligned}
&Pr(\neg B|\neg E, A) \\
&\quad = \tfrac{Pr(\neg B, \neg E, A)}{Pr(\neg E, A)} \\
&\quad = \tfrac{0.999*0998*0.01}{0.998*(0.001*0.94+0.999*0.01)} \\
&\quad = 0.914
\end{aligned}
$$