Probabilisitc Programming Exercises 2 – Jan Boerman, Jasper van Rooijen

Exercise 1

```
x := 10;
y := 20;
while (x > 0 || y > 0) {
        { x := x-5 } [1/2] { { x := x*2 } [2/3] { { y := y-2 } [3/5] { y := y+1 } } }
}
```

Exercise 2

$$For\ target\ State\ 6, we\ consider$$

$$\sum\nolimits_{?} = \{s_1, s_2, s_3, s_4\}$$

$$(I - A) = \begin{pmatrix} 1 & -1/2 & 0 & -1/2 \\ -1/3 & 1 & -1/3 & 0 \\ 0 & -3/4 & 1 & -1/4 \\ 0 & 0 & -4/5 & 1 \end{pmatrix}$$

$$x = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$$

$$b = \begin{pmatrix} 0 \\ 1/3 \\ 0 \\ 0 \end{pmatrix}$$

Such that holds:

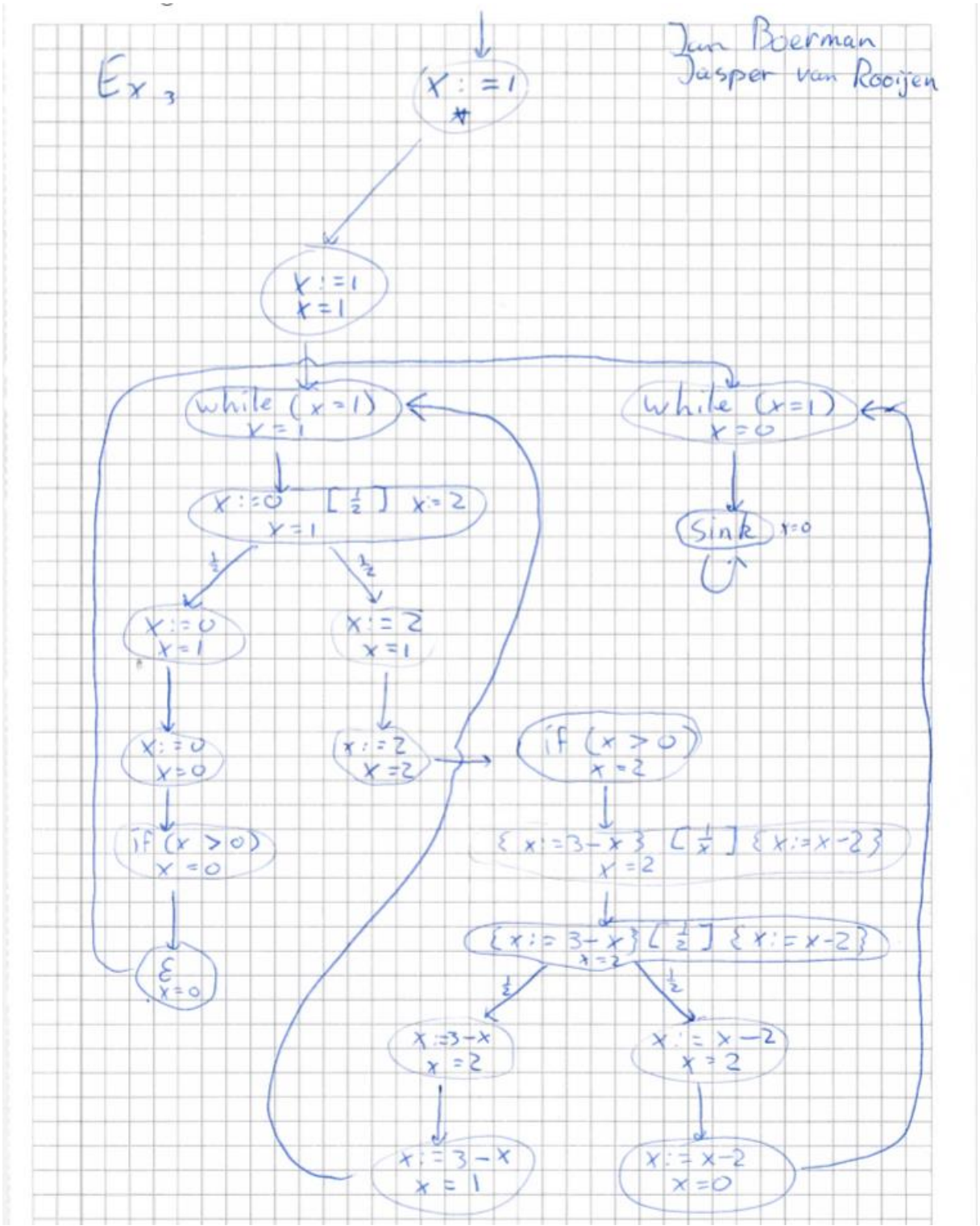
$$(I - A) * x = b$$

Gaussian elimination yields:

$$\begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} 14/19 \\ 16/19 \\ 15/19 \\ 12/19 \end{pmatrix}$$

Thus the probability of reaching state 6 from state 1 is the probability as yielded by the elimination: 14/19, thus 73,7%.

Exercise 3


Ex 3
Jan Boerman
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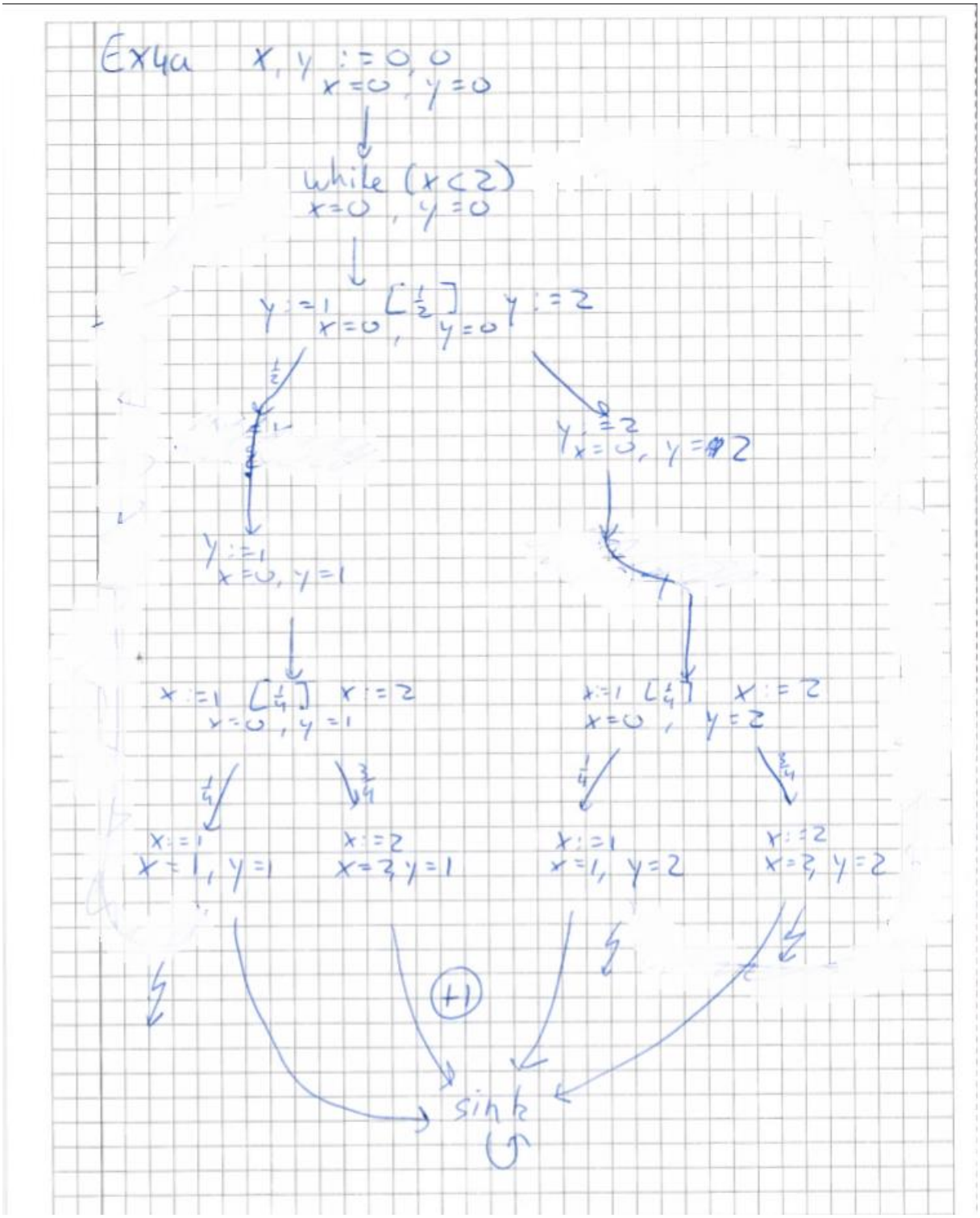
x := 1
x := 1
x = 1
while (x = 1)
x = 1
while (x = 1)
x = 0
x := 0 [1/2] x := 2
x = 1
Sink x=0
1/2
1/2
x := 0
x = 1
x := 2
x = 1
x := 0
x = 0
x := 2
x = 2
if (x > 0)
x = 0
if (x > 0)
x = 2
{x := 3 - x} [1/x] {x := x - 2}
x = 2
{x := 3 - x} [1/2] {x := x - 2}
x = 2
ε
x = 0
1/2
1/2
x := 3 - x
x = 2
x := x - 2
x = 2
x := 3 - x
x = 1
x := x - 2
x = 0

Exercise 4a



Exercise 4b

The expected value is defined as (probability of a valid path * value for that path) / (1 – probability of an invalid path)
Thus the expected value of x + y
= ((1/2)*(3/4)*(2+1)) / (1 – ((1/2)*(1/4) + (1/2)*(1/4) + (1/2)*(3/4)))
= ((3/8)*3) / (1 – (1/8 + 1/8 + 3/8)) = (3*(3/8)) / (1 – 5/8)
= (3 * (3/8)) / (3/8)
= 3