

Exercise Sheet 1

General remarks:

- **Due date:** February 8th (before the exercise class).
- Please submit your solutions to Enno Ruijters (e-mail: e.j.j.ruijters@utwente.nl). We kindly ask you to hand in your solutions in groups of about two students.
- Solutions must be written in English.
- The solution to the exercises will be discussed during the exercise class.
- If you have any questions regarding the lecture or the exercise, feel free to write us an email or visit us.

Exercise 1 (Birthday Paradoxon)

20%

- [15%] What is the probability that in a room filled with 23 people, at least one pair of persons has the same birthday? Write a WEBPPL program to determine this probability.
- [5%] How many people in the same room are at least required such that the probability of one pair of persons having the same birthday is above 0.9?

Exercise 2 (Reasoning about Reasoning)

30%

Two agents play a game in which each agent needs to name a number between 0 and 9. They win if their numbers add up to 13. The first player knows this, and she knows that the second player gets to see the number the first player chooses. However, the second player mistakenly thinks that the two win if their numbers add up to any number greater than 8. The first player knows this as well.

Implement a WEBPPL program for player one to determine which number she should choose.

For this exercise, you may use all features of standard programming languages, such as assignments, recursion, etc. Moreover, to sample a number between 0 and 9 you can use the expression `"sample(RandomInteger(n:10))"`. However, you are not allowed to use explicit conditioning constructs, e.g. `condition(...)`.

Exercise 3 (Conditioning)**25%**

You are running a factory that produces 10 products every day. Many of the produced products are defective and must be discarded. There are two levels of tests that can be performed:

- A normal test, which passes a product with probability 0.66.
- A rigorous test, which passes a product with probability 0.33.

At each step, the part inspectors apply the following two rules to decide which test is applied:

- The test for the first product is determined by a coin flip.
 - For all other products, if the previous product passed the test, the inspectors become suspicious and apply the rigorous test; if the test failed, they relax and apply the normal test.
- (a) [20%] Write a WEBPPL program to model the distribution of products passing the test on a single day.
- (b) [5%] Assume that some governmental regulatory body intervenes if 7 or more products pass on a single day. Similarly, the factory management intervenes if all products fail to pass the test. How does the probability distribution change if the inspectors remain honest, but both of the above rules are satisfied?

Exercise 4 (Expectations)**25%**

Assume that N children reach for one of N candies, with each child choosing a candy bar independently and uniformly at random. If a candy is chosen by exactly one child, the child becomes happy and both the candy and the child drop out. The remaining children then repeat the process for another round until every child has a candy bar. Use WEBPPL to determine an approximate bound on the expected number of rounds until every child has a candy bar.