Math Assignment

for the position of (Junior) Mathematician Gaming

BEECOOLGAMING

In this task, we are going to analyze a simple slot machine game. The game is played on a 4×4 matrix, where each column represents a different reel. The file `data.xlsx` contains two different sets of four reels, where each of the reels has a fixed length of $n = 100$ symbols. When a reel is spinned, a random reel excerpt of four consecutive symbols is shown on the corresponding matrix column. More precisely, a stop position P is chosen uniformly at random from $\{0, 1, ..., , n - 1\}$ and the corresponding reel matrix column is populated with the consecutive symbols at positions P `mod` $n, (P + 1)$ `mod` $n, (P + 2)$ `mod` n and $(P + 3)$ `mod` n.

In our game, the reel symbols can be either the `O` symbol or the `X` symbol. When the game is played, the player places a bet of \$1 and the four reels are independently spinned. The amount W that the player wins is equal to $W = \$X^2$, where X is the total number of `X` symbols that appear on the matrix. For example, if there are three `X` symbols that land on the matrix, the player wins an award of \$9 on their \$1 bet. If there is only one `X` symbol on the reel matrix, the player wins an award of \$1, in which case they only recover their initial bet of \$1.

O	O	X	O
X	O	X	O
O	X	O	O
O	O	O	X

Figure 1: An example with five `X` symbols on the reel matrix and an award of \$25.

Note. You can assume that the matrix dimensions of 4×4 stay fixed and use the height and width as constants where necessary. The prize award rule $W = \$X^2$ also remains fixed.

Q1. For this question, we will use the reels given in the worksheet `RS_01` in `data.xlsx`. Note that in this particular case, all `X` symbols are at least three other symbols apart, so there can be at most one `X` symbol that lands on a particular reel. For each reel $i \in \{1, 2, 3, 4\}$, let p_i be the probability that an `X` symbol lands on the i-th reel and is therefore present on the i-th column in the matrix. Write a formula for the theoretical average win amount $\mathbf{E}[W]$ as a function of the probabilities (p_1, p_2, p_3, p_4) and compute the actual value of $\mathbf{E}[W]$ for `RS_01`.

Q2. For this question, we will use the reels given in the worksheet `RS_02` in `data.xlsx`. Note that in this case there can be more than one `X` symbol landing on a particular reel and the formula from the previous question would not hold in general anymore. Write a function in a programming language of your choice /Python/Java/C/C++/C#/ that takes as input an arbitrary set of four reels, and returns the theoretical average win amount $\mathbf{E}[W]$ for the input reel set. Using this function, compute the actual value of $\mathbf{E}[W]$ for the `RS_02` reel set.

Q3. Write a function that takes as input an arbitrary set of four reels and a positive integer M, simulates the slot game for the specified number of simulations M and outputs an estimate of the expected win amount $\mathbf{E}[W]$ and its volatility $\sigma(W) = \sqrt{\mathbf{Var}[W]}$.

Q4. What is the maximum possible value $w_{\max}$ that W can take and what is the probability $\mathbf{P}\{W = w_{\max}\}$ that the player wins the top prize, for both the reel sets `RS_01` and `RS_02` ?