Evolutionary Computation

Assignment 10

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https://github.com/JankowskiDaniel/evolutionary-computation/tree/AL/assignment10

Problem description

The task involves analyzing three columns of integers, each row corresponding to a single node. The initial two columns designate the x and y coordinates, pinpointing the nodes' locations on a plane, while the third column specifies the cost associated with each node. The objective is to meticulously choose an exact half of the total nodes (in cases where the total node count is an odd number, the count of nodes to be selected is adjusted upward to the nearest whole number) to construct a Hamiltonian cycle, which is essentially a continuous loop that passes through each member of the selected set of nodes. The criterion for this selection is that the aggregate of the complete path's length and the cumulative cost of the chosen nodes should be as low as possible.

To quantify the distances between nodes, we employ the Euclidean distance formula, and the resulting figures are rounded off to the nearest integer in a standard mathematical fashion. Moreover, as part of the distance between nodes, we consider the cost of the destination node. This ensures that cost has a significant impact on the final results.

In this assignment, we improved the Hybrid Evolutionary Algorithm, by optimizing the functions that we used to get more epochs, and by adding in every second epoch the perturb operator performed on one of the parents and passed as offspring, and in the other epoch we used our previous operator used in Hybrid Evolutionary Algorithm. The perturb operator is a combination of multiple edges swap.

Pseudocode of implemented algorithms

```
calculate_distance_matrix(coords, costs):
        dist_matrix = [][]
        FOR i IN RANGE(len(coords)):
                FOR j IN RANGE(len(coords)):
                        dist_matrix[i][j]  =  round(sqrt((coords[i].x  -  coords[j].x)**2  +
                        (coords[i].y - coords[j].y)**2)
        RETURN dist_matrix

objective_function(solution, dist_matrix, costs):
        total_score = 0
        n = len(solution)
        FOR x in range(n):
                        total_score += dist_matrix[solution[x - 1]][solution[x]]
                        total_score += costs[solution[x]]
        RETURN total_score

generate_random_solution(n):
    RETURN random.sample(range(0, n * 2), n)

generate_init_population(n, size):
        population = [generate_random_solution(n) FOR _ IN RANGE(size)]
        RETURN population
```

```
select_random_parents(population):
        parents = random.sample(population, 2)
        return parents[0], parents[1]
```

We've decided to implement a custom recombination operator. The logic is that we select at random subpaths from both parents that are then inserted to the offspring solution.

```
recombine_subpath_operator(parent1, parent2):
        offspring = []
        n = len(parent1)
        num_nodes = 20
        WHILE len(offspring) < n:
                random_start = random.randint(0, len(parent1) - num_nodes)
                random_subpath = parent1[random_start : random_start + num_nodes]

                # remove already selected nodes from both parents
                parent1 = [el FOR el IN parent1 IF el NOT IN random_subpath]
                parent2 = [el FOR el IN parent2 IF el NOT IN random_subpath]

                parent1, parent2 = parent2, parent1
        RETURN offspring
```

The population size has been set to 30. The offspring solution was added to the population only if its score was better than the score of the current worst solution and if the offspring's score wasn't already present in the population (to diverse solutions inside the population)

Highlighted parts are changed ones.

```
run_algorithm(dist_matrix, costs, avg_runtime):
        population = generate_init_population(100, 30)
        population = [(sol, objective_function(sol, dist_matrix, costs)) FOR sol IN
                                                                                population]
        all_scores = [x[1] FOR x IN population]
        worst_solution = max(population, key=lambda x: x[1])
        epochs = 0
        start_time = time()
        WHILE time.time() - start_time < avg_runtime:
                epochs += 1
                parent1, parent2 = select_random_parents(population)
                IF epochs%2==0:
                        offspring = recombine_subpath_operator(parent1[0], parent2[0])
                ELSE:
                        offspring = perturb(parent1[0])
                solution, score = SteepestLocalSearch(offspring)
                IF score < worst_solution[1] AND score NOT IN all_scores:
                        population.remove(worst_solution)
                        population.append((solution, score))
                        all_scores.remove(worst_solution[1])
                        all_scores.append(score)
                        worst_solution = max(population, key=lamda x: x[1])
        end_time = time.time()
        runtime = end_time - start_time
        best_solution, best_score = min(population, key=lambda x: x[1])
        RETURN best_solution, best_score, runtime, epochs

perturb(current_solution):
    l = len(current_solution)
    FOR x in range(4):
        i = random.randint(2,l-2)
        j = random.randint(0,i-1)
        k = random.randint(j+2,l)
        m = random.randint(0,k-1)

        new_solution = (current_solution[:j ]
                     + current_solution[j :i ][::-1]
                     + current_solution[i :])
        current_solution = new_solution
        new_solution = (current_solution[:m ]
                     + current_solution[m :k ][::-1]
                     + current_solution[k :])

        current_solution = new_solution
```

```
RETRUN current_solution
```

Results

Method	Instance A	Instance B	Instance C	Instance D
Greedy LS, random solution, two-edges + inter route	77,014(74,663-79,803)	69,990(67,877-74,141)	50,998 (49,340-53,141)	48,068 (45,336-51,629)
Greedy LS, random solution, two-nodes + inter route	90,940(84,816-99,390)	85,570(77,908-97,299)	63,929 (58,135-70,886)	62,175 (54,310-71,108)
Greedy LS, best solution from 2-regret with weighted sum, two-edges + inter route	75,792 (74,221-79,688)	71,266 (67,384-77,120)	52350,15(48,931-55,758)	51,013 (45,212-59,478)
Greedy LS, best solution from 2-regret with weighted sum, two-nodes + inter route	75,932(74,344-79,315)	71,839 (67,384-77,565)	52,638 (49,649-56,472)	51,248(45,097-60,185)
Steepest LS, best solution from 2-regret with weighted sum, two-edges + inter route	75,728(74,091-79,220)	71,233 (67,384-77,057)	52,299 (49,098-5,5665)	50,977(45,097-59,478)
Steepest LS, best solution from 2-regret with weighted sum, two-nodes + inter route	75,880(74,280-79,220)	71,894(67,384-77,420)	52,607 (49,460-56,472)	51,247 (45,097-60,185)
Candidates LS, random solution, two-edges + inter route	81,129(76,609-86,447)	73,977(69,300-80,189)	51,588(49,120-54,801)	48,429(45,385-51,392)
Steepest LS, random solution, two-nodes + inter route	92,714(84,218-103,034)	87,666(79,356-97,895)	65,679(59,604-73,386)	64,162(54,716-75,351)
Steepest LS, random solution, two-edges + inter route	78,017 (74,874-82,619)	71,337(67,909-76,199)	51,485 (49,235-53,755)	48,225 (45,673-51,639)
Deltas from previous iteration, random solution, two-edges + inter route	78,192(75,149-82,556)	71,709(68,307-76,210)	51,940(49,347-55,591)	48,509(45,966-52,016)
Multi Start Local Search random solution, two-edges + inter route	75,447(74,773-76,051)	68,523(67,810-69,028)	49,567(49,141-50,190)	45,267(45,870-46,275)
Iterated Local Search random solution, two-edges + inter route	73,114(72,894-73,445)	66,239(66,137-66,422)	**47,259(46,805-47,686)**	44,131(43,690-44,784)
Large Scale Search without LS	78,788(76,120-81,680)	72,062(70,211-75,098)	51,986(50,094-53,606)	48,619(46,564-50,932)

Large Scale Search with LS	76,682(74,766-79,213)	70,725(68,804-73,873)	51,275(49,389-53,318)	48,453(46,008-50,428)
Hybrid Evolutionary Algorithm	73,087(72,864-73,462)	**66,270(66,136-66,548)**	47,392(46,998-47,929)	**43,307(43,062-43,598)**
Improved HEA	**73,041 (72,855-73,244)**	66,277 (66,148- 66,544)	47,192 (46,833- 47,782)	43,372 (43,214-43,650)

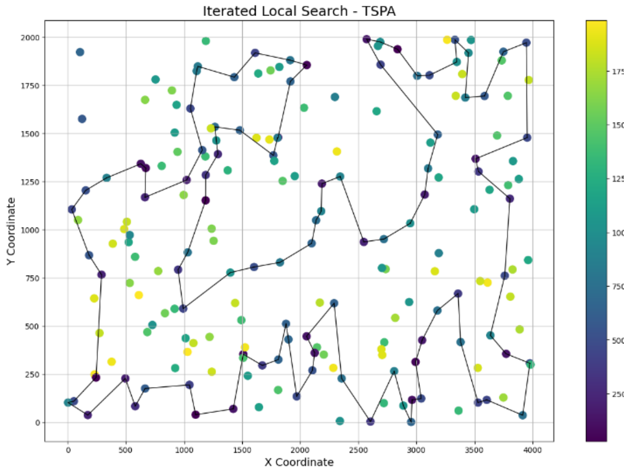
Runtimes

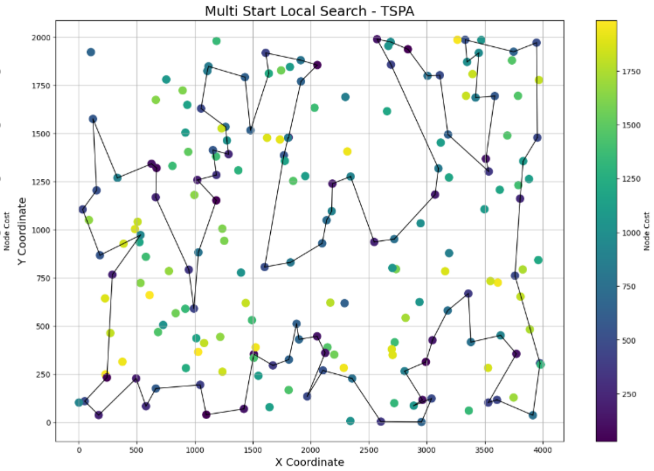
Method	Instance A	Instance B	Instance C	Instance D
Greedy LS, random solution, two-edges + inter route	1.56(1.06-2.63)	1.95(1.19-3.48)	1.25(0.77-2.28)	1.18(0.72-1.99)
Greedy LS, random solution, two-nodes + inter route	1.68(1.03-2.98)	1.95(0.81-6.66)	1.38(0.79-2.21)	1.37(0.77-2.36)
Greedy LS, best solution from 2-regret with weighted sum, two-edges + inter route	0.67(0.51-0.97)	0.7(0.5-1.18)	0.66(0.5-0.93)	0.65(0.51-0.89)
Greedy LS, best solution from 2-regret with weighted sum, two-nodes + inter route	0.63(0.46-0.89)	0.69(0.53-1.15)	0.68(0.49-1.24)	0.67(0.54-1.18)
Steepest LS, best solution from 2-regret with weighted sum, two-edges + inter route	0.85(0.55-1.53)	0.95(0.53-1.78)	0.94(0.57-1.6)	1(0.58-1.38)
Steepest LS, best solution from 2-regret with weighted sum, two-nodes + inter route	0.88(0.58-1.71)	0.83(0.53-1.57)	0.89(0.5-1.58)	1.03(0.67-1.5)
Candidate LS, random solution, two-edges + inter route	4.43(3.95-6.41)	4.52(3.99-5.70)	4.53(3.83-6.44)	4.58(4.06-5.88)
Steepest LS, random solution, two-nodes + inter route	6.82(5.46-8.96)	6.63(4.89-10.51)	6.8(5.41-9.2)	0.69(0.5-1.18)
Steepest LS, random solution, two-edges + inter route	5.46(4.47-7.46)	5.64(4.51-7.16)	5.41(4.72-6.54)	5.64(4.76-6.88)
Deltas from previous iteration, random solution, two-edges + inter route	1.34(1.06-2.13)	1.80(0.80-2.31)	1.82 (1.05-2.51)	1.88(1.23-2.44)

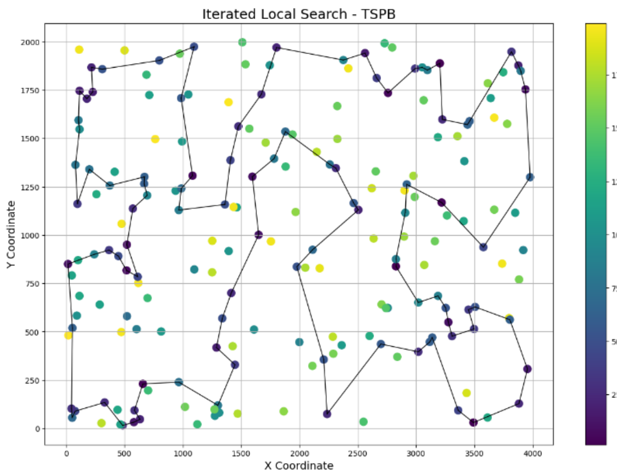
Multi Start Local Search random solution, two-edges + inter route**	379.31(338.99-403.86)	308.32(294.82-326.69)	301.18(290.45-320.35)	334.86(320.82-346.04)
Iterated Local Search random solution, two-edges + inter route**	379.40(379.32-379.60)	308.42(308.32-308.70)	301.29(301.20-301.44)	334.98(334.86-335.23)
Large Scale Search without LS	379.36(379.31-379.46)	308.38(308.33-308.45)	301.23(301.18-301.30)	334.91(334.88-334.96)
Large Scale Search with LS	379.50(379.32-379.77)	308.48(308.34-308.83)	301.35(301.19-301.78)	335.03(334.88-335.28)
Hybrid Evolutionary Algorithm	379.43(379.32-379.58)	308.42(308.32-308.55)	301.35(301.35-301.90)	334.96(334.86-335.13)
Improved HEA	379.34 (379.33-379.46)	308.41 (308.33-308.52)	301.30 (301.19-301.50)	334.93 (334.87-335.13)

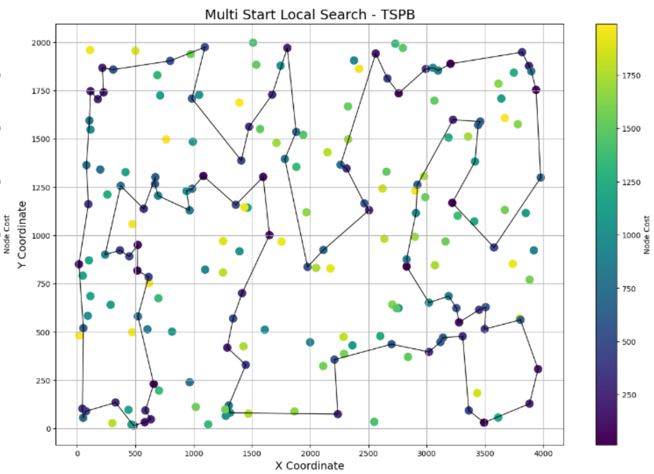
Iterations

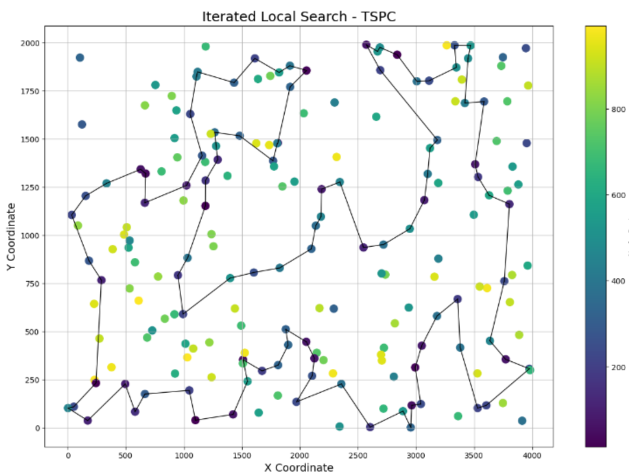
Iterated Local Search random solution, two-edges + inter route	1923(1878-1986)	1534(1478-1579)	1517(1448-1556)	1668(1520-1798)
Large Scale Search without LS	3499(2726-5522)	2794(2248-4449)	2849(2365-4436)	3393(2712-5659)
Large Scale Search with LS	1184(771-1987)	1171(519-2411)	1039(504-1803)	1142(509-2294)
Hybrid Evolutionary Algorithm	1388(1179-1553)	1211(918-1485)	1050(510-1346)	1344(1122-1591)
Improved HEA	2188 (1855-2433)	1814 (1592-1981)	1589 (1363-1817)	1719 (1508-2060)

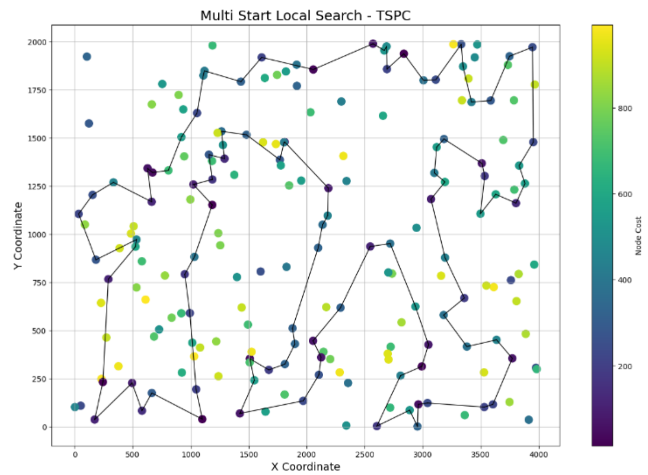

Iterated Local Search - TSPA
Y Coordinate
X Coordinate
Node Cost

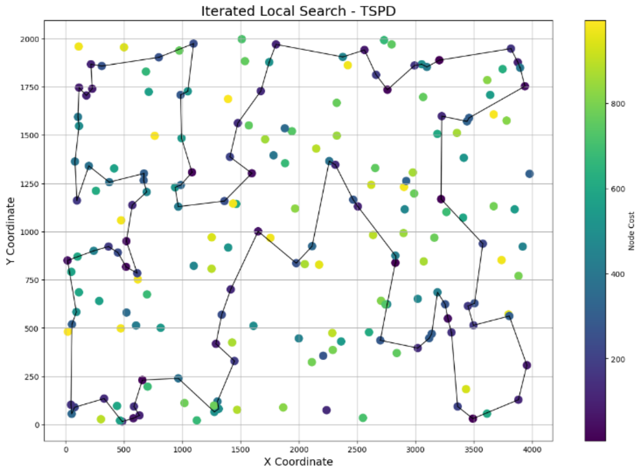

Multi Start Local Search - TSPA
Y Coordinate
X Coordinate
Node Cost

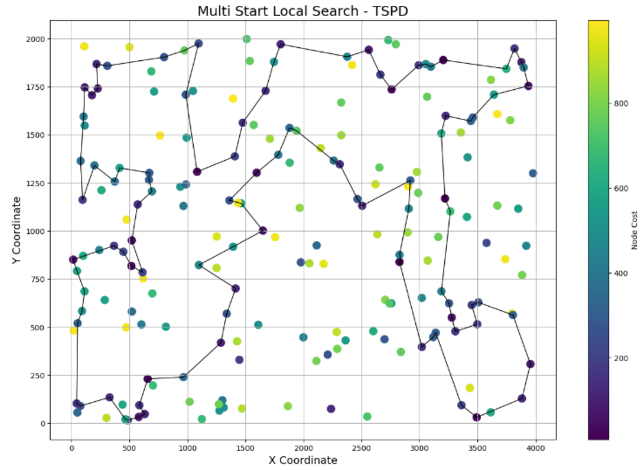

Iterated Local Search - TSPB
Y Coordinate
X Coordinate
Node Cost

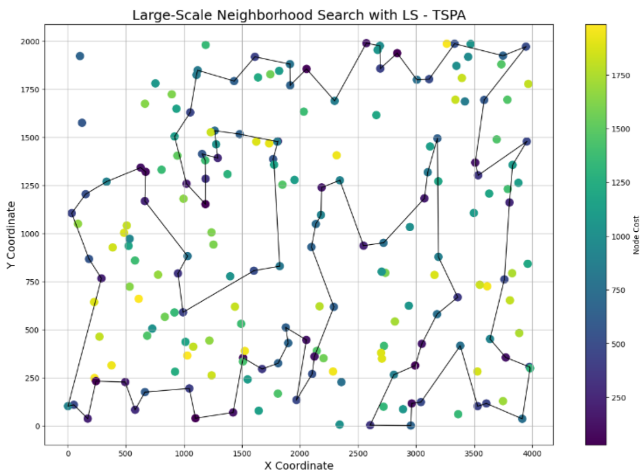

Multi Start Local Search - TSPB
Y Coordinate
X Coordinate
Node Cost

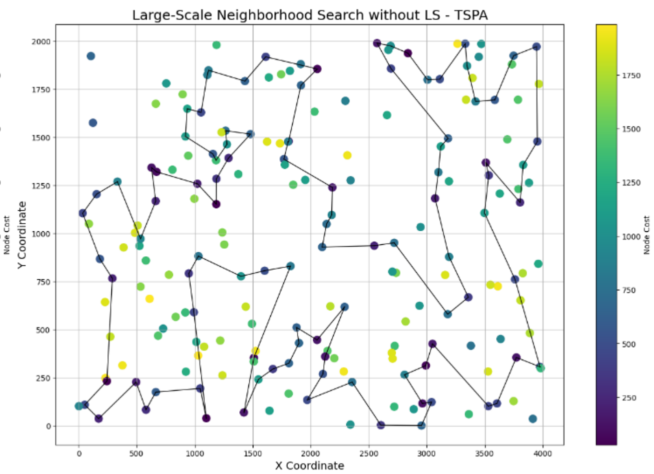

Iterated Local Search - TSPC
Y Coordinate
X Coordinate
Node Cost

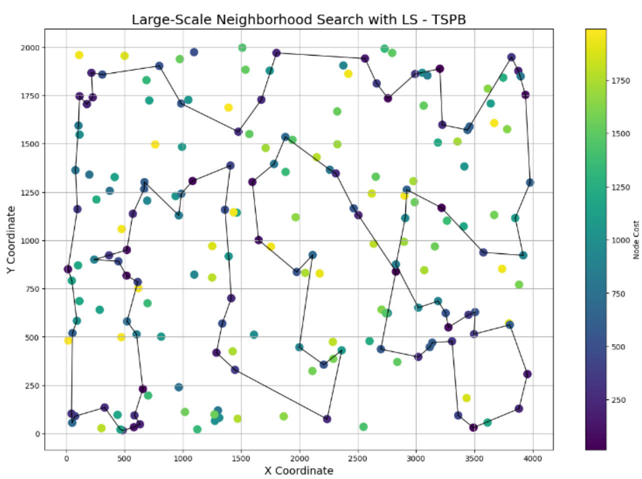

Multi Start Local Search - TSPC
Y Coordinate
X Coordinate
Node Cost

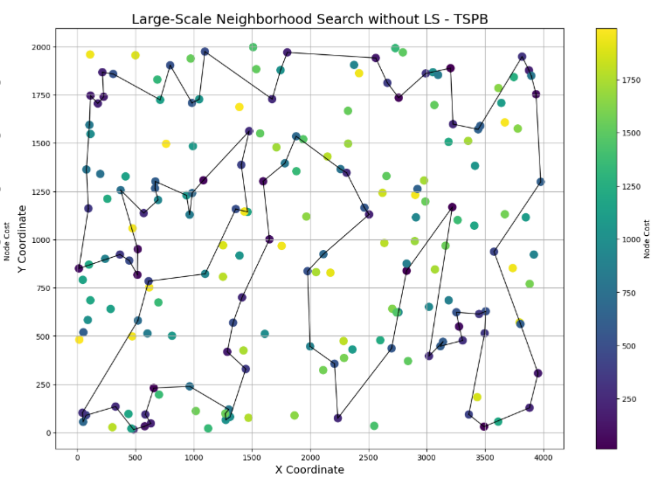

Iterated Local Search - TSPD
X Coordinate
Y Coordinate
Node Cost

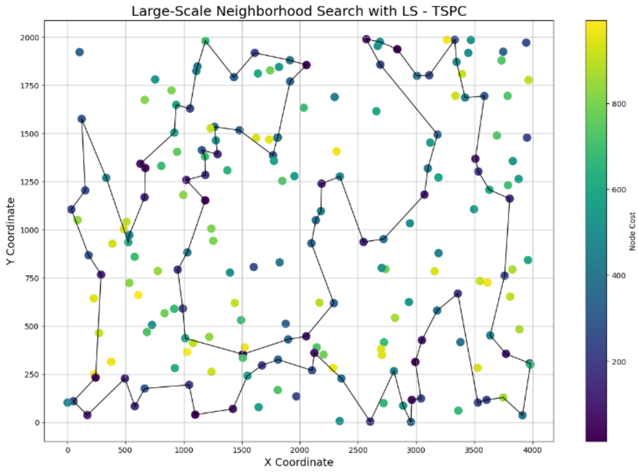

Multi Start Local Search - TSPD
X Coordinate
Y Coordinate
Node Cost

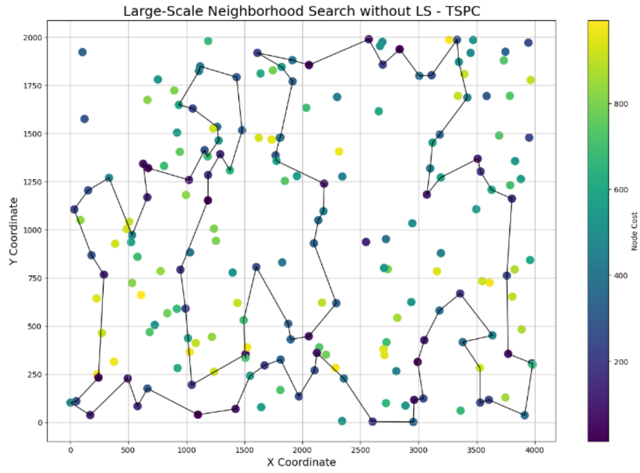

Large-Scale Neighborhood Search with LS - TSPA
X Coordinate
Y Coordinate
Node Cost

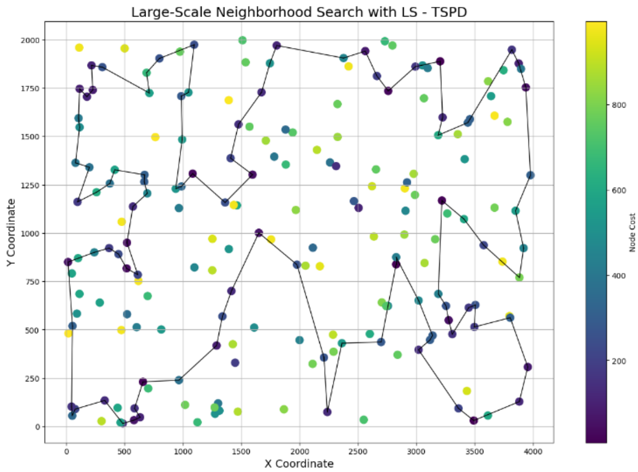

Large-Scale Neighborhood Search without LS - TSPA
X Coordinate
Y Coordinate
Node Cost

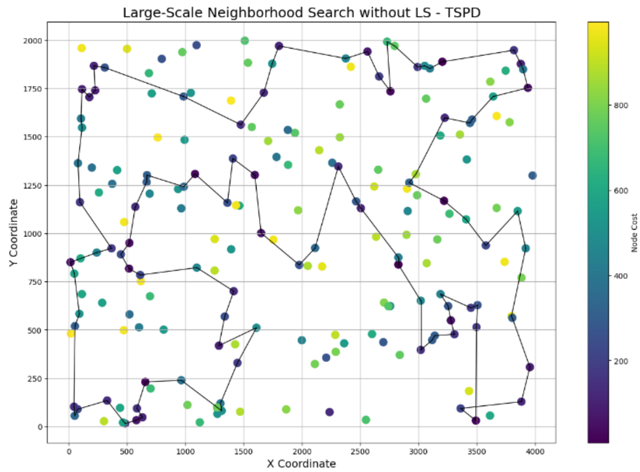

Large-Scale Neighborhood Search with LS - TSPB
X Coordinate
Y Coordinate
Node Cost

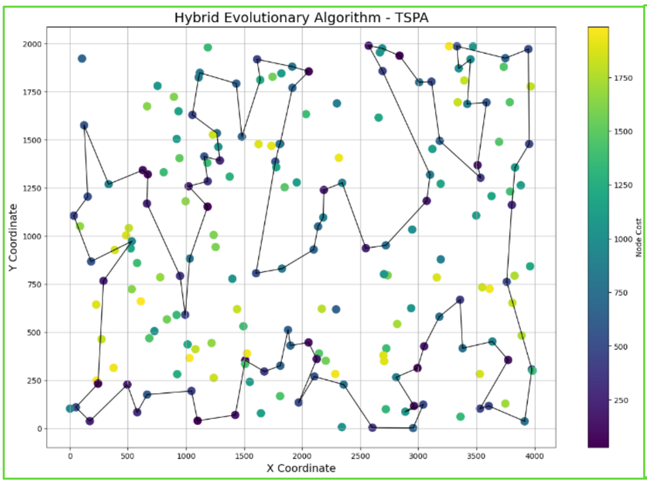

Large-Scale Neighborhood Search without LS - TSPB
X Coordinate
Y Coordinate
Node Cost

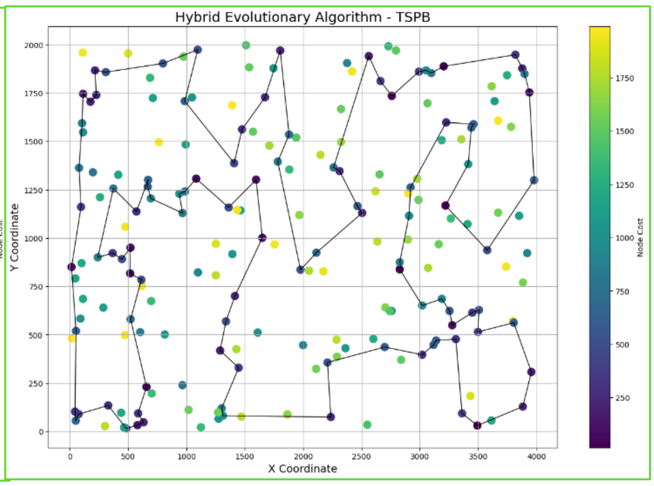

Large-Scale Neighborhood Search with LS - TSPC
X Coordinate
Y Coordinate
Node Cost

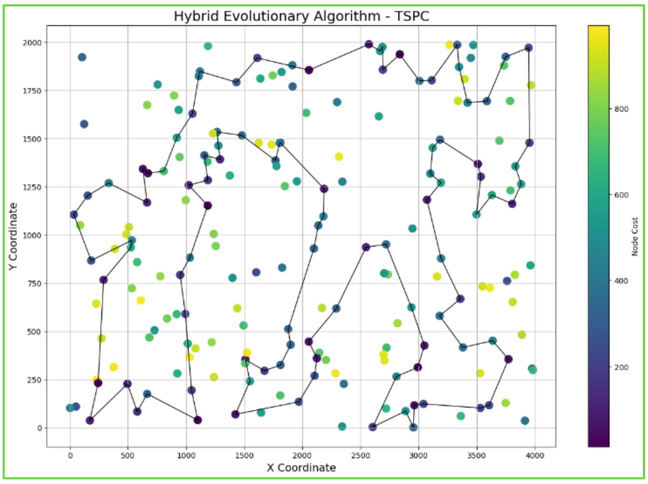

Large-Scale Neighborhood Search without LS - TSPC
X Coordinate
Y Coordinate
Node Cost

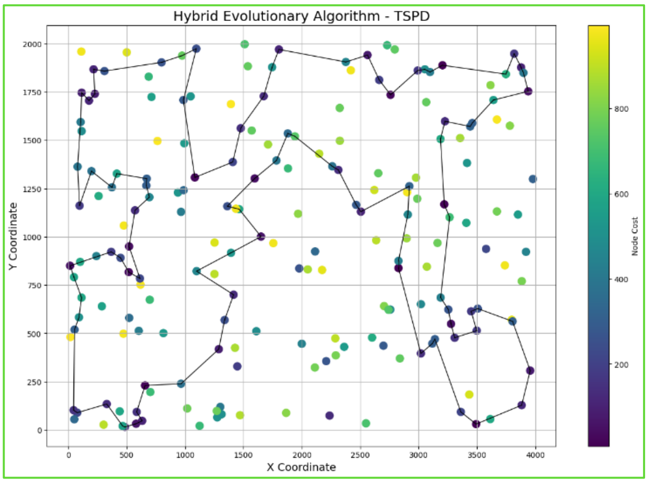

Large-Scale Neighborhood Search with LS - TSPD
X Coordinate
Y Coordinate
Node Cost

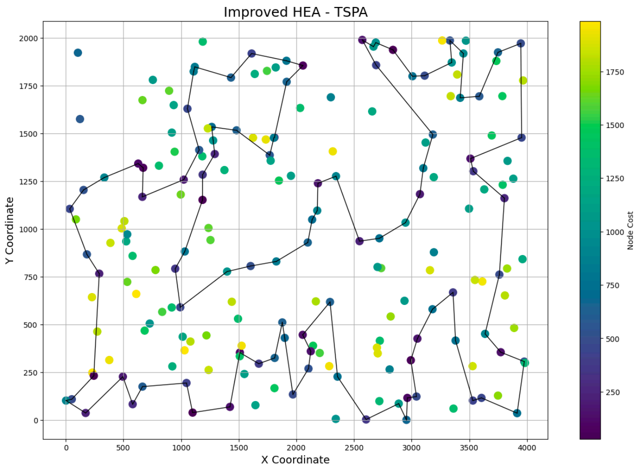

Large-Scale Neighborhood Search without LS - TSPD
X Coordinate
Y Coordinate
Node Cost

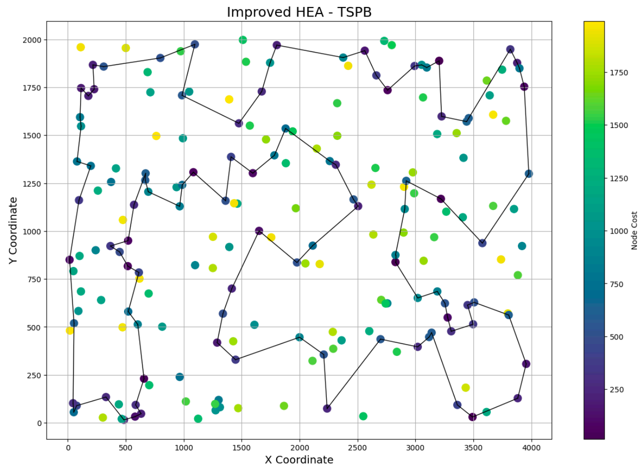

Hybrid Evolutionary Algorithm - TSPA
X Coordinate
Y Coordinate
Node Cost

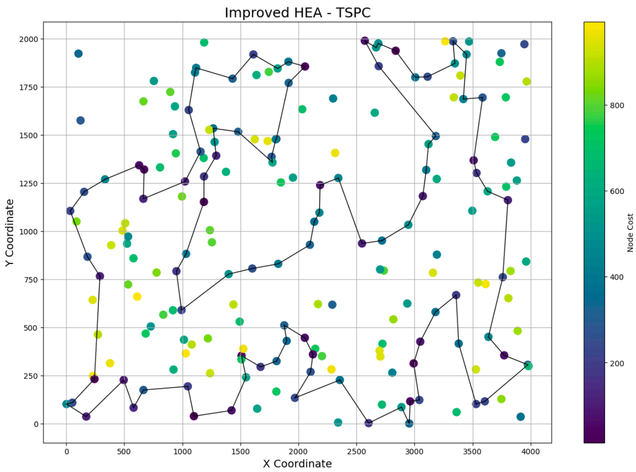

Hybrid Evolutionary Algorithm - TSPB
X Coordinate
Y Coordinate
Node Cost

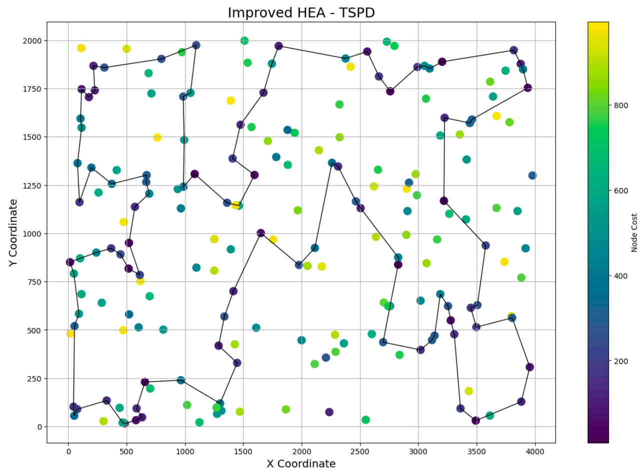












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Conclusions

The results obtained from the improved Hybrid Evolutionary Algorithm are really close to the original HEA in some instances better. We significantly increased the number of epochs. Despite the introduction of the new operator and optimization of some functions, which led to the increased number of epochs the results remained more or less the same. It may suggest that we are close to optimum. The changes are subtle, but still differences between graphs are distinguishable.