Freie Universität Berlin
FB Mathematik und Informatik
WS 2021

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Monday 18th October, 2021

Optimization I

Exercise Sheet 1

Submission: Friday 29th October, 2021, 16:00 CET before the tutorial

Exercise 1.1 **5+5 Points**

Let $G = (V, E)$ be a connected graph and let I be the set of independent sets in G.

1. Show that (V, I) is an independence system.
2. Let B_1, B_2 be bases of (V, I). Show that $\frac{|B_1|}{|B_2|}$ can become arbitrarily large and that (V, I) is not a matroid.

Exercise 1.2 **10 Points**

Let $G = (V, E)$ be a connected graph. The graphic matroid M_G associated to G has E as its ground set and $F \subseteq E$ is independent if and only if $G[F]$ contains no cycle. Denote the set of independent sets of M_G by $\mathcal{I}_G$.

1. Show that M_G is indeed a matroid.

2. How do the bases and circuits of M_G look like?

3. Give a formula for the rank function of M_G.

Exercise 1.3 **5+5 Points**

Let $M = (E, \mathcal{I})$ be a matroid and $\mathcal{I}^* := \{S \subseteq E : \exists \text{ basis } B \text{ for } M \text{ s.t. } B \cap S = \varnothing\}$. Prove that $M^* = (E, \mathcal{I}^*)$ is a matroid.

Exercise 1.4 **10 Points**

Consider the following alternative definition of a matroid.

Definition. *A matroid $(E, \mathcal{B})$ is a pair of a ground set E and a set of bases $\mathcal{B} \subseteq 2^E$ with the following properties*

(B1) The set of bases $\mathcal{B}$ is non-empty.
(B2) For any two bases $A, B \in \mathcal{B}$ and $a \in A \backslash B$, there is $b \in B \backslash A$ such that $A \backslash \{a\} \cup \{b\}$ is a basis.

We call all subsets $S \subseteq B$ for some $B \in \mathcal{B}$ independent sets.

Show that this definition is equivalent to any of the definitions of a matroid from the lecture.