

$$f = \sum_{i=1}^n c_i \underbrace{\varphi_i(x)}_{\text{FE basis}} \in V_h$$

$$\|f\|_{L^2}^2 = \int_b \left(\sum_i c_i \varphi_i \right)^2 dx$$

$$= \sum_{i,j} c_i c_j \underbrace{\int_b \varphi_i \varphi_j dx}_{= M_{ij}}$$

$$\stackrel{V_h}{=} c^T M c$$

$$\Rightarrow \|f\|_{L^2}^2 = \int c^T M c$$

$$z = \sum_{i=1}^n d_i \varphi_i$$

$$\|f - z\|_{L^2} = \sqrt{(c - d)^T M (c - d)}$$