Ann. Rev. Fluid Mech. 1978. 10: 221–46

THE STRUCTURE OF VORTEX BREAKDOWN

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INTRODUCTION

Vortex flows are subject to a number of major structural changes involving very large disturbances when a characteristic ratio of azimuthal to axial velocity components is varied. Vortex breakdowns are among the structural forms that may occur.

As used here, the term "vortex breakdown" refers to a disturbance characterized by the formation of an internal stagnation point on the vortex axis, followed by reversed flow in a region of limited axial extent. Two forms of vortex breakdown predominate, one called "near-axisymmetric" (sometimes "axisymmetric," or "bubble-like"), and the other called "spiral." They are shown in the photographs in Figure 1. When a dye filament is introduced on the vortex axis, the spiral form is marked by a kink in the filament, followed by a corkscrew-shaped twisting of the dye; the bubble form has a nearly axisymmetric envelope resembling a solid body of revolution.

Vortex-breakdown flow fields (of either type) may be divided into three spatial regimes:

1. The approach flow consists of a concentrated vortex core embedded in a flow that often may be approximated as irrotational. Changes in the approach flow with axial distance are slow and predictable (that is, according to intuition), and the flow either is laminar or has relatively low turbulent intensities. Axial-velocity profiles in approach flows are observed to be jetlike, with speeds on the axis exceeding those outside the core by an amount comparable to the maximum azimuthal speeds.

2. The breakdown region, characterized by rapid changes in axial direction, follows; it occupies an axial interval on the order of 5 vortex-core diameters in length, subdivided into three subintervals of approximately equal extent. In the first subinterval, the approach flow is decelerated and a stagnation point is formed on the vortex axis; in the second, flow reversal occurs near the axis for the spiral as well as the bubblelike breakdown. The radius of bubblelike breakdowns is nearly

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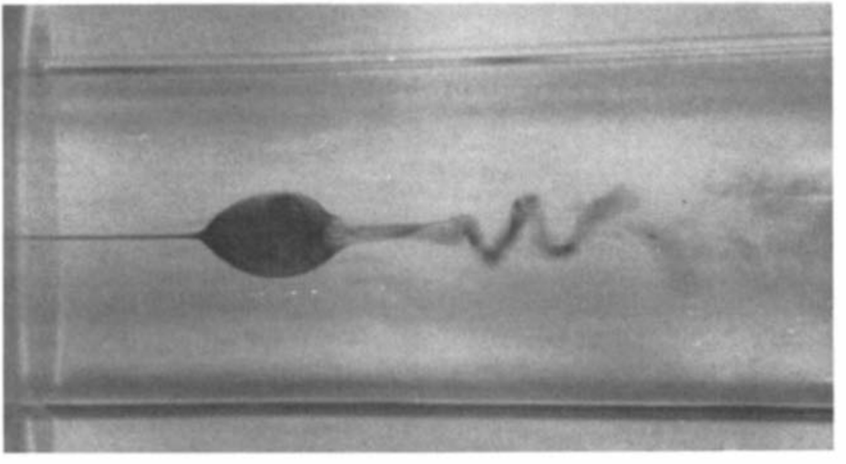
equal to that of the vortex core (defined by the location at which the azimuthal velocity in the approach flow is maximum). The original direction of axial flow is restored in the third subinterval, which is marked by a large increase in turbulent intensity (or, for laminar-approach flows, by the first unmistakable signs of transition to turbulence).

3. A new vortex structure with an expanded core is established downstream of the breakdown zone; like its upstream counterpart, axial variations are gradual.



(a)

(b)

Figure 1 Photographs of vortex breakdowns, with the flow generally from left to right. (*a*) Bubble form; (*b*) Spiral form.

Axial-velocity profiles closely resemble a conventional wake behind a solid obstacle, with centerline speeds less than those outside the vortex core, and the flow is invariably turbulent.

Reversed axial flows of a nearly columnar nature are common in theory (e.g. Donaldson & Sullivan 1960) and are often observed in rapidly swirling flows in devices like cyclone chambers. Vortex breakdown, however, is set apart by its free stagnation point and region of flow reversal embedded within a flow that otherwise has only one direction. It is therefore a phenomenon intermediate between weakly swirling flows, which exhibit no flow reversal, and rapidly swirling flows with columnar flow reversals. As an "intermediate" phenomenon, vortex breakdown is inherently nonlinear, and this is the source of major theoretical difficulties.

Peckham & Atkinson (1957) first documented the breakdown of leading-edge vortices above wings with highly swept leading edges when the angle of incidence exceeded a critical value. Naturally occurring vortex breakdown in aircraft trailing vortices has also been observed, and the possibility of artificially stimulating breakdown to enhance the dissipation of these vortices is frequently mentioned (Olsen, Goldburg & Rogers 1971). Much theoretical work (e.g. Squire 1960, Ludwieg 1962, 1965, Gartshore 1962, 1963, Hall 1967, Bossel 1969, 1971, Benjamin 1962, Mager 1972) has been motivated by these aerodynamical problems. Theoretical goals for such applications are to characterize vortex conditions that favor breakdown, to predict its location, and to infer the vortex structure downstream. Since the transitional breakdown region is very small compared to distances of interest in aerodynamical vortices, the structure of the transition region is not of great interest in itself; in this view the approach and wake regions may be thought of as joined by a sudden jump, and the establishment of jump conditions capable of connecting the two is of significant interest. A similar viewpoint would be adequate for some other circumstances, such as surging in draft tubes of Francis-type turbines (Cassidy 1969, Cassidy & Falvey 1970) or tornadoes (Barcilon 1967), in which vortex breakdown occurs or is thought to occur.

Combustion chambers for gas turbine engines and other combustion devices often employ swirl to establish a recirculation zone to serve as a fluid-dynamical flame holder; these zones appear to be examples of the vortex-breakdown phenomenon. Accounts of this useful application may be found in papers by Gore & Ranz (1964), Yetter & Gouldin (1977), and Syred & Beer (1974). A photograph of the recirculating combustion zone in Swithenbank & Chigier (1969) closely resembles the photograph in Figure 1a.

Applications that depend upon the structure of the breakdown region itself clearly require theories that aspire to a more complete description of the phenomenon. Some theories, such as those of Benjamin (1967), Leibovich (1968), Bossel (1969), Leibovich (1970), and Randall & Leibovich (1973) seek to provide additional information about this zone, and numerical studies of the Navier-Stokes equations by Kopecky & Torrance (1973) and Grabowski & Berger (1976) attempt detailed predictions of its structure.

This survey concentrates on work published since the 1972 review by Hall in this series. It should be added that, although Elle (1960) and Lambourne & Bryer (1961) give some evidence that vortex breakdown occurs at speeds up to transonic, all available systematic studies have been carried out at low speeds; consequently only incompressible motions will be considered.

EXPERIMENTS

Although vortex breakdown was discovered in vortices produced by flow separation from the leading edges of highly swept wings (see Lambourne & Bryer 1961, Hummel 1965), experimental emphasis soon shifted to more easily controllable experiments in tubes.

With the exception of Chanaud's (1965) experiments, swirl in tube facilities has been imparted to the fluid as it moves radially inward over adjustable vanes. Harvey (1962) and Kirkpatrick (1964) used the same apparatus and conducted experiments in air; Sarpkaya (1971a,b, 1974), Faler & Leibovich (1977a,b), and Garg (1977) used basically similar apparatus for experiments in water. All of these devices therefore produce vortices of a similar kind; except at very low Reynolds numbers where wall effects are non-negligible, the generated approach-flow vortices have a jetlike axial-velocity profile and an azimuthal velocity V that is well represented by the Burgers vortex,

$$V = \frac{K}{r}[1 - \exp(-\alpha r^2)]. \tag{1}$$

The parameters K and α vary slowly with axial distance.

Measurements reported by Garg (1977) show that α increases nearly linearly with Reynolds number, which is consistent with Harvey's (1962) observation that the size of the vortex core is determined by the thickness of the boundary-layer shed from the centerbody of the swirl generator.

An important consequence of these facts is that data are apparently available only for one family of velocity profiles. These are similar to those obtained in leading-edge vortices, as described by Hall (1966, p. 86). Batchelor (1964) demonstrated the need for axial flow in trailing-tip vortices, including the likelihood of a jetlike axial flow near the wing; Moore & Saffman (1973) discuss this question further, and Singh & Uberoi (1976) have measured jetlike axial motion in trailing vortices very near (within one chord of) the trailing edge of a wing tip. Vortices produced by vanes in experimental tube facilities may therefore also resemble vortices near wing tips.

Most experimental data taken since Hall's (1972) review have been in tubes, and this review deals primarily with such cases.

Qualitative Observations

LOW-REYNOLDS-NUMBER OR LOW-SWIRL PROGENITORS Vortex breakdowns are at the end of an evolutionary chain of large-amplitude forms of disturbances in flows with a significant swirling component. Sarpkaya (1971a,b, 1974) distinguished bubble and spiral forms of vortex breakdown reported earlier by others, and a new form that he called the "double-helix." These are among six transitions between flow states that Faler & Leibovich (1977a) observed as swirl increased at fixed Reynolds number (based on tube radius), or as Reynolds number increased at fixed swirl-vanes setting. Both operations have the same effect on the type and position of disturbance obtained, since both increase axial vorticity, peak swirl, and the axial velocity overshoot in the upstream vortex core. That increasing vane angle has this effect is obvious: circulation about the vortex core is thereby increased. Increasing Reynolds number at *fixed* vane angle leads to a decrease in vortex-core size which, with fixed circulation, leads to an increase in core vorticity. This implies a lower pressure, and hence increased axial velocity on the vortex axis.

The order of succession of the disturbance forms is invariant and repeatable, reminiscent of transitions observed in other swirling flows [cf Coles (1965), where 74 transitions are documented in circular Couette flow; or Nagib (1972), where transitions in circular Couette flow with axial flow are investigated].

The flow states have distinctive morphologies. Each displays a considerable degree of structural stability, although broad ranges of conditions exist in which the flows are bistable, and transitions between adjacent flow states may occur spontaneously. Such bistable regimes were also seen by Sarpkaya, who described them as hysteresis regimes; however, the phenomena seem to occur in the same way regardless of how the flow settings are arrived at, and therefore are not true hysteresis effects.

The first large-amplitude structural change (type 6 in the Faler-Leibovich classification), as Reynolds number or swirl increases, is marked by a large, gentle spiral deflection of the center streamline. The "double-helix" (type 5) evolves from this form by a shearing and splitting of the centerline filament, and disturbance forms 5 and 6 are the only ones that are stationary in time. Succeeding disturbances contain an internal stagnation point, and according to our definition are therefore vortex breakdowns; all are asymmetric in azimuth, and all are marked by very regular periodic oscillations. Oscillations in types 4 and 3 disturbances may be thought of as waves standing in a preferred azimuthal (θ) plane of the form $\exp(im\theta)\exp(i\omega t)$. The ultimate and penultimate forms in the sequence are the "bubble" and "spiral" breakdowns shown in Figure 1. From viewing angles that are nearly perpendicular to their preferred planes, dye patterns in the type 4 disturbance look like those in bubble breakdowns, and patterns in type 3 forms look like those in spiral breakdowns. The asymmetric dye features of bubble and spiral breakdowns have no preferred plane, but instead rotate about the axis. As shown by Lambourne & Bryer (1961), rotation of these features is not necessarily associated with rotation of fluid particles: one may link the asymmetries and temporal oscillations by ascribing both to waves propagating in azimuth (θ) with the form $\exp[i(m\theta-\omega t)]$. All further discussion in this review is confined to bubble and spiral breakdowns.

In addition to having regular oscillations, the entire breakdown form is generally unsteady in position, moving back and forth in an unpredictable way along the tube axis. These axial excursions, which have been reported by all observers, may be very small, but generally are on the order of a few diameters of the upsteam vortex core, and take place about some definite position determined by configuration of the apparatus, swirl, and flow rate.

BREAKDOWN RESPONSE TO INCREASE IN SWIRL LEVEL Once a spiral breakdown is formed, increase of swirl (either by increase of vane angle or Reynolds number) causes upstream movement of the breakdown. When swirl is sufficiently large, a spiral form may spontaneously (without apparent alteration of experimental conditions) transform into a bubble form: when this happens, the mean position of the resulting bubble is always several vortex-core diameters upstream of that of the original spiral mode. A range of swirl levels exists in which these transformations are reversible: the new bubble may be spontaneously replaced by a spiral.

For sufficiently large swirl, only the bubble form is seen. Continued increase in swirl causes upstream movement of the bubble until it reaches the upstream boundary of the apparatus. According to Harvey (1962), if the swirl is increased beyond this level a columnar vortex results, with backflow along the entire axis of the tube. (This did not occur in the experiments of Faler and Leibovich, nor in reports of Sarpkaya's experiments. Either the vane-angle range was too small to produce columnar backflows or exit conditions in these experiments preclude backflows extending throughout the tube.)

EFFECTS OF PRESSURE GRADIENTS Vortex breakdown is promoted by adverse pressure gradients (pressure increasing in the direction of flow). Pressure gradients may be impressed upon the vortex core by a deceleration of the outer flow (e.g. by shaping the walls of a confining tube); from pressure differences along a vortex core caused by a sudden expansion if the tube abruptly ends; or, in a leading-edge vortex, from the pressure rise associated with the trailing edge of a swept wing.

Hall (1972) summarizes the role of an applied pressure gradient: when an adverse pressure gradient is increased, less swirl is required to maintain a breakdown, or, if the same level of swirl is maintained, the breakdown is moved upstream.

Sarpkaya (1974) initiated a systematic study of the effect of an adverse pressure gradient. The location of breakdown was measured as a function of flow rate and vane angle for four tubes having interior walls with differing angles of divergence. Hall's account of the general effects of an adverse pressure gradient was confirmed, although the adverse gradients that could be produced were limited by onset of boundary-layer separation on the tube walls.

TRANSIENT MOTIONS The mean breakdown position for any fixed set of flow conditions is reasonably repeatable. Sarpkaya (1971a) presents a fascinating description of the axial movements of vortex breakdowns subjected to sudden alterations of either flow rate or vane angle. Instead of moving monotonically from one "equilibrium" mean position to another, the initial motion was in the direction opposite to that expected: a rapid movement toward the final equilibrium position followed, resulting in an overshoot and then a relaxation towards the equilibrium position required by the new flow settings.

These events were confirmed by Faler & Leibovich (1977a). Laser-doppler anemometer measurements also revealed existence of a pulse convected down the tube that is attributed to the collection of starting or stopping vortices generated when either the vane angle or flow speed is suddenly changed. The peculiar bubble motions are qualitatively accounted for by Faler & Leibovich (1977a) by considering quasi-steady effects caused by local accelerations in the pulse.

It is possible that transient formation of multiple axisymmetric breakdowns observed by Sarpkaya (1971a) occur in these pulses in response to associated local accelerations.

Velocity Measurements

Harvey (1962) succinctly describes two distinct and serious problems encountered in attempts to measure velocity in vortex breakdown flows. First, random variations in the position of the vortex breakdown make measurements of any kind extremely difficult in the immediate vicinity of the breakdown. Second, vortex breakdown is highly sensitive to external disturbances, and introduction of material probes near the breakdown can completely alter the character of the flow.

As a result of these difficulties, until recently velocity measurements were available only at stations sufficiently distant from the breakdown location to avoid probe distortion effects. Data in the immediate neighborhood of vortex breakdown was limited to information obtainable by flow visualization, such as swirl-angle distributions in the approach flow (determined by the ratio of azimuthal to axial velocity components) (Harvey 1962, Sarpkaya 1971a) and the centerline axial-velocity component just upstream of breakdown (Sarpkaya 1971a).

Recent measurements by Faler (1976), Faler & Leibovich (1977b), and Garg (1977) have provided additional detail, including power spectra and a velocity map of the recirculation region of the bubble type of breakdown. Laser-doppler anemometry circumvents probe difficulties and choice of special flow conditions yield a bubble fixed in position long enough to take measurements. Precautions taken and a discussion of experimental errors may be found in Faler (1976). The following descriptions are taken from Faler & Leibovich (1977b) and from Garg (1977).

THE APPROACH FLOW With one exception, all approach flows were either laminar and steady, or turbulent, but without evidence of periodic oscillations. At a given axial location, the radial distributions of the axial velocity component $W(r)$ were fitted well by the function

$$W(r) = W_1 + W_2 \exp(-\alpha r^2), \tag{2}$$

and the azimuthal velocity component by (1) with the same parameter α. The parameter W_2 increased with the maximum swirl velocity.

Examples of swirl and axial velocity profiles in the approach flow for a number of vane angles are shown in Figure 2. The Reynolds number, based upon flow rate and tube diameter, was 6000: for reference purposes, the Reynolds number based upon W_1 and the core diameter as determined by the point of maximum azimuthal velocity was about 1400. A breakdown of bubble form occurred downstream for the profile indicated by triangles, and spiral breakdowns occurred in the other two cases.

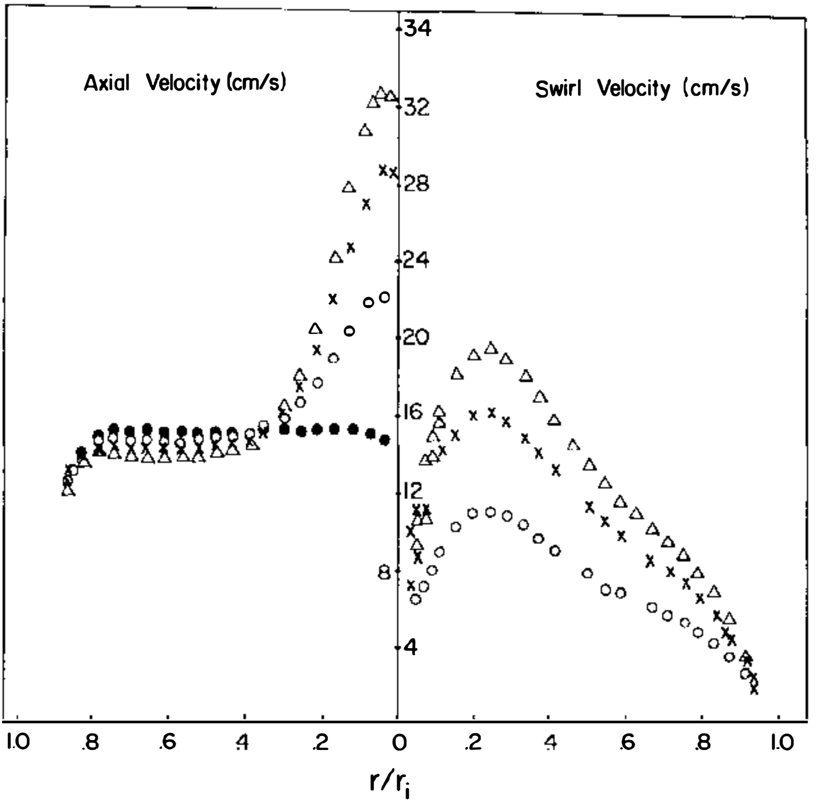


Figure 2 Examples of velocity profiles in the approach flow for fixed flow rate and different vane angles. The solid circles represent axial velocity at zero vane angle.

THE BREAKDOWN REGION Detailed measurements in the interior of a recirculation zone are available for only one vortex breakdown (Faler and Leibovich 1977b); the flow studied, chosen for the steadiness of breakdown location, occurred at a Reynolds number of 2560, based on flow rate and tube diameter.

Deceleration of the approach flow Axial velocity profiles just upstream of the recirculation zone show a rapid fall of centerline velocity. All flows measured upstream of the stagnation point were laminar, and, so long as the breakdown position remained stationary, all were steady.

Recirculation zone Motion in the recirculation zone is unsteady, with regular low-frequency oscillations. Fluctuation magnitudes vary considerably; in some parts of the recirculation zone they are negligibly small, and in other parts they are strong and dominate the motion. Visual evidence of the existence of periodic fluctuations has been reported also by Sarpkaya (1971a).

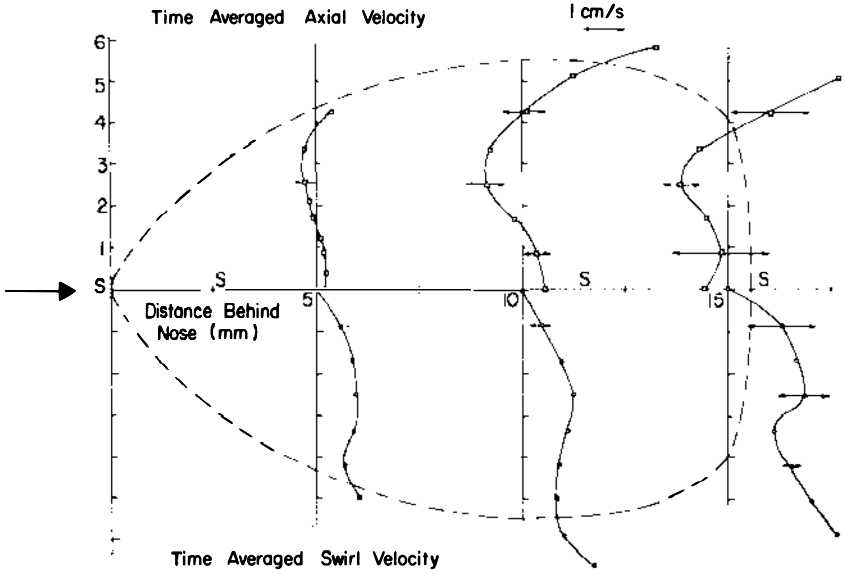


Figure 3 Mean-velocity profiles inside the recirculation zone. The bubble envelope is indicated by the broken line. Upper half gives data on axial velocity, the lower half shows the azimuthal velocity component. The horizontal bars indicate the peak-to-peak range of fluctuations about the mean.

Mean velocities and peak-to-peak fluctuations in the recirculation zone are summarized in Figure 3 with the bubble envelope superimposed. The upper portion gives axial-velocity data, and the lower portion contains data on the azimuthal velocity. Horizontal bars superimposed on the velocity profiles indicate maximum peak-to-peak fluctuations about the mean: the dominant period of oscillation was approximately 2 Hz.

All speeds in the recirculation zone are small compared with the approach flow: for example, the maximum mean axial velocity in the recirculation zone is 15% of the centerline speed in the approach flow, and the maximum mean centerline speed in the inner cell is only 4% of its value in the approach flow.

Streamlines for the mean flow, constructed from the mean axial velocity component, are shown in Figure 4. A total of four stagnation points are found on the axis, and are marked by the symbol "S" in the figure. A multiple-cell structure is indicated with an inner cell embedded inside the recirculation zone. This "streamline" pattern gives an impression of the instantaneous motion in parts of the flow where fluctuations are somewhat smaller than the local mean quantities.

Although the existence of the inner cell is revealed in a mean streamline pattern constructed from a flow with sizeable fluctuations, it may also be seen in the instantaneous motion. The direction of the instantaneous, as well as the mean, flow on the axis between the two interior stagnation points is in the direction indicated

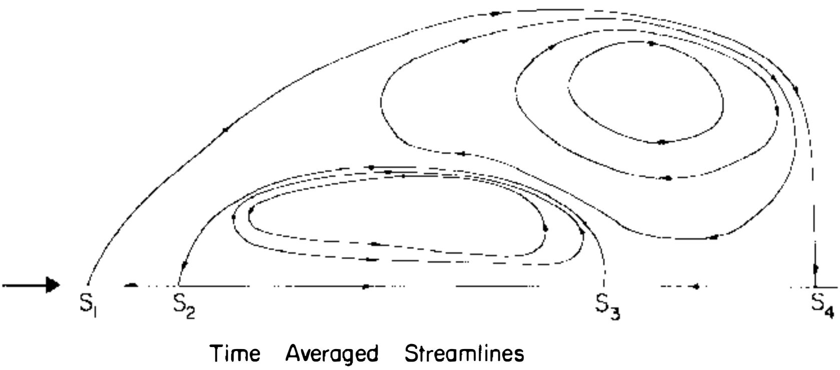


Figure 4 Streamlines constructed from the mean axial velocity reveal a two-celled internal bubble structure. Four stagnation points on the axis, indicated by "S," occur for the mean flow.

in Figure 4, showing the presence of a two-celled structure. Mean and instantaneous axial velocities on the axis between the first and second, and between the third and fourth stagnation points, are reversed.

A plane through the third stagnation point divides the recirculation zone. Upstream of this plane, fluctuations are typically smaller than the local mean velocitics, sometimes negligibly small; fluctuations at many points in the flow downstream of this plane are larger than the mean, sometimes several times as large. Exchange of fluid across the rear boundary of the bubble envelope occurs through the large periodic oscillations.

Low speeds in the inner cell, and between the first and second stagnation point, imply that dye in this region should disappear more slowly than in the rest of the recirculation zone if the supply of dye is interrupted. Visual confirmation of the cxistence of thc inner cell bascd upon these remarks is shown in photographs taken aftcr the supply of dyc was shut off. The dark filamcnt of dyc marking thc inncr cell in Figure 5 terminates precisely at the rear stagnation point of thc inner cell, as detcrmined from the mcan streamline pattern of Figure 4; the dye filament forms a kink at this stagnation point, and the vestige of a rotating corkscrew pattern characteristic of a spiral breakdown may be seen downstream. Thus, the visual evidence suggests the existence of an embryonic spiral breakdown *inside* the bubble form.

Visual evidence of the inner cell may also be found in Figure 6 or Sarpkaya (1971a), and in a short note by Bellamy-Knights (1976). Bellamy-Knights concludes that flow on the axis of inner structure is not reversed, in agreement with Faler & Leibovich (1977b), but does not observe a stagnation point terminating the inner cell.

Axial flow recovery A spiral breakdown typically occurs shortly downstream of the recirculation zone of a bubble breakdown form and presumably has its own region of flow reversal. Within a few diameters of the vortex core, turbulent wake flow is then established. Fluctuations in the recovery region remain very strong and regular.

WAKE REGION Extensive measurements in the wakes of vortex breakdowns, of both bubble and the spiral types, are reported by Garg (1977). The wake is invariably turbulent, but low-frequency oscillations remain strongly in evidence.

Mean wake profiles Garg's data was collected on six flows at higher Reynolds numbers than the experiments of Faler & Leibovich (1977a,b). Three flows contained the bubble form of breakdown, and three contained the spiral form. Qualitative features of the breakdowns appear to be independent of Reynolds number.

Batchelor (1964) showed that the velocity in a weak decaying trailing vortex is approximately given by functions (1) and (2) with W_2 negative. These functions in

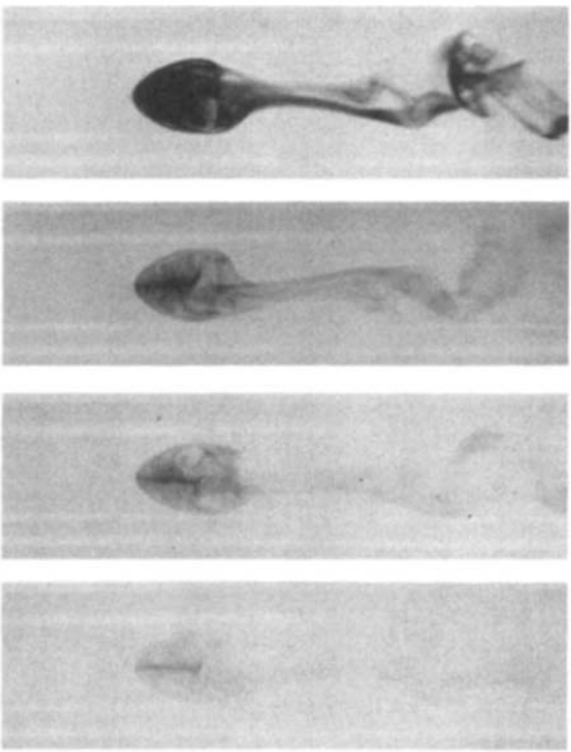


Figure 5 Dye concentrations in the inner cell are heavier and serve to indicate multiple-cell structure. This sequence of photographs was taken over a 12-second interval after the dye supply was interrupted. Dye remains in the inner cell longer than elsewhere in the bubble.

fact supply a reasonable fit to the wake data, and therefore adequately represent profiles in both the jetlike approach flow (W_2 positive) and in the wake (W_2 negative).

The vortex-core diameter in the wake of spiral breakdowns is typically about two times as large as the core of the approach vortex according to Garg's measurements. The expansion of the vortex core is larger than this for "axisymmetric" breakdowns; the bubble may therefore be regarded as the stronger form of breakdown, and the spiral the milder form.

Table 1 gives a summary of Garg's (1977) approach and mean wake flows in terms of the exponential least-squares best-fit parameters (α, W_1, W_2, K). Included in the Table is the ratio of the wake-core radius to that in the approach flow, and the parameter q, defined as $K\sqrt{\alpha}/W_2$. The q parameter, introduced by Lessen, Singh & Paillet (1974), equals 1.57 V_{max}/W_2, where V_{max} is the maximum swirl in the vortex (1); q is a measure of the ratio of axial to azimuthal vorticity in the core of the vortex (1, 2) [at the point of maximum azimuthal velocity, $q = 1.12 \times$ (axial vorticity) $\div$ (azimuthal vorticity)]. Garg finds the parameter α to be sensitive to small errors in the location of the vortex centerline, but the parameters (W_1, W_2, K, q) are quite insensitive. Consequently a smoothed value of α is given in Table 1 for approach flows.

Table 1 Summary data for approach-wake pairs in the experiments of Garg (1977)[a]

Reynolds number and breakdown type	Location	α cm^{-2}	W_1 $cm\ s^{-1}$	W_2 $cm\ s^{-1}$	K $cm^2\ s^{-1}$	q	Core expansion ratio
1920	Approach	7.95	24.5	21.75	15.87	1.98	
Spiral	Wake	2.95	18.0	13.0	10.96	1.45	1.64
1920	Approach	7.95	23.8	37.6	20.06	1.59	
Bubble	Wake	1.33	22.3	21.0	17.54	0.97	2.45
2812	Approach	8.5	38.1	37.0	23.91	1.86	
Spiral	Wake	2.68	27.1	20.1	17.54	1.43	1.78
2812	Approach	8.5	36.2	50.5	27.8	1.62	
Bubble	Wake	2.18	28.0	24.3	19.47	1.18	1.97
3348	Approach	13.1	54.7	57.1	32.0	1.85	
Spiral	Wake	3.0	37.6	30.7	24.2	1.38	2.1
3348	Approach	13.1	53.6	79.1	33.1	1.64	
Bubble	Wake	1.72	45.7	41.8	33.1	1.04	2.76

[a] Parameters obtained by a best fit of data to equations (1) and (2) for approach-wake pairs in Garg's (1977) experiments. Reynolds number is based upon W_1 and upon core diameter d_c, determined by the position of maximum swirl and inferred from the table according to the rule $d_c = 2.24\ \alpha^{-1/2}$. Flows with the same Reynolds number have the same approach core size, and the vortex core expansion of an approach flow of given size is smaller for spiral breakdowns.

Fluctuations and stability Garg (1977) presents over 500 power spectra for the axial and azimuthal velocity fluctuations at numerous points in his flows. Fluctuations are somewhat more regular and more intense in the breakdown region than in the wake, and are stronger in the bubble form of breakdown than they are in the spiral form.

It is likely that the oscillations arise from an instability of the mean flow. The inviscid stability of the profiles defined by (1, 2) has been investigated by Lessen et al (1974) for nonaxisymmetric disturbances with normal modes of form $\exp[i(\kappa z + m\theta - \omega t)]$ and by Howard & Gupta (1962) for axisymmetric ($m = 0$) disturbances. Stability in both cases depends only upon the parameter q; flows are stable to axisymmetric disturbances provided $q > 0.4$, which always seems to be the case in experiments leading to vortex breakdown. According to Lessen et al, stability to *all* infinitesimal disturbances is assured if $q > 1.5$, while instability to nonaxisymmetric disturbances is obtained for smaller values. All experimental approach flows are stable by this criterion, but the mean flow measured in the wake is always unstable to nonaxisymmetric perturbations.

The dominant peak of the measured spectra may be compared with the real part of the frequency corresponding to the most unstable wave number at that station in the wake. Lessen et al (1974) give sufficient detail to apply their results only to the case of $m = -1$ azimuthal wave number. Garg (1977) has made this comparison. Peaks were always found in the measured spectra very close to the frequencies calculated from the theory, and in all but one case, the calculated frequencies coincided with the spectral peak of lowest frequency (usually the dominant one). There is no way to determine whether the $m = -1$ mode actually occurs in the experiment, but it would be surprising if the close agreement obtained were fortuitous.

Singh & Uberoi's (1976) experiment on trailing tip vortices reveal similar phenomena; their description suggests that vortex breakdown occurred about 2.4 chord lengths downstream of the wing, with a wakelike region downstream. Fluctuations for distances in the wake up to 40 chord lengths from the wing were dominated by periodic oscillations experimentally determined to be $|m| = 1$ modes.

Breakdown in Trailing Vortices and Tornados

Sudden expansion of vortex cores, accompanied by increased turbulence levels, have been reported in aircraft trailing vortices and are sometimes evident in photographs of tornados.

Barcilon (1967) describes this process as it occurs in a laboratory model of a tornado. Breakdown of trailing vortices occurred in flight tests reported by Chevalier (1973), in model experiments reported by Bilanin & Widnall (1973), and in Singh & Uberoi's (1976) experiments. Figure 6 is adapted from a photograph of breakdown in trailing vortices taken by Bilanin and Widnall.

Although the abrupt expansion of vortex cores in these circumstances has been attributed to vortex breakdown, details about the flows are unavailable (e.g. whether there is a stagnation point), and the relationship with breakdown in tubes and in leading-edge vortices is not known.

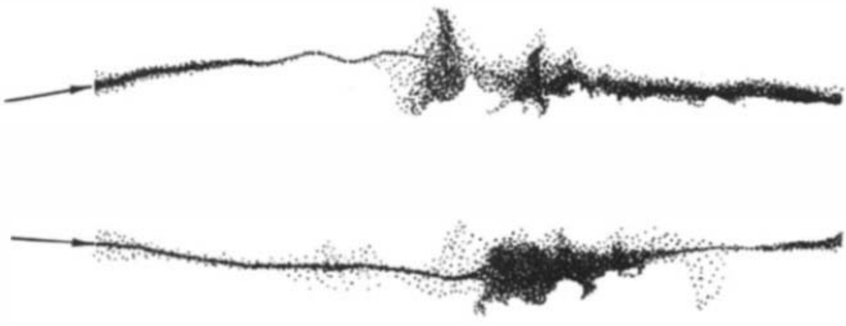


Figure 6 Breakdown in trailing vortices behind lifting wing towed through water. The vortex core contains an axial velocity component from left to right. Vortex bursting is apparent.

THEORIES

Steady solutions with regions of closed streamlines resembling vortex breakdown abound. Let the flow be described in cylindrical (r, θ, z) coordinates with velocity components (u, v, w). If the flow is assumed inviscid and axisymmetric, the equation governing the motion is

$$\psi_{rr} - r^{-1}\psi_r + \psi_{zz} = r^2 H'(\psi) - KK'(\psi), \tag{3}$$

where ψ is the Stokes stream function and $(\ )' \equiv d/d\psi$. In (3), H is the total head, which by Bernoulli's theorem is a function only of ψ, and $K = rv$, the circulation about the axis of symmetry (z), is a function of ψ alone by Kelvin's theorem. The functions H and K are arbitrary, and must be prescribed to specify a given flow: for an axially unbounded streaming flow, they are conveniently described at upstream infinity, where flow may be assumed to be free from axial gradients. The functional forms so determined will apply to all regions of the flow threaded by streamlines originating at upstream infinity.

When the flow upstream has solid-body rotation and uniform axial velocity, the disturbance stream function satisfies a linear equation, although the solutions in this special case represent disturbances of arbitrary magnitude. Solutions containing regions of closed streamlines and waves, like the "rotors" and lee waves obtained in analogous stratified-flow problems, have been found by Fraenkel (1956), Chow (1969), and Bossel (1969). Taylor's (1922) classical solution for a solid sphere can be converted into a solution with closed streamlines with a spherical dividing stream surface, as shown by Moore & Leibovich (1971).

As a phenomenon, embedded backflow regions would seem to have enough theoretical examples to be regarded as commonplace, and perhaps even "understood." Unfortunately, this is far from true. The rigidly rotating environment is very different from the concentrated-vortex approach flows in which observed breakdown takes place, and, of course, no axisymmetric theory can account for both the spiral and bubble form of breakdown. Even putting aside these difficulties, and the mathematically troublesome nonuniqueness associated with wave propagation and upstream influence [see McIntyre (1972), which contains a bibliography on the question], calculations based upon Equation (3) give physically absurd results for the structure of the recirculation zone. This may be seen in the context of general admissible functions $H(\psi)$ and $K(\psi)$. In a physically realistic approach flow, $K = 0$ on the rotation axis (on which $\psi = 0$ may be taken), and the axial vorticity on the axis is normally nonzero, so that $K'(0) \neq 0$. If a recirculation zone forms near the axis, the bounding streamline divides regions of positive and negative ψ; since K'

zone. In a steady flow, this is impossible even if viscosity is permitted, since an unavailable torque would be required to maintain the reversed swirl.

One has the option, exploited by Leibovich (1968), of specifying different functional forms for the interior of the recirculation. The difficulty of reversed swirl can be circumvented this way, but the nonuniqueness inherent in specifying more than one form for $H'(\psi)$ and $K(\psi)$ obviously complicates matters even further.

Even though the internal structure of the bubble will be wrong, and this must affect the bubble shape, solutions of (3) obtained with a single set of functions (H', K)

It is easy, for example, to generate bubble shapes resembling the bubble envelope, and the external flow around the breakdown may be represented reasonably well by such solutions.

In summary, although solutions of (3) have been applied to vortex breakdown by Leibovich (1968) and by Bossel (1969) (without invoking the necessary changes of K and H'

information; the solutions obviously do not encompass the spiral form or deal with transitions between bubble and spiral forms, and they cannot give unambiguous criteria for either the onset or the position of breakdown.

Working Hypotheses

The predominant stance taken in theoretical research has been to ignore asymmetric features. The principal mechanisms shaping vortex breakdown are assumed to exist in axially symmetric flow, and asymmetries are assumed to be secondary effects resulting from an instability of the basic asymmetric phenomenon to nonaxisymmetric perturbations. From this point of view, first explicitly enunciated by Benjamin (1962), there is an ideal axisymmetric breakdown phenomenon that could, at least in principal, be experimentally realized.

The second point of view, advanced primarily by Ludwieg, emphasizes the asymmetrical features observed in breakdown. From this viewpoint, instability to nonaxisymmetric disturbances is the principal mechanism of breakdown and not a secondary effect. The hypothesis has suffered from the fact that the Ludwieg's (1962) stability criterion was derived for a geometry differing greatly from that encountered in vortex breakdown flows. The criterion is not a valid stability boundary for relevant vortex flows and predicts instability for essentially all approach flows. In addition, the analysis of Lessen et al (1974), which does seem to apply, invariably shows that approach flows leading to vortex breakdown are *stable* to all infinitesimal disturbances (including asymmetric ones). Furthermore, vortex breakdown flows, with oscillations existing only downstream of a very sharply defined point, appear very different from familiar hydrodynamic instabilities. As a result, Ludwieg's main idea (which is *not* dependent upon his stability criterion) has not been widely accepted or fully explored.

Although the role of spiral disturbance has not found favor as a primary mechanism for breakdown, the story is far from complete. All breakdowns *are* asymmetric, all belong to an inherently asymmetric hierarchy (as the swirl is increased), and the internal structure even of the bubble form seems controlled by nonaxisymmetric fluctuations. Basic questions about vortex breakdown clearly will remain unanswered until nonaxisymmetric disturbances are better understood. (For example, a theoretical description of the transition from spiral to bubble breakdown modes cannot be constructed neglecting nonaxisymmetric motions.)

Theoretical Elements

Recurring themes in theoretical work on vortex breakdown are the quasi-cylindrical approximation, which is the common starting point, and propagation of axially symmetric waves.

Axial gradients in approach flows seen in experiments are much weaker than radial gradients; the transverse (radial) velocity component is small compared with axial and azimuthal components; the pressure is in approximate balance with centrifugal forces; and the motion is axisymmetric. The quasicylindrical approximation exploits these features of the flow and leads to governing equations (cf Hall 1972) with a strong resemblance to shallow-water wave theory, or to boundary-layer theory. Not surprisingly, the various theories have drawn analogies to phenomena, such as hydraulic jumps or solitary waves on one hand, or boundary-layer separation on the other, that are familiar in these classical theories.

If viscosity is neglected, the quasi-cylindrical approximation has solutions in which the axial velocity $W(r)$ and the swirl velocity $V(r)$ are arbitrary functions of r alone. Stability of this basic flow to axisymmetric disturbances is guaranteed if Howard & Gupta's (1962) "Richardson number" criterion $r^{-2}(dW/dr)^{-2}V[d(rV)/dr] \geqq 1/4$ is satisfied. Propagation of neutral axisymmetric waves is always possible in stable vortices, and it can be shown that the extreme values of the phase speed occur for waves of extreme length ($k \rightarrow 0$). Vortex flows may be classified (Benjamin 1962) as "supercritical" if wave propagation is only possible with phase speed in the downstream direction ($c > 0$), or "subcritical" if waves can also propagate with phase speeds in the upstream direction ($c < 0$). "Critical" flow, on which standing waves ($c = 0$) of extreme length are possible, separates the two categories. Increasing swirl generally enhances the ability of waves to propagate against the flow; thus a supercritical flow is driven towards subcritical by increasing swirl.

Squire (1960) proposed that breakdown first is possible when perturbations may spread upstream from a downstream source of disturbance. Based upon phase speeds, this seemed possible for $c < 0$, and $c = 0$ is the marginal case. Benjamin (1962), however, showed that the group velocity for waves of extreme length and zero phase speed was positive (downstream): such waves would in fact be swept downstream, and he dismissed Squire's proposal on these grounds. Since flows of interest in vortex breakdown have properties that vary (slowly) with axial location, and have supercritical regions followed by subcritical regions, the merits of this criticism are questionable.

Although Benjamin's objection to Squire's proposal is based upon group-velocity arguments, his flow classification scheme is cast in terms of the phase velocity. To exclude upstream propagation of disturbances, one should prove that the group velocity of all (axisymmetric) waves in supercritical flow is directed downstream. Otherwise, upstream propagation of wave packets, containing wavelets propagating through the packet in the downstream direction, may be possible. This possibility can be eliminated quite generally for flows with smooth velocity profiles (Leibovich, unpublished). Nevertheless, the classification remains seriously incomplete since the propagation characteristics of azimuthally asymmetric waves are not included (nor known).

Recent Theoretical Work

Contributions appearing since 1971 include wave-propagation theories by Randall & Leibovich (1973), Bilanin (1973), and Huang (1974), and an analysis by Mager (1972) based upon the quasi-cyclindrical approximation.

MAGER'S ANALYSIS Mager's (1972) treatment of the quasi-cylindrical equations by momentum-integral methods parallels earlier work by Gartshore (1962, 1963). Vortices embedded in an irrotational flow with given circulation and uniform axial velocity are examined. Since the pressure gradient in the outer flow vanishes, the vortex changes form only in response to viscous effects: at Reynolds numbers occurring in practice, hundreds of vortex-core diameters are required for significant changes due to viscosity.

The neglect of imposed pressure gradients is a major simplification, and the momentum-integral equations can be integrated, leading to algebraic equations. These depend upon axial distance z only parametrically, so a complete treatment is possible.

If a dimensionless constant θ_1 is smaller than a certain value θ_1^*, Mager finds that the vortex core decays continuously with z. If $\theta_1 > \theta_1^*$, a singularity occurs at a well-determined point, and the solution cannot be further continued. The singularity corresponds to the failure of the quasi-cylindrical approximation of the kind previously discovered by Gartshore (1962), Hall (1967), and Bossel (1971): viscosity is now known (Ludwieg 1970, Hall 1972) to be unnecessary for the existence of this singularity, and the condition for its occurrence coincides with Benjamin's (1962) critical condition.

The solution upstream of the singularity is double-valued; initial conditions determine the solution branch. For $\theta_1 > \theta_1^*$, Mager postulates the possibility of a "crossover" from the upper to the lower branch. A "crossover" is similar to Benjamin's (1962) inviscid "finite-transition" between dynamically conjugate states. Benjamin's transition requires that (*a*) the upstream "primary" flow be supercritical, (*b*) the downstream "conjugate" flow be subcritical, (*c*) the mutually conjugate flows share the same distribution of total head $H(\psi)$ (apart from a possible constant difference), and the same distribution of angular momentum $K(\psi)$, and (*d*) the downstream conjugate has a larger "flow force" $[= 2\pi \int (p+\rho w^2)r\,dr]$ than the primary flow. Mager shows that his upper and lower branches are supercritical and subcritical, respectively; conditions (*b*) and (*c*) involve details not contained in an integral analysis. Further considerations of the flow force lead Mager to conclude that "finite transitions" and "crossovers" are equivalent.

Since finite transition and failure (by virtue of a singularity) of the quasi-cylindrical approximation are known possibilities, it is not surprising to find both features in a quasi-cylindrical analysis. Mager's work is interesting since both phenomena are explictly displayed. The behavior of his solutions leads Mager to suggest that crossover may be identified with occurrence of bubblelike breakdown, and the singularity with spiral breakdown. The inference of structural detail from an analysis containing no structure, however, seems an unusually bold step.

Since finite transition and the failure of the quasi-cylindrical approximation can be attributed to inertial effects, viscosity is not essential to these possible breakdown mechanisms. Its effect is probably secondary; together with an adverse pressure gradient, viscous forces tend to drive a supercritical flow towards critical conditions.

WAVE THEORIES Squire's early idea that waves may propagate upstream from a downstream disturbance source (e.g. created by the tube exit, or trailing edge of a swept wing) is the starting point of wave theories. Wave-propagation characteristics change as a vortex responds to an imposed pressure gradient. Upstream propagation of wave energy in subcritical regions is permitted by group-velocity considerations for infinitesimal waves; waves cannot penetrate upstream of the point at which critical conditions obtain. The possibility that disturbances build up to large values at this critical "barrier," resulting in vortex breakdown, is then suggested.

The theory of Randall & Leibovich (1973) is based upon an expansion for long axisymmetric waves given in Leibovich & Randall (1973). Allowance is made for acceleration of the outer flow by considering motions in a slowly diverging tube; the expansion, valid near the critical condition, may be described as "transcritical." Finite-amplitude wave propagation is shown to be governed by a modified Korteweg-de Vries equation, with coefficients that vary slowly with axial location and a term that leads either to wave amplification or damping. Deceleration of the basic flow causes energy transfer from the basic flow to the wave; acceleration causes energy transfer in the opposite direction. A stationary-wave equilibrium, in which the energy gained from the basic flow is balanced by viscous dissipation, may exist only between two well-defined axial locations in decelerated portions of the flow (adverse-pressure gradient); to persist, these waves must remain "trapped" in this permissible zone. It is significant that there are no free adjustable parameters in the theory once the tube geometry and basic flow are prescribed and the existence of a stationary wave is postulated. When applied to a flow in which Sarpkaya (1971a) found a bubble form of breakdown, the trapped-wave amplitude was found to be necessarily large, resulting in formation of a stagnation point and reversed flow;

bubble shape, size, and some other flow features are consistent with Sarpkaya's observations.

Despite interesting correspondences between structural features in the trapped-wave theory and those observed in experiment, Randall & Leibovich (1973) point out that there are deficiencies in the theory. Reversed swirls predicted in the bubble interior are physically impossible, and, although the theory has a rational validity for *weakly* nonlinear disturbances, the disturbances predicted are necessarily *large*. [This self-generated inconsistency may, however, help to explain what Hall (1972) describes as a "physical preference for large perturbations" associated with the breakdown phenomenon.] Furthermore, there is no reason to assume that the critical station acts as a barrier to upstream propagation of asymmetric waves; their exclusion is difficult to justify, except on grounds of the existence of an ideal axially-symmetric motion.[1]

Bilanin (1973) has considered a "trapped-wave" theory of vortex breakdown conceptually similar to that of Randall & Leibovich (1973), but quite different in approach. Landahl's (1972) general theory of wave focussing and wave breakdown provides the framework for the analysis, and Bilanin allows nonaxisymmetric wave motion. Vortex breakdown is associated with a finite-amplitude trapped wave, but the structure of this wave is not determined.

The need to invoke viscous dissipation probably should be regarded as a shortcoming of the Randall-Leibovich theory. A more significant form of "dissipation" is possible, however, that may conceivably replace viscosity. Nonaxisymmetric waves can exchange energy with axisymmetric wave modes in a conservative system, and the exchange can result in an energy loss (or gain) to the axisymmetric mode. Huang (1974) has explored the nonlinear interaction process; the axisymmetric mode amplitude is shown to satisfy a Korteweg-de Vries equation with a sink (or source) term involving the energy density of nonaxisymmetric waves. Some consequences of this interaction process are discussed in Huang's thesis, but the treatment is formal, and not sufficiently comprehensive to allow conclusions to be drawn about its role in vortex breakdown.

SOME COMPARISONS BETWEEN EXPERIMENT AND THEORY

A proper determination of the utility of differing theories should begin with detailed solutions constructed using experimentally determined boundary conditions. Such solutions are not available in the literature, so the quantitative discussion here is confined to theoretical aspects requiring a minimum of computation. Other comparisons are qualitative, and inevitably somewhat subjective.

One may classify flows as supercritical, etc, with little computational effort.

[1] C. Y. Tsai and S. E. Widnall (private communication) have computed wave characteristics of the azimuthally asymmetric $m = \pm 1$ modes for some supercritical approach flows using data in Table 1 of this review. They find group velocities are directed downstream, but phase velocities may be in either direction depending on value of the wave number.

According to Hall (1972), Kirkpatrick's (1964) measurements showed that his flow, far upstream of breakdown, was supercritical. Computations done for initial stations and wake profiles for all data gathered by Faler & Leibovich (1977a,b) and by Garg (1977) in all cases show the initial stations are supercritical and the wakes subcritical. This is in accord with Benjamin's (1962) finite-transition theory, and with the wave theories initiated by Squire (1960).

The approach flow, as defined here, consists of the region upstream of the breakdown zone in which axial gradients are small; the criticality classification, which ignores axial gradients, therefore has an approximate validity. Faler & Leibovich (1977b) have measured velocity profiles at a sequence of stations in the same approach flow. All stations were supercritical, and displayed a fall towards criticality consistent with wave theories and with Hall's "critical retardation" (discussed below).

Gartshore (1962), Hall (1967), Bossel (1971), and Mager (1972) associated vortex breakdown with the development of large axial gradients in step-by-step calculations using the viscous quasi-cylindrical approximation. The appearance of large axial gradients violates the assumptions of the quasi-cylindrical approximation in the same way the approach to separation violates the boundary-layer approximation. Reynolds numbers in most experiments, however, are sufficiently large to insure that viscous forces are negligible, so viscosity is unlikely to cause the approximation to fail. Ludwieg (1970) has pointed out, however, that solutions of the inviscid quasi-cylindrical approximation are singular at the critical condition. Following up on this thought, Hall (1972) proposed the eclectic concept of "critical retardation." This idea asserts that breakdown is related to the formation of a singularity at critical conditions; the location often may be computed, step-by-step, through a sequence of supercritical flows using the *inviscid* quasi-cylindrical approximation. Experimental data are consistent with "critical retardation": not only are upstream flows supercritical, but breakdown locations seem reasonably close to positions predicted to be critical using the inviscid quasi-cylindrical approximation.

Sarpkaya (1974) used Mager's equations, with deceleration of the outer flow taken into account, to estimate the location of breakdown in his experimental apparatus; breakdown position was compared to the location of the singularity. By a suitable choice of initial-profile parameters (not determined experimentally), the singularity could be made close to the observed point of breakdown in test cases, but the calculation was found to be sensitive to the choice of initial profile. The best profile parameters were used to compute breakdown locations for other experimental conditions of Reynolds number and swirl level, and the results were thought acceptable. In retrospect, however, it is clear that Sarpkaya's calculations do not satisfactorily test the correspondence between the location of Mager's singularity and breakdown. Initial-profile parameters should be experimentally determined; furthermore, a change of Reynolds number or swirl causes significant change in approach-flow velocity profiles. The comparison of solutions obtained using only one set of profile parameters with experiments over a range of conditions is therefore inappropriate.

Faler & Leibovich (1977a) found the heuristic use of wave propagation concepts to be useful in correlating the position and movement of vortex breakdowns under both steady and transient experimental conditions. To the (important) extent that theory should serve to develop intuition, the wave theories can take credit for their ability to connect events occurring in Faler & Leibovich's (1977a) experiments.

Randall & Leibovich (1973) computed flow fields predicted by the trapped-wave theory using swirl-angle data reported by Sarpkaya (1971a); the axial velocity was assumed to be uniform. On that basis, reasonable agreement was found with the size, location, and wall-pressure histories determined in Sarpkaya's experiment. An interesting qualitative point of agreement between this theory and experiment is the existence of a definite range of breakdown locations. Sarpkaya (1974) compared breakdown location predicted by the trapped-wave theory with observations. He noted, as did Randall and Leibovich, that the computed position is very sensitive to tube shape, and that only one curve of Reynolds number vs position was possible instead of the family of curves obtained experimentally for different vane angles. However, axial velocity profiles are far from uniform, and parameters in the velocity profiles leading to bubble breakdowns depend strongly on Reynolds number and vane angle, although swirl angles (determined by the ratio V/W) do not. Had appropriate experimental input profiles been available, Randall and Leibovich's theory would have yielded a family of breakdown positions.

If Benjamin's (1962) finite-transition theory provided a jump condition allowing the (subcritical) wake structure to be inferred from prescribed (supercritical) approach flows, then it would clearly be superior to Squire's (1960) criterion. Evidently, Benjamin (1962) thought that this might sometimes be the case, but that if the transition is strong so that turbulence results, he realized that "this dissipative process cancels the original dynamical relationships determining the subcritical conjugate." These "dynamical relationships" mean the functional forms $K(\psi)$ and $H'(\psi)$
steady nor axially symmetric, but one might wonder if Benjamin's subcritical conjugate might describe the mean flow properties (which are axially symmetric) in the wake. The functions K and H'
for the data in Table 1; the results demonstrate that the necessary dynamical relationships indeed are destroyed. Apparently, breakdown is never sufficiently mild to allow the subcritical conjugate to determine the flow properties in the wake.

NUMERICAL EXPERIMENTS

The full Navier-Stokes equations have been solved numerically by three sets of investigators, subject only to restrictions of axial symmetry, steadiness, and incompressibility. This work is of great importance to the determination of the internal structure in the breakdown region since, as we have already seen, existing theories are unable to treat this region properly.

The numerical solutions cover a range of Reynolds numbers from four to several hundred and very different initial profiles, yet bubble-like flow reversals occur in the entire range. Lavan, Nielsen & Fejer (1969) considered low-Reynolds-number flow passing from a rigidly rotating circular pipe into a stationary pipe of the same diameter; Kopecky & Torrance (1973) considered flow in a cylindrical tube with initial velocity distributions given by Equation (1) and uniform axial velocity; and Grabowski & Berger (1976) considered a vortex core embedded in an irrotational flow with initial velocity distributions given by Mager's (1972) family of polynomial profiles.

Taken together, the numerical experiments convincingly demonstrate that the Navier-Stokes equations *do* have axially symmetric solutions with embedded regions of closed stream surfaces resembling the bubble form of vortex breakdown. The physical experiments show, just as convincingly, that these solutions must be unstable to nonaxisymmetric disturbances. The extent to which the numerical solutions represent the structure of experimentally observed flows is, nevertheless, an important question. If the representation is good, then one may conclude that nonaxisymmetric motions may safely be ignored (on a time-averaged basis) when considering the structure of the recirculation zone.

Lavan et al (1969) deal with a geometry and a Reynolds number range very different from those in vortex-breakdown experiments, and devote little attention to the structure of the recirculation zone. As a result, the present discussion is confined to the remaining two numerical experiments; both consider comparable Reynolds-number ranges (in the hundreds) and more relevant geometries. As in the case with theories, comparisons between physical and numerical experiments are clouded since the numerical work assumes geometries and initial velocity profiles that differ from the physical experiments. Despite this fact, and the fact that the Reynolds numbers treated are lower than those covered by reported experimental data, some conclusions are possible.

Kopecky & Torrance (1973) treat a geometry and initial conditions that resemble vortex-breakdown experiments in tubes more closely than the cases considered by Grabowski & Berger (1976), but the calculations by the latter authors are more extensive, and have greater spatial resolution. Results of the two studies are broadly similar, however, so we may draw from either as the need arises.

For a given flow rate, small or moderate swirl in numerical experiments causes a deceleration on the axis, with an associated local swelling of the stream surfaces. When the swirl is sufficiently large, a stagnation point appears on the axis, followed by a small region of closed stream surfaces. Further increase of swirl causes the region of closed stream surfaces to grow in size. In Grabowski & Berger's calculations, a long second deceleration region follows the recirculation region, and for cases giving large "breakdowns," the downstream deceleration zone becomes an elongated second region of reversed flow. Kopecky and Torrance never find multiple flow reversals, but their closed streamline zones are quite elongated. It seems plausible to conjecture that the true structure of Kopecky and Torrance's bubble is similar to Grabowski and Berger's, but that the crude mesh used in the former case caused the "bubble" and second deceleration zone (or bubble) to merge.

For large swirl, Grabowski and Berger's bubble appears to begin to lift off the axis: although the bubble is large, the interval of reversed flow on the axis shrinks in size and the first stagnation point appears downstream of the forward part of the bubble.

Using results presented by Leibovich (1970), one can show that all of the initial flows considered by Kopecky and Torrance are *subcritical*, and, using Mager's (1972) results, Grabowski and Berger determine that flow reversals result from both subcritical and supercritical initial flows.

Comparison with Physical Experiments

Some qualitative points of agreement between physical and numerical experiments are apparent from the description above. There are, however, a number of ways in which the numerical and physical experiments are quite different, and this suggests that the assumption of steady axisymmetric motion may not be adequate to compute the detailed structure of vortex breakdown flows.

All breakdown flows for which velocity data exist are supercritical upstream and subcritical downstream, but the results of numerical experiment seem indifferent to this classification. The significance of this difference is not known, but it is a marked discrepancy.

It is possible that the criticality classification, which requires an approach flow with weak axial gradients, cannot be meaningfully applied to the numerical experiments. All results presented by both Kopecky and Torrance and by Grabowski and Berger for flows having reversals contain strong axial gradients right up to the initial axial station; there is no "approach" flow. This effect is severe enough in Kopecky and Torrance's work to deserve their comment; it apparently is more serious in Grabowski and Berger's work under nominally subcritical conditions. The "upstream influence" of the bubble suggests that the choice of boundary condition at the inlet plane may impose an artificial constraint on the flow. A relaxation of this inlet boundary condition may lead to an upstream propagation of the bubble (which could then leave the computational domain) leaving a smooth subcritical flow everywhere. (In the nominally subcritical cases the postulated behavior is essentially compelled by subcriticality, but the disturbance cannot pass through the inlet plane if radial velocities are prohibited there.) Whatever behavior might follow a relaxation of the inlet boundary condition, the contamination of that condition by upstream influence makes the interpretation of available solutions uncertain.

The numerical experiments do not reveal the two-celled interval structure seen in physical experiments. The existence of the embryonic "spiral" breakdown inside the bubble is closely tied to the inner recirculation cell, and it is unlikely that the mean internal structure in physical experiments can be described unless the periodic asymmetric motions in the interior are accounted for.

The numerical experiments produce bubbles that increase continuously in size as the swirl is increased, and show only mild bulges for low swirl. These bulges are never seen in experiments, except as transients that soon disappear. Furthermore, the radius of experimentally observed bubbles always seem to be approximately equal to the radius of the vortex core in the approach flow. Smaller bubbles never seem to be seen; at lower swirls, a spiral breakdown appears downstream instead.

Physical experiments show that the vortex-core radius in the wake of a breakdown of bubble form is twice (or more) as large as that of the approach flow. Core expansions in the numerical experiments and in the wave theories are very slight. This is not surprising, since entrainment made possible both by asymmetric coherent fluctuations and by turbulence occurring in real flows are not taken into account in either approach.

CONCLUSIONS

To a significant extent, the assumption of axial-symmetry has produced useful results. The classification of flows as supercritical or subcritical, a step that assumes symmetry, has proved universally useful. Experiments show that vortex breakdown is always preceded by an upstream supercritical flow and followed by a subcritical wake, and the bubble form, at least, is located at the dividing (critical) point. Step-by-step calculations using the quasi-cylindrical approximation, often with the neglect of viscosity, can describe the development of a supercritical flow. Although the evidence is scanty, the location at which critical conditions appear in these calculations seems to provide an acceptable estimate of the location of breakdown. Both theory and numerical experiments concerned with detailed behavior yield structures bearing a resemblance to the bubble form of breakdown, and predict a response of bubble location to changes in upstream flow conditions in qualitative agreement with physical experiments.

Despite points of agreement, however, the comparison between experiment and attempts at prediction is less than encouraging. The spiral form of breakdown is excluded by hypothesis by axisymmetric formulations: important structural features, even of the bubble form of breakdown, are not accessible to existing theories, and are incorrectly represented by numerical experiments. A satisfactory understanding of the structure of vortex breakdown and of the transitions between apparently distinct forms seems to demand that consequences of asymmetry, largely ignored in the past, be faced.

ACKNOWLEDGMENTS

Professors S. E. Widnall and F. K. Moore read a draft of this paper, and I am indebted to them for constructive criticisms. Professor Widnall supplied slides of breakdown in wing-tip trailing vortices; Mrs. Brenda Hagnagy converted one of these into Figure 6, and also prepared the other figures. Mrs. Barbara Benedict typed more drafts of this paper than I care to admit, and somehow managed to remain cheerful and patient in the process. This work was supported by the National Aeronautics and Space Administration through Grant NSG 3019, technically monitored by the Lewis Research Center.

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