STATS 3006 Mathematical Statistics III
Assignment 1
2018

Assignment 1 is due by 5:00pm Monday 19 March 2018.

Assignments are to be submitted online on MyUni.

1. Suppose $X \sim \text{geom}(p)$ with probability function

$$p(x) = p(1-p)^x \text{ for } x = 0, 1, 2, \ldots$$

   Prove that $\text{var}(X) = (1-p)/p^2$.

2. Consider the Binomial distribution with probability function

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x = 0, 1, 2, \ldots, n.$$

   (a) Show directly that $E(X) = np$. That is, without using the MGF.

   (b) Show directly that $\text{var}(X) = np(1-p)$. That is, without using the MGF.

   (c) Consider the moment generating function, $M_n(t)$, for the binomial distribution with parameters n and p_n and suppose

$$n \to \infty;\ p_n \to 0 \text{ such that } np_n = \mu > 0.$$

   Find $\lim_{n\to\infty} M_n(t)$ and interpret the result.

3. Suppose $X \sim \text{Gamma}(\alpha, \lambda)$.

   (a) Show directly that $E(X) = \alpha/\lambda$. That is, without using the MGF.

   (b) Show directly that $\text{var}(X) = \alpha/\lambda^2$. That is, without using the MGF.

   (c) Show that the MGF is given by

$$M(t) = \left(\frac{\lambda}{\lambda - t}\right)^\alpha$$

   for $t < \lambda$.

4. Suppose $U \sim U(0,1)$ and let $X = \sqrt{U}$.

   (a) Find the PDF of X.

   (b) Calculate $E(X)$ directly from its PDF and also from the distribution of U and check that the two answers agree.

5. Supposed $U \sim U(0,1)$ and let $X = 3U + 2$.

   (a) Find the MGF of X.

   (b) Hence, identify the distribution of X.

6. Suppose $Z \sim N(0,1)$.

   (a) Show that $E(Z) = 0$

(b) Show that $\text{var}(Z) = 1$.

(c) Derive the moment generating function, $M(t)$.

7. Suppose $X \sim N(\mu, \sigma^2)$ and let $Y = aX + b$ for constants $a,\ b$ with $a \neq 0$. Prove that $Y \sim N(a\mu + b, a^2\sigma^2)$.

Honours Questions

The following questions are compulsory for Honours and Masters students and may be attempted for bonus marks by all students.

8. Suppose X has the Cauchy distribution. Find the distribution of $Y = 1/X$.

9. Consider the Poisson process with rate λ and suppose it is given that there is exactly 1 occurrence in the interval $[0, t)$. Show that conditionally on this information, the exact time, X, of the occurrence is $U(0, t)$.
Hint: Find the conditional CDF of X using the usual definition of conditional probability.

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