CRICOS PROVIDER 00123M

School of Computer Science

COMP SCI 1103/2103 Algorithm Design & Data Structure

Graphs and Trees

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seekLIGHT

Review

- A linked list is a data structure that is composed of objects that have:
  - values
  - a pointer to another object of the linked list type
- We start with a single element and form a chain of elements, each one pointing to the next element using a pointer.

- Can be used to implement
  - Stacks
  - Queues

Some structures are more complex

- A linked list may consist of nodes that include more than one pointer.

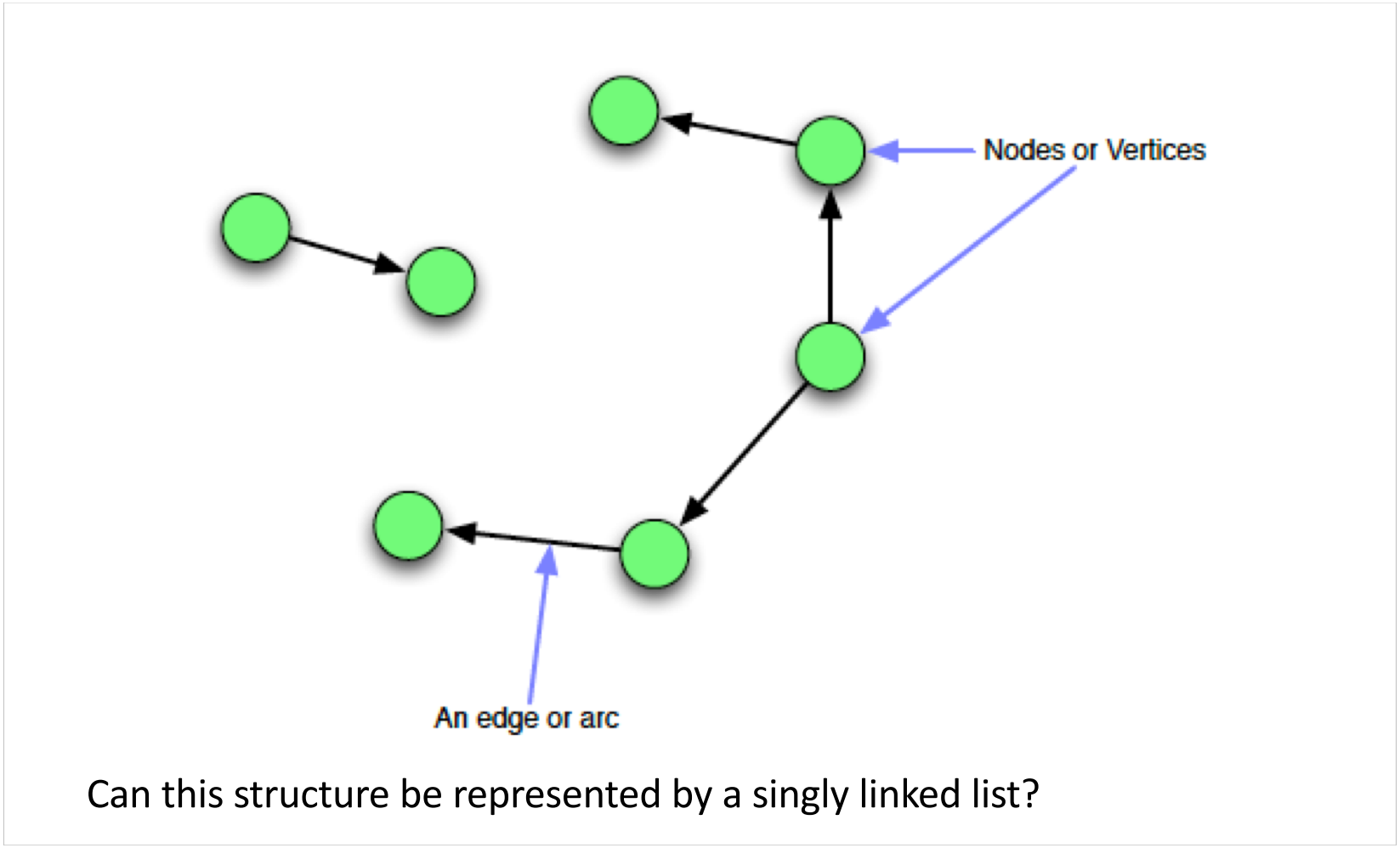
```
struct Node {
int data;
Node *link;
Node *otherLink;
}
```

- This can form a doubly linked list, but can also allow us to create very different structures.

Graph

- A graph is a collection of points (vertices or nodes) where some of the points are connected by line segments (edges or arcs).
- Graphs are a useful mathematical structure and are heavily used in networking, algorithmic studies and advanced computation.
- G= (V,E),
  - V=$\{v_1, v_2, \ldots, v_n\}$,
  - E=$\{e_1, e_2, \ldots, e_m\}$,
  - $e_i = (v_j, v_k)$ for directed and undirected
  - $e_i = \{v_j, v_k\}$ for undirected graphs
- Maximum number of edges in an undirected graph?

Graph Example



Can this structure be represented by a singly linked list?

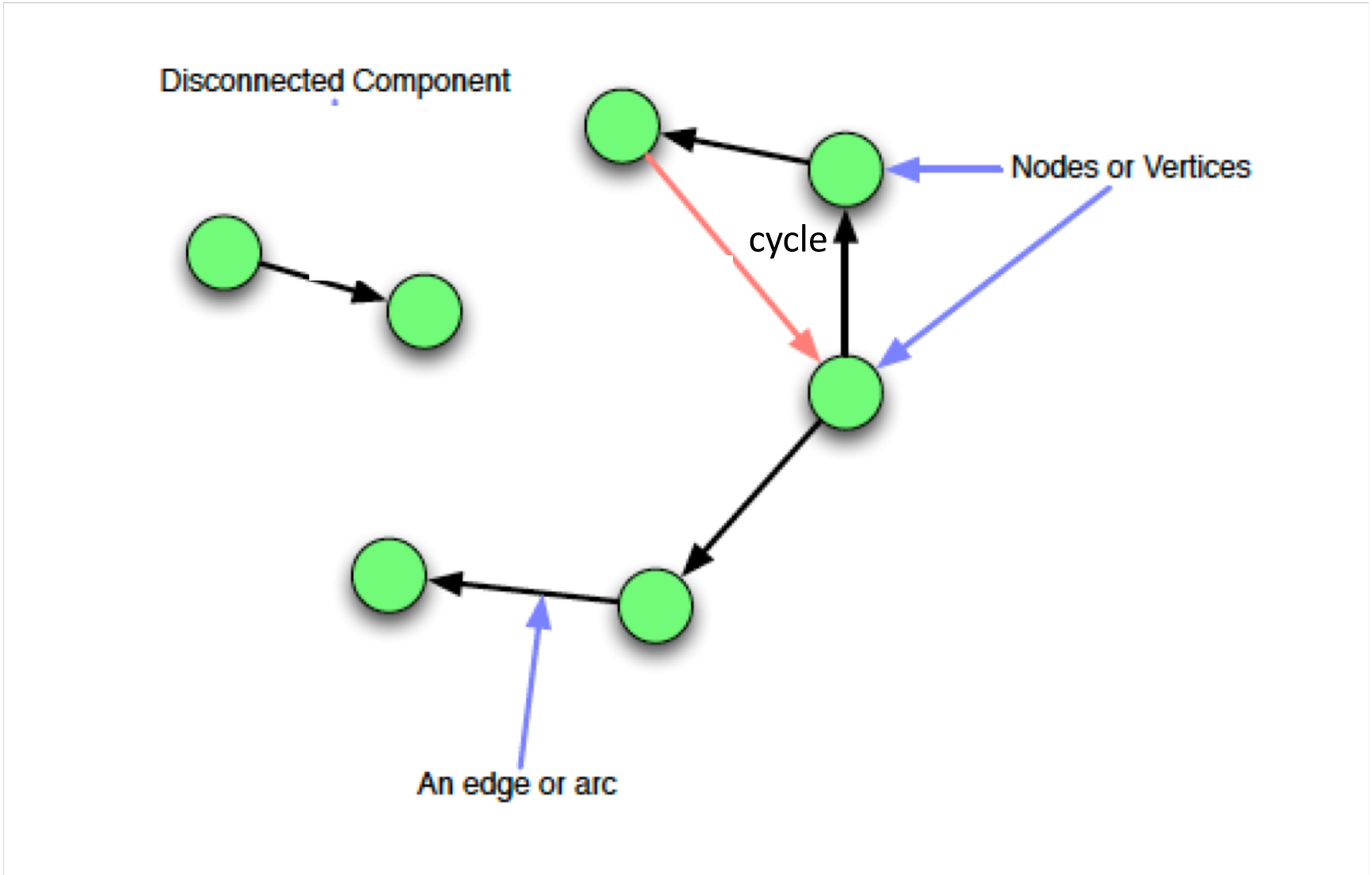
Can you represent any arbitrary graph by a doubly linked list?

How can we represent graphs in general?

Graphs

- Graphs don't have to have all of their points connected.
  - Isolated nodes/components
- Graphs can have edges that connect nodes in a cycle - if you follow the link, you'll keep going around in a circle.
  - What would the problem be if we used any of our existing linked list code to implement graphs?
    - In a lot of linked list functions you iterate in a loop until finding a null pointer!

Graph Example II



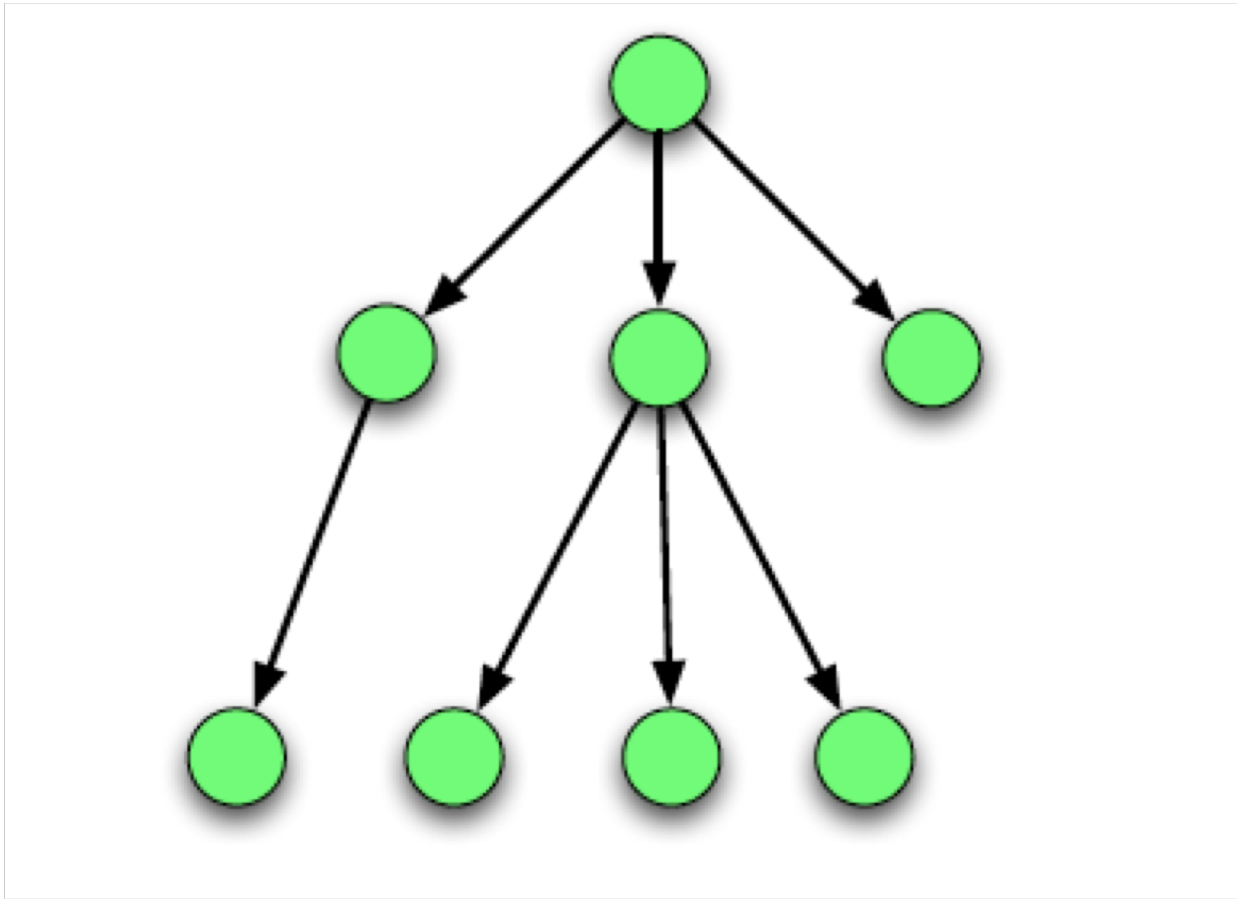
Trees

- Graphs with certain properties are called trees.
- Trees are a subset of Graphs.
  - Trees must have all of their nodes connected.
  - Trees cannot contain cycles.
  - In other words, trees are connected, acyclic graphs.
- A tree can be defined in several ways. One natural way to define a tree is using recursion.
- A graph that has no cycles but is not connected is a forest. (Because there's more than one tree…)
- n nodes, how many edges?

Directed Rooted Tree

- A tree that consists of a distinguished node r called the root and zero or more nonempty (sub)trees, each of whose roots are connected by a directed edge from r.
  - Recursion is more obvious for this definition (directed trees).
  - In this course when we say tree, we mean this!

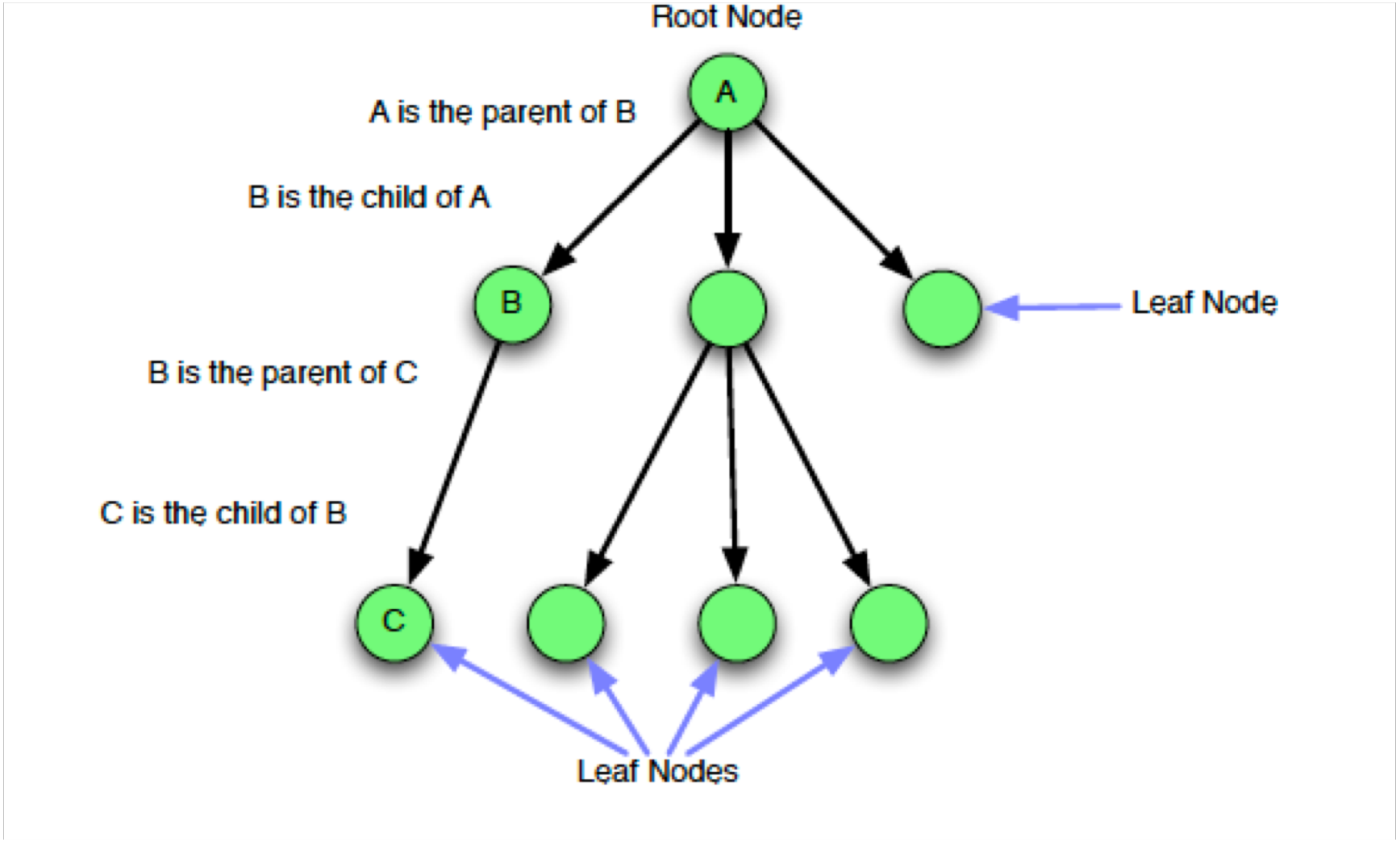
Example



Tree Terminology

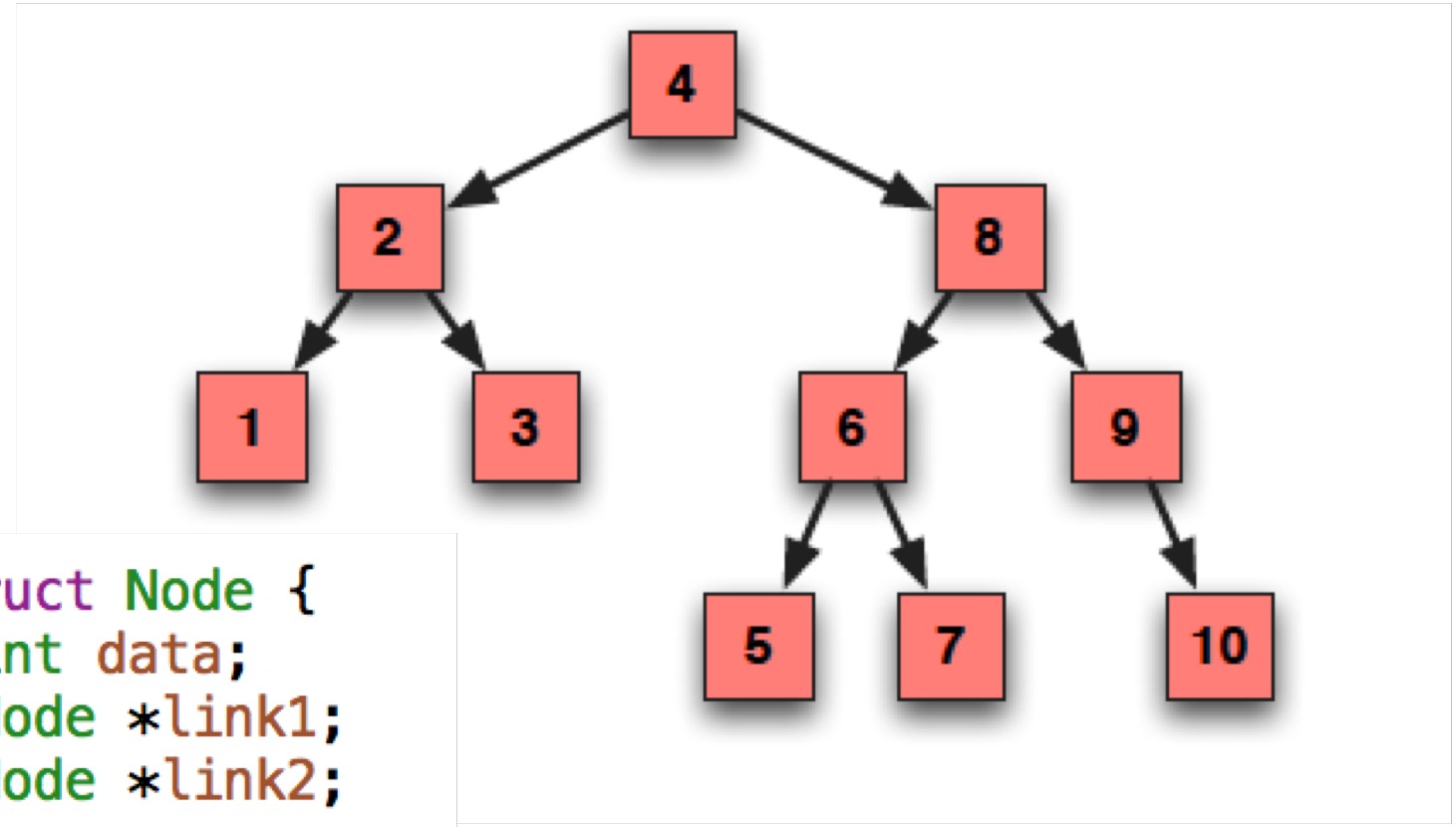
- Trees have a **root**, normally drawn at the top.
- If a node n_1 has a link to another node n_2, then n_1 is the **parent** of n_2 and n_2 is the **child** of n_1.
- If a node has no children, it is called a **leaf**.
- The **depth of a node** n is the length of the path from the root to n. The **depth of a tree** n is the maximum depth of its nodes.
- The **height of a node** the number of edges on the longest path between that node and a leaf. **The height of a tree** is height of its root.

Tree Example II



Binary Trees

- Binary Trees are trees that have 0, 1 or 2 children.



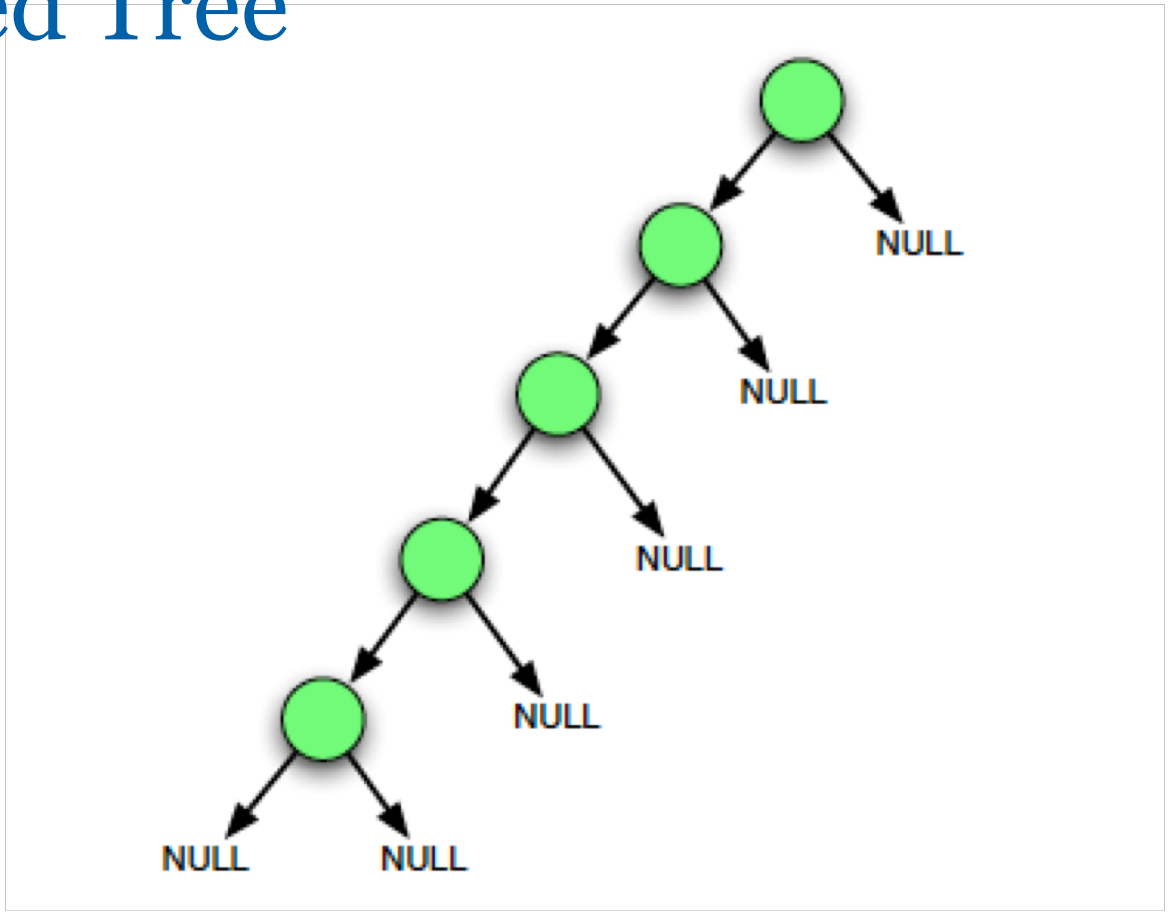

```
struct Node {
   int data;
   Node *link1;
   Node *link2;
}
```

Balanced Trees

```
struct Node {
  int data;
  Node *link1;
  Node *link2;
}
```

- Good to construct the tree such that the height of the tree is minimized
- How would you make a tree of 5 elements?
- What would it look if you had a tree of 5 elements where link2 was always NULL?

Unbalanced Tree



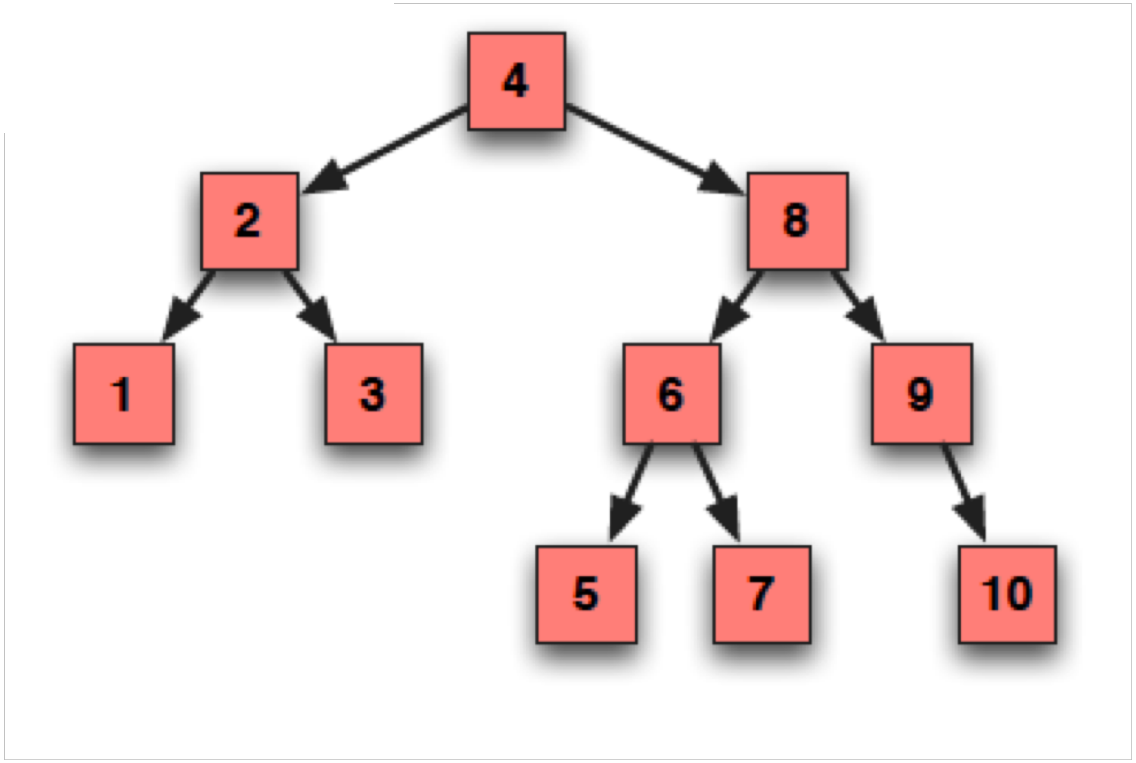
- Remind you of anything? (Hint: take out the NULLs)
  - Linked list!
- If our aim is to minimize the depth, this is a worst case

Advanced Trees

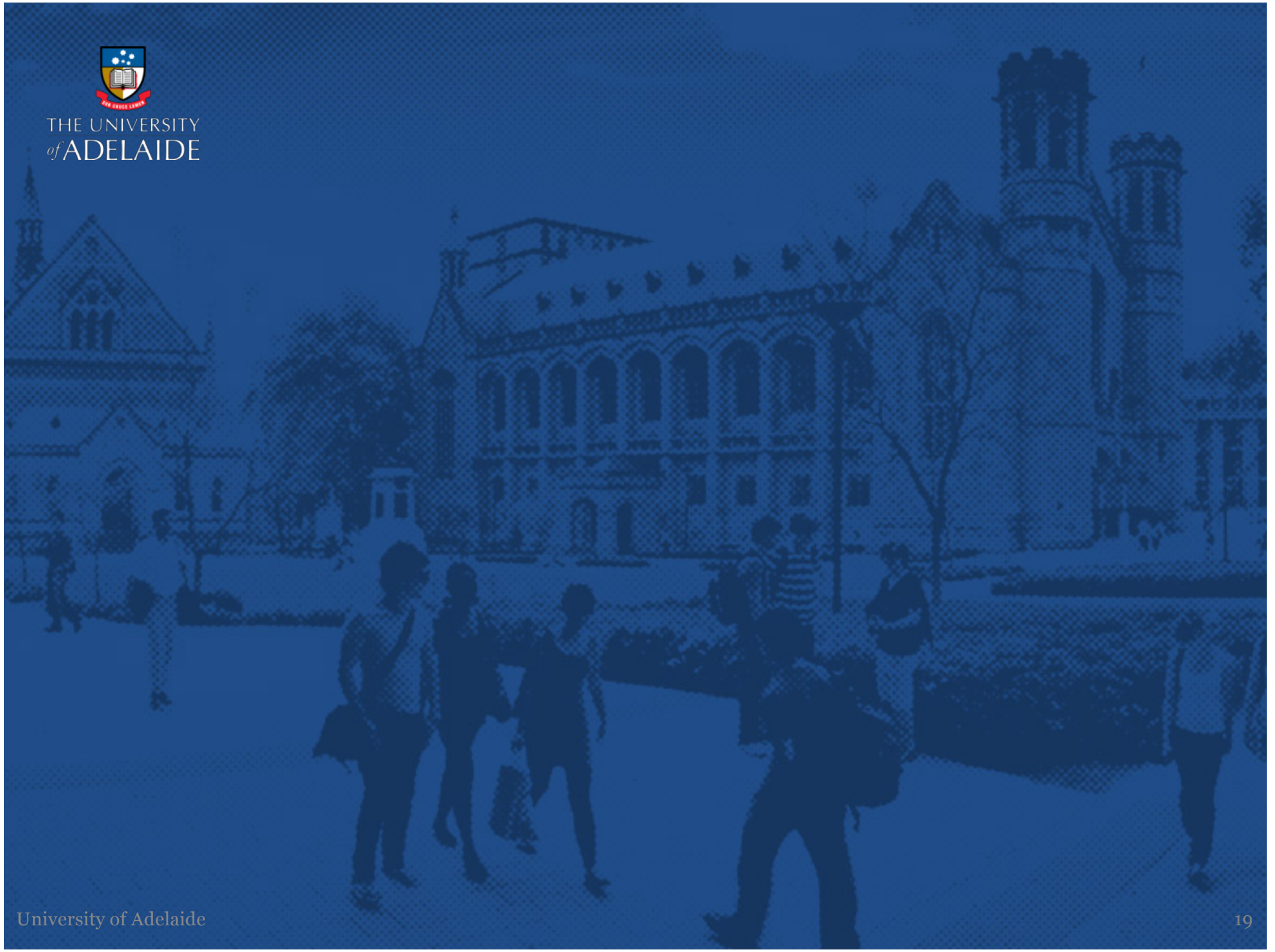
- Some trees are set up to limit their worst case depth in relation to their size. Restricting which items go where in the tree also helps.
- A (fully or partially) ordered and balanced tree can have much better search performance (O(log n)).
- Next week, and also in later courses, you will learn a lot more about graphs and trees.

An Ordered, Balanced Binary Tree

- Each node has all elements less than its value in its left-hand child.



- Search for an item
  - O(log n) – better than linked list!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