STATS 3005 Time Series III
Tutorial 3
2018

1. Consider the AR(1) process
$$Y_t = \alpha Y_{t-1} + Z_t$$
with $|\alpha| < 1$.
   (a) Express Y_t in the form
$$\sum_{j=0}^{\infty} a_j Z_{t-j}.$$
   (b) Use the expansion from 1a to find γ_0 and γ_k for each integer $k > 0$.
   (c) Find the autocorrelation function ρ_k.

2. Consider an AR(1) process
$$Y_t = \alpha Y_{t-1} + Z_t \tag{1}$$
with $|\alpha| > 1$.
   (a) Show that the process is not *causal*.
   (b) Show that the process can be expressed equivalently as
$$Y_t = \delta Y_{t+1} + U_t$$
   where $\delta = 1/\alpha$ and $U_t = -Z_{t+1}/\alpha$.
   (c) Deduce that there exists a stationary process satisfying equation (1). Obtain an expression for its autocorrelation function and explain the nature of the violation of causality.

3. Consider the MA(2) process,
$$Y_t = Z_t + 0.5Z_{t-1} - 0.25Z_{t-2}.$$
   (a) Find the autocorrelation function for this process.
   (b) Determine whether the process is invertible.

4. Show that the AR(3) process
$$Y_t = Y_{t-1} + cY_{t-2} - cY_{t-3} + Z_t$$
is non-stationary for all values of c.

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