Random Processes III 2018: Assignment 5,

to be submitted by 1pm on Friday 26^{th} October.

[39 marks in total]

Question 0. [4 marks]

Make sure that in all your answers you:

(a) use full and complete sentences,

(b) include units where necessary,

(c) give ranges/constraints for parameters in formulae, and

(d) make sure your layout and mathematical arguments are clear and concise.

Question 1. [18 marks]

Consider a simple circuit-switched loss network consisting of 8 nodes (labelled A, B, C, D, E, F, G and H), and 9 links (labelled from 1 to 9), as shown in Figure 1, where the edges are labelled with ID:Capacity.

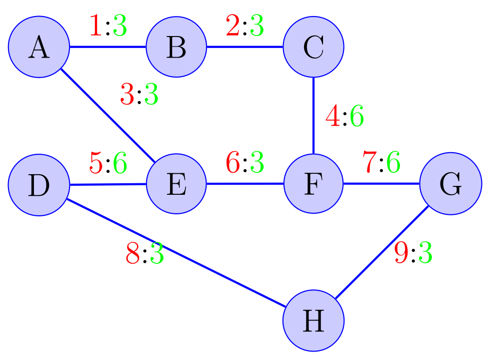

Figure 1: Circuit-switched loss network for Question 2.

There are four routes in the network; we assume that calls arrive to these routes as independent Poisson processes, and that all calls use 1 circuit on each link and have an exponentially distributed holding time with unit mean and use 1 circuit on each link it uses. Their arrival rates and links used are listed in the following table.
Note: for loss networks, we always assume an infinite number of servers.

Route label	Route	Arrival Rate	Links Used
1	A–C	1	1, 2
2	C–H	2	4, 7, 9
3	D–G	2	5, 6, 7
4	A–H	1	3, 5, 8

(a) By defining all necessary notation, write down an appropriate state space for a CTMC representation of this circuit-switched network.

(b) Write down an expression for the equilibrium distribution for this network.

(c) Write down an expression for the blocking probability of calls on Route 2 (that is between nodes C and H)

(d) Write down the expressions required to define the Erlang Fixed Point approximation for a circuit-switched network, including an expression for the blocking probability on a route. *Note: You are not being asked to solve anything.*

(e) What assumption is made about the links on the network, in order to justify the Erlang Fixed Point approximation? Comment on the validity of this assumption, and whether it affects the approximation.

(f) Evaluate the approximate blocking probabilities of calls on Route 2 through use of the Erlang Fixed Point Method, as outlined at the end of Lecture 20. Provide code.

Question 2. [10 marks] [2011 Exam] A CTMC model of a particular queue has states $n \in S = \{0, 1, 2, 3\}$, where n represents the number of customers in the queue. Assume that the arrival rate into the queue when it is in state n, λ_n, is given by

$$\lambda_n = (5 - n)\lambda, n = 0, 1, 2, 3,$$

and that all arrivals to the queue when it is in state 0, 1 or 2 are accepted, and all arrivals to the queue when it is in state 3 are lost. Finally, note that the equilibrium distribution is given by $\pi = (2/5, 1/5, 1/5, 1/5)$.

(a) What is the equilibrium probability that there are 3 customers in the queue? (Hint: this is easier than you think!)

(b) What is the equilibrium probability that an arrival sees 3 customers in the queue, in other words, what is the blocking probability?

(c) What is the average length of the queue?

(d) Determine the average time that a customer spends in the system.

Question 3. [7 marks] Consider a counting process, $N(t)$, which counts the number of times a particular component is replaced. The lifetime of the component has a CDF of $F(t)$. Every time the component is replaced, there is a probability p that it will fail instantly; otherwise, the lifetime is assumed to be exponential with rate λ.

(a) Show that the CDF $F(t)$ is given by

$$F(t) = 1 - (1 - p)e^{-\lambda t}.$$

(b) Determine the Laplace-Stieltjes Transform, $\widehat{F}(s)$, of $F(t)$.

(c) Determine the Laplace-Stieltjes Transform, $\widehat{M}(s)$, of $M(t)$, and consequently determine $M(t)$.