THE UNIVERSITY *of* ADELAIDE

Examination in School of Mathematical Sciences

Semester 2, 2017

005675 STATS 3005 Time Series III

Official Reading Time:	10 mins
Writing Time:	120 mins
Total Duration:	130 mins

NUMBER OF QUESTIONS: 6 TOTAL MARKS: 75

Instructions

- Attempt all questions.
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.

Materials

- 1 Blue book is provided.
- Formulae sheets are provided.
- Calculators are permitted.

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO.

1. (a) Briefly explain why time series data require special statistical methods of analysis.

   (b) Let $\{Y(t)\}$ be a random process with mean $\mu(t)$. Define the autocovariance function, $\gamma(s,t)$.

   (c) Define what it means to say that $\{Y(t)\}$ is marginally stationary.

   (d) Define what it means to say that $\{Y(t)\}$ is second-order stationary.

   (e) Let $\{Y_t\}$ be a Gaussian white noise sequence with mean zero and variance σ^2.

   (i) Show that $\{Y_t\}$ is second-order stationary.

   (ii) Find the normalised spectrum, $f^*(\omega)$, of $\{Y_t\}$.

[11 marks]

2. (a) Let $\{Y_t\}$ be a second-order stationary random sequence with mean μ and autocovariance function $\gamma_Y(k)$, and let $D_t = Y_t - Y_{t-1}$.
   Find the autocovariance function $\gamma_D(k)$ in terms of $\gamma_Y(k)$.

   (b) Suppose $\{Z_t\}$ is a white noise sequence with mean zero and variance σ_Z^2, and

$$S_t = \frac{(Z_{t-1} + Z_t + Z_{t+1})}{3}$$

   is a simple three-point moving average. Let $\{R_t\}$ be the residual sequence defined by $R_t = Z_t - S_t$.

   (i) Find the mean and variance of $\{R_t\}$.

   (ii) Hence find the autocorrelation function for $\{R_t\}$. [**Hint**: Find the autocovariance function of $\{R_t\}$ for $k = 1, 2$.]

   (c) Comment on the implications of the result in Question 2(b)(ii) for examining the correlogram of a residual series following trend removal using the three-point moving average.

[15 marks]

3. (a) Define a *linear filter*.

   (b) The first-order moving average process, $MA(1)$, is defined by

$$Y_t = Z_t + \beta Z_{t-1}, \tag{1}$$

   where $|\beta| < 1$ and $\{Z_t\}$ is white noise with mean zero and variance σ_Z^2.

   (i) Express the process (1) in backward shift operator notation, B.

   (ii) Express Z_t as a linear filter of Y_t.

   (iii) Is this $MA(1)$ process invertible? Justify your answer.

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(c) Show that the spectrum for this process is given by

$$f(\omega) = \sigma_Z^2(1 + 2\beta\cos(\omega) + \beta^2).$$

[13 marks]

4. Consider the autoregressive process of order p, that is, the $AR(p)$ process

$$\phi(B)Y_t = Z_t,$$

where B is the backward shift operator and $\phi(B) = 1 - \sum_{j=1}^{p} \alpha_j B^j$ is such that all roots lie outside the unit circle in the complex plane.

(a) Show that the autocorrelations ρ_k satisfy the Yule-Walker equations,

$$\rho_k = \sum_{j=1}^{p} \alpha_j \rho_{k-j}, \text{ for } k = 1, 2, \ldots$$

(b) Consider the $AR(2)$ process

$$Y_t = \frac{1}{12}Y_{t-1} + \frac{1}{12}Y_{t-2} + Z_t.$$

(i) Verify that the process is second-order stationary.

(ii) Write down the auxiliary equation and find its roots.

(iii) Write down the general form of the solution to the auxiliary equation in this case.

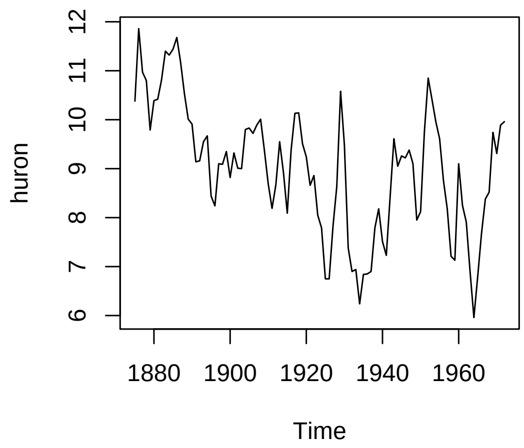
(iv) Show that the autocorrelation function is given by

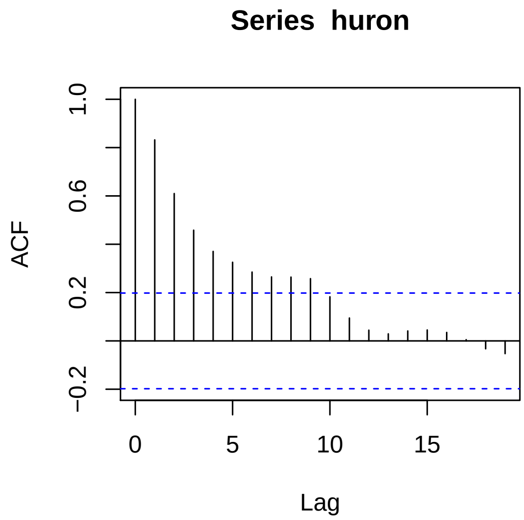
$$\rho_k = \frac{45}{77}\left(\frac{1}{3}\right)^k + \frac{32}{77}\left(\frac{-1}{4}\right)^k \quad , k = 1, 2, \ldots$$

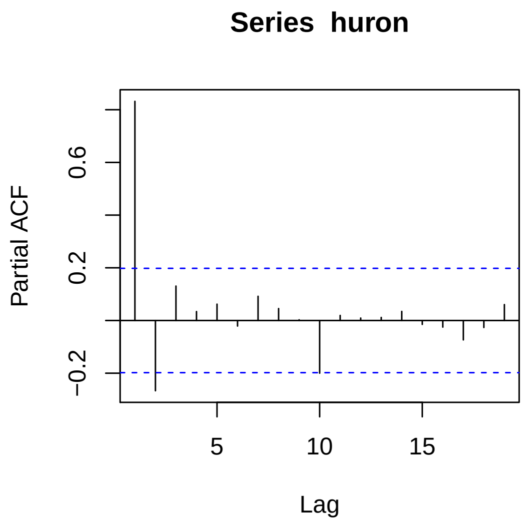
[14 marks]

5. The level of Lake Huron in feet was recorded annually from 1875 to 1972.

   (a) Shown below is a plot of the data, the estimated ACF and the estimated PACF. On the basis of these plots, suggest a suitable ARMA model. Give reasons for your answer.







   (b) Shown below are summary statistics from a number of ARIMA models that were fitted to the data. Identify the model you think would be preferable and any other models that might also be plausible. Give reasons for your answer.

Model	Model fit statistics
ARIMA$(1,0,0)$	`log likelihood = -106.6,  aic = 219.2`
ARIMA$(2,0,0)$	`log likelihood = -103.6,  aic = 215.27`
ARIMA$(2,0,1)$	`log likelihood = -103.2,  aic = 216.48`
ARIMA$(3,0,0)$	`log likelihood = -103.0,  aic = 216.04`
ARIMA$(3,0,1)$	`log likelihood = -102.9,  aic = 217.8`

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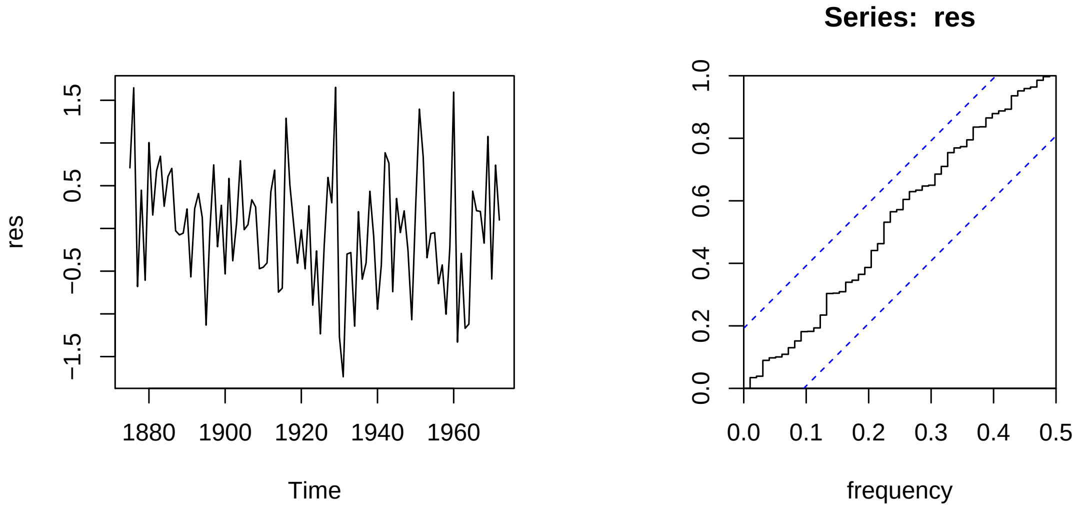
(c) On the basis of the plots shown below, does the ARIMA$(2,0,0)$ model provide an adequate description of the data? Explain your answer and comment on any notable features of the data.

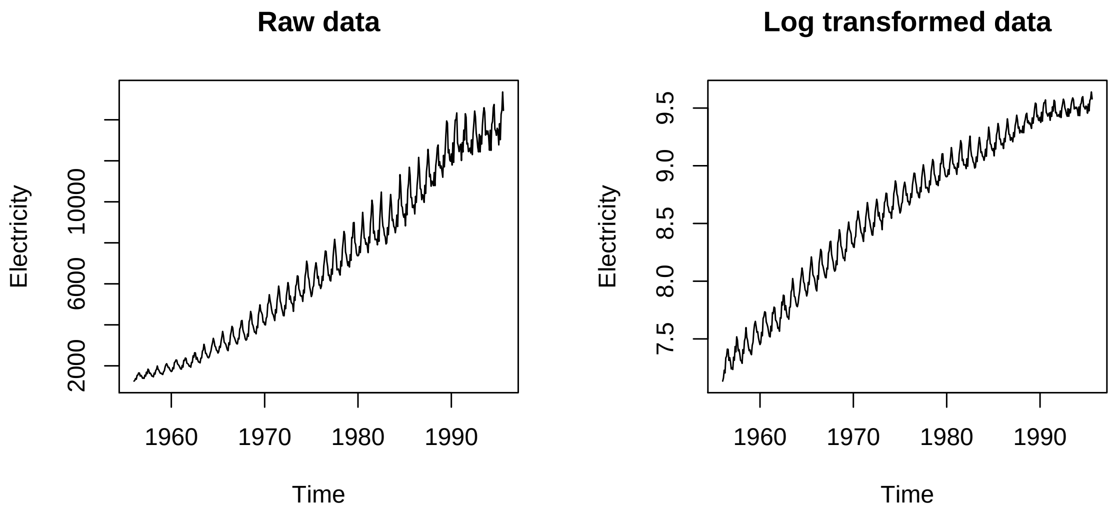
```
res=residuals(arima(huron, order=c(2,0,0)))
plot(res)
cpgram(res)
```



[9 marks]

6. The production of electricity in Australia was recorded monthly from January 1956 to August 1995. Plots of the raw and log-transformed data are shown below.

```
plot(elec, main="Raw data")
plot(log(elec), main="Log transformed data")
```

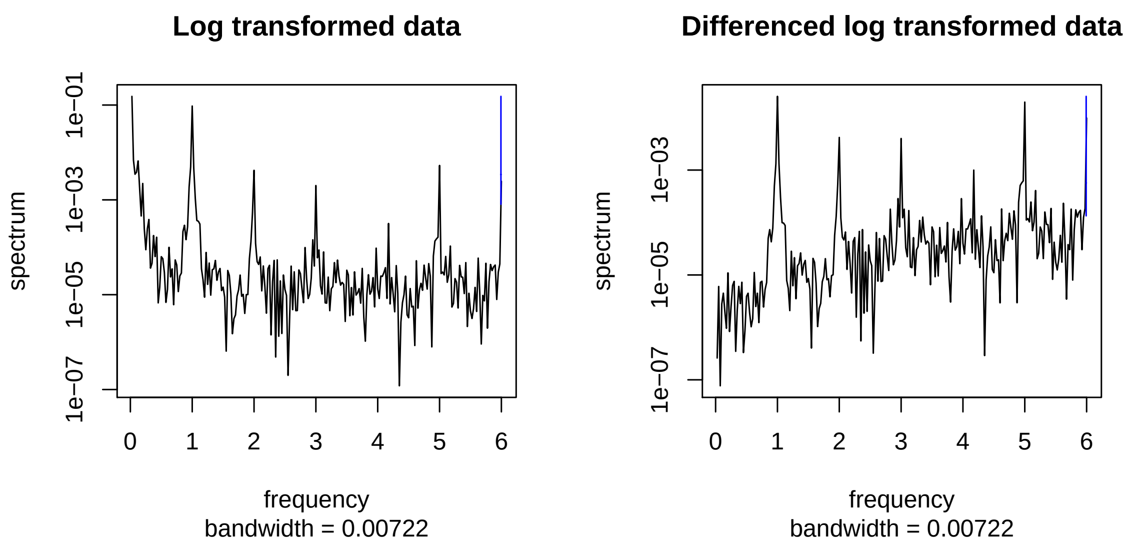


(a) Based on the plot of the raw data, do the data appear stationary?

(b) Explain why a log transformation should be considered for the data.

(c) The data were log transformed and first differences calculated. Shown below are the periodogram of the log transformed data and a periodogram of their differences.

```
spectrum(log(elec), main="Log transformed data")
spectrum(diff(log(elec)), main="Differenced log transformed data")
```



(i) Based on the periodogram, does the log-transformed series appear stationary? Explain your answer.

(ii) Based on the periodogram, does the differenced, log-transformed series appear stationary? Explain your answer.

(iii) Describe and interpret the difference between the two periodograms.

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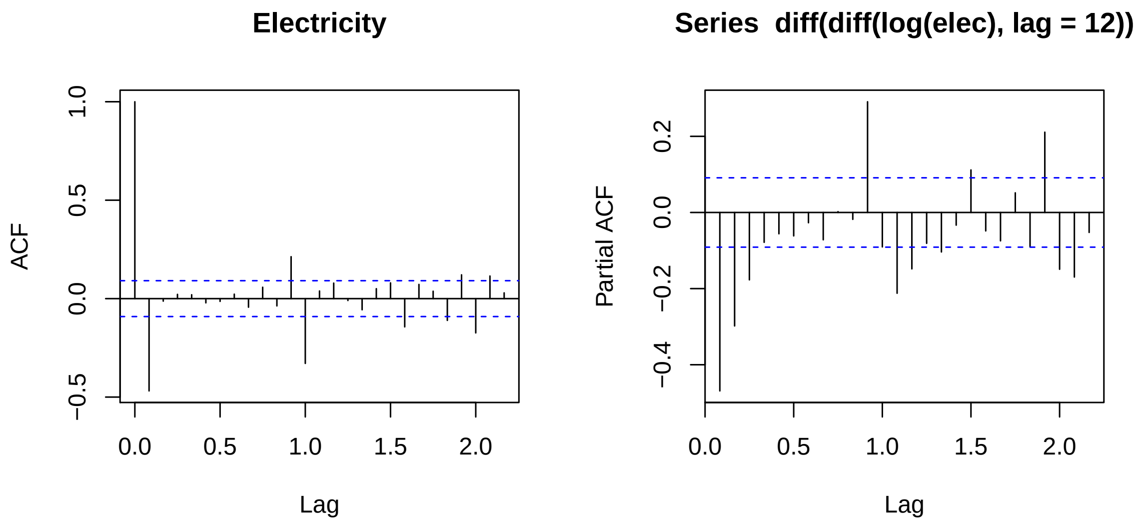
(d) Shown below are the ACF and PACF for the log transformed data differenced twice at lags 1 and 12. Explain why a SARIMA model with $(p = 0, d = 1, q = 1)$ and $(P = 0, D = 1, Q = 1)$ may be indicated as a suitable model to consider in the first instance.

```
acf(diff(diff(log(elec),lag=12)))
pacf(diff(diff(log(elec),lag=12)))
```



[13 marks]

Formula Page for Time Series

Moving averages: A moving average of order $2p+1$ for a time series $\{y_t : t = 1, 2, \ldots, n\}$ is a time series defined by

$$s_t = \sum_{j=-p}^{p} w_j y_{t+j}, \quad t = p+1, \ldots, n-p$$

Sines and cosines:

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

Periodogram ordinates:

$$I(\omega) = \frac{1}{n} \left\{ \left(\sum_{t=1}^{n} y_t \cos(\omega t) \right)^2 + \left(\sum_{t=1}^{n} y_t \sin(\omega t) \right)^2 \right\}$$

where $0 < \omega \leq \pi$.

Yule-Walker equations for AR(p) process:

$$\rho_k = \sum_{j=1}^{p} \alpha_j \rho_{k-j}, \quad k = 1, 2, \ldots$$

ARMA(p, q) process:

$$Y_t - \alpha_1 Y_{t-1} - \cdots - \alpha_p Y_{t-p} = Z_t + \beta_1 Z_{t-1} + \cdots + \beta_q Z_{t-q}$$

Spectrum and normalized spectrum:

$$f(\omega) = \gamma_0 + 2 \sum_{k=1}^{\infty} \gamma_k \cos(k\omega)$$

$$f^*(\omega) = 1 + 2 \sum_{k=1}^{\infty} \rho_k \cos(k\omega)$$

Final page