INFORMATION SHEET FOR LAPLACE TRANSFORMS

Formula	Name, Comments
$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$ $f(t) = \mathcal{L}^{-1}\{F(s)\}$	Definition of Transform Inverse Transform
$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$	Linearity
$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$ $\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0)$ $\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$ $\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}\mathcal{L}(f)$	Differentiation of Function Integration of Function
$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$ $\mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$	s-Shifting (1st Shifting Theorem)
$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$ $\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$	t-Shifting (2nd Shifting Theorem)
$\mathcal{L}\{tf(t)\} = -F'(s)$ $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(\tilde{s})d\tilde{s}$	Differentiation of Transform Integration of Transform
$(f*g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$ $= \int_0^t f(t-\tau)g(\tau)d\tau$ $\mathcal{L}(f*g) = \mathcal{L}(f)\mathcal{L}(g)$	Convolution
$\mathcal{L}(f) = \frac{1}{1-e^{-\ell s}} \int_0^\ell e^{-st} f(t)dt$	f Periodic with Period ℓ

Table of Laplace Transforms

$F(s) = \mathcal{L}\{f(t)\}$	$f(t)$
$1/s$	1
$1/s^2$	t
$1/s^n \quad (n = 1, 2, \ldots)$	$t^{n-1}/(n-1)!$
$1/\sqrt{s}$	$1/\sqrt{\pi t}$
$1/s^{3/2}$	$2\sqrt{t/\pi}$
$1/s^a \quad (a > 0)$	$t^{a-1}/\Gamma(a)$
$\dfrac{1}{s-a}$	e^{at}
$\dfrac{1}{(s-a)^2}$	te^{at}
$\dfrac{1}{(s-a)^n} \quad (n = 1, 2, \ldots)$	$\dfrac{1}{(n-1)!}\, t^{n-1}e^{at}$
$\dfrac{1}{(s-a)^k} \quad (k > 0)$	$\dfrac{1}{\Gamma(k)}\, t^{k-1}e^{at}$
$\dfrac{1}{(s-a)(s-b)} \quad (a \neq b)$	$\dfrac{1}{(a-b)}\,(e^{at} - e^{bt})$
$\dfrac{s}{(s-a)(s-b)} \quad (a \neq b)$	$\dfrac{1}{(a-b)}\,(ae^{at} - be^{bt})$

$F(s) = \mathcal{L}\{f(t)\}$	$f(t)$
$\dfrac{1}{s^2+\omega^2}$	$\dfrac{1}{\omega}\,\sin\omega t$
$\dfrac{s}{s^2+\omega^2}$	$\cos\omega t$
$\dfrac{1}{s^2-a^2}$	$\dfrac{1}{a}\,\sinh at$
$\dfrac{s}{s^2-a^2}$	$\cosh at$
$\dfrac{1}{(s-a)^2+\omega^2}$	$\dfrac{1}{\omega}\,e^{at}\sin\omega t$
$\dfrac{s-a}{(s-a)^2+\omega^2}$	$e^{at}\cos\omega t$
$\dfrac{1}{s(s^2+\omega^2)}$	$\dfrac{1}{\omega^2}(1-\cos\omega t)$
$\dfrac{1}{s^2(s^2+\omega^2)}$	$\dfrac{1}{\omega^3}\,(\omega t - \sin\omega t)$
$\dfrac{1}{(s^2+\omega^2)^2}$	$\dfrac{1}{2\omega^3}\,(\sin\omega t - \omega t\cos\omega t)$
$\dfrac{s}{(s^2+\omega^2)^2}$	$\dfrac{t}{2\omega}\,\sin\omega t$
$\dfrac{s^2}{(s^2+\omega^2)^2}$	$\dfrac{1}{2\omega}\,(\sin\omega t + \omega t\cos\omega t)$
$\dfrac{s}{(s^2+a^2)(s^2+b^2)} \quad (a^2 \neq b^2)$	$\dfrac{1}{b^2-a^2}\,(\cos at - \cos bt)$
e^{-as}/s	$u(t-a)$
e^{-as}	$\delta(t-a)$