School of Mathematical Sciences

# Modelling with ODEs

Semester 1, 2019

**Assignment 2**

**Due 5pm Monday, Week 8: Submit via MyUni**

**You will be marked on the presentation of your answers (including clarity of explanations)!**

1. Consider the linear 2D system

$$\dot{x} = 2\,x - 2\,y \tag{1a}$$
$$\dot{y} = 2\,x - 3\,y. \tag{1b}$$

   (a) Write the system in matrix–vector form.

   (b) Calculate the eigenvalues and eigenvectors of the system.

   (c) Find the solution for initial values $x(0) = x_0$ and $y(0) = y_0$.

   (d) Classify the steady state with reason.

   (e) State the steady and unsteady directions with reason.

   (f) Produce the phase portrait for the system, i.e. typical solutions in the phase plane, and include the stable and unstable directions. You can use MatLab/other technology or sketch by hand.

2. Consider the two population interaction model

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha\,x + \beta\,x\,y \\ \gamma\,y + \delta\,x\,y \end{pmatrix} \quad \text{for} \quad \begin{cases} \alpha, \gamma > 0 \\ \beta, \delta < 0. \end{cases} \tag{2}$$

   (a) Write down the two steady states (no need to calculate). For each of the steady states, determine the nature of the associated linearised problems.*

   (b) For each of the steady states of the nonlinear system (2), state with reason whether the behaviour is equivalent to the linearised problem.

   (c) State, with reason, if each of the states is biologically relevant.

   (d) Produce a phase portrait for system (2) with $\alpha = \gamma = 1$ and $\beta = \delta = -1$, in the biologically relevant region of the phase plane (either using MatLab/other technology or by hand).

   (e) Give a biological interpretation of the phase portrait.

*You can use results from lectures for part (a).