School of Mathematical Sciences

THE UNIVERSITY OF ADELAIDE AUSTRALIA

Assessment Cover Sheet

Student Name	Andrew Martin
Student ID	a1704466
Assignment Number	2
Course	PDEs
Tutorial Group	N/A

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PDEs Assignment 2

Andrew Martin

August 17, 2017

Question 1:
I was given a $\frac{4}{5}$ for mathematical writing in assignment 1. The mark was deducted due to a lack of explaining steps and derivations in some cases where the steps taken may have been unclear, and a formula similar to one derived in lectures was not fully explained.

Question 2:
Considering the PDE:

$$\rho(x)\frac{\delta^2 u}{\delta t^2} = S_0(x)\frac{\delta^2 u}{\delta x^2} - \alpha(x)u - \beta(x)\frac{\delta u}{\delta t}$$

The coefficients ρ, S_0 , α, and β are all functions of x. Show that separation of variables can only work if $\beta(x) = c\rho(x)$ for some constant c.

Question 3:
Spherically symmetric solutions $u(r,t)$, where r is radius from the origin, of the wave PDE in 3D:

$$\frac{\delta^2 u}{\delta t^2} = c^2 \left[\frac{\delta^2 u}{\delta r^2} + \frac{2\delta u}{r\delta r} \right]$$

(a)
Use separation of variables i.e. $u = R(r)T(t)$, to find an eigen-problem for the radial spacial structure $R(r)$.

(b)
Rearrange the ODE for $R(r)$ to write it in Sturm-Liouville form.

(c)
Show the the boundary conditions imply the linear operator in the Sturm-Liouville form is self-adjoint.

(d)
What does the infinity of real eigenvalues for this eigenproblem imply for the time dynamics $T(t)$?

Question 4:
Consider the regular Sturm-Liouville eigenproblem for $0 \leq x \leq 1$,

$$u'' + \lambda u = 0 \qquad u(0) = 0 \qquad 3u(1) + u'(1) = 0$$

Verify the following properties via writing the eigenvalues as $\lambda = k^2$ where the values of k are the positive solutions of the transcendental equation $\tan(k) = \frac{-k}{3}$. (consider the graphs of $y = \tan(k)$ and $y = \frac{-k}{3}$).

(a)
There are an infinite number of eigenvalues with no largest one.

(b)
The nth eigenfunction has $n-1$ zeros over $0 < x < 1$ with boundary conditions that $u(0,t)$ and $u(1,t)$ are specified.

(c)
Write down the orthogonality condition for the eigenfunctions.
Orthogonality of functions over the domain $[a, b]$ that for $m \neq n$:

$$< v_m, v_n >= 0$$

I.e.

$$\int_a^b v_m v_n r dx = 0$$

In this case, the domain $[a, b]$ is the domain $[0, 1]$. So with respect to some weight function $r(x)$ the orthogonality implies

$$\int_0^1 v_m v_n r dx = 0$$

Question 5:
Explore computationally approximating solutions $u(x,t)$ to the linear advection-diffusion PDE $u_t = -3u_x + u_{xx}$ on the domain $0 < x < 1$ with boundary conditions that $u(0,t)$ and $u(1,t)$ are specified.

(a)
Using Lagrange's remainder theorem, confirm that $f'(x) = \frac{[-f(x-h)+f(x+h)]}{2h} + Ch^2$ and find the expression for C.

(b)
The simplest discretised approximation to this advection-diffusion PDE is to have one grid point interior to the spatial domain: defining the grid $x_1 = 0$, $x_2 = 1/2$, and $x_3 = 1$, derive the approximate ODE for $u_2(t) := u(\frac{1}{2}, t) = b$

(c)
Modify Algorithm 4.1 to computationally approximate solutions to the advection-diffusion PDE over time $0 \leq t \leq 1$ with boundary values $u(0,t) = 0$ and $u(1,t) = sin(9t)$, and any reasonable initial condition of your choice.
Submit code and graphical output here: