Assignment II

Worth 10% of course assessment; due by 1pm on Friday 10th May, 2019.

Relevant lectures: Lectures 1 – 18.

Individual marks are noted in [] at start of each question; total marks for assignment is 50.

Please provide an explanation/discussion with all answers, and code where appropriate.

Q1: Specifying models, and deterministic approximations
[16 marks]

In the lectures we looked at the basic SIR model. Now consider what would happen if we also add in demography (births and deaths). Assume that recovered individuals die at a rate μ and are immediately reborn as a susceptible.

(i) Specify the CTMC version of this model.

(ii) Derive a deterministic approximation to the stochastic dynamics.

(iii) What is the long term behaviour of the deterministic approximation?

(iv) Simulate the stochastic model and compare the long term dynamics with the deterministic version. Consider the parameters: $R_0 = 15$, $1/\gamma = 13$ days, $1/\mu = 60$ years, and frequency-dependent mixing.

(v) Calculate $E(I(t))$ conditional on the disease not going extinct and compare with the deterministic model.

(vi) How would the model change if all individuals can die at a rate μ and susceptible individuals are born independently at rate μ proportional to the total population?

Q2: Degree of advancement, and branching processes
[18 marks]

(i) Write code that generates the complete Q matrix for an SIR model (population size N) using the degree-of-advancement representation.

(ii) For $N = 15$, $\beta = 1.6$ and $\gamma = 1$, and starting with initial condition of $(S(0), I(0)) = (N-1, 1)$, produce some figures showing the probability mass function of I(t) at times 0, 1, 5 and 50. Use an implicit-Euler method to numerically solve the forward equation.

(iii) What is the probability that exactly 12 people are infected over the course of the epidemic? (Use the parameters from part (ii).)

(vi) For $N = 100$, $\beta = 1.6$ and $\gamma = 1$, plot the expected value of $I(t)$ as a function of time by solving the forward equation numerically, and compare to the deterministic approximation we derived in lectures.

(v) For $N = 50, 100$ and 500, $\beta = 1.6$ and $\gamma = 1$, calculate the probability of a minor outbreak using the same methods as in part (iii). How do these compare to the results derived using the branching process approximation?

Q3: Path integrals

[16 marks]

(i) In class, you evaluated R_0 and the probability mass function of secondary infections arising from a single individual with an exponentially-distributed infectious period, in an infinite population. Investigate how these two quantities (R_0 and the pmf of secondary infections) changes, if in place of an exponentially-distributed infectious period the individual has an Erlang-2-distributed infectious period with the same mean (i.e., $1/\gamma$).

(ii) Consider the SIR CTMC model of disease dynamics, in a population with $N = 20$ individuals. Assume that infectious individuals require care during their infectious period. Wards in the care facility are such that four individuals can be in each ward. Each individual has a per unit cost of \$$a$ per unit time whilst infectious, and each ward has a cost of \$$b$ per unit time whilst open. Assuming the CEO of the care facility is operating to minimise cost, that $a = 2$, $b = 5$, the effective transmission rate parameter $\beta = 0.6$, and that the average infectious period is 3 days, what is the expected cost of caring for individuals during an outbreak?