APP MATH 3020 Stochastic Decision Theory
Assignment 1

Due: Friday, 17 August, 2018, 4 p.m. (Week 4). **Total marks: 38**

Question 1 2 marks

Make sure that in all your answers you

½ (a) use full and complete sentences.

½ (b) include units where necessary.

½ (c) use logical arguments in your answers and proofs.

½ (d) structure your answers and assignment clearly and precisely.

Question 2 10 marks

Consider the following linear program:

$$
\text{(P)} \qquad \begin{array}{ll} \text{maximise} & z = -5x_1 + x_2 - 4x_3 \\ \text{such that} & 2x_1 + 2x_2 - 4x_3 = 1 \\ & 2x_1 + 2x_2 + 2x_3 = 4 \\ & x_1,\, x_2,\, x_3 \geq 0. \end{array}
$$

4 (a) Construct its dual using the general relationship between primal and dual (instead of rewriting the above LP in standard form and then writing the dual of the LP in the standard form).

6 (b) Provide the optimal solutions of both the primal and dual, using `linprog.m`. Include your MATLAB code and the output of the code.

Question 3 $\boxed{\text{16 marks}}$

Consider the following linear program:

$$
\begin{array}{lll}
\text{(P)} & \text{maximise} & z = 4x_1 + 8x_2 + 5x_3 \\
& \text{such that} & x_1 \geq 10 \; (= b_1) \\
& & x_2 \geq 9 \; (= b_2) \\
& & x_3 \geq 3 \; (= b_3) \\
& & 2x_1 + 3x_2 + x_3 \leq 80 \; (= b_4) \\
& & x_1 + 2x_2 + 2x_3 \leq 70 \; (= b_5) \\
& & x_1,\, x_2,\, x_3 \geq 0.
\end{array}
$$

Suppose we replace b_i with $b_i(\boldsymbol{\zeta})$ for $i = 1, 2, \ldots, 5$, with

$$
\boldsymbol{\zeta} = \begin{cases} \varepsilon_1 & \text{with probability } 0.3, \\ \varepsilon_2 & \text{with probability } 0.5, \\ \varepsilon_3 & \text{with probability } 0.2, \end{cases}
$$

where

$$
\boldsymbol{b}(\boldsymbol{\zeta}) = \begin{pmatrix} b_1(\boldsymbol{\zeta}) \\ b_2(\boldsymbol{\zeta}) \\ b_3(\boldsymbol{\zeta}) \\ b_4(\boldsymbol{\zeta}) \\ b_5(\boldsymbol{\zeta}) \end{pmatrix}, \quad \text{with} \quad \boldsymbol{b}(\varepsilon_1) = \begin{pmatrix} 8 \\ 6 \\ 1 \\ 80 \\ 70 \end{pmatrix}, \quad \boldsymbol{b}(\varepsilon_2) = \begin{pmatrix} 10 \\ 10 \\ 3 \\ 80 \\ 70 \end{pmatrix} \quad \text{and} \quad \boldsymbol{b}(\varepsilon_3) = \begin{pmatrix} 13 \\ 11 \\ 6 \\ 80 \\ 70 \end{pmatrix}.
$$

Assume that we can meet any shortfall in demand through recourse at market, but we must pay q_1, q_2 and q_3 (units of currency) per unit of x_1, x_2 and x_3 purchased at market, respectively.

$\boxed{6}$ (a) Formulate and write down for this problem, a two-stage stochastic linear program (SLP) with fixed recourse.

$\boxed{4}$ (b) Write the above SLP in the extended form.

$\boxed{6}$ (c) Solve the recourse DEP in part (b) for $q_1 = 10, q_2 = 1$ and $q_3 = 20$. Provide MATLAB code and output of the code, with an interpretation.

Question 4 $\boxed{\text{10 marks}}$

Suppose X is a continuous random variable with density function

$$
f(x) = \lambda^2 x e^{-\lambda x} \quad \text{for } x \geq 0.
$$

Suppose $\lambda = 3$.

$\boxed{2}$ (a) Find the 99% CI for X.

$\boxed{5}$ (b) Determine a discrete approximation for X, with 10 realisations. Explain clearly how you obtain the realisations and their respective probabilities. (Include computer code if you have used a computer to help generate your realisations.)

$\boxed{2}$ (c) Propose a mathematical formula/method for assessing the error of your discretisation.

$\boxed{1}$ (d) Given the above, suggest how you would reduce the approximation error.