APP MATH 3020 Stochastic Decision Theory
Assignment 2

Due: Monday, 17 September, 2018, 10 a.m. $\boxed{\textbf{Total marks: 41}}$

Question 1 $\boxed{2 \text{ marks}}$

Make sure that in all your answers you

$\boxed{1/2}$ (a) use full and complete sentences.

$\boxed{1/2}$ (b) include units where necessary.

$\boxed{1/2}$ (c) use logical arguments in your answers and proofs.

$\boxed{1/2}$ (d) structure your answers and assignment clearly and precisely.

Question 2 $\boxed{19 \text{ marks}}$

Consider the problem:

$$\begin{array}{lrcrcrcl} \min & z & = & 3x_1 & + & 2x_2 & + & \mathbb{E}_{\boldsymbol{\xi}}[15y_1 + 18y_2] \\ \text{s.t.} & & & 3y_1 & + & 2y_2 & \le & x_1 \\ & & & 2y_1 & + & 5y_2 & \le & x_2 \\ & & & 0.6d_1 & \le & y_1 & \le & d_1 \\ & & & 0.8d_2 & \le & y_2 & \le & d_2 \\ & & & & & x_i & \ge & 0 \quad \text{for } i = 1, 2 \\ & & & & & y_i & \ge & 0 \quad \text{for } i = 1, 2, \end{array}$$

where the random vector $\boldsymbol{\xi} = (d_1, d_2)^\top$ has two realisations:

$$\begin{aligned} \boldsymbol{\varepsilon}_1 &= (d_1, d_2) = (4, 4)^\top \quad \text{with probability } 0.5, \\ \boldsymbol{\varepsilon}_2 &= (d_1, d_2) = (6, 8)^\top \quad \text{with probability } 0.5. \end{aligned}$$

$\boxed{19}$ (a) Perform Steps 0, 1, and 2 of the L-shaped algorithm on the above problem. For the purpose of this assignment, do each step only once; that is, stop when the algorithm asks you to either go back to Step 1 or proceed to Step 3. Explain all your working and provide all MATLAB code and relevant output.

Question 3 $\boxed{16 \text{ marks}}$

The Adelaide University Mathematics Society is having a BBQ; there are loads of free food. You stand in line to get a hot dog, and n students are in front of you. With probability p the student at the head of the queue will finish being served in the next minute, independently of what happens in all other minutes. At the beginning of every minute, you need to decide to continue waiting or leave.

Suppose there is a cost of \$$a$ for every minute spent waiting, and the hot dog is \$$b$. Let J_n denote the expected reward obtained by employing an optimal waiting policy when there are n people ahead of you in the queue.

$\boxed{5}$ (a) Write the optimality equation for $n = 0$ and $n > 0$, with justification.

$\boxed{3}$ (b) Show that the optimality equation can be rewritten as

$$J_n = \max\{J_{n-1} - a/p, 0\} \qquad \text{for } n > 0.$$

$\boxed{2}$ (c) By induction, the above implies $J_n \leq J_{n-1}$. Explain why intuitively this is expected.

$\boxed{6}$ (d) Determine the threshold N such that the form of the optimal policy is to wait only if $n \leq N$. Find J_n in terms of a, b and p.

Question 4 $\boxed{\text{4 marks}}$

The bad news is you have just been bitten by a poisonous spider. The good news is you have m potion bottles which might just save your life. If you drink the ith bottle, there is a probability of α_i that you will stay alive, a probability of β_i that you will die instantaneously, and a probability of $1 - \alpha_i - \beta_i$ that the potion will do absolutely nothing.

$\boxed{4}$ (a) Determine the order in which you should drink the potion bottles to maximise your probability of staying alive, in terms of the ratios α_i/β_i.