Mathematical Biology (App Topic D)

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1 Introduction - Basic Ideas

1.1 Dimensional Analysis

Units on both sides of an equation have to be the same - they have to be dimensionally consistent.

For example Coulomb's Law - for the force between two particles with charges q_1 and q_2 respectively

$$F = k_e \frac{q_1 q_2}{r^2}$$

Where q_1, q_2 have units Q (Coulombs). r will have units $[L]$ - square braces will mean units And $F \implies \frac{[M][L]}{[T]^2}$ So

$$\frac{[M][L]}{[T]^2} = \frac{[k_e][Q]^2}{[L]^2}$$

$$[k_e] = \frac{[M][L]^3}{[T]^2[Q]^2}$$

As an aside - we will denote temperature as $[\Theta]$

This means we can write any physical relationship as a dimensionless equation in form

$$f(\boldsymbol{\beta}) = 0$$

Where β_i are dimensionless groups. E.g. if we have time, length, gravity and mass, we can form the dimensionless group

$$\frac{gT^2}{L}$$

since the units of them will cancel out (g has units L/T^2).

The mass can't be involved in this relationship since it can't form a dimensionless group. It is possible that we have 'forgotten' some other term which could contain mass also.

Meaning

$$f(\frac{gT^2}{L}) = 0$$

I.e. gT^2/L is a zero of the function, and hence

$$\frac{gT^2}{L} = k$$

Or we could rewrite as

$$T = \sqrt{\frac{kL}{G}}$$

Consider the power of an atomic bomb. We want to know the energy, we know that the function will have to relate to E (units $[ML/T^2]$),R (units $[L]$),t (units $[T]$), and the air density ρ (units $[M/L^3]$).

$$
\begin{aligned}
[E] &= \frac{[M][L]^2}{[T]^2} \\
\frac{[E]}{[\rho]} &= \frac{[L]^5}{[T]^2} \\
\frac{[E][t^2]}{[\rho]} &= [L]^5 \\
\frac{[E][t^2]}{[\rho][R^5]} &= [1]
\end{aligned}
$$

Where [1] is dimensionless

And hence

$$\frac{[E][t^2]}{[\rho][R^5]} = k$$

We have graphs of the R at various T

$$R^5 = \frac{E}{k\rho}t^2$$

$$\implies R^5 \propto t^2$$

Since we know E is constant, k constant and ρ won't really change early on.

So if we take the logs

$$5\log R = 2\log t + const$$

$$\log t = \frac{5}{2}\log R + const$$

Of course this doesn't tell us what k is, and this is the only problem.