Revision Quiz

Q1. What are modelling, analysis and computation?

Q2. What is a well-posed problem? As a bonus, who is credited with this concept?

Q3. What are some important considerations when finding numerical solutions.

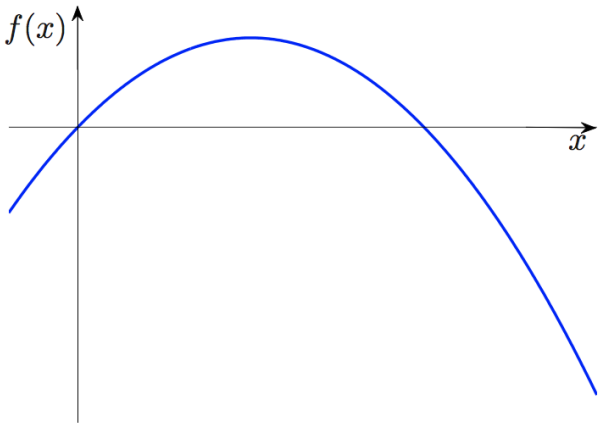
Q4. Given that most real problems must be solved numerically, what benefits are provided by exact solutions of idealised problems?

Q5. What is an autonomous ODE?

Q6. What is a fixed point and what other names can be used for it?

Q7. What is phase-line analysis of an ODE?

Q8. The following plot shows the function $f(x)$ for the ODE $dx/dt = f(x)$. Perform a phase-line analysis.



Q9. Let x_* be a fixed point of the ODE $dx/dt = f(x)$.

(a) If $f'(x_*) > 0$ is the point stable or unstable?

(b) If $f'(x_*) < 0$ is the point stable or unstable?

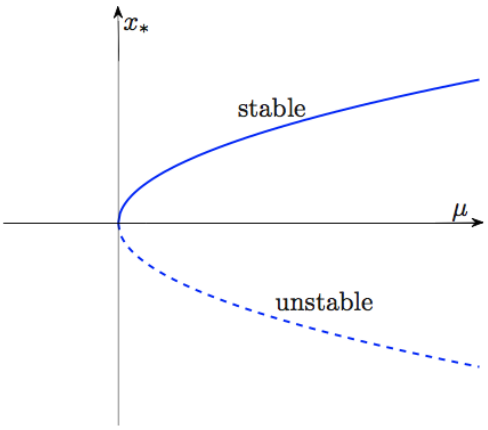
(c) If $f'(x_*) = 0$ is the point stable or unstable?

Q10. What is a bifurcation?

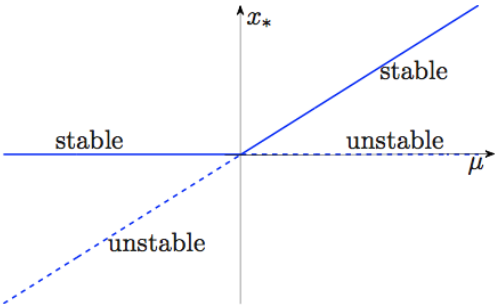
Q11. What is a bifurcation diagram?

Q12. What types of bifurcations are depicted in the bifurcation diagrams shown below?

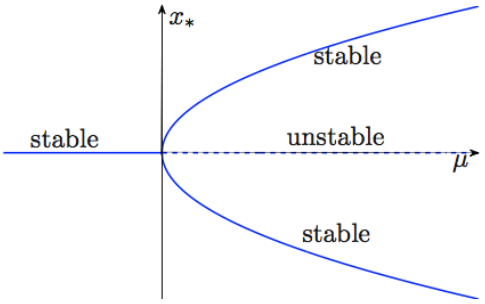
(a)



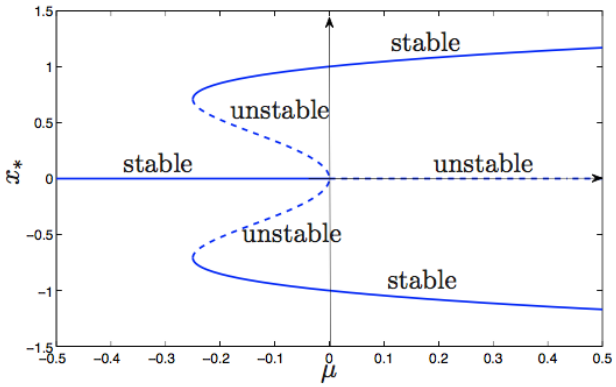
(b)



(c)



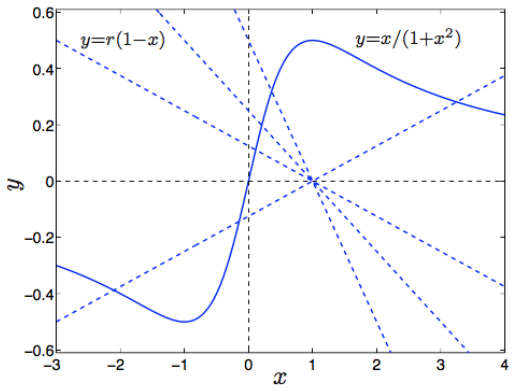
Q13. Show that hysteresis is possible for a system with the following bifurcation diagram.



Q14. For the ODE

$$\frac{dx}{dt} = rx(1-x) - \frac{x^2}{1+x^2}$$

one steady state is $x = 0$. The other steady states can be determined from the diagram below that shows the curve $y = x/(1+x^2)$ (solid) and lines $y = r(1-x)$ (dashed). Mark them and determine their stability for both $r > 0$ and $r < 0$.



Q15. If you are told that the trivial steady state for the previous question is stable for $r < 0$ and unstable for $r > 0$, plot the bifurcation diagram. Note: what is the non-trivial steady state as $r \to \infty$?

Q16. Does the existence and uniqueness theorem tell you when a unique solution does not exist?

Q17. What does it mean for a function $f(x)$ to be Lipschitz continuous on the interval $J = [x_0, x_1]$?

Q18. What is an ill-conditioned problem?

Q19. What is an unstable algorithm?

Q20. Would you expect to get a good solution to an ill-conditioned problem if you were using a stable algorithm?

Q21. What is the difference between an explicit and implicit finite difference method?

Q22. A finite difference formula must be consistent with the ODE being solved by it. What does this mean?

Q23. What is convergence?

Q24. What is Lax's equivalence theorem?

Q25. What test problem is used to generate stability diagrams?

Q26. What is unconditional and conditional stability?

Q27. What is a stiff problem?

Q28. What should be thought about when solving a stiff problem numerically?

Q29. What is a strictly diagonally dominant matrix?

Q30. For an $n \times n$ matrix system of equations, what does strict diagonal dominance of the matrix guarantee?

Q31. What type of model is this

$$\dot{x} = \alpha x + \beta xy, \quad \dot{y} = \gamma y - \delta xy$$

if $\alpha, \beta, \gamma, \delta$ are all positive constants.

Q32. What type of model is this

$$\dot{x} = \alpha x - \beta xy, \quad \dot{y} = \gamma y - \delta xy$$

if $\alpha, \beta, \gamma, \delta$ are all positive constants.

Q33. For the model

$$\dot{x} = f(x, y), \quad \dot{y} = g(x, y)$$

what are the x and y nullclines?

Q34. For the model

$$\dot{x} = f(x, y), \quad \dot{y} = g(x, y),$$

suppose that the x and y nullclines in part coincide (i.e. lie on top of one another). What does this signal?

Q35. How do we examine the stability of steady states of a nonlinear 2D model?

Q36. What is required for asymptotic stability of a steady state?

Q37. What does stability of a steady state mean physically?

Q38. What is a limit cycle? If it is attracting, what does it imply about surround solutions?