APP MTH 3001 Applied Probability III
Class Exercise 2, 2018
Due: 3pm, 23 March 2018, via Canvas (PDF only).

1. Consider the "Two Gamblers" example discussed in lectures.
   This corresponds to a random walk on $\mathcal{S} = \{0, 1, \ldots, N\}$, with transition probabilities $p_{0,0} = p_{N,N} = 1$, and

$$p_{i,j} = \begin{cases} p & \text{if } \ j = i+1 \\ q & \text{if } \ j = i-1 \\ 0 & \text{otherwise.} \end{cases}$$

   In lectures, we obtained an expression for the hitting probabilities
   $u_i = \mathrm{P}\left(X_n \text{ reaches state 0 before state N} \mid X_0 = i\right), i \in \mathcal{S}$.

   Consider the probabilities $v_i = \mathrm{P}\left(X_n \text{ reaches state } N \text{ before state } 0 \mid X_0 = i\right), i \in \mathcal{S}$ and use the same general method for solving second order homogeneous difference equations to find the $v_i, i \in \mathcal{S}$. Find $u_i + v_i$ and explain what it means.

2. | **Definition:** the random variables X and W are *independent* if the events $\{X = x_i\}$ and $\{W = w_j\}$ are independent for all i and j. |
   |---|

   (a) Show, from first principles, that for independent random variables X and W,
$$E(XW) = E(X)E(W).$$
   (b) Show, from first principles, that for independent random variables X and W,
$$Var(X + W) = Var(X) + Var(W).$$

   Note: In all of Q2, first principles means that you shouldn't use the definition of Covariance or any of its properties.

3. Let Y be a random variable having the binomial distribution with parameters n and p. Recall that Y has probability mass function

$$f_Y(k) = \mathrm{P}\left(Y = k\right) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad k = 0, 1, 2, \ldots, n.$$

   Prove that the binomial distribution converges to the Poisson distribution with parameter λ if $n \to \infty$ and $p \to 0$ in such a way that $\lambda = np$ remains constant.

   Hint: use the identity
$$\lim_{n\to\infty} \left(1 - \frac{\lambda}{n}\right)^n = e^{-\lambda}.$$

4. Create a visualisation of some aspect of piece of music given to your group for the group project. This can be as simple as a single figure, e.g., a time-series-like "piano roll" plot, a plot of the distribution of note values/lengths/intervals, an autocorrelation plot, a network, ... (you can get more creative if you wish!)
   Be sure to explain clearly what your visualisation shows, and use it to make a comment on your piece of music. All group members must submit different visualisations.
   Hint: try importing your MusicXML file into Matlab, converting into a MIDI-friendly "Notematrix", and then exploring the various functions in the Matlab MIDI toolbox. The documentation for that package is very useful here.