APP MTH 3002 Fluid Mechanics III
Assignment 5
Due: 12 noon, Friday 8 June.

When presenting your solutions to the assignment, please include some explanation in words to accompany your calculations. It is not necessary to write a lengthy description, just a few sentences to link the steps in your calculation. Messy, illegible or inadequately explained solutions may be penalised. The marks awarded for each part are indicated in boxes.

This assignment has 2 questions, for a total of 30 marks.

---

1. Consider an accelerating sphere moving at speed $U(t)$ along the z-axis through an otherwise quiescent incompressible inviscid fluid. The velocity potential of the flow in a *stationary* reference frame is

$$\phi = -\frac{a^3 U(t)}{2} \frac{(z - Z(t))}{[x^2 + y^2 + (z - Z(t))^2]^{3/2}},$$

where $U(t)$ is the speed of the sphere at time t, $Z(t)$ is the position of the centre of the sphere along the z-axis and a is the radius of the sphere.

$\boxed{6}$ (a) Suppose that the centre of the sphere passes through the origin at time t_0. Show that

$$\phi|_{t=t_0} = -\frac{a^3 U_0}{2} \frac{\cos\theta}{r^2},$$

$$\left.\frac{\partial \phi}{\partial t}\right|_{t=t_0} = -\frac{a^3 U_0'}{2} \frac{\cos\theta}{r^2} + \frac{a^3 U_0^2}{2} \frac{(1 - 3\cos^2\theta)}{r^3},$$

where $U_0 = U(t_0)$, $U_0' = U'(t_0)$ and (r, θ, φ) are the coordinates of our usual spherical coordinate system.

$\boxed{1}$ (b) Using spherical coordinates, calculate the velocity of the fluid at $t = t_0$.

$\boxed{2}$ (c) Verify that the velocity found in part (b) satisfies the impermeability condition on the surface of the sphere $r = a$ at $t = t_0$.

$\boxed{10}$ (d) The drag on the sphere is $D = -\boldsymbol{F} \cdot \boldsymbol{k}$, where $\boldsymbol{F}$ is the force

$$\boldsymbol{F} = -\int_{\mathcal{S}} p\, \hat{\boldsymbol{n}}\, \mathrm{d}S,$$

$\mathcal{S}$ denotes the surface of the sphere, p is the pressure and $\hat{\boldsymbol{n}}$ is the normal to the sphere. Assuming that there are no external forces, use an appropriate form of Bernoulli's equation, together with the results obtained in parts (a) and (b), to calculate the drag on the sphere at $t = t_0$.

Hint: You may use technology to evaluate the integrals. If you do, please list all the results so that the reader can follow the calculation.

2. Consider a pair of parallel line vortices propagating at constant speed in an incompressible irrotational fluid. Suppose that we observe this flow from a reference frame that moves with the two vortices. In this frame, the flow is steady and consists of one vortex circulating in a clockwise sense at $z = \mathrm{i}d$ with circulation $-\Gamma$, another vortex circulating in an anticlockwise sense at $z = -\mathrm{i}d$ with circulation Γ, and a uniform flow of speed $\Gamma/(4\pi d)$ in the positive x-direction. Here, $z = x + \mathrm{i}y$ is a point on the complex plane, d is half the distance between the vortices and $\Gamma > 0$.

$\boxed{3}$ (a) Write down the complex potential for this flow.

$\boxed{1}$ (b) Find the complex velocity.

$\boxed{3}$ (c) Find the stagnation points.

$\boxed{4}$ (d) Find and plot the stream function.