

Formula Page for Time Series

Moving averages: A moving average of order $2m + 1$ for a time series $\{y_t : t = 1, 2, \dots, n\}$ is a time series defined by

$$s_t = \sum_{j=-m}^m w_j y_{t+j}, \quad t = m + 1, \dots, n - m$$

Sines and cosines:

$$\begin{aligned} \sin(a + b) &= \sin a \cos b + \cos a \sin b \\ \sin(a - b) &= \sin a \cos b - \cos a \sin b \\ \cos(a + b) &= \cos a \cos b - \sin a \sin b \\ \cos(a - b) &= \cos a \cos b + \sin a \sin b \end{aligned}$$

Periodogram ordinates:

$$I(\omega) = \frac{1}{n} \left\{ \left(\sum_{t=1}^n y_t \cos(\omega t) \right)^2 + \left(\sum_{t=1}^n y_t \sin(\omega t) \right)^2 \right\}$$

for $\omega = 2\pi j/n$ where $j < n/2$ is a positive integer.

Yule-Walker equations for AR(p) process:

$$\rho_k = \sum_{j=1}^p \alpha_j \rho_{k-j}, \quad k = 1, 2, \dots$$

ARMA(p, q) process:

$$Y_t - \alpha_1 Y_{t-1} - \dots - \alpha_p Y_{t-p} = Z_t + \beta_1 Z_{t-1} + \dots + \beta_q Z_{t-q}$$

Spectrum and normalized spectrum:

$$\begin{aligned} f(\omega) &= \gamma_0 + 2 \sum_{k=1}^{\infty} \gamma_k \cos(k\omega) \\ f^*(\omega) &= 1 + 2 \sum_{k=1}^{\infty} \rho_k \cos(k\omega) \end{aligned}$$