THE UNIVERSITY of ADELAIDE

Examination in School of Mathematical Sciences

Semester 2, 2016

005675 STATS 3005 Time Series III

Official Reading Time:	10 mins
Writing Time:	120 mins
Total Duration:	130 mins

NUMBER OF QUESTIONS: 6 TOTAL MARKS: 75

Instructions

- Attempt all questions.
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.

Materials

- 1 Blue book is provided.
- Calculators are permitted.

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO.

1. Consider the process $Y_t = t + Z_t$, where $\{Z_t\}$ is a white noise process with $E(Z_t) = 0$ and $\text{var}(Z_t) = \sigma^2$, and let $U_t = Y_t - Y_{t-1}$ be the differenced series.
   (a) Is $\{Y_t\}$ a second order stationary process? Explain your answer.
   (b) Find $E(U_t)$ and $\text{var}(U_t)$.
   (c) Find $\text{cov}(U_t, U_{t-1})$.
   (d) Show that $\text{cov}(U_t, U_{t-k}) = 0$ for $k > 1$.
   (e) Is the process $\{U_t\}$ second order stationary? Explain your answer.

[12 marks]

2. Consider the periodogram $I(\omega)$ defined by

$$I(\omega) = \frac{1}{n}\left\{\left(\sum_{t=1}^{n} y_t \cos(\omega t)\right)^2 + \left(\sum_{t=1}^{n} y_t \sin \omega t)\right)^2\right\}$$

for $\omega = 2\pi j/n$ where $j < n/2$ is a positive integer.

   (a) Show that

$$I(\omega) = \frac{1}{n}\left\{\left(\sum_{t=1}^{n}(y_t - \bar{y})\cos(\omega t)\right)^2 + \left(\sum_{t=1}^{n}(y_t - \bar{y})\sin \omega t)\right)^2\right\}.$$

   Hint: You may use, without proof, the trigonometric identities

$$\sum_{t=1}^{n}\cos(2\pi jt/n) = \sum_{t=1}^{n}\sin(2\pi jt/n) = 0 \text{ for } j < n/2.$$

   (b) Hence, or otherwise, show that

$$I(\omega) = \frac{1}{n}\sum_{s=1}^{n}\sum_{t=1}^{n}(y_s - \bar{y})(y_t - \bar{y})\cos(\omega(t - s))$$

   for $\omega = 2\pi j/n$ where $j < n/2$ is a positive integer.
   Hint: You may use, without proof, the trigonometric identity

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b).$$

(c) Hence, or otherwise, show that

$$I(\omega) = g_0 + 2\sum_{k=1}^{n-1} g_k \cos(k\omega)$$

where g_k

$$g_k = \frac{1}{n}\sum_{t=k+1}^{n}(y_t - \bar{y})(y_{t-k} - \bar{y})$$

is the sample autocovariance.
Hint: You may use, without proof, the identity

$$\sum_{s=1}^{n}\sum_{t=1}^{n} a_{st} = \sum_{t=1}^{n} a_{tt} + \sum_{k=1}^{n-1}\sum_{t=k+1}^{n}(a_{t,t-k} + a_{t-k,t}).$$

(d) Suppose the periodogram was calculated from an observed white noise series. Explain what you would expect to see.

[14 marks]

3. (a) Give the definition of an ARMA(p, q) process.

(b) Consider the process

$$Y_t = \alpha Y_{t-1} + Z_t + \beta Z_{t-1}$$

where Z_t is a white noise process, with $Z_t \sim N(0, \sigma^2)$.

(i) For what values of α and β is Y_t a valid ARMA process.

(ii) Calculate the spectrum, $f_y(\omega)$ for this process.

(c) Consider the process

$$Y_t = 0.5Y_{t-1} + 0.5Y_{t-2} + Z_t + 0.5Z_{t-1}$$

where $\{Z_t\}$ is a white noise process with $Z_t \sim N(0, \sigma^2)$.

(i) Is this process stationary? Explain your answer.

(ii) What type of process is Y_t? Explain your answer.

[13 marks]

Please turn over for page 4

4. Consider the $AR(p)$ process
$$\phi(B)Y_t = Z_t$$
where $\phi(z) = 1 - \sum_{j=1}^{p} \alpha_j z^j$ is such that all roots lie outside the unit circle on the complex plane.

   (a) Show that the autocorrelations ρ_k satisfy the Yule-Walker equations,
$$\rho_k = \sum_{j=1}^{p} \alpha_j \rho_{k-j} \text{ for } j = 0, 1, 2, \ldots$$

   (b) Consider the $AR(2)$ process
$$Y_t = \frac{2}{3}Y_{t-1} - \frac{1}{9}Y_{t-2} + Z_t.$$

      (i) Verify that the process is stationary.

      (ii) Write down the auxiliary equation and find its roots.

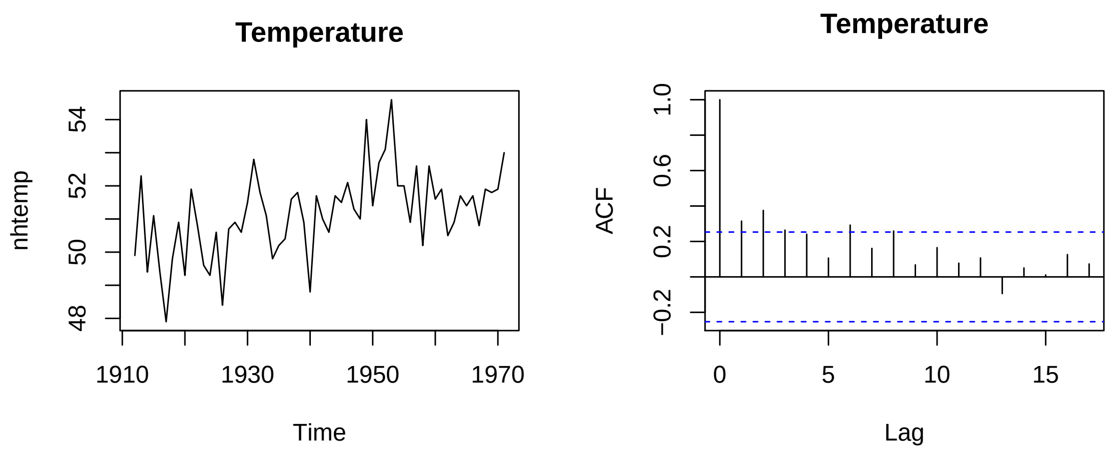
      (iii) Write down the general form of the solution to the auxiliary equation in this case.

      (iv) Show that the autocorrelation function is given by
$$\rho_k = \left(1 + \frac{4}{5}k\right)\left(\frac{1}{3}\right)^k.$$

[14 marks]

5. The mean annual temperature in °F for New Haven, Connecticut was recorded from 1912 to 1971.

   (a) Based on the plot of the data and the estimated ACF shown below, does the series appear to be stationary? Explain your answer.

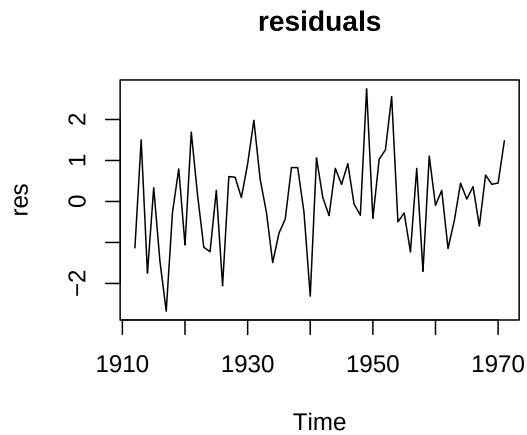


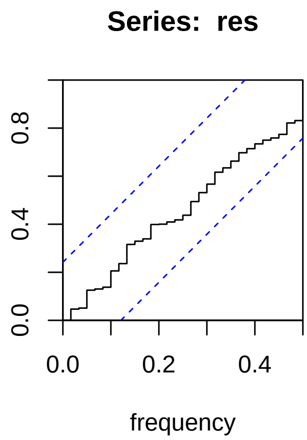
(b) Below are summary statistics from a number of ARMA models. Identify the your preferred model and any other models that might be plausible. Explain your answer.

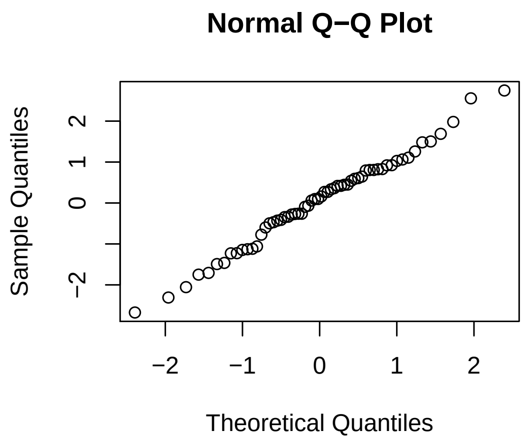
Model	Model fit statistics
ARMA(2, 2)	`log likelihood = -89.69,  aic = 191.38`
ARMA(1, 1)	`log likelihood = -92.15,  aic = 192.29`
ARMA(2, 0)	`log likelihood = -92.47,  aic = 192.94`
ARMA(2, 1)	`log likelihood = -92.00,  aic = 193.99`
ARMA(0, 2)	`log likelihood = -94.37,  aic = 196.74`
ARMA(1, 0)	`log likelihood = -95.51,  aic = 197.01`
ARMA(0, 1)	`log likelihood = -96.79,  aic = 199.58`

(c) Based on the plots shown below, does the ARMA(1, 1) model provide an adequate description of the data? Explain your answer.

```
res=residuals(arima(nhtemp,order=c(1,0,1)))
plot(res,main="residuals"); cpgram(res); qqnorm(res)
```





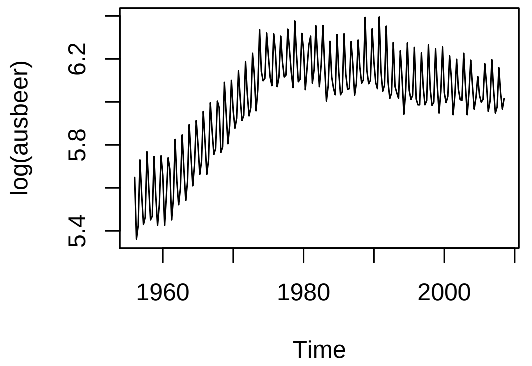


[9 marks]

6. The production of beer in Australia was recorded quarterly from 1956 to 2008.

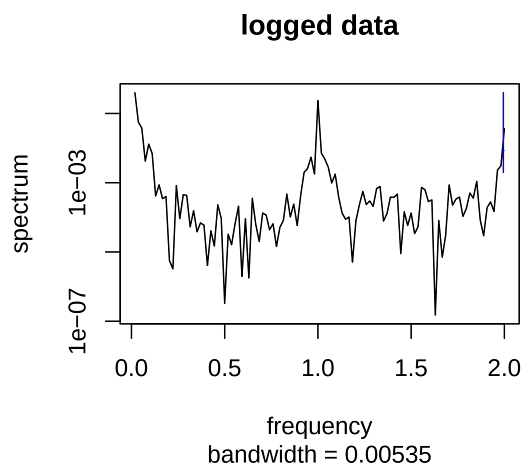
   (a) Based on the plot shown below, do the log transformed data appear stationary?

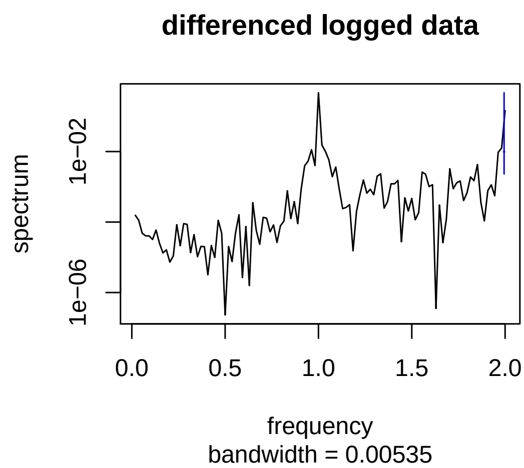
```
plot(log(ausbeer))
```



   (b) Shown below are periodograms of the log transformed data and the first differences of the log transformed data. Interpret and compare the two periodograms.

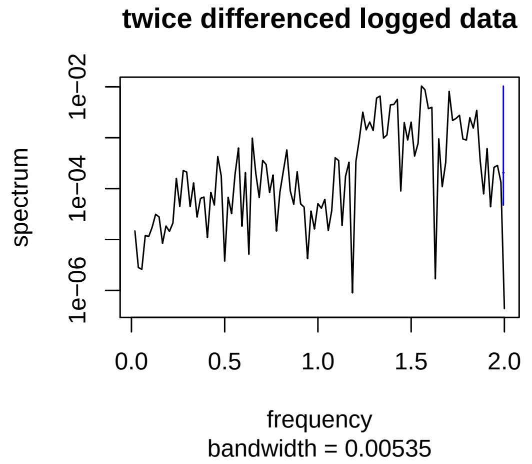
```
spectrum(log(ausbeer),main="logged data")
spectrum(diff(log(ausbeer)),main="differenced logged data")
```





(c) Below is a periodogram of the log transformed series differenced twice, once at lag one and once at lag 4.

```
spectrum(diff(diff(log(ausbeer),lag=4)),main="twice differenced logged data")
```



(i) Compare this periodogram to those in part (b) above.

(ii) Explain the significance of taking differences at lag 4 for these data.

(iii) Is the twice differenced series compatible with white noise? Explain your answer.

[13 marks]