STATS 3005 Time Series III
Practical 4
2018

1 Outline

At the end of this practical you should be able to:

- Interpret the periodogram and cumulative periodogram for real data.
- Use simulation to investigate various AR, MA and ARMA processes.

2 Analysis of deaths due to lung disease

The data on monthly deaths due to lung disease in the UK is available as the R object, `ldeaths`.

1. Obtain a time series plot of the data and comment on the salient features of the plot.
2. Obtain a correlogram and also a periodogram. Of these plots, which do you consider to be more informative for these data? Explain.
3. Use the `cpgram` function to obtain the cumulative periodogram for the data.
   (a) On the basis of the cumulative periodogram, is white noise a plausible model for this series. Explain why or why not.
   (b) Obtain an unlogged periodogram of the data using the `spectrum` function with the optional argument `log="no"` and compare the features of this graph to the cumulative periodogram.

3 Simulation

1. A simple method to simulate an MA process in `R` is as follows.

```
z=rnorm(1001)
y=rep(0,1001)
b=0.5
for(i in 2:1000) y[i]=z[i]+b*z[i-1]
y=y[-1]
y=ts(y)
```

   (a) Modify the above commands to simulate a series of 1000 observations from the MA(1) process with $\beta_1 = 0.95$.
   (b) Obtain the correlogram of the simulated data. Calculate the theoretical autocorrelations for these data and compare them to the correlogram.
   (c) Obtain the spectrum and the cumulative periodogram for the data and comment.

2. A simple method to simulate an AR process in R is as follows.

```
z=rnorm(2000)
y=rep(0,2000)
a=0.5
for(i in 2:2000) y[i]=a*y[i-1]+z[i]
y=ts(y[1001:2000])
```

   (a) Why do you think the first 1000 observations were discarded.

   (b) Modify above commands to simulate a series of 10000 observations from the AR(1) process with $\alpha_1 = 0.9$.

   (c) Obtain the correlogram of the simulated data. Calculate the theoretical autocorrelations for these data and compare them to the correlogram.

   (d) Obtain the spectrum and the cumulative periodogram for the data and comment.

3. Based on the above method, simulate a series of length 1000 from the AR(2) process, $Y_t = -0.5Y_{t-1} + 0.4Y_{t-2} + Z_t$.

   (a) Examine the simulated series and comment on whether it appears to be stationary.

   (b) Write down the second degree polynomial $\phi(u)$ for this process. Use the `polyroot` functions to find its roots and the `Mod` function to check that both roots exceed 1 in magnitude.

4. Simulate a series of length 1000 from the process, $Y_t = -0.5Y_{t-1} + 0.505Y_{t-2} + Z_t$.

   (a) Examine the simulated series and comment on whether it appears to be stationary.

   (b) Write down the second degree polynomial $\phi(u)$ for this process. Investigate the roots of ϕ and comment.

5. By combining the preceding methods, simulate a series of length 10000 from the ARMA(2,2) process with $\alpha_1 = 0.5$, $\alpha_2 = 0.25$, $\beta_1 = \beta_2 = 0.5$. Examine the time series plot and correlogram for this series.

6. Repeat question 5 using the inbuilt `arima.sim` function. Check that the correlogram is similar to that in question 5.

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