# Assignment I

Worth 5% of course assessment; due by 1pm on Tuesday 26th March 2019.

Relevant lectures: Lectures 1– 4.

Individual marks are noted in [] at start of each question; total marks for assignment is 28.

Please provide an explanation/discussion with all answers, and code where appropriate.

**Q1: A very simple ODE model for a sexually-transmitted infection** [18 marks]

Chlamydia (*Chlamydia trachomatis*) is a sexually transmitted disease for which there is no immunity following infection. We wish to model the number of people infected with Chlamydia, in a population of size $N$, through time using a simple deterministic model. We will assume that each infected individual recovers at the same constant rate $\gamma$. When appropriate, assume $N = 2000$, $q = 0.75$ and $\gamma = 0.68$; and for frequency-dependent transmission $c = 2$, or for density-dependent transmission $A = 400$ and $\kappa = 0.4$ (using variable names as in lectures).

(i) Discuss which of frequency-dependent or density-dependent transmission you think is most appropriate for modelling Chlamydia?

(ii) Specify your deterministic model for modelling Chlamydia.

(iii) Find the equilibrium points of the model and assess their stability. Discuss your findings in relation to the first Theorem in the course.

(iv) Numerically solve (say, using ODE45 in MATLAB) the model for (at least) two different choices of parameter values.

(v) Derive the solution for the proportion of infectious individuals at time $t$. Compare with your numerical solutions in (iv).

(vi) What does the rate of contacts $c$ (if frequency-dependent transmission) or scaling rate constant $\kappa$ (if density-dependent transmission) need to be reduced to so that Chlamydia is eradicated from the population?

**Q2: Investigate the error in the approximation to the SIR model**
[10 marks]

In class, we derived the approximation (under certain conditions)

$$r(t) \approx \frac{\rho^2}{s_0}\left(\frac{s_0}{\rho} - 1\right) + \frac{\alpha\rho^2}{s_0}\tanh\left(\frac{\gamma\alpha t}{2} - \phi\right)$$

where $\phi = \tanh^{-1}\left[(1/\alpha)((s_0/\rho) - 1)\right]$, and $s(0) = s_0$.

(i) Specify the corresponding explicit approximations for $s(t)$ and $i(t)$ using relationships derived in class.

(ii) Investigate whether these appear to be reasonable approximations? Can the accuracy be explained in terms of the conditions used in its derivation? Support your answer with plots.

(iii) What restrictions does this place on the 'type' of epidemic that can be modelled; i.e. qualitatively what combination of parameters and times are we restricted to?