School of Mathematical Sciences

Modelling with ODEs

Semester 1, 2019

Assignment 1

Due 5pm Monday, Week 4: Submit via MyUni

You will be marked on the presentation of your answers!

A model for the amount of vegetation, $V(t)$, in a field grazed by a herd of H cows is

$$\frac{dV}{dt} = \underbrace{R\,V\left(1-\frac{V}{K}\right)}_{\text{logistic growth}} - \underbrace{\frac{H\,V}{V_0+V}}_{G(V)=\text{grazing}}, \tag{1}$$

where H, R, K and V_0 are positive constants.

1. (a) Sketch the grazing term $G(V)$.
   (b) Give an interpretation of the grazing term in the model. Explain the different parameters.
2. (a) Determine scalings $V = V_c\,\hat{V}$ and $t = t_c\,\hat{t}$ that result in the nondimensional version of ODE (1)

$$\frac{d\hat{V}}{d\hat{t}} = \hat{V}\left(1-\frac{\hat{V}}{k}\right) - \frac{\hat{V}}{v_0+\hat{V}}, \tag{2}$$

   and give the nondimensional parameters k and v_0 in terms of the original parameters.
   (b) Find another scaling that produces a nondimensional version of ODE (1) with two parameters.
3. For $k = 1$ and $v_0 = 3$:
   (a) Find the relevant fixed points $\hat{V} = \hat{V}_*$ of the nondimensional ODE (2).
   (b) What are the corresponding fixed points of the dimensional ODE for $V_0 = 6$ and $H = 7$.
   (c) Perform a phase-line analysis to determine the stability of the fixed points from part (a), i.e. sketch the right-hand side of ODE (2) and arrows around the fixed points. State the stabilities of the fixed points.
   (d) Use the MATLAB `quiver` function to plot the vector field.⋆
   (e) Solve ODE (2) numerically (e.g. using Euler's method), and plot at least two solutions on the vector field to show the stability/instability of the fixed points.⋆

⋆Attach your code to your solutions. It will be marked.