Random Processes III 2018: Tutorial 2,

please come to the tutorial on Friday 17^{th} August having attempted these questions. Solutions to these questions will not be uploaded to MyUni.

Problem 1

Consider the general two-state CTMC $(X(t), t \geq 0)$ with generator

$$Q = \begin{pmatrix} -a & a \\ b & -b \end{pmatrix},$$

with $0 \leq a, b < \infty$.

(a) Write down the state space of this CTMC.

(b) Evaluate the eigenvalues, and left and right eigenvectors of Q.

(c) Use part (b) to specify the transition function of this CTMC.

Problem 2

A CTMC $(X(t), t \geq 0)$ has the following infinitesimal generator

$$Q = \begin{pmatrix} -3 & 3 & 0 \\ 2 & -4 & 2 \\ 0 & 6 & -6 \end{pmatrix}.$$

(a) Write down the state space of this CTMC.

(b) Write down and solve the equilibrium equations for $X(t)$. That is, solve $\pi^Q Q = \mathbf{0}$.

(c) Give the probability transition matrix $\mathbb{P}$ for the associated Jump Chain (i.e., the DTMC governing the movement between states, when ignoring the time spent in each state). Hence, solve the equilibrium equations for the Jump Chain. That is, solve $\pi^J = \pi^J \mathbb{P}$.

(d) Can you make sense of the relationship between these two equilibrium distributions?