APP MTH 3020 Stochastic Decision Theory

Tutorial 1

Week 3, Friday, August 10

1. Suppose we roll two (standard, six-sided) dice.
   (a) Specify the probability mass function for the sum of the numbers on their faces.
   (b) Specify the probability mass function for the minimum of the numbers on their faces.
   (c) Evaluate the expected value of the sum.
   (d) Evaluate the expected value of the minimum.

2. Prove **Jensen's Inequality** (Theorem 1.6 in the lecture notes) for the case of discrete random variables, which states that: If $h(x)$ be a convex function and X a discrete random variable, then

$$\mathbb{E}\left[h(X)\right] \geq h\left(\mathbb{E}[X]\right).$$

3. A furniture maker makes two products, P_1 and P_2, where there is a total production limit of a total of 1500 units of furniture products P_1 and P_2. Both carpentry and finishing are required resources for the manufacturing process. The requirements measured in hours per unit are known and shown in the below table, along with the profit per unit of product.

Product parameters	P_1	P_2
Carpentry hours	4	8
Finishing hours	3	2
profit per unit	15	25

Our problem is to select the product mix to maximise total profit, but the availability of the resources are unknown. Rather, we have two equally likely estimates of the hours available for each resource:

$$\text{Available carpentry hours} = \begin{cases} 4950 & \text{with probability } p_{c_1} = 0.5 \\ 5850 & \text{with probability } p_{c_2} = 0.5, \end{cases}$$

$$\text{Available finishing hours} = \begin{cases} 3636 & \text{with probability } p_{f_1} = 0.5 \\ 4064 & \text{with probability } p_{f_2} = 0.5. \end{cases}$$

   (a) Write down the expected time available for carpentry, and the expected time available for finishing.
   (b) Write down the LP using the **expected time availabilities** for the RHS of your problem constraints.
   (c) Write down the dual to this LP.
   (d) Verify that 1250 units of P_1 and 50 units of P_2 is a solution to the original LP, with a profit of $\$20,000$.
   (e) Given that a solution to the dual may also be found such that its objective function is $\$20,000$, is this solution an optimal solution?
   (f) A solution to the averaged value LP is not very acceptable because it does not allow for the stochastic variation of available carpentry and finishing hours. Assume that additional carpentry hours may be purchased at \$5/hr and that extra finishing hours may be purchased at \$12/hr. Also assume that any unused base carpentry hours are wasted and must be costed at \$4/hr and similarly any unused base hours of finishing must be costed at \$9/hr.
      (i) Write down the expanded version of a recourse model considering all realisations.
      (ii) Solve this problem in expanded form and give an interpretation of the solution.