Random Processes III 2018: Tutorial 1,

please come to the tutorial on Friday 3rd August having attempted these questions. Solutions to these questions will not be uploaded to MyUni.

Problem 1: The migratory behaviour of a bird

A particular species of bird migrates between three islands. The precise reason for the migration patterns is unknown. A *null* model for this behaviour might be Markovian migration, in that the next island visited only depends upon the island it currently occupies, and not on how it got there.

A field ecologist put a GPS tracker on a bird, and provides the following information:

- From island A, the bird migrates to B with probability 1/2 and to C with probability 1/2.
- From island B, the bird migrates to A with probability 3/4 and to C with probability 1/4.
- From island C, the bird always migrates to A.
- The bird stays at each island for an average of 1/5 hours when on island A, 1/5 hours when on island B, and 1/4 hours when on island C.

Construct a continuous-time Markov chain (CTMC) to model this process, where the state $X(t)$ represents the island currently occupied by the bird at $t \geq 0$, by following these steps.

(a) Write down the state space of this CTMC.

(b) Draw a state transition diagram corresponding to this CTMC.

(c) Specify the generator of this CTMC.

Problem 2: Models for $2, 3, \ldots$ birds on 3 islands

Let us consider how to extend the model in Problem 1 to handle 2, or several, species.

(a) Write down the state space for a CTMC where the state $X(t)$ is bivariate, representing, respectively, the location of bird 1 and bird 2.

(b) Now repeat (a) for three birds.

(c) How does the size of the state space scale with the number of birds, B?

(d) Can you think of a different way of specifying the state of the CTMC, such that the state space is not as large, with potentially some loss of information? What is the size of the state space in this description when modelling $B = 2$, and $B = 3$, birds, respectively?

  (i) What information was lost?

  (ii) What is the size of the state space for B birds in the new representation?

Problem 3: Accuracy of the probability estimated in Lecture 4

In Lecture 4, we estimated the probability of there being 2 broken printers 180 days after starting with both printers working. Figure 4 of Lecture 4 indicates that the estimate is highly variable for a small number of simulations, and appears to converge as the number of simulations increases. But how can we have some confidence that our estimate is good for a chosen number of simulations, or, alternatively, how many simulations should we do to be able to provide a certain level of confidence in our estimate?

(a) How might you use simulation to establish confidence? Could you estimate a *standard error* in the estimate?

(b) Consider tossing a fair coin, but you don't know it is fair. If you toss the coin N times, how would you estimate the probability of getting a head?

  (i) The standard error of this estimate is simply the square-root of the variance of this estimate. Calculate the expression for the standard error of an estimate of the proportion of heads from N Bernoulli trials.

  (ii) How might you use this expression to inform the number of coin tosses?

  (iii) Explain how this coin example can assist in the original Problem, of having confidence in the estimated probability from Lecture 4.