School of Mathematical Sciences

Modelling with ODEs

Semester 1, 2019

Assignment 2

Due 5pm Monday, Week 6: Submit via MyUni

You will be marked on the presentation of your answers (including clarity of explanations)!

1. Consider the following ODEs:

   (a) $\dot{x} = r + x - \log(1+x)$; (b) $\dot{x} = x - r\,x\,(1-x)$; (c) $\dot{x} = r\,x - 4\,x^3$.

   For each ODE:

   - Find the bifurcation value $\bar{r}$. You may find it helpful to use MATLAB.
   - State the type of bifurcation with reason.
   - Produce the bifurcation diagram, with the stable and unstable branches indicated.

2. In Tutorial 2 you studied the ODE

$$\frac{dx}{d\tau} = s - r\,x + \frac{x^2}{1+x^2}, \tag{1}$$

   which models the dynamics of a (nondimensional) gene product $x(\tau)$, activated by a (nondimensional) biochemical substance $s \geq 0$, and with parameter $r > 0$.

   (a) Let $r = 0.4$, and assume that initially there is no gene product, i.e. $x(0) = 0$. Suppose that the biochemical substance is introduced by slowly increasing s from zero up to 0.2.

      i. Explain what happens to $x(\tau)$ and why.

      ii. Explain (with reasons) what happens if the biochemical substance is then slowly decreased back to zero.

   (b) Consider ODE (1) for two varying parameters, $s \geq 0$ and $r > 0$.

      i. Calculate the bifurcation curves.

      ii. Plot the bifurcation curves in the (r, s)-plane.

      iii. Determine the number of steady states and their stability in each region of your plot, and describe what happens on the bifurcation curves.