# School of Mathematical Sciences

# Assignment Cover Sheet

THE UNIVERSITY of ADELAIDE

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| Student Name | |
| Student ID | |
| Assessment Title | **Assignment 3** |
| Due Date | Thursday, 10 October, 2019 @ 12:00 noon |
| Course / Program | APP MTH 3022–Optimal Functions & Nanomechanics |
| Date Submitted | |
| **OFFICE USE ONLY**<br>Date Received | |

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Signed ............................................................................ Date ..........................................................

# OFN Assignment 4

Andrew Martin

September 20, 2019

1. (a) $dA$ is obtained using the cross product of the tangent vectors of the parametrisation

$$\begin{aligned}\frac{\partial r}{\partial \theta} &= (-b\sin\theta\sin\phi, b\cos\theta\sin\phi, 0)\\ \frac{\partial r}{\partial \phi} &= (b\cos\theta\cos\phi, b\sin\theta\cos\phi, -c\sin\phi)\end{aligned}$$

$$dA = \left|\frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial \phi}\right| d\theta d\phi$$

$$\frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial \phi} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -b\sin\theta\sin\phi & b\cos\theta\sin\phi & 0 \\ b\cos\theta\cos\phi & b\sin\theta\cos\phi & -c\sin\phi \end{vmatrix}$$

$$\begin{aligned}&= \left(-bc\cos\theta\sin^2\phi, bc\sin\theta\sin^2\phi, -b^2\sin^2\theta\sin\phi\cos\phi - b^2\cos\theta^2\sin\phi\cos\phi\right)\\ &= \left(-bc\cos\theta\sin^2\phi, bc\sin\theta\sin^2\phi, -b^2\sin\phi\cos\phi\right)\\ &= b\sin\phi\,(-c\cos\theta\sin\phi, c\sin\theta\sin\phi, -b\cos\phi)\end{aligned}$$

$$\begin{aligned}dA &= b\sin\phi\sqrt{(-c\cos\theta\sin\phi)^2 + (c\sin\theta\sin\phi)^2 + (-b\cos\phi)^2}d\theta d\phi\\ &= b\sin\phi\sqrt{c^2\cos^2\theta\sin^2\phi + c^2\sin^2\theta\sin^2\phi + b^2\cos^2\phi}d\theta d\phi\\ &= b\sin\phi\sqrt{c^2\sin^2\phi + b^2\cos^2\phi}d\theta d\phi\\ &= b\sin^2\phi\sqrt{c^2 - b^2}d\theta d\phi\end{aligned}$$

   (b)

   (c)

2. (a)

   (b)

3.

# School of Mathematical Sciences

## APP MTH 3022/7106 - Optimal Functions and Nanomechanics

### Assignment 4 question sheet

*Due: Thursday, 10 October, at 12 noon (in the hand-in box on level 6)*

*When presenting your solutions to the assignment, please include some explanation in words to accompany your calculations. It is not necessary to write a lengthy description, just a few sentences to link the steps in your calculation. Messy, illegible or inadequately explained solutions may be penalised. The marks awarded for each part are indicated in brackets.*

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*All students are to attempt Questions 1–3.*

1. A spheroidal surface $\mathcal{P}$ is given parametrically by the position vector $\boldsymbol{r}(\theta, \phi)$ as

$$\boldsymbol{r}(\theta, \phi) = (b\cos\theta\sin\phi, b\sin\theta\sin\phi, c\cos\phi),$$

where $-\pi < \theta \leqslant \pi$, and $0 \leqslant \phi \leqslant \pi$ and the constant $b$ is the minor semi-axis length and $c$ is the major semi-axis length.

   (a) Derive an expression for the scalar surface element $\mathrm{d}A$ for $\mathcal{P}$.

   (b) Integrate your answer from part (a) to find the surface area of $\mathcal{P}$ as a function of $b$ and $c$.
   *Hint: it is easiest to express this in terms of the usual hypergeometric function.*

   (c) One isomer of the $C_{70}$ fullerene may be modelled as a spheroid with a minor semi-axis length of $b = 3.59$ Å and major semi-axis length of $c = 4.17$ Å. Use your answer from part (b) to derive a reasonable approximation (to four decimal places) of the mean surface density of carbon atoms for this molecule.

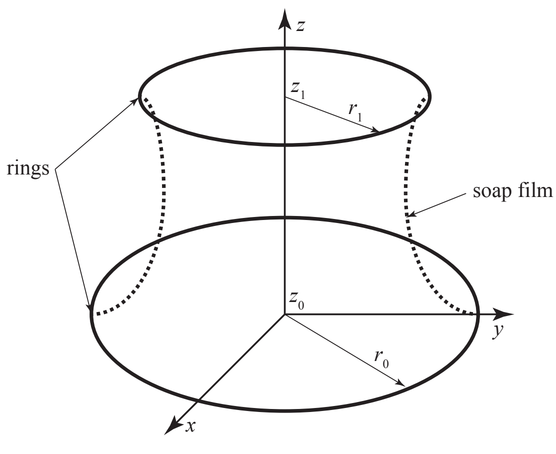
[8 marks]

2. Consider a soap film suspended between two parallel concentric, but displaced rings of radius $r_0$ and $r_1$ at an offset in the $z$-direction of $z_0$ and $z_1$ (see the figure for a clearer view). Ignoring gravity and other external forces, the shape of the soap film will minimise the surface area.

   (a) Use the Calculus of Variations to determine the profile that the soap film will adopt.

   (b) Plot the resulting profile for $(z_0, r_0) = (0, 9)$ and $(z_1, r_1) = (10, 10)$.

[8 marks]



3. Consider the problem of finding the extremal of the functional

$$F\{y\} = \int_0^1 x(y'^2 - \lambda y^2)\,dx,$$

subject to $y(0)$ being finite and $y(1) = 0$. Assuming that the extremal will be an even function of $x$, we may choose as basis functions

$$\phi_n(x) = 1 - x^{2n},$$

and propose an approximate series solution of the form

$$y(x) \approx \bar{y}(x) = c_1\phi_1(x) + c_2\phi_2(x) + \cdots .$$

Use Ritz's method with $\bar{y}(x) = c_1\phi_1(x) + c_2\phi_2(x)$, to derive a constraint on $\lambda$ that must be satisfied for a non-trivial solution to exist. Give the numerical value (to four significant digits) of the smallest $\lambda$ that satisfies the constraint.

[8 marks]