APP MATH 3020 Stochastic Decision Theory
Assignment 5

Due: Monday, 29 October, 2018, 10 a.m. $\boxed{\textbf{Total marks: 31}}$

Question 1 $\boxed{\text{2 marks}}$

Make sure that in all your answers you

$\boxed{1/2}$ (a) use full and complete sentences.

$\boxed{1/2}$ (b) include units where necessary.

$\boxed{1/2}$ (c) use logical arguments in your answers and proofs.

$\boxed{1/2}$ (d) structure your answers and assignment clearly and precisely.

Question 2 $\boxed{\text{26 marks}}$

Squirrel and Koala live far apart from each other, and talk together daily over the telephone about what they have done that day. Squirrel is only interested in three activities: foraging, eating, and singing. What he does is determined entirely by his mood. Based on his daily activity, Koala tries to guess what Squirrel's mood that day must have been.

Koala believes that Squirrel's mood operates as a DTMC $\{X_t\}_{t\geq 0}$. There are two states, Happy and Sad, but she cannot observe them directly. Let Happy =: 1 and Sad=: 2. Then we have the following transition probability matrix:

$$P = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}.$$

On Day 0, Squirrel is happy with probability 0.9 and sad with probability 0.1. We have

$$\Pr(\text{Foraging} \mid \text{Happy}) = 0.2, \quad \Pr(\text{Eating} \mid \text{Happy}) = 0.3, \quad \Pr(\text{Singing} \mid \text{Happy}) = 0.5,$$
$$\Pr(\text{Foraging} \mid \text{Sad}) = 0.3, \quad \Pr(\text{Eating} \mid \text{Sad}) = 0.6, \quad \Pr(\text{Singing} \mid \text{Sad}) = 0.1.$$

Suppose over the period from Day 0 to Day 3 Squirrel tells Koala the following sequence of activities: {Foraging, Singing, Eating, Foraging}. Determine

$\boxed{9}$ (a) the marginal probability of this observation sequence,

$\boxed{6}$ (b) the most likely underlying mood on Day 2,

$\boxed{9}$ (c) the most likely underlying mood sequence from Day 0 to Day 3.

$\boxed{2}$ (d) Is $\Pr(X_1 = \text{Happy} \mid X_0 = \text{Sad}) = \Pr(X_1 = \text{Happy} \mid X_0 = \text{Sad}, Y_0 = \text{Eating})$? As always, justify your answer.

Question 3 $\boxed{\text{3 marks}}$

Consider a Hidden Markov chain $\{X_t, Y_t\}_{t\geq 0}$ with model parameters Λ and an observed sequence $\boldsymbol{y} = (y_0, \dots, y_T)$.

$\boxed{3}$ (a) Show that, for $i = 1, \dots, N$ and for $t = 0, \dots, T-1$,

$$\Pr(X_t = i, \boldsymbol{y} \mid \Lambda) = \alpha_t(i)\beta_t(i),$$

where $\alpha_t(i) := \Pr((y_0, \dots, y_t), X_t = i \mid \Lambda)$ and $\beta_t(i) := \Pr((y_{t+1}, \dots, y_T) \mid X_t = i, \Lambda)$.