APP MTH 3020 Stochastic Decision Theory

Tutorial 2

Week 5, Friday, August 24

1. Suppose the second-stage constraints of a two-stage problem are given by

$$W\boldsymbol{y} = \begin{pmatrix} 1 & 3 & -1 & 0 \\ 2 & -1 & 2 & 1 \end{pmatrix} \boldsymbol{y} = \begin{pmatrix} -6 \\ -4 \end{pmatrix} \boldsymbol{\zeta} + \begin{pmatrix} 5 & -1 & 0 \\ 0 & 2 & 4 \end{pmatrix} \boldsymbol{x}$$

where $\boldsymbol{\zeta}$ is a random variable with support $S_{\boldsymbol{\zeta}} = \{0, 1\}$.
Assume that in the second-stage program we minimise $\boldsymbol{q}^\top \boldsymbol{y}$. Write down the LP(s) (both primal and dual formulations) needed to check if a given $\boldsymbol{x}$ produces a feasible second-stage program.

2. (a) Is W in **Question 1** a complete recourse matrix? Justify.
   (b) Is the following matrix W a complete recourse matrix? Justify.

$$W = \begin{pmatrix} 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}.$$

   (c) What is the implication of a complete recourse matrix?
   (d) Give a numerical example of a simple recourse matrix.
   (e) What is the implication of a simple recourse matrix?

3. Consider a farmer who has a total of 500 acres of land available for growing *wheat*, *corn*, and *sugar beets*, with the planting costs per acre being \$150, \$230, and \$260, respectively.
   The farmer needs at least 200 tons (T) of wheat and 240 T of corn for cattle feed which can be grown on the farm or bought from a wholesaler. The purchase prices per ton are \$238 for wheat and \$210 for corn. The amount produced in excess will be sold at prices of \$170 per ton for wheat and \$150 per ton for corn. For sugar beets there is a quota on production which is 6000 T for the farmer. Any amount of sugar beets up to the quota can be sold at \$36 per ton, the amount in excess of the quota is limited to \$10 per ton.
   The farmer knows the average yield on his land is 2.5 T, 3.0 T and 20.0 T per acre for wheat, corn and sugar beets. The data are shown the following table.

	Wheat	Corn	Sugar Beets
Yield (T/acre)	2.5	3.0	20.0
Planting cost (\$/acre)	150	230	260
Purchase price (\$/T)	238	210	–
Selling price (\$/T)	170	150	36 (under 6000 T) 10 (above 6000 T)
Minimum requirement (T)			–
Total available land : 500 acres			

   (a) Formulate, and solve, an LP to find a solution to the farmer's problem.
   (b) The yield is actually sensitive to the weather and the previous problem and solution can be described as that based on average yields. The yields may vary by a margin of $\pm 20\%$. Find the optimal solution for both the below average and above average yield scenarios.
   (c) What is the average profit of all three scenarios?

(d) The farmer now realises he cannot make a perfect decision that is best in all circumstances. Decisions on land assignment need to be made immediately, but sales and purchases that depend on the yields are taken later. To maximise his long run profit, he seeks a solution which maxmises his expected profit.

Assuming each of the three previous scenarios occur w.p. 1/3 each, that is, the yields are under by 20%, average or over by 20%. Formulate the farmer's problem as a two-stage SLP with recourse. Write in expanded form.

(e) Solve this recourse DEP.