APP MATH 3020 Stochastic Decision Theory
Assignment 4

Due: Monday, 15 October, 2018, 10 a.m. **Total marks: 33**

Question 1 2 marks

Make sure that in all your answers you

½ (a) use full and complete sentences.

½ (b) include units where necessary.

½ (c) use logical arguments in your answers and proofs.

½ (d) structure your answers and assignment clearly and precisely.

Question 2 10 marks

Each day, Squirrel the Singer is asked to take on a new singing gig. The gigs are independently distributed over 3 possible types; on a given day, the offered type is i with probability $\alpha_i \in (0,1)$ for $i = 1, \ldots, 3$. Upon completion, gigs of type i pay r_i dollars. Once Squirrel has accepted a gig, she may accept no other gigs until that gig is complete. The probability that a gig of type i takes k days is $(1-p_i)^{k-1}p_i$, for $k = 1, 2, \ldots$, where $p_i \in (0,1)$.

7 (a) Evaluate the average reward, g_1, of a stationary policy in which Squirrel accepts only gigs of type 1.

3 (b) Apply one step of the Policy Improvement Algorithm to determine an improved policy, clearly stating what the improved policy is.

Question 3 10 marks

Heather receives \$10 for every chess game that she wins. Playing costs her \$$c$ per hour. The total number of chess games that Heather can play is T. The probability of winning one game in the next hour is $\omega(r)$, where $\omega(r)$ is an increasing function of r, the remaining number of games. There is zero probability of winning more than one game in an hour. Heather wants to maximise her net expected profit.

4 (a) Specify, with justification, Heather's stopping rule.

6 (b) If $T = 12$, $\omega(r) = 1 - e^{-r/5}$ and $c = \$0.5$, determine Heather's expected profit and detail the stopping rule.

Question 4 11 marks

You are moving overseas soon! Suppose you need to sell your car (a twenty-year-old aqua Mirage) and have 10 weeks in which to advertise and sell it. You receive one offer per week; these offers are independent with a value of j dollars with probability p_j, for $j = 1, \ldots, 10$. Any offer not immediately accepted, can be accepted at a later date. Every week that the Mirage remains unsold, it costs you c dollars per week.

The state space is $\mathcal{S} = \{1, 2, \ldots, 10\}$, where state i corresponds to the highest offer to date. There are only two actions you might take when in state i, to either accept the best offer to date with value i or not accept the best offer to date and continue with costs c.

$\boxed{6}$ (a) Give the transition probabilities when continuing on and not accepting the best offer to date, with justification.

$\boxed{5}$ (b) What is the optimal policy for selling your car?