School of Mathematical Sciences

APP MTH 3022 - Optimal Functions and Nanomechanics III

Assignment 1 question sheet

Due: Thursday, 15 August, at 12 noon (in the hand-in box on level 6)

When presenting your solutions to the assignment, please include some explanation in words to accompany your calculations. It is not necessary to write a lengthy description, just a few sentences to link the steps in your calculation. Messy, illegible or inadequately explained solutions may be penalised. The marks awarded for each part are indicated in brackets.

All students are to attempt Questions 1–3.

1. Use the method of Lagrange multipliers to maximise the volume of a rectangular prism subject to the constraints of having a total surface area of 2 m^2 and a total edge length of 12 m. Give the lengths of the three sides and the maximum volume attained.

   [8 marks]

2. Consider the functional

$$F\{y\} = \int_0^1 xy^2 y'^3 \, dx.$$

   (a) If we constrain our investigation of possible functions for y to $y(x) = x^\epsilon$, what is the value of $\epsilon > 1/5$ which leads to an extremum of F.

   (b) What value does the functional take for this value of ϵ.

   (c) Is this a maximum or a minimum? Justify your answer.

   [8 marks]

3. Consider the multivariate function

$$f(x_1, x_2, x_3) = \cosh(x_1)\cos(x_2)e^{x_2 x_3}.$$

   (a) Using the nD formula given in lectures, generate the Taylor series expansion for f about the point $(x_1, x_2, x_3) = (0, 0, 0)$, including all terms up to order two.

   (b) If you were to continue to find higher terms in the expansion, would there be any terms of order three? Justify your answer.

   [8 marks]