Random Processes III 2018: Tutorial 4,

please come to the tutorial on Friday 14^{th} September having attempted this sheet. Solutions to these questions will not be uploaded to MyUni.

Problem 1

Consider an infinite buffered switch, which is fed by a Poisson stream of packets of rate λ. The switch initially processes packets at rate $\mu < \lambda$. However, when the queue length rises to a threshold level K, the switch starts processing packets at rate $2\mu > \lambda$. It continues to do this until the number of packets in the buffer reaches zero, at which time the processing speed reverts to μ again.

(i) Using an appropriate CTMC model, show that if there are less than K packets at the start, then the number of packets in the buffer will rise to K with probability 1.

(ii) Similarly, show that with probability one the number of packets in the buffer will fall to zero again, once it has reached K.

(iii) Calculate the expected time that elapses between the point at which the number of packets reaches K and the point at which it reaches zero again.

Problem 2

In Lecture 14, we derived – via a first step analysis – a system of linear equations for the expected hitting times of a particular state given an initial state i, t_i. Now consider the problem where we accrue a cost $\$_k$ per unit time whilst we are in state k, and we'd like to evaluate the expected total cost incurred up to reaching a particular state given an initial state i, c_i. Derive the corresponding system of linear equations, following the working of Lecture 14 for expected hitting times.

Problem 3

Suppose two single-server queues have 3 waiting rooms: one of size R_1 for customers of type 1 which go to queue 1, one of size R_2 for customers of type 2 which go to queue 2. There is also an overflow waiting room of size R_3, which can hold customers of types 1 and 2, which overflow from the other waiting rooms. These customers move to their relevant waiting room if a space in that room becomes available.

Let λ_1 and λ_2 be the Poisson arrival rates and μ_1 and μ_2 be the exponential service rates, respectively, for queue 1 and queue 2.

(i) Give restrictions on the state space.

(ii) Write down the equilibrium distribution for the joint distribution of the number of customers at each queue in this system.