APP MTH 3002 Fluid Mechanics III
Assignment 2
Due: 12 noon, Friday 6 April.

When presenting your solutions to the assignment, please include some explanation in words to accompany your calculations. It is not necessary to write a lengthy description, just a few sentences to link the steps in your calculation. Messy, illegible or inadequately explained solutions may be penalised. The marks awarded for each part are indicated in boxes.

This assignment has 4 questions, for a total of 30 marks.

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1. Suppose that $\boldsymbol{r} = (x_1, x_2, x_3)$, $r = \|\boldsymbol{r}\| = (x_i x_i)^{1/2}$ and $\boldsymbol{u} = f(r)\,\boldsymbol{r}$, where $f(r)$ is an arbitrary differentiable function.

[1] (a) Find an expression for $\partial x_i / \partial x_j$ in suffix notation.

[3] (b) Use suffix notation to find an expression for $\partial r / \partial x_j$.

[3] (c) Use suffix notation to find an expression for $\nabla \cdot \boldsymbol{u} = \partial u_i / \partial x_i$ in terms of $f(r)$ and its derivatives.

[1] (d) Suppose that $\nabla \cdot \boldsymbol{u} = 0$. Find the function $f(r)$ and hence $\boldsymbol{u}$.

[4] 2. Use suffix notation to show that

$$\boldsymbol{u} \times (\nabla \times \boldsymbol{u}) = \tfrac{1}{2}\nabla(\boldsymbol{u} \cdot \boldsymbol{u}) - \boldsymbol{u} \cdot \nabla \boldsymbol{u}.$$

3. Consider the two-dimensional flow

$$\boldsymbol{u}(\boldsymbol{x}, t) = e^{-2\nu t} \cos x \sin y\, \boldsymbol{i} - e^{-2\nu t} \sin x \cos y\, \boldsymbol{j},$$

where ν is a constant called the kinematic viscosity.

[1] (a) Is this flow steady? Justify your answer.

[1] (b) Determine the components of the velocity gradient tensor $\partial u_j / \partial x_i$.

[1] (c) Is this flow incompressible? Justify your answer.

[1] (d) Determine the components of the rate-of-strain tensor.

[1] (e) Determine the components of the rate-of-rotation tensor.

[2] (f) Use your answers to parts (d) and (e) to *describe in words* the local flow near the points

$$\boldsymbol{x} = (0, 0) \quad \text{and} \quad \boldsymbol{x} = (\tfrac{\pi}{2}, \tfrac{\pi}{2}).$$

[1] (g) Determine the vorticity.

[3] (h) Find the stream function for this flow.

[1] (i) Use your expression for the stream function to plot streamlines in the region $-2\pi \leq x \leq 2\pi$ and $-2\pi \leq y \leq 2\pi$.

6 4. Consider a flow with velocity

$$\boldsymbol{u}(x,y) = U(x)f(\eta)\,\boldsymbol{i} + v(x,y)\,\boldsymbol{j}$$

where $\eta = y/\delta(x)$ is a *similarity* variable, and $\delta(x)$ and $U(x)$ are unspecified differentiable functions.

If the flow is incompressible, two-dimensional and $v(x,0) = 0$, use the conservation of mass equation to show that

$$v(x,y) = -\delta\frac{\mathrm{d}U}{\mathrm{d}x}\left(2\eta f(\eta) - \int_0^\eta f(s)\,\mathrm{d}s\right)$$

You are given that $U^2\delta = \text{constant}$, which means that

$$2\delta\frac{\mathrm{d}U}{\mathrm{d}x} + U\frac{\mathrm{d}\delta}{\mathrm{d}x} = 0.$$