

School of Mathematical Sciences  
MODELLING WITH ODES  
Semester 1, 2019

**Assignment 4**

**Due 5pm Wednesday, Week 12: Submit via MyUni**

**You will be marked on the presentation of your answers (including clarity of explanations)!**

1. Consider the IVP

$$\frac{dy}{dx} = x - y + 1 \quad \text{with} \quad y(1) = 2.$$

- (a) Express the IVP as an integral equation, and write MATLAB code to calculate the Picard iterates. Submit your code and the first three iterates it produces.
- (b) Calculate the exact solution. Using MATLAB or otherwise, find the Taylor series of the exact solution, and hence comment on the relationship between the Picard iterates and the exact solution.
- (c) Show that the Picard-Lindelöf theorem applies to the IVP, and find the largest  $x$ -interval on which it guarantees a unique solution.

2. Consider the forward difference formula for the third derivative

$$x_j''' = \sum_{i=0}^3 a_i x_{j+i} + O(h^m).$$

- (a) Calculate the coefficients  $a_i$ .
- (b) Calculate the order of the truncation error  $m$ .
- (c) Perform a simple check of the coefficients that ensures the finite difference formula is correct for constant functions.

3. For the IVP

$$x' = f(t, x) \quad \text{with} \quad x(0) = a,$$

recall the leapfrog method is

$$x_{n+1} = x_{n-1} + 2h f_n.$$

- (a) Suppose you use the explicit Euler method to compute  $x_1$ . Does the use of Euler's method for the first step compromise the global error of the leapfrog method? Explain your answer.

- (b) Write a Matlab code to solve the IVP

$$x' = -x \quad \text{with} \quad x(0) = 1, \quad (1)$$

using the leapfrog method with Euler's method used to find  $x_1$ . Use your code to calculate the absolute error  $e(h)$  at  $t = 1$  for a range of step sizes  $h$ . Plot  $\log(e)$  vs.  $\log(h)$ , and explain how this confirms the order of accuracy of the leapfrog method.

- (c) You are given that for IVPs of the form (1), solutions given by the leapfrog method can be written

$$x_n = c_+ \xi_+^n + c_- \xi_-^n \quad \text{for} \quad n = 2, \dots$$

where

$$\xi_{\pm} = -h \pm \sqrt{1 + h^2},$$

and  $c_{\pm}$  are constants. Explain why this means that the leapfrog method is not suitable to investigate the long-time behaviour of the solution of IVP (1). Use your code from part (b) to confirm the problem in calculating long-term solutions of IVP (1) using the leapfrog method.