APP MATH 3020 Stochastic Decision Theory
Assignment 2

Due: Monday, 3 September, 2018, 10 a.m. (Week 7). **Total marks: 44**

Question 1 2 marks

Make sure that in all your answers you

½ (a) use full and complete sentences.

½ (b) include units where necessary.

½ (c) use logical arguments in your answers and proofs.

½ (d) structure your answers and assignment clearly and precisely.

Question 2 17 marks

Consider the two-stage SLP

$$\begin{aligned} \max \quad & \left\{ 2x_1 + 2x_2 + \sum_{k=1}^{3} p_k \boldsymbol{q}^\top y^{(k)} \right\} \\ \text{subject to} \quad & x_1 + x_2 \leq 10 \\ & x_1 + 2x_2 - y_1^{(k)} + y_2^{(k)} + 4y_3^{(k)} = h_1^{(k)} \quad \text{for } k = 1, 2, 3, \\ & x_1 - x_2 + 2y_1^{(k)} + 3y_2^{(k)} + 2y_3^{(k)} = h_2^{(k)} \quad \text{for } k = 1, 2, 3, \\ & x_j, y_\nu^{(k)} \geq 0 \quad \text{for } j = 1, 2; k = 1, 2, 3; \nu = 1, 2, 3, \end{aligned}$$

with $\boldsymbol{q} = (-2, -3, -2)^\top, \boldsymbol{h}^{(1)} = (5, 4)^\top, \boldsymbol{h}^{(2)} = (3, 5)^\top, \boldsymbol{h}^{(3)} = (2, 2)^\top$.

6 (a) Does this SLP the complete recourse property? Justify.

8 (b) Determine the induced constraints and the corresponding induced feasibility set.

3 (c) Have this SLP the relatively complete recourse property? Justify.

Question 3 10 marks

Suppose that the recourse function $Q(x, \zeta)$ is given by

$$Q(x, \zeta) = \min_y \{ y \mid y \geq \zeta, y \geq x \}$$

where $x > 0$ and the random variable ζ has the density function

$$f_\zeta(\omega) = \frac{2}{\omega^3}, \quad \omega \geq 1.$$

6 (a) Determine the closed form expression for $\mathbb{E}_\zeta[Q(x, \zeta)]$.

4 (b) Consequently, solve

$$\min_{x \geq 0} \left\{ x + \mathbb{E}_\zeta[Q(x, \zeta)] \right\}.$$

Question 4 15 marks

Qantas would like to determine how to best partition their new plane for a direct Adelaide-Paris route. The plane can accomodate 200 economy passengers. A portion of the plane can be reserved for first class seats, each of which takes up twice the space of an economy seat. A business section can also be considered, but each business seat takes up 1.5 times as much space as an economy seat. Once the plane is designed to section into these three classes, it is permanent.

A first class seat and a business seat generate three times and two times, respectively, as much profit as does an economy seat.

Clearly, for each flight the plane will not necessarily be full in every section. There are three scenarios, each happens with equal probability: (a) weekday morning and evening traffic, (b) weekend traffic, and (c) weekday midday traffic.

For Scenario (a), it is expected that up to 20 first class, 50 business, and 200 economy seats can be sold. For Scenario (b), the numbers are 10, 25 , and 175, respectively. For Scenario (c), 5, 10, and 150, respectively. We assume that they cannot sell more tickets than seats, for each section. (In reality overbooking happens all the time!)

8 (a) Formulate the above problem as a stochastic linear program, where Qantas wants to maximize the expected profit.

7 (b) Solve the above SLP using MATLAB. In addition to your solution, include code, output, and an interpretation of the solution.