Random Processes III 2018: Tutorial 5,

please come to the tutorial on Friday 12^{th} October having attempted this sheet. Solutions to these questions will not be uploaded to MyUni.

Problem 1

Consider an open Jackson network of three queues, each staffed with a single server that can serve at rate μ. Calls arrive to the first queue from outside the network according to a Poisson process at rate λ and, when their service is complete, are routed according to the matrix

$$[\gamma_{ij}] = \begin{pmatrix} 0 & 1/8 & 7/8 \\ 3/4 & 0 & 1/4 \\ 1/2 & 1/4 & 0 \end{pmatrix}.$$

(i) Solve the traffic equations to determine the average arrival rate at each queue.

(ii) Write down an expression for the equilibrium distribution for the network.

(iii) Assume that $\lambda = 9/2$ and $\mu = 10$. Comment on the stability of the network and the existence of the equilibrium distribution.

Problem 2

Consider a queue with a Poisson arrival process of rate λ and exponentially distributed service time with parameter μ. Let the residence time be the total of the waiting time and the service time.

(i) If the queue is an M/M/∞ queue, what is the distribution of the residence time?

(ii) If the queue is an M/M/N queue, what is the mean waiting time? Hence, what is the mean residence time?

(iii) If the queue is an M/M/1 queue, what is the mean waiting time? Hence, what is the mean residence time?

(iv) How does the answer to part (iii) relate to the conditional expected waiting time in such a queue, given that you have to wait? Can you explain this relationship?

Problem 3

A hospital operates like a very complex queueing system with patients moving between areas and waiting for services.

(i) Consider the hospital that operates at 90% occupancy and admits, on average, 100 people per day. Assuming that it has 500 beds, use Little's Law to determine the average stay of a patient.

(ii) Now consider the Emergency department of this hospital. Here, the staff record the waiting time between arrival of the patient at the Emergency Department and the start of treatment. Assuming that the average waiting time is 3 hours and that the Emergency Department starts to treat on average 10 new patients per hour, determine the average number of patients waiting for service. List any assumptions you need to make to justify your answer.

(iii) Finally, consider the single resuscitation room within the Emergency Department. This operates like an M/G/1/1 queue. Show that the probability that the resuscitation room is occupied is given by $\dfrac{\lambda}{\mu+\lambda}$, where λ is the arrival rate of patients requiring use of the resuscitation room and $1/\mu$ is the mean time that a patient occupies the resuscitation room.