Practical Asymptotics (APP MTH 4051/7087)
Assignment 3 (5%)

Due 10 May 2019

1. Consider the following boundary value problem.

$$\epsilon \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + (\cosh x)\frac{\mathrm{d}y}{\mathrm{d}x} - y = 0,$$

subject to $y(0) = y(1) = 1$, for $\epsilon \to 0$ over the interval $0 \leq x \leq 1$.

   (a) Find a leading-order composite solution to this problem.

   (b) Apply a leading-order WKB ansatz to find a different approximate solution.

   (c) Compare these approximations with a numerical solution and comment briefly. How well do the approximate solutions satisfy the outer boundary condition?

2. This question involves an **internal boundary layer**, a region of rapid variation located away from the edges of the domain. Find leading-order outer and inner solutions to the following problem:

$$\epsilon \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} + xy = 0,$$

subject to $y(-2) = -4$ and $y(2) = 2$, for $\epsilon \to 0$ over the interval $-2 \leq x \leq 2$. As part of your solution you should identify where the internal layer is located (discussing in detail why there is no boundary layer at $x = \pm 2$).

Compare the inner and outer solutions with a numerical solution.

(Note: A composite solution is not required here, although coming up with one might be fun!)

[Hints: it would be a **very** good idea to look at a numerical solution before starting your analysis. Different outer solutions are required each side of the internal layer (perhaps call these y_L and y_R) which require their own matching conditions.]