Group project: Turning Charlie Parker into Ornette Coleman

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April 28, 2019

Report submitted for **APP MTH 3001** at the School of Mathematical Sciences, University of Adelaide



Project Area: Applied Probability

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Executive Summary

The purpose of this report is to generate new jazz improvisations in the style of jazz sax-ophonist Charlie Parker. This is achieved by using three different stochastic, Markovian processes to produce new improvisations based on of the original pieces within Charlie Parker's Omnibook. Each new improvisation uses the same 'lead' timing and length as original piece, while randomly choosing notes.

The first process is formed based only on the order of notes used within the original piece, where the proportion of times that one note transitions to the next (in one step) is used to form a probability for that transition. Each note within the new improvisation is then chosen randomly, dependant only on the previous note played. This improvisation is then created using the same chord progression and timing as the original piece. This approach lead to a piece with a good underlying flow as it mimicked the original chord progression, but suffered from note clashing due to certain notes not partnering with particular chords.

The second process is formed by considering that certain notes 'work' with certain chords. The approach was to, create note-chord pairs, such that the probability matrix looked at the transition of each note-chord pair within the original piece. Each new note-chord pair was then chosen randomly dependent on the previous note-chord pair played. This improvisation is then created using the timing of the 'lead' part from the original piece. This improvisation removed the clashing from the first process, but due to the chords been allocated randomly, the improvisation lacked flow, with almost an aggressive feel to the solo, which is uncharacteristic of the genre.

The third process looked to build on the strengths of the previous processes by choosing notes based on note-chord pairs, but maintaining the original chord progression of the piece. This involved looking to allocate notes based on the pre-allocated timing of chords. For each chord, a probability matrix is formed by the transitions between notes for that particular chord within the original piece. The notes are then chosen by randomly allocating a section for each chord through Markovian process for each unique chord. Overall, this produced a solo which had a strong flow as seen in the first process, but also significantly reducing the clashes within the notes and chords as seen within the second process. One issue was that some transitions had to be randomly generated to avoid particular transition issues. However, this approach significantly improved the listening quality of the piece.

Finally, a 'human' element was added to the third process, by removing notes which didn't sound aesthetically pleasing to hear due to note clashes. As well as this, the note 'velocities' or volumes were changed for the improvisation, to add depth to the playing. This element this created a smoother, cleaner sound which made for a 'good' sounding improvisation.

Overall, the third process produced a solo which was most enjoyable to listen to, due to the combination of the chord progression of the original song partnered with notes that matched the chords. However adding the Human element to the solo produced the cleanest, and most realistic improvisation.

1 Project Aim

The objective of this report is to generate new jazz improvisations in the style of the jazz music of Charlie Parker. Namely, based off the solo in the piece, "Moose the Mooche". Despite the jazz improvisations being created through technical and algorithmic means, the aim of the final product is to have created a jazz solo that is not only technically correct but also has an appealing to listen to.

2 Initial Set up

In order to manipulate the data with MATLAB, the data first has to be transferred to a file which MATLAB can read and understand. The data representing the original piece is initially within a MusicXML (.mxml) file which is a file format that stores information on the notes, times and volumes. This data is converted into a MATLAB .mat file, using the mxl toolbox, which can then be accessed on MATLAB. Within MATLAB, the data is the loaded in, where the chords and lead melody sections are separated into their own note matrices - where the columns contained start-time, duration, midi channel, pitch, velocity, start-time (seconds), and duration (seconds), respectively.

Notes are denoted within the pitch column with a number between 0 and 127, where each value represents a different octave + semitone. There are 12 semi-tones within each octave, where each greater value of octave has a greater pitch than the previous. Hence note 38 would be represented by 3 octaves + 3 semitone, whilst note 83 would be represented by 6 Octaves + 12 semitones and would be a higher pitch, where 60 is a middle C (on a piano).

Both the lead and chord note matrices are represented by rows which indicate the note played at a particular time during the song. It was found that 24 separate semitones where played in various sequences throughout the song, ranging from note note 54 (represented as 4 octaves + 7 semitones) to note 77 (6 Octaves + 6 semitones). In order to simplify probability matrix, the value of each note from note 54 to 77 is shifted down by 53 to create a range of notes from 1 to 24. It was also found that 16 separate chords were played throughout the piece. The total amount of notes played throughout the piece was 373.

3 First Process

3.1 Background

The first process aims to create a new improvisation of the original piece based on only the order of notes from the lead melody within the original piece of music. Due to this, the state space is set to represent only the notes played within the original piece. As these notes where shifted and represented by numbers 1 to 24, the state space S is the range of notes after the conversion in the set up, i.e.:

$$S = \{1, 2, ..., 23, 24\}$$

The next aim is to form a probability transition matrix that represents the proportion of time one note transitions to the another over the whole original piece. By creating this matrix, we are able to run a stochastic process which is Markovian. It is Markovian as the next note within the new improvisation is only depends on the the current note, and independent of all other notes played.

From this, a random DTMC walk is conducted by running through the total amount of notes played within the original piece of music, which in this case was 373. This sequence of 373 notes represents the sequence of notes played within the newly formed solo. The new improvisation is then backed with the same timing and the same chord progression as the original piece.

3.2 Methods

The method to form the transition probability matrix (represented by \mathbb{P} -Matrix) was initially to form a 24×24 zero matrix. Each row and column would represent one of the 24 semi-tones that were played throughout the piece of music, or similarly each element within the state space S. The rows represent the note that was previously played, whilst the columns represent probability of choosing the note played next. The number of times semitone j immediately follows note i within the whole original piece, excluding the final note, is then calculated, with this number being represented by row i, column j of the previously formed zero matrix. Each row of the matrix was then normalised to 1 by summing each row, then dividing each element in that specific row by that number. This allowed us to construct a \mathbb{P} -Matrix, where each row summed to 1.

The final step in forming the new solo was to simulate a random DTMC walk through the length of the number of notes in the original song. This produces a new sequence of notes that make up a new solo based on the original sequence of notes. These new notes are re-shifted up to match the original range of notes from note 54 to 77, and matched with the same timing and chord progression. Hence from this, our new improvisation is formed, outputting a midi file.

3.3 Analysis

3.3.1 Last Note of the Original Song

The manner in which we dealt with the last note of the original song was to consider the transition to the last note but to ignore the transition away from the last note as the solo ends there. This is due to there being no note to transition too, hence the transition is non-existent. The limitation to this method is that the \mathbb{P} -Matrix has one less transition to be formed around. Voiding this one transition may cause us to form an inaccurate \mathbb{P} -matrix which means that the final solo is based on inaccurate data.

An alternative method is to make the final note transition to the pedal note of the song - or finding the first note played after the solo, which in the case of the original song is an F. This creates an addition transition to form the P-matrix. The limitation to this is that the root note is a guess to what the next note is, and not an actual representation of the evolution of the song. The fundamental consequence is similar to the previous case where a wrong P-matrix is formed. The solo created due to this would then also be based on wrong data.

3.3.2 \mathbb{P} -Matrix

The formed \mathbb{P} -Matrix has no zero columns indicating that each note is transitioned into at least once from a previous note. Similarly each row sums to 1, as expected. If S_+ denotes a subset of S where the proportions is strictly positive, then $S_+ = S$. Due to this, the probabilities of $P(X_1 = K|X_o = i)$ for $i \in S_+$ is just represented by the initial \mathbb{P} -Matrix. For the case of analysis, we set the value of K to note 21. The probabilities for note 17 $(P(X_1 = 21|X_o = i))$ can be seen in Figure 1a. The probabilities of after the 10^{th} step $(P(X_{10} = K|X_o = i))$ can be found by taking the \mathbb{P} -matrix to the power of 10. Hence again by making K=21, $P(X_{10} = 21|X_o = i)$ can be seen in Figure 2

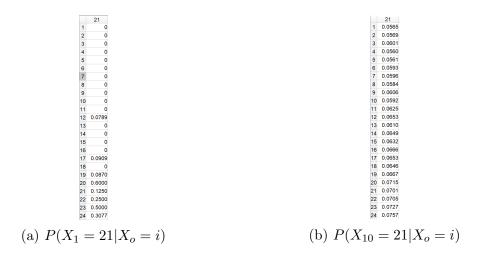


Figure 1: The probabilities of playing note 21 after 1 and 10 steps

The general probabilities of ending up in each specific note is after 10 steps seen

in Figure 2, where the rows represent i and the columns represent the different values of K. It can be seen that regardless of what note is initially played, there is some probability of being in any state. This implies that any note can be reached after 10 steps from any initial note. This also indicates that the all the states in S are recurrent. This is further shown in the state diagram of the \mathbb{P} -Matrix in Figure 3 where no states appear to be either ephemeral or transient.

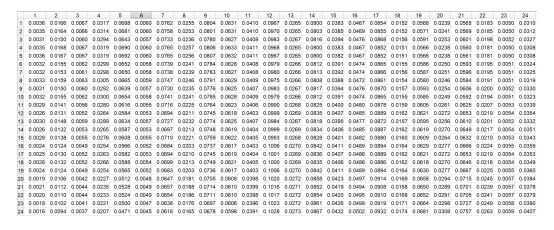


Figure 2: The probabilities of $P(X_{10} = K | X_o = i)$ for $i \in S_+$

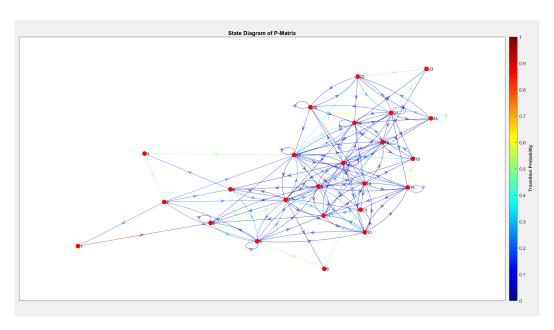


Figure 3: The state diagram of the P-Matrix

3.3.3 Expected Hitting Time

The generic formula for finding the mean hitting time is:

$$k_i^{\mathcal{A}} = \begin{cases} 0 & \text{for } i \in \mathcal{A} \\ 1 + \sum_{j \notin \mathcal{A}} p_{i,j} k_j^{\mathcal{A}} & \text{for } i \notin \mathcal{A} \end{cases}$$

When looking at the case for mean hitting time of K, which is state 21 in our case, this formula becomes:

$$k_i^{21} = \begin{cases} 0 & for \ i = 21\\ 1 + \sum_{j \neq 21} p_{ij} k_j^{21} & for \ i \neq 21 \end{cases}$$

This represents all (k_i) 's for all (i)'s in (S), not including note 21. The solved equations for can be seen in Figure 4. The equations were solved using Matlab (see Appendix A.1 for code).

1	27.5231
2	27.0079
3	25.2547
4	27.5527
5	27.5807
6	25.7942
7	25.8815
8	26.5231
9	25.2497
10	26.3688
11	24.5261
12	20.8701
13	25.3506
14	22.0076
15	24.4948
16	21.9123
17	21.5016
18	24.0666
19	20.2679
20	7.2614
21	C
22	13.5834
23	7.7917
24	8.6335

Figure 4: The Expected Hitting times of state 21 from state i

These values are sensible because as we know, the state can be reached from every other state (so all of these times exist and are $< \infty$).

3.4 Resultant Improvisation

The improvisation generated through this approach has a seemingly good flow and feel to it, however there is a significant number of clashes between the produced notes and the backing chords. However the strongest parts to the improvisation is the chord progression and timing, which were not produced by the original process. Whilst the produced notes have various sections which sound good, too much of the song is effected by the clashing of notes to be considered and overly good and enjoyable improvisation.

4 Second Process

4.1 Background

The second process aims to take into account how notes and chords are paired throughout the original song, as well as how these 'note-chord' pairs transition to other note-chord pairs. Due to this, the state space for this process is represented by each of the 24 semi-tones paired with each of the 16 unique chords, which creates 384 pairs. Each unique is chord is given an index from 0 to 15. Due to this state space T is given by:

$$T = \{1, 2, ..., 384\}$$

Elements 1 to 24 are represented by each note paired with index 0 chord, elements 25 to 48 are represented by each note paired with the index 1 chord, etc.

The probability transition matrix (P-Matrix) is then formed looking at the proportion of note-chord pair transitions. Similarly to the first process, this is Markovian. This is due to the fact that the next note-chord pair that is produced is only dependent on the previous note-chord played, and independent of all previous note-chord pairs. From this, a random DTMC walk is conducted over the 373 steps, producing a note-chord pair for each step, and creating the basis of the improvisation. The improvisation is then formed based on the produced notes-chord pairs with the timing of the original piece.

4.2 Method

The manner in which producing the \mathbb{P} -Matrix was to first create a 384 \times 384 zero matrix which would initially represent the \mathbb{P} -Matrix. As for first process, each row and column represent one element from the state space T, with the rows representing the note-chord currently being played and the columns represent the new note-chord pair that could be next played. Each chord within the original piece is then iterated through. Based on the start and end time for the chord, each lead note transition in that time causes the appropriate element in the set up \mathbb{P} -Matrix to be incremented by 1 for $(note_i, chord_i) \to (note_j, chord_j)$. After this, rows which have values in are normalised to 1 in the same way as the first process.

After this, the final step in forming the improvisation was to simulate the random DTMC walk through the 373 steps. This produced a new combination/sequence of 373 note-chord pairs that would create the new improvisation. The note and chord values are then shifted back to their original value, i.e. the notes were shifted back to the range of 54-77 whilst the chords converted from their index into its notes. The same timing used within the original piece is also used for the new improvisation. The improvisation is then created using a midi file.

4.3 Analysis

4.3.1 Mean Expected Hitting Times

The formula for finding the mean hitting time is the same is in the first approach. In order to interpret the mean hitting time for this process, we need to remove the rows and columns which have no values in them. This is because the matlab script would be unable to produce the mean hitting times otherwise. Hence the K we are looking for is the (K^{th}) row that has an initial transition in it. 152 out of the 384 note-chord pairs pass this. For this case we will look at K=1, which represent the mean hitting time of the first non-zero note-chord pair (i.e ordered pair (1,1)) from the other note-chord pairs. For this case, we have

$$k_i^1 = \begin{cases} 0 & for \ i = 1\\ 1 + \sum_{j \neq 1} p_{ij} k_j^1 & for \ i \neq 1 \end{cases}$$

This represents all (k_i) 's for all (i)'s in 151 viable note-chord pairs, not including note 1. The solved equations for K=1 can be seen in the Appendix A.2. The equations were solved using Matlab (See Appendix A.1).

4.4 Resultant Improvisation

The resultant improvisation fixes the main issue from the first process, such that the notes no longer significantly clash with the backing chords. However due to the notechord pairs being chosen randomly, there is seemingly no flow to the improvisation. This is due to the constant changing of backing chords with no particular patten, with the resultant product having a somewhat violent and choppy feel. This is the opposite to the first process where the chord progression from the original piece gives that improvisation a better flow and feel to it. Overall, it appears the importance of having a flowing chord progression as seen within the first process is more important to the end product than having notes that don't constantly clash with the backing chords.

5 Third Process

5.1 Background

The third process aims to build on the strengths of both the first and second processes. It uses the premise that notes 'work' better with certain chords, forming the note-chord pairs as seen in the second process, but the chord progression follows that from the original piece. This implies that each new note-chord pair is chosen randomly dependent on the previous note-chord pair, but the chord from the new pair is already known from the original chord progression. This implies that for each step within the stochastic process, the chord being played is always known.

The way the probability matrix is formed is to store it as a 3D matrix. The idea is that for each chord, a probability matrix is created looking at the proportion of transformations between notes for that specific chord. This is done for all 16 chords, and combined into one probability matrix. Hence each chord will have a unique probability matrix that will interpret randomly what the next note will be. This causes the Markov chain to be time-inhomogeneous as the probability matrix changes when the chord changes. This would cause the probability from moving from note i to j potentially different at two different steps, hence there is the possibility that $P(X_{n+1} = j | X_n = i) \neq P(X_1 = j | X_0 = i)$.

The improvisation is then formed by iterating through the chords, simulating lead notes for that specific chord until the next chord is played. Overall this produces an improvisation which has the same chord progression and timing as the original piece, whilst playing notes that fit the chord progression.

5.2 Methods

The initial set up was to find the chords played in the song, and then to create the \mathbb{P} -Matrix. This approach is similar to the second process, where a chord space is generated in the same manner. 16 different probability matrices are then formed, one representing each unique chord. These individual probability matrices are then built in the same manner as the first process, where they are a 24 by 24 matrix where each column and row represent the state space S. Then it iterates through the notes, with the transition from note i to j incremented by 1 in the probability matrix for that specific chord. As before, for each unique probability matrix, the rows are normalised if there are data points within them. In general, this forms a 3D (\mathbb{P} -Matrix of dimension 24 by 24 by 16, representing the current note, the next note and the chord played.

The manner in which the improvisation is simulated is by simulating a section for each chord within the chord progression. Initially a random note is chosen to start a random Walk for the first chord. The random walk is of ran for the amount of notes within that chord. The final note played for that chord is then used to start the simulation of the next chord. If there is no possible transition for this note, a random note associated with non-zero columns will be chosen to start the next random walk for that chord. This is repeated for each chord within the chord progression of the

original piece. These notes are then combined in order and converted back to the original values from 54-77. The same timing used within the original piece is also used for the new improvisation. The midi file is then created using this improvisation.

5.3 Resultant Solo

The resultant solo has a relatively similar feel to the first solo, however it is clear that the amount of clashes that occur are vastly reduced - though there are still some 'bad' combinations. This indicates that it combined the best parts/fixes the worst part of the first and second process, matching the flow of the chord progression of the first process with the lack of note clashes in the second process. There are still sparse note clashes within the piece which may be due to the existence of accidental notes within the original piece, which are unable to represented correctly in the computer generated improvisation. However, overall the end product is a significant improvement over the first two processes.

6 Extension

6.1 Introduction

To extend the resultant improvisation created from the third process, a 'human' component is added to the solo. The manufactured solos have sections which don't sound aesthetically pleasing to the hear; various note clashes due to the notes being chosen randomly and the incorrect use of accidentals. 3 main changes were made to the improvisation produced in process 3. The first was to change the velocities (volumes) throughout the piece of music, the second was to remove 'bad' notes that clashed with certain chords and finally, the instrument used to play the improvisation was changed. The aim of adding the human element is to produce music which is more enjoyable to listen to, whilst fixing the sections within the third process improvisation that can be improved

6.2 Method

The first change was to look at changing the velocities (volumes) within the piece of music. The was used to create contrasts within section of the piece, and add depth to the playing. This was done by observing periods of trills, 'licks' and other fast parts, and then applying a velocity curve to the notes to add emphasis. Overall this adds some depth of the improvisation, improving the final end product.

The next part was to remove the 'bad' notes that clashed in the improvisation. The aim of this was to limit the amount of unnecessary clashes within the improvisation to produce a cleaner sounding end product - without removing accidentals entirely. This was achieved by listening for obviously clashing notes, and then shifting them to a suitable note in the scale. There was no exact method to this as it was based on subjective interpretation. Overall, this step created a cleaner, more aesthetically pleasing end product.

Finally, the instrument that played the final improvisation was changed. The default instrument did not cooperate with the changing velocities. First a saxophone was tried - but unfortunately it was not the best quality. We instead used a synthesiser, so that the velocity changes could be observed. This gave the music a different aspect, creating a seemingly smoother end product. The saxophone sound package was also attempted to try and mimic Charlie Parker's original piece, however the end product sounded unrealistic as was discarded.

6.3 Resultant Piece

Overall, the three changes significantly improve the final end product of the improvisation by reducing and eliminating the key weaknesses within the improvisation within the third process. This produces a improvisation which not only sounds good, but successfully mimics the quality of music produced by Charlie Parker.

7 Limitations

One of the biggest limitations that impacts each process is the use of accidentals in the original piece of music. Accidentals are notes which don't appear in-key, and hence appear to slightly clash with the backing chord on purpose. Due to the processes producing the solos by random discrete time Markov chain walks, the accidentals are unable to be produced in the manner of the original song, but will be placed randomly throughout. This causes bad clashes between the notes and the chords within the solo that reduces the listening quality.

In an attempt to minimise the limitation the accidentals have on the improvised solos, the generated outputs have been manually "cleaned" by erasing the notes that seem to lower the listening quality, this method in itself is a limitation as it is based off human opinion which is not consistent among every human.

Another limitation for the second and third process is that not all note-chord pairs 'work'. There is also insufficient data from one piece of music to prove an accurate and realistic \mathbb{P} -matrix. Having more data to work with would allow for more potential note-chord pair transitions that produce a superior end product.

A further limitation is that the timing of the notes has been kept the same as the original piece. The objective was to generate a new solo, but this software purely looks to map each note to a new note - irrespective of timing, rests and 'feel'. Whilst the objective was to try to create a solo that mirrors the original piece, the changing of timing can create a uniqueness within each different solo.

8 Conclusion

The ability to produce new improvisations based on a piece within Charlie Parker's Omnibook proved to be a challenging but rewarding experience. Using a variety of Markovian and MATLAB techniques, 3 different and unique improvisations of the original work were produced. It was found that the third improvisation which kept the original chord progression as well as the randomly selecting notes which go with the chord playing produced the best overall sound. Finally, the process of adding a 'human' element allowed the flaws within the third improvisation to be cleaned up in order to produce a reasonable improvisation which successfully mimics the style of Charlie Parker.

A Appendices

A.1 Expected Hitting Time

EHT.m - generating the Expected Hitting times

```
1 %EHT.m should be run after any one of the approach files
2 %Generates the expected hitting time for the ktosolve 'th non-
     zero entry
з %May 2018
4 %Andrew Martin, James Schoff, Thomas Carey
5 %will solve for the ktosolve'th NONZERO note
6 approach 2=0;
7 %This is a changed version of the matrix from the approach
8 % removes all zero/nan columns and rows.
9 absorbprobmatrix = markovMatrix.P;
  absorbprobmatrix (~any (absorbprobmatrix, 2),:) = [];
  absorbprobmatrix (:, ~any (absorbprobmatrix, 1)) = [];
  numNotes = length (absorbprobmatrix);
13
_{14} K =sym ('k', [1, numNotes]);
  %the one we want to solve the E(hitting time) for
  ktosolve = 1;
  absorbprobmatrix (ktosolve,:) = zeros (1, numNotes);
  absorbprobmatrix(ktosolve, ktosolve) = 1;
19
  syms eqnarray
  for(j = 1:numNotes)
       eqnarray(j) = K(j) - sum(absorbprobmatrix(j,:).*K) ==1;
22
  eqnarray(ktosolve) = 0;
  [A,B] = \text{equationsToMatrix}(\text{eqnarray}, [K(1:\text{ktosolve}-1), K(\text{ktosolve})]
     +1:end));
27 %linear solve the equations
  %since 'k21' == 0 we just subtract it so we can convert to
     double
  expHittingTime = linsolve(A,B) - ('k'+string(ktosolve));
  expHittingTime=double(expHittingTime);
  expHittingTime = [expHittingTime(1:ktosolve-1); 0;
     expHittingTime(ktosolve:length(expHittingTime))];
```

A.2 Expecting Hitting Time for the Second Process

2 52.43374 42 142.8785 82 121.6084 122 165.826 3 109.2241 43 143.1612 83 132.8471 123 161.826 4 147.9446 44 143.3269 84 133.8471 124 158.826 5 123.1717 45 144.0198 85 133.8471 125 160.826 6 163.3121 46 136.998 86 134.8471 126 159.826 7 150.7694 47 147.8554 87 152.3244 127 160.826 8 149.7694 48 140.9511 88 153.3244 128 163.829 9 150.7694 49 140.9511 89 119.1979 130 155.3991 11 149.4318 51 151.0857 91 123.1979 131 156.3991 12 146.5131 52 153.0857 92 118.1979 132 156.326	4	0	2.0	142.0705	04	110 211	121	105 220
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14 149.9116 54 131.3336 94 119.1979 134 160.6126 15 149.3244 55 132.3336 95 122.1979 135 161.6126 16 150.3101 56 133.3336 96 123.1979 136 157.826 17 151.5187 57 124.3215 97 175.3548 137 158.826 18 143.5994 58 128.3255 98 174.3548 138 159.6126 19 145.9843 59 129.3255 99 173.3548 139 154.3991 20 144.9843 60 146.0857 100 173.8548 140 152.7325 21 144.1477 61 147.0857 101 174.3548 141 154.0658 22 144.2551 62 113.3093 102 174.8548 142 151.7325 23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 <td>12</td> <td>146.5131</td> <td>52</td> <td>153.0857</td> <td>92</td> <td>118.1979</td> <td>132</td> <td>159.6126</td>	12	146.5131	52	153.0857	92	118.1979	132	159.6126
15 149.3244 55 132.3336 95 122.1979 135 161.6126 16 150.3101 56 133.3336 96 123.1979 136 157.826 17 151.5187 57 124.3215 97 175.3548 137 158.826 18 143.5994 58 128.3255 98 174.3548 138 159.6126 19 145.9843 59 129.3255 99 173.3548 139 154.3991 20 144.9843 60 146.0857 100 173.8548 140 152.7325 21 144.1477 61 147.0857 101 174.3548 141 154.0658 22 144.2551 62 113.3093 102 174.8548 142 151.7325 23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 <td>13</td> <td>147.5131</td> <td>53</td> <td>130.3309</td> <td></td> <td>118.1979</td> <td>133</td> <td>158.6126</td>	13	147.5131	53	130.3309		118.1979	133	158.6126
16 150.3101 56 133.3336 96 123.1979 136 157.826 17 151.5187 57 124.3215 97 175.3548 137 158.826 18 143.5994 58 128.3255 98 174.3548 138 159.6126 19 145.9843 59 129.3255 99 173.3548 139 154.3991 20 144.9843 60 146.0857 100 173.8548 140 152.7325 21 144.1477 61 147.0857 101 174.3548 141 154.0658 22 144.2551 62 113.3093 102 174.8548 142 151.7325 23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 <td>14</td> <td>149.9116</td> <td>54</td> <td>131.3336</td> <td>94</td> <td>119.1979</td> <td>134</td> <td>160.6126</td>	14	149.9116	54	131.3336	94	119.1979	134	160.6126
17 151.5187 57 124.3215 97 175.3548 137 158.826 18 143.5994 58 128.3255 98 174.3548 138 159.6126 19 145.9843 59 129.3255 99 173.3548 139 154.3991 20 144.9843 60 146.0857 100 173.8548 140 152.7325 21 144.1477 61 147.0857 101 174.3548 141 154.0658 22 144.2551 62 113.3093 102 174.8548 142 151.7325 23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147<	15	149.3244	55	132.3336	95	122.1979	135	161.6126
18 143.5994 58 128.3255 98 174.3548 138 159.6126 19 145.9843 59 129.3255 99 173.3548 139 154.3991 20 144.9843 60 146.0857 100 173.8548 140 152.7325 21 144.1477 61 147.0857 101 174.3548 141 154.0658 22 144.2551 62 113.093 102 174.8548 142 151.7325 23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148	16	150.3101	56	133.3336	96	123.1979	136	157.826
19 145.9843 59 129.3255 99 173.3548 139 154.3991 20 144.9843 60 146.0857 100 173.8548 140 152.7325 21 144.1477 61 147.0857 101 174.3548 141 154.0658 22 144.2551 62 113.3093 102 174.8548 142 151.7325 23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3701 70 57.43374 110 166.8548 1	17	151.5187	57	124.3215	97	175.3548	137	158.826
20 144.9843 60 146.0857 100 173.8548 140 152.7325 21 144.1477 61 147.0857 101 174.3548 141 154.0658 22 144.2551 62 113.3093 102 174.8548 142 151.7325 23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548	18	143.5994	58	128.3255	98	174.3548	138	159.6126
21 144.1477 61 147.0857 101 174.3548 141 154.0658 22 144.2551 62 113.3093 102 174.8548 142 151.7325 23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 1	19	145.9843	59	129.3255	99	173.3548	139	154.3991
22 144.2551 62 113.3093 102 174.8548 142 151.7325 23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 1	20	144.9843	60	146.0857	100	173.8548	140	152.7325
23 146.9446 63 120.3147 103 175.3548 143 149.253 24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 1	21	144.1477	61	147.0857	101	174.3548	141	154.0658
24 145.5908 64 130.3363 104 168.3548 144 150.253 25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 34 150.8554 74 155.3244 114 163.1977 35 144.16	22	144.2551	62	113.3093	102	174.8548	142	151.7325
25 146.5908 65 131.3363 105 166.8548 145 148.8787 26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 34 150.8554 74 155.3244 114 163.1977 35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.32	23	146.9446	63	120.3147	103	175.3548	143	149.253
26 145.9588 66 132.3363 106 165.8548 146 113.2241 27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 162.1977 34 150.8554 74 155.3244 114 163.1977 161.826 35 142.5994 76 153.3244 116 161.1977 17 36 142.5994 76 153.3244 116 161.826 18 38 141.1361<	24	145.5908	64	130.3363	104	168.3548	144	150.253
27 146.5908 67 56.43374 107 169.3548 147 110.2241 28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 34 150.8554 74 155.3244 114 163.1977 35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39	25	146.5908	65	131.3363	105	166.8548	145	148.8787
28 144.5044 68 55.43374 108 166.5119 148 111.2241 29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 34 150.8554 74 155.3244 114 163.1977 35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	26	145.9588	66	132.3363	106	165.8548	146	113.2241
29 146.3505 69 54.43374 109 170.3548 149 112.2241 30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 34 150.8554 74 155.3244 114 163.1977 35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	27	146.5908	67	56.43374	107	169.3548	147	110.2241
30 146.3741 70 57.43374 110 166.8548 150 145.2551 31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 34 150.8554 74 155.3244 114 163.1977 35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	28	144.5044	68	55.43374	108	166.5119	148	111.2241
31 147.149 71 58.43374 111 165.5119 151 146.2551 32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 34 150.8554 74 155.3244 114 163.1977 35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	29	146.3505	69	54.43374	109	170.3548	149	112.2241
32 151.8554 72 153.3244 112 167.8548 152 147.2551 33 147.3741 73 154.3244 113 162.1977 34 150.8554 74 155.3244 114 163.1977 35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	30	146.3741	70	57.43374	110	166.8548	150	145.2551
33 147.3741 73 154.3244 113 162.1977 34 150.8554 74 155.3244 114 163.1977 35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	31	147.149	71	58.43374	111	165.5119	151	146.2551
34 150.8554 74 155.3244 114 163.1977 35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	32	151.8554	72	153.3244	112	167.8548	152	147.2551
35 144.1612 75 152.3244 115 162.1977 36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	33	147.3741	73	154.3244	113	162.1977		
36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	34	150.8554	74	155.3244	114	163.1977		
36 142.5994 76 153.3244 116 161.1977 37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826	35	144.1612	75	152.3244	115	162.1977		
37 137.998 77 120.1979 117 161.826 38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826								
38 141.1361 78 123.1979 118 164.5119 39 140.1361 79 124.1979 119 164.826								
39 140.1361 79 124.1979 119 164.826			78	123.1979				
	39			124.1979				
		137.8571		123.4032	120	164.326		