Math 20C, Final Exam
June 14, 2023

Name:

PID:

1. You have 180 minutes to complete this exam.
2. Write your name on every page.
3. There are 9 questions in this exam.
4. You may use one page (both sides) of **handwritten** notes.
5. No books or calculators are allowed.
6. No cellular phones or any other electronic devices are allowed on desktops.
7. All work must be your individual efforts.
8. **Always justify your answers and show all your work.** Write your answers and all accompanying work *neatly*

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1. Consider the path

$$\mathbf{r}(t) = \langle\, 3 + \sin(2t)\,,\, t^2 - 5t + 10\,,\, \frac{1}{1+3t}\,\rangle$$

(a) (4 pts) Find an equation of the tangent line to the path $\mathbf{r}(t)$ at $t = 0$.

(b) (2 pts) Does the tangent line in part (a) intersect the xz-plane? If so, find the intersection point.

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2. (6 pts) Find an equation of the plane containing the lines

$$\mathbf{r}_1(t) = \langle 2 - t, t, 2t - 1 \rangle \quad \text{and} \quad \mathbf{r}_2(s) = \langle 2 + 3s, 3 - 3s, -2 - 6s \rangle$$

Give your answer in the form $ax + by + cz = d$.

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3. Find the limit if it exists or show that the limit does not exist.

(a) (4 pts) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4y^2}{x^6+y^6}$

(b) (4 pts) $\lim\limits_{(x,y)\to(0,0)} \sin\Big(\dfrac{1}{x^2+y^2}\Big)\ln(x^2+y^4+1)$

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4. (6 pts) Find the point on the surface $4x - y^2 - z^2 = 0$ where the tangent plane is parallel to the plane

$$2x + y + 4z = 5$$

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5. Let $f(x, y) = \ln(x^3 + 2) + y\cos(x^2 y)$.

   (a) (4 pts) Find the rate of change of $f(x, y)$ at $(-1, 0)$ in the direction of $\mathbf{u} = \langle -3, 4\rangle$. (Note: $\mathbf{u}$ is not a unit vector).

   (b) (4 pts) Find **all** direction(s) in which the rate of change of $f(x, y)$ at the point $(-1, 0)$ is 0. Give your final answer as unit vector(s).

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6. (4 pts) Let f be a differentiable function. Let $z = f(2s + 6t)$. Show that

$$3\frac{\partial z}{\partial s} - \frac{\partial z}{\partial t} = 0.$$

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7. (7 pts) Find the global (absolute) maximum and minimum values of

$$f(x, y) = x^2 - 3y^2 + 4y$$

over the region $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$.

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8. (5 pts) Evaluate

$$\iint_D xy \, dA$$

where D is the region bounded by $y = \sqrt{x}$ and $x - 2y = 0$.

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9. (5 pts) Evaluate

$$\int_0^2 \int_{x^2}^4 x \cos(y^2)\, dy\, dx$$