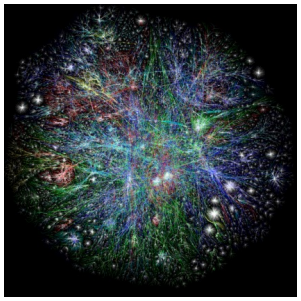
CS 225

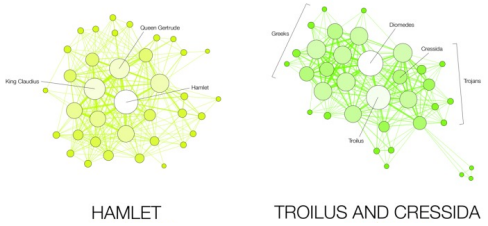
Data Structures

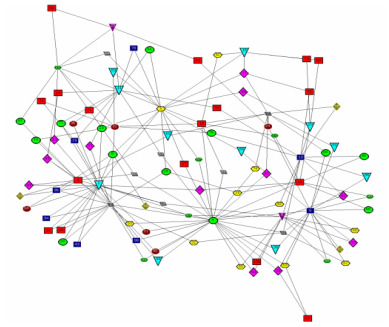
March 31 – Graph Traversals

G Carl Evans

Graphs

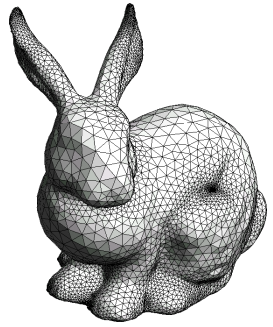


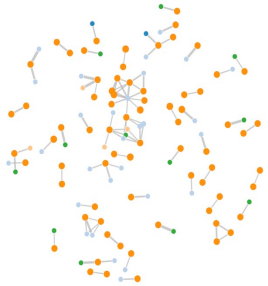


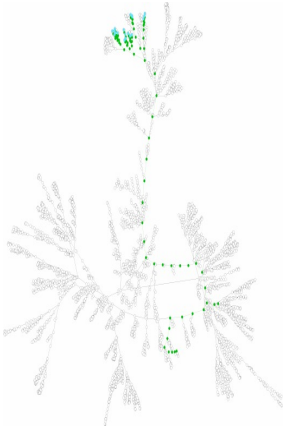


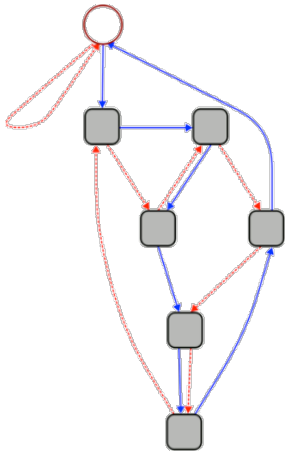
To study all of these structures:

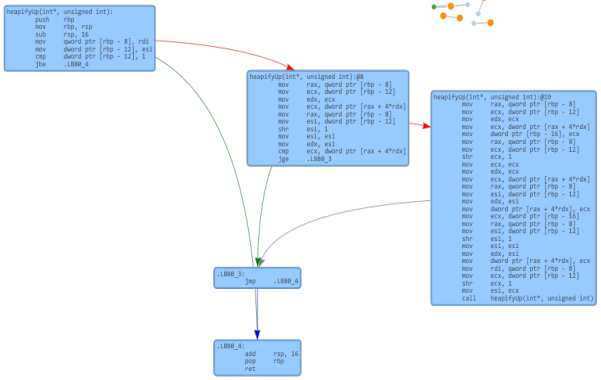
1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms



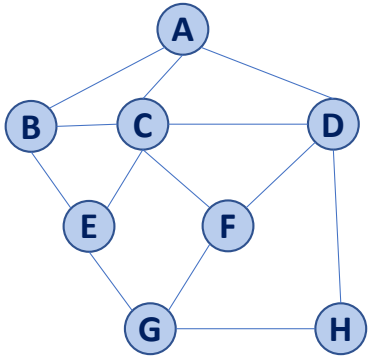






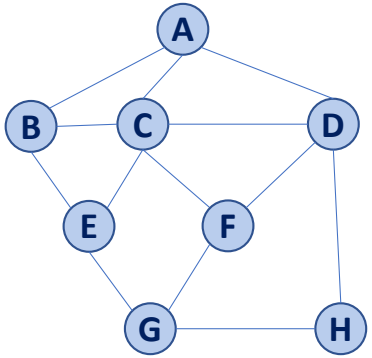


Traversal: BFS



v	d	P	Adjacent Edges
A			
B			
C			
D			
E			
F			
G			
H			

Traversal: BFS



v	d	P	Adjacent Edges
A	0	-	C B D
B			A C E
C			B A D E F
D			A C F H
E			B C G
F			C D G
G			E F H
H			D G

A

BFS Observations

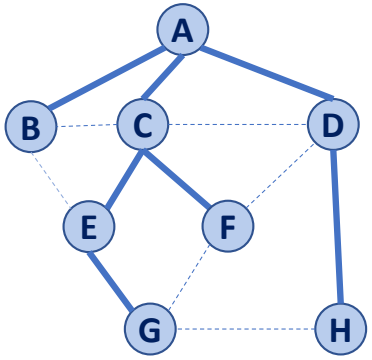
Q: What is a shortest path from **A** to **H**?

Q: What is a shortest path from **E** to **H**?

Q: How does a cross edge relate to **d**?

Q: What structure is made from discovery edges?

v	d	P	Adjacent Edges
A	0	-	**C B D**
B	1	**A**	**A C E**
C	1	**A**	**B A D E F**
D	1	**A**	**A C F H**
E	2	**C**	**B C G**
F	2	**C**	**C D G**
G	3	**E**	**E F H**
H	2	**D**	**D G**



BFS Observations

Obs. 1: BFS can be used to count components.

Obs. 2: BFS can be used to detect cycles.

Obs. 3: In BFS, **d** provides the shortest distance to every vertex.

Obs. 4: In BFS, the endpoints of a cross edge never differ in distance, **d**, by more than 1:

$$|d(u) - d(v)| = 1$$

```
 1  BFS(G):
 2    Input: Graph, G
 3    Output: A labeling of the edges on
 4        G as discovery and cross edges
 5
 6    foreach (Vertex v : G.vertices()):
 7      setLabel(v, UNEXPLORED)
 8    foreach (Edge e : G.edges()):
 9      setLabel(e, UNEXPLORED)
10    foreach (Vertex v : G.vertices()):
11      if getLabel(v) == UNEXPLORED:
12         BFS(G, v)
```

```
14  BFS(G, v):
15    Queue q
16    q.enqueue(v,NULL)
17
18    while !q.empty()
19      [v,e] = q.dequeue()
20      if( getLabel(v) == UNEXPORED)
21          setLabel(v, VISITED)
22          setLabel(e, DISCOVERY)
23          foreach (Vertex w : G.adjacent(v)):
24            q.enqueue(w,Edge(v,w))
25      else
26          setLabel(e, CROSS)
27
```

BFS Analysis

Q: Does our implementation handle disjoint graphs? If so, what code handles this?

- ***How do we use this to count components?***

Q: Does our implementation detect a cycle?

- ***How do we update our code to detect a cycle?***

Q: What is the running time?

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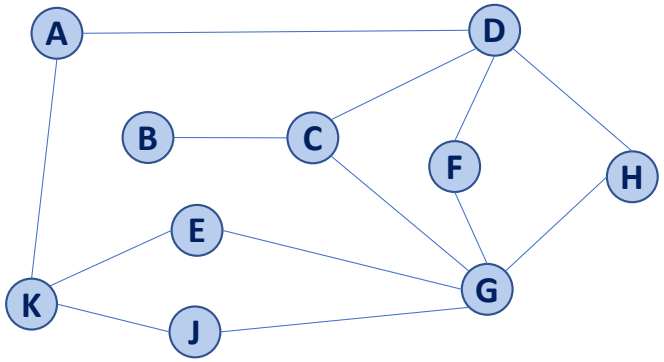
```

Traversal: DFS



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```
14  BFS(G, v):
15    Queue q
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20      if( getLabel(v) == UNEXPORED)
21          setLabel(v, VISITED)
22          setLabel(e, DISCOVERY)
23          foreach (Vertex w : G.adjacent(v)):
24            q.enqueue(w,Edge(v,w))
25      else
26          setLabel(e, CROSS)
27
```

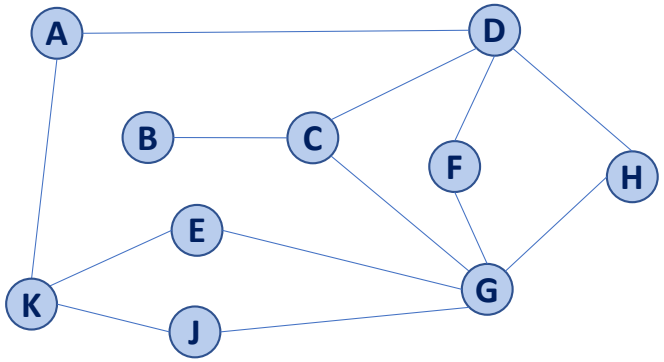
```
 1  DFS(G):
 2    Input: Graph, G
 3    Output: A labeling of the edges on
 4        G as discovery and back edges
 5
 6    foreach (Vertex v : G.vertices()):
 7      setLabel(v, UNEXPLORED)
 8    foreach (Edge e : G.edges()):
 9      setLabel(e, UNEXPLORED)
10    foreach (Vertex v : G.vertices()):
11      if getLabel(v) == UNEXPLORED:
12         DFS(G, v)
```

```
14  DFS(G, v):
15    Stack q
16    q.enqueue(v,NULL)
17
18    while !q.empty()
19      [v,e] = q.dequeue()
20      if( getLabel(v) == UNEXPORED)
21          setLabel(v, VISITED)
22          setLabel(e, DISCOVERY)
23          foreach (Vertex w : G.adjacent(v)):
24            q.enqueue(w,Edge(v,w))
25      else
26          setLabel(e, BACK)
27
```

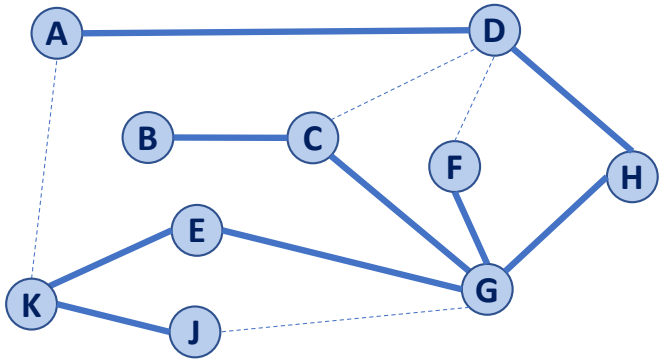
```
1  DFS(G):
2    Input: Graph, G
3    Output: A labeling of the edges on
4        G as discovery and back edges
5
6    foreach (Vertex v : G.vertices()):
7      setLabel(v, UNEXPLORED)
8    foreach (Edge e : G.edges()):
9      setLabel(e, UNEXPLORED)
10   foreach (Vertex v : G.vertices()):
11     if getLabel(v) == UNEXPLORED:
12       DFS(G, v, null)
```

```
14 DFS(G, v, e):
15   Stack q
16   q.enqueue(v)
17
18   while !q.empty()
19     [v,p] = q.dequeue()
20     if( getLabel(v) == UNEXPORED)
21       setLabel(v, VISITED)
22       setLabel(p, DISCOVERY)
23       foreach (Vertex w : G.adjacent(v)):
24         q.enqueue(w,v) DFS(G, w, Edge(w,v))
25     else
26       setLabel(e, BACK)
27
```

Traversal: DFS



Traversal: DFS



Discovery Edge

Back Edge

Running time of DFS

Labeling:

- Vertex:

- Edge:

Queries:

- Vertex:

- Edge: