Other Complexity Notations

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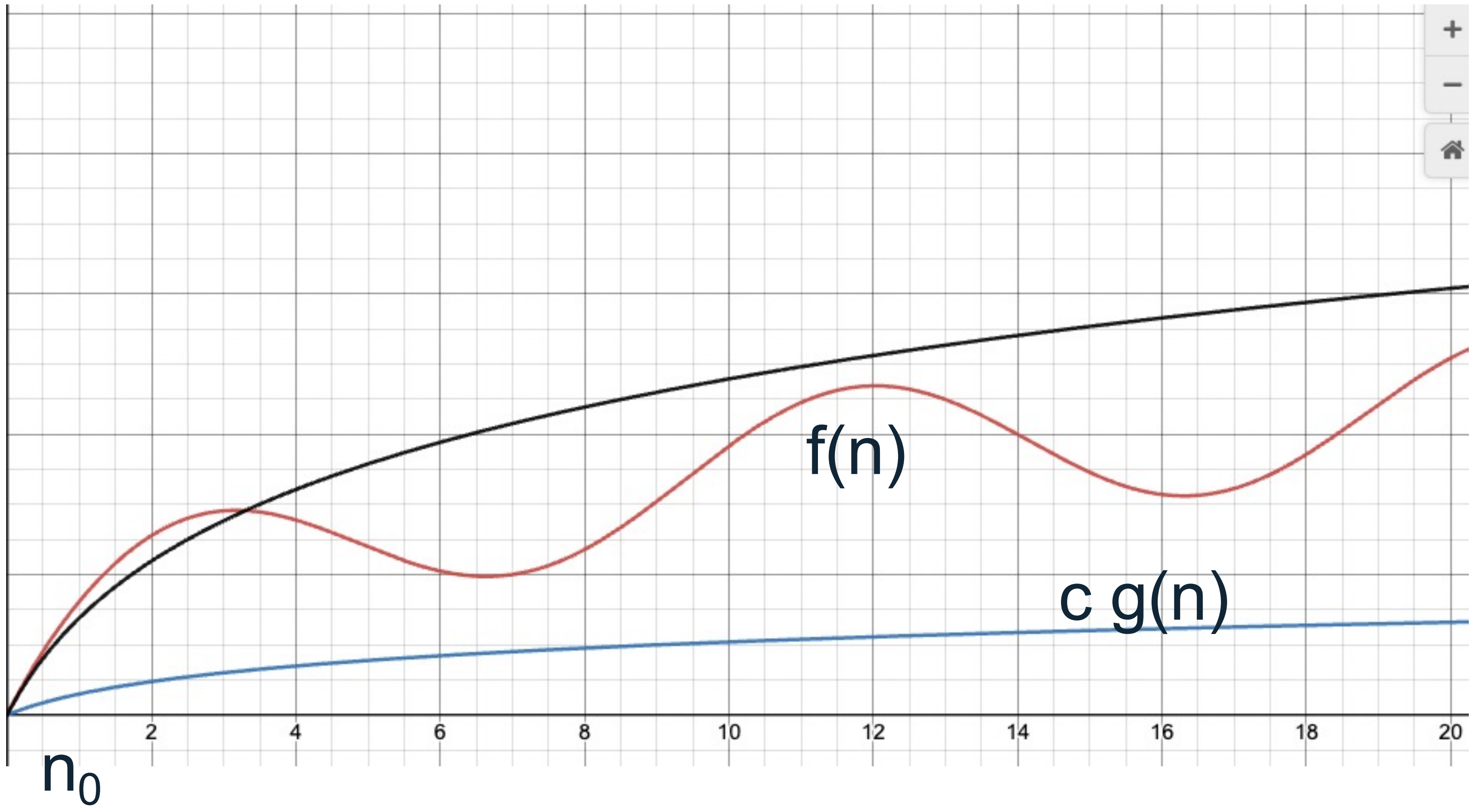
Big Ω– Lower bound

We say that $f(n)$ is in $\Omega(g(n))$ iff (if and only if)

$$\exists c \in R^{+}, \exists n_0 \in N, \ such\ that\ \forall n \geq n_0 : f(n) \geq cg(n).$$



Examples:

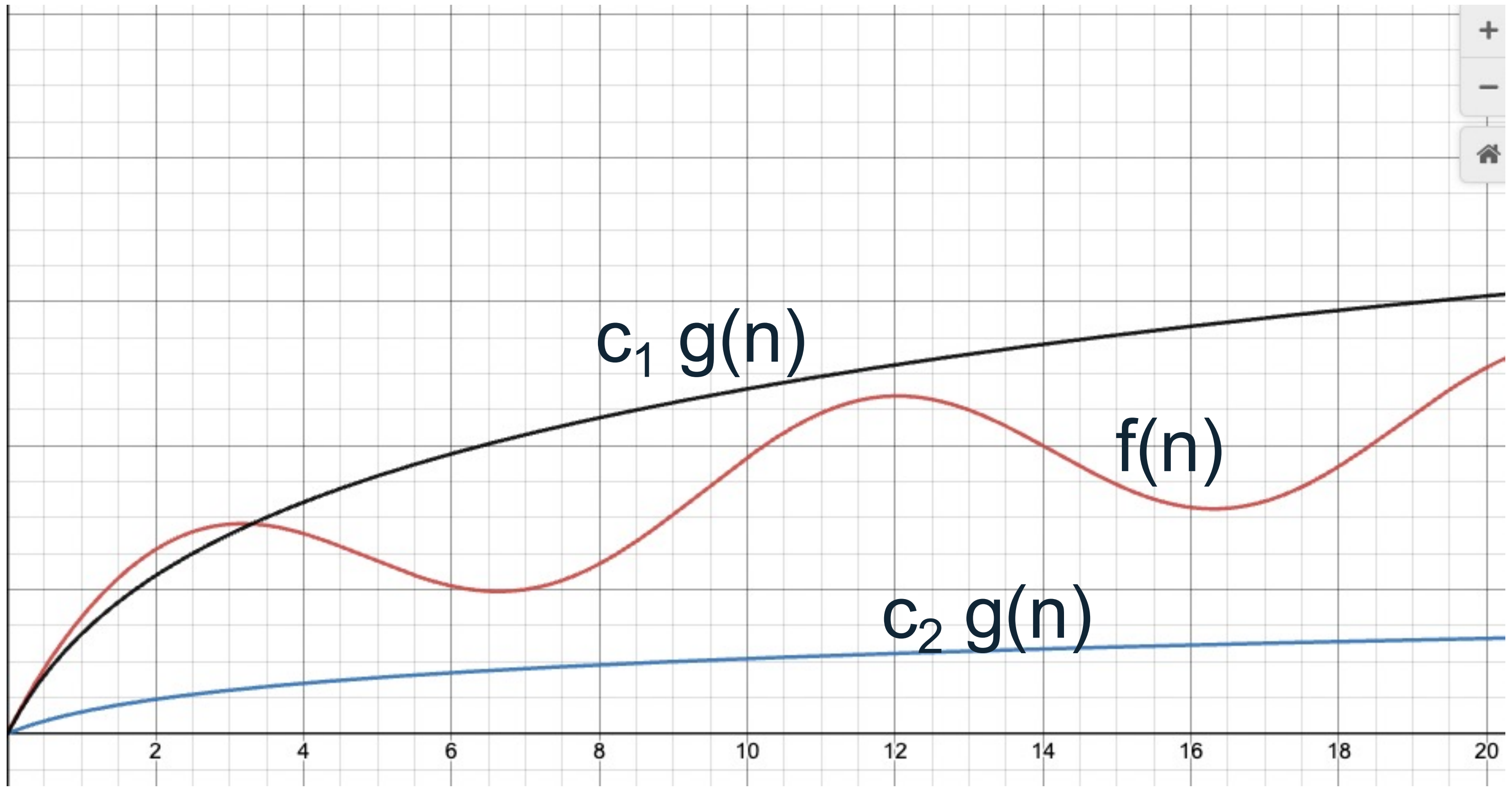
n= $\Omega(2n)$,
because n$\geq c * 2n, c = 0.5$.

$2n^{0.5} = \Omega(\log 500n)$

Big Θ Tight bound

$f(n) = \Theta(g(n))$ iff (if and only if):

$f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.



Examples:

$n = \Theta(2n)$

Polynomials of degree k are in $\Theta(n^k)$:

$a_k n^k \leq a_k n^k + \ldots + a_1 n + a_0 \leq (a_k + \ldots + a_1 + a_0) n^k$

Little o Upper bound

$$\forall c \in R^{+}, \exists n_0 \in N, such\ that\ \forall n \geq n_0\ such\ that\ f(n) < cg(n)\ (\left|\frac{f(n)}{g(n)}\right| < c).$$

In other words:

$\lim\limits_{n\to\infty} \frac{f(n)}{g(n)}$ =0.

Examples:

$\mathrm{n}=\mathrm{o}(\mathrm{n}^2)$.

$\log \mathrm{n}=\mathrm{o}(\mathrm{n})$.

If f(n)=o(g(n)) then f(n)=O(g(n)).

However, Little o does not allow the same growth rate.

Summary of the notations

- **Big O** and **Little o** are upper bounds. **Little o** is stronger than **Big O** because it does not allow for the same growth rate (g(n) grows faster than f(n)).

- **Big Ω** is a lower bound.

- **Big Θ** is a tight bound (upper and lower together).

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