Data Structures and Algorithms
Bloom Filters

CS 225
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April 25, 2025

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From slides by Brad Solomon

Memory-Constrained Data Structures

What method would you use to build a search index on a collection of objects *in a memory-constrained environment*?

Constrained by Big Data (Large N)

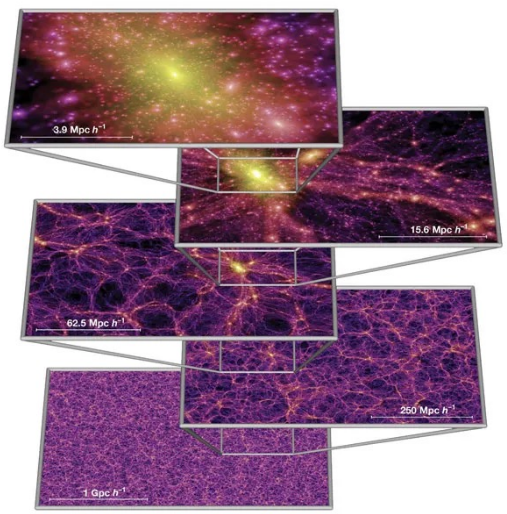


Image: https://doi.org/10.1038/nature03597

Sky Survey Projects	**Data Volume**
DPOSS (The Palomar Digital Sky Survey)	3 TB
2MASS (The Two Micron All-Sky Survey)	10 TB
GBT (Green Bank Telescope)	20 PB
GALEX (The Galaxy Evolution Explorer)	30 TB
SDSS (The Sloan Digital Sky Survey)	40 TB
SkyMapper Southern Sky Survey	500 TB
PanSTARRS (The Panoramic Survey Telescope and Rapid Response System)	~ 40 PB expected
LSST (The Large Synoptic Survey Telescope)	~ 200 PB expected
SKA (The Square Kilometer Array)	~ 4.6 EB expected

Table: http://doi.org/10.5334/dsj-2015-011

Estimated total volume of one array: 4.6 EB

Bloom Filter: Insertion

S = { 16, 8, 4, 13, 29, 11, 22 }

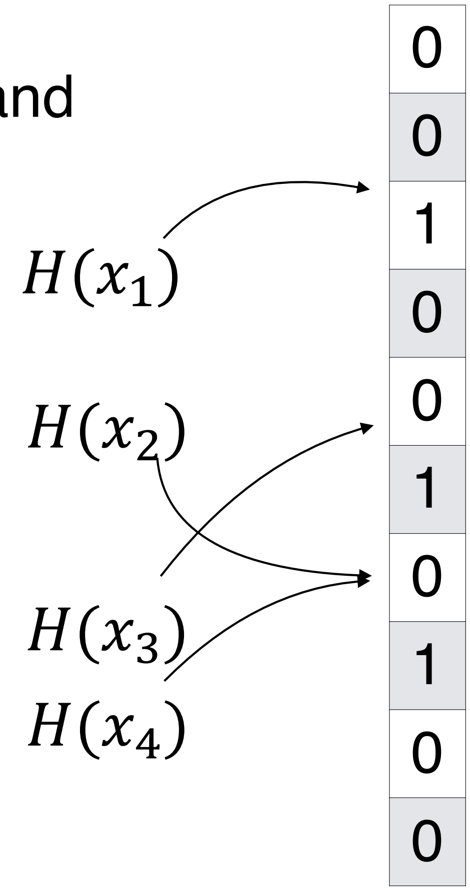
h(k) = k % 7

0	0
1	0
2	0
3	0
4	0
5	0
6	0

Bloom Filter: Insertion

An item is inserted into a bloom filter by hashing and then setting the hash-valued bit to 1

If the bit was already one, it stays 1



Bloom Filter: Search

S = { 16, 8, 4, 13, 29, 11, 22 }

h(k) = k % 7

0	0
1	1
2	1
3	0
4	1
5	0
6	1

`_find(16)`

`_find(20)`

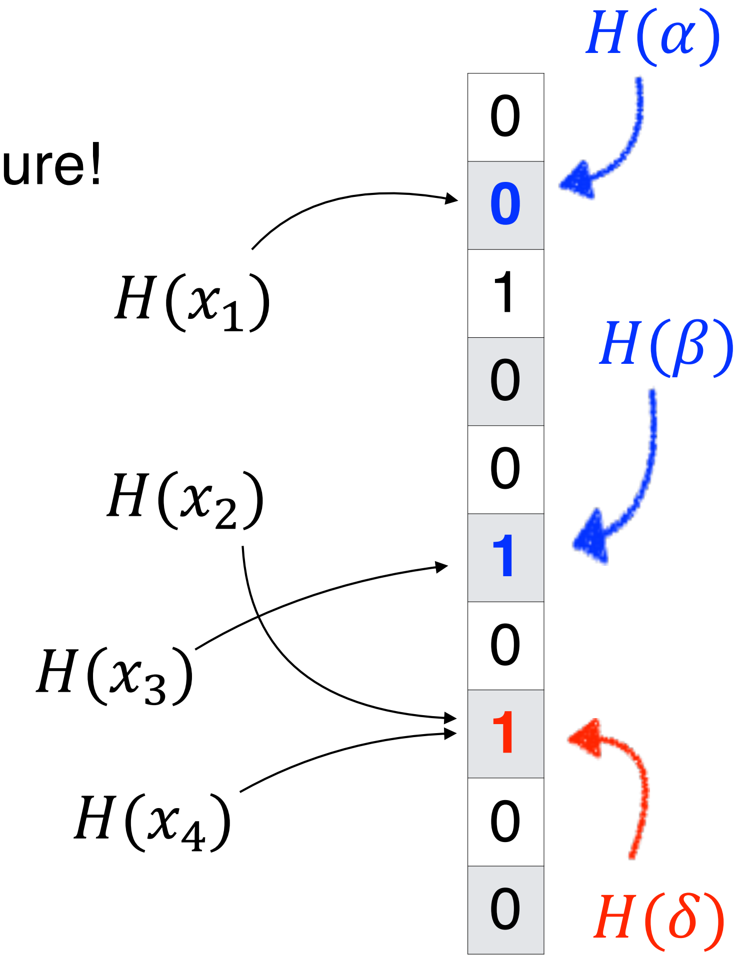
`_find(3)`

Bloom Filter: Search

The bloom filter is a *probabilistic* data structure!

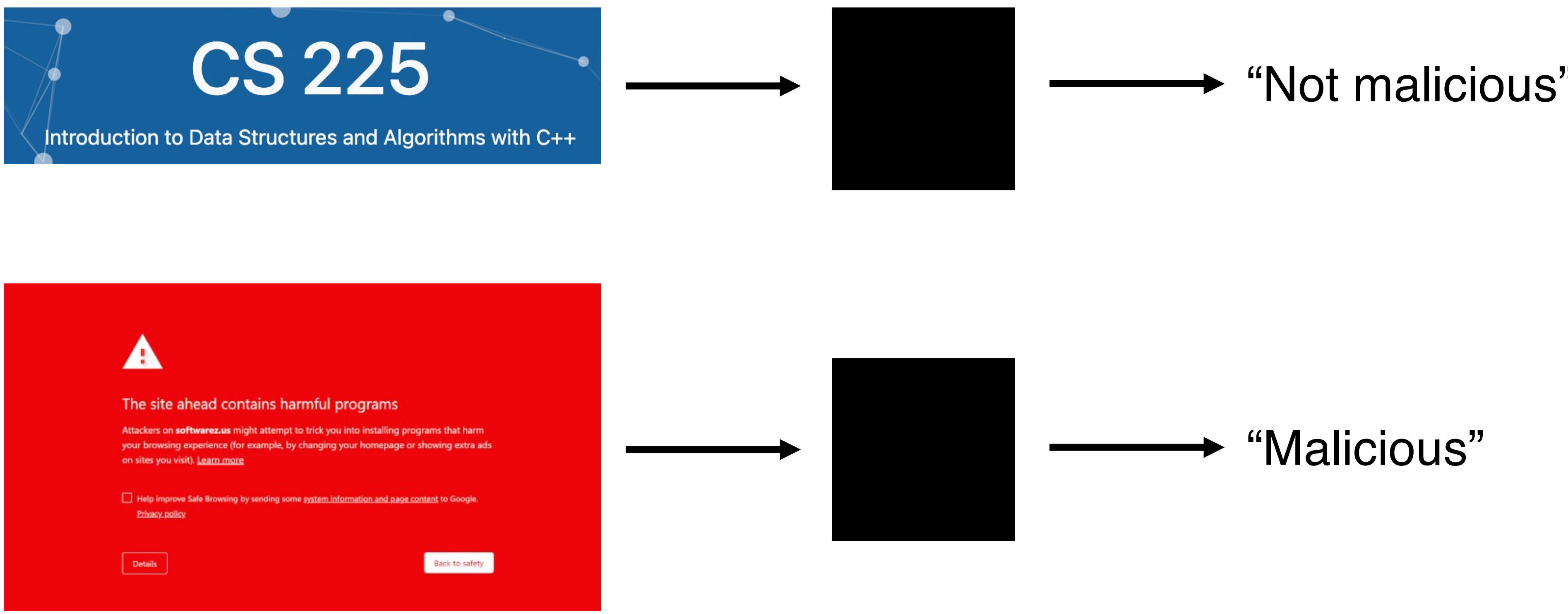
If the value in the BF is 0:

If the value in the BF is 1:



Probabilistic Accuracy: Malicious Websites

Imagine we have a detection oracle that identifies if a site is malicious



Probabilistic Accuracy: Malicious Websites

Imagine we have a detection oracle that identifies if a site is malicious

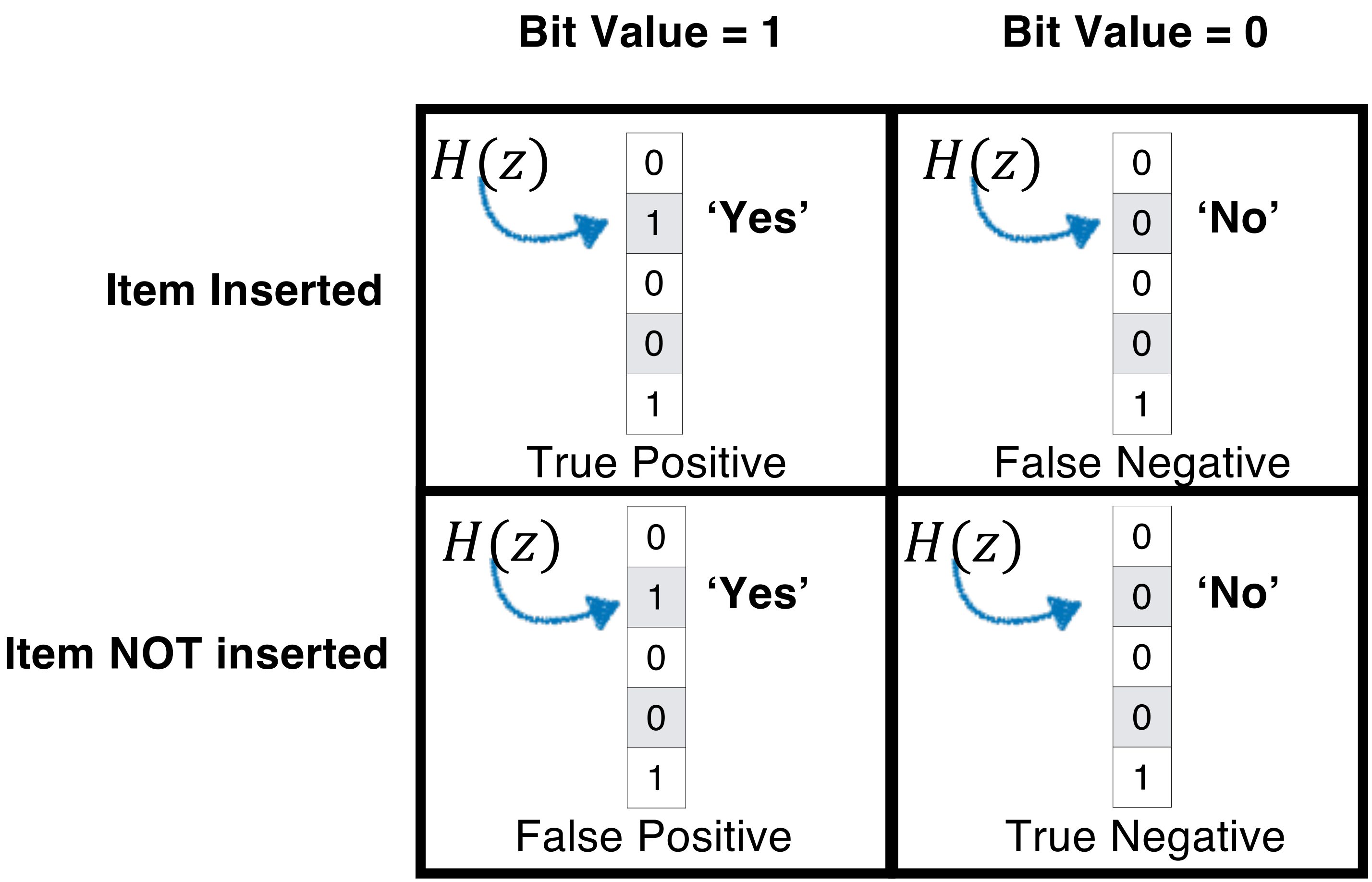
True Positive:

False Positive:

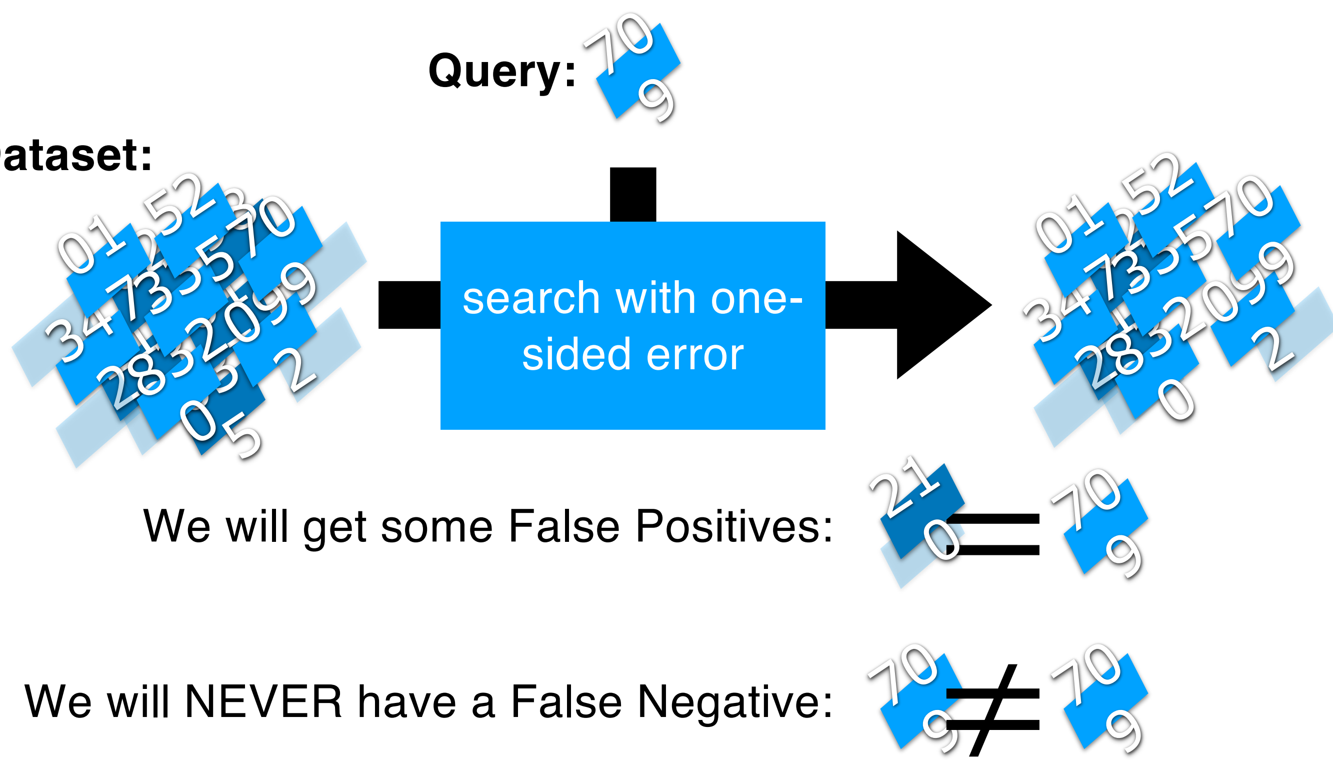
False Negative:

True Negative:

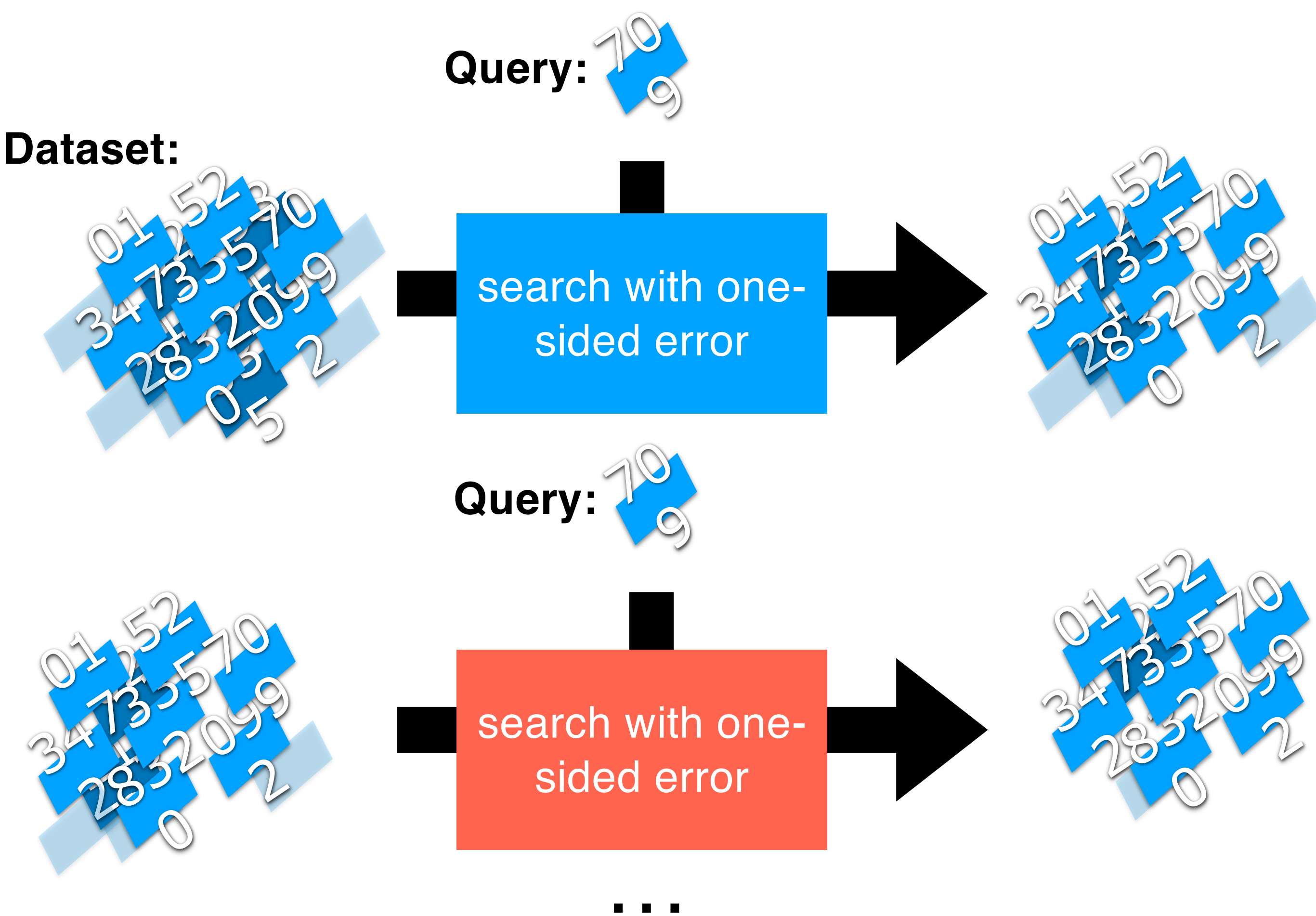
Imagine we have a **bloom filter** that **stores malicious sites…**



Probabilistic Accuracy: One-sided error



Probabilistic Accuracy: One-sided error

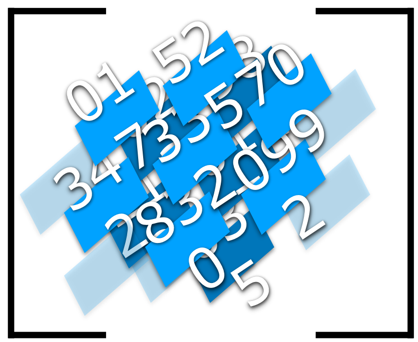


Bloom Filter: Repeated Trials

Use many hashes/filters; add each item to each filter

h_1

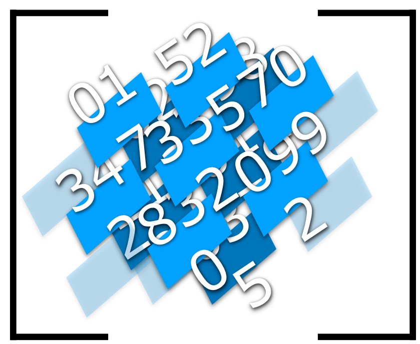
0
1
0
1
0
0
0
1
0
1
1
0
1
0
1
1
0
1
0
1



Bloom Filter: Repeated Trials

Use many hashes/filters; add each item to each filter

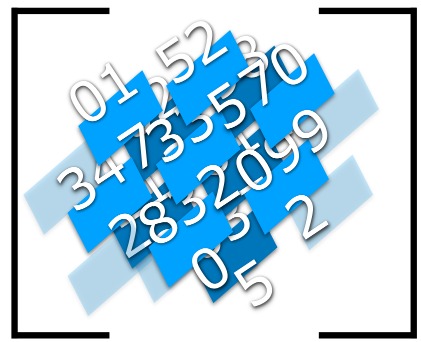
h_1	h_2
0	0
1	0
0	0
1	1
0	0
0	0
0	0
1	1
0	0
1	1
1	0
0	0
1	1
0	1
1	1
1	0
0	0
1	1
0	0
1	0



Bloom Filter: Repeated Trials

Use many hashes/filters; add each item to each filter

h_1	h_2	h_3
0	0	0
1	0	1
0	0	1
1	1	1
0	0	0
0	0	0
0	0	1
1	1	1
0	0	0
1	1	1
1	0	1
0	0	0
1	1	1
0	1	0
1	1	1
1	0	1
0	0	0
1	1	1
0	0	0
1	0	1



Bloom Filter: Repeated Trials

Use many hashes/filters; add each item to each filter

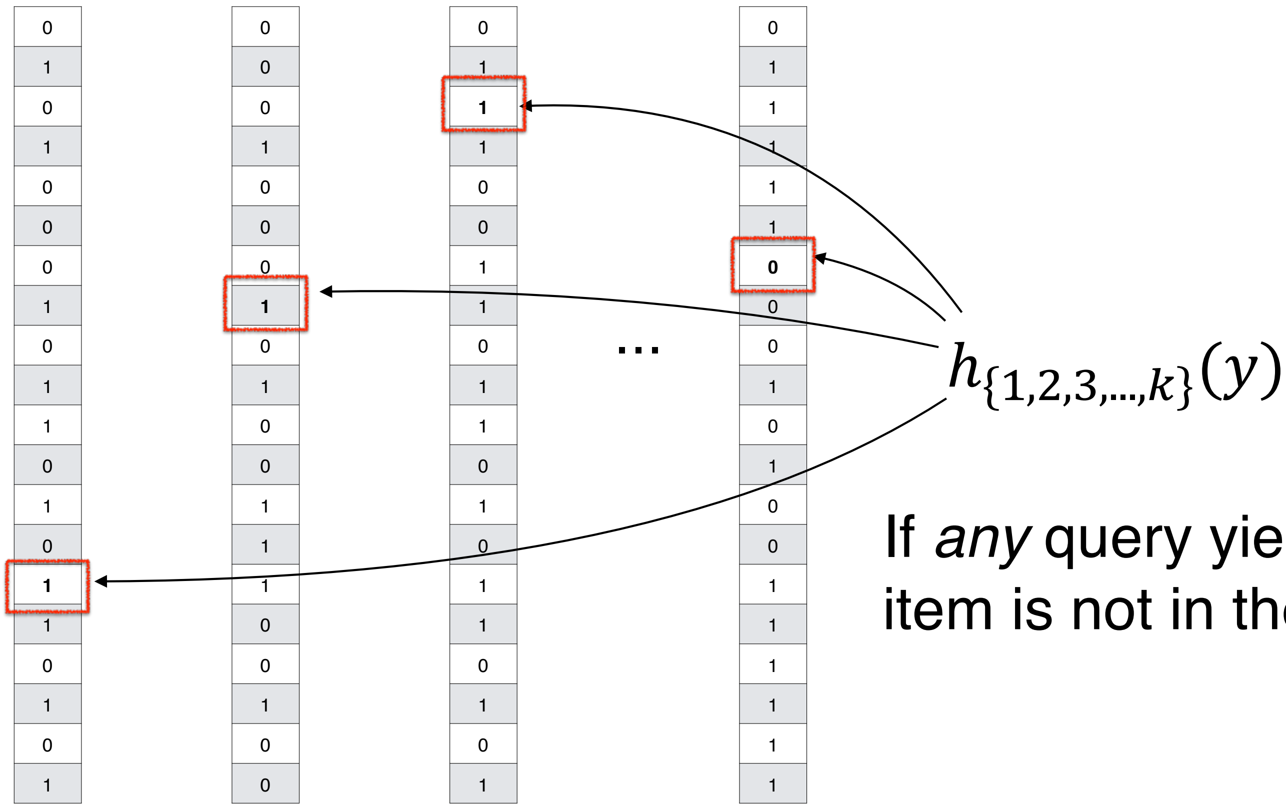
h_1	h_2	h_3	$\cdots$	h_k
0	0	0		0
1	0	1		1
0	0	1		1
1	1	1		1
0	0	0		1
0	0	0		1
0	0	1		0
1	1	1		0
0	0	0		0
1	1	1		1
1	0	1		0
0	0	0		1
1	1	1		0
0	1	0		0
1	1	1		1
1	0	1		1
0	0	0		1
1	1	1		1
0	0	0		1
1	0	1		1

Bloom Filter: Repeated Trials

0	0	0		0
1	0	1		1
0	0	1		1
1	1	1		1
0	0	0		1
0	0	0		1
0	0	1		0
1	1	1		0
0	0	0	…	0
1	1	1		1
1	0	1		0
0	0	0		1
1	1	1		0
0	1	0		0
1	1	1		1
1	0	1		1
0	0	0		1
1	1	1		1
0	0	0		1
1	0	1		1

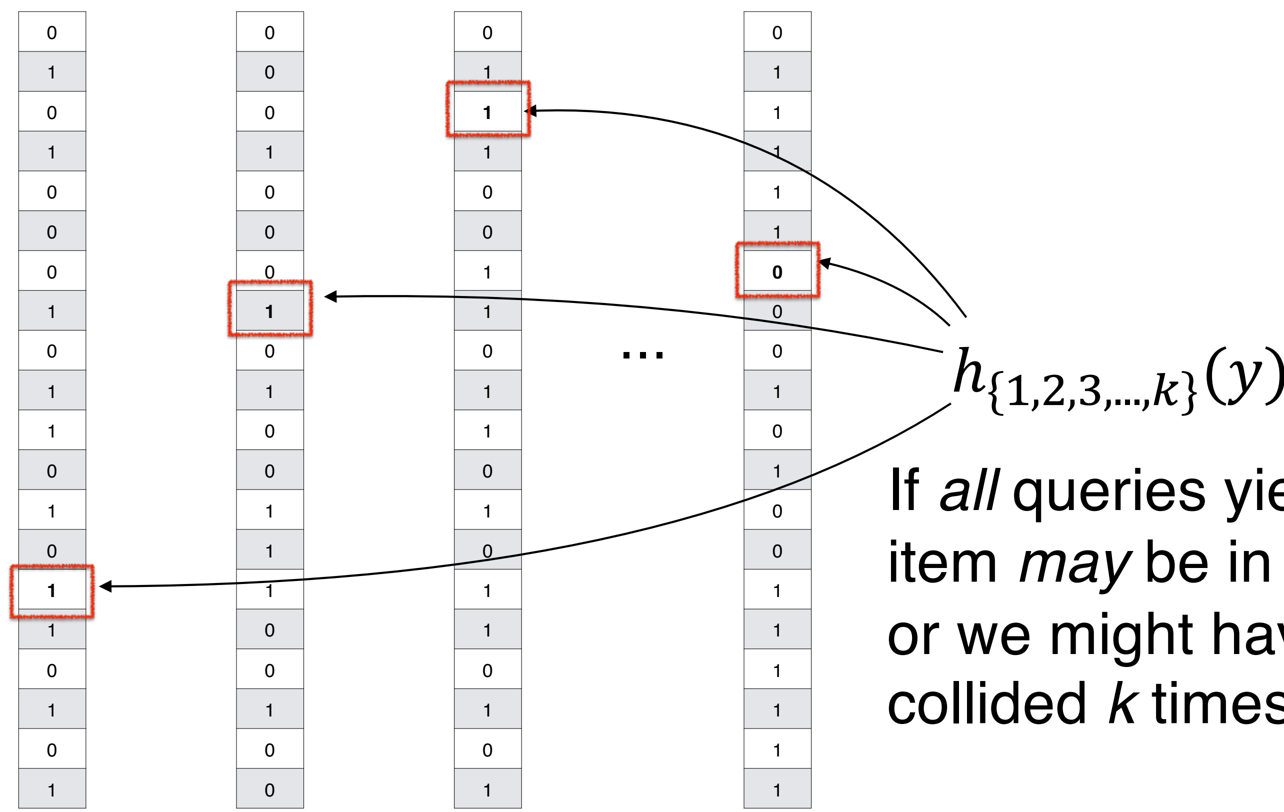
$$h_{\{1,2,3,\ldots,k\}}(y)$$

Bloom Filter: Repeated Trials



If *any* query yields 0, item is not in the set

Bloom Filter: Repeated Trials



If *all* queries yield 1, item *may* be in the set; or we might have collided *k* times

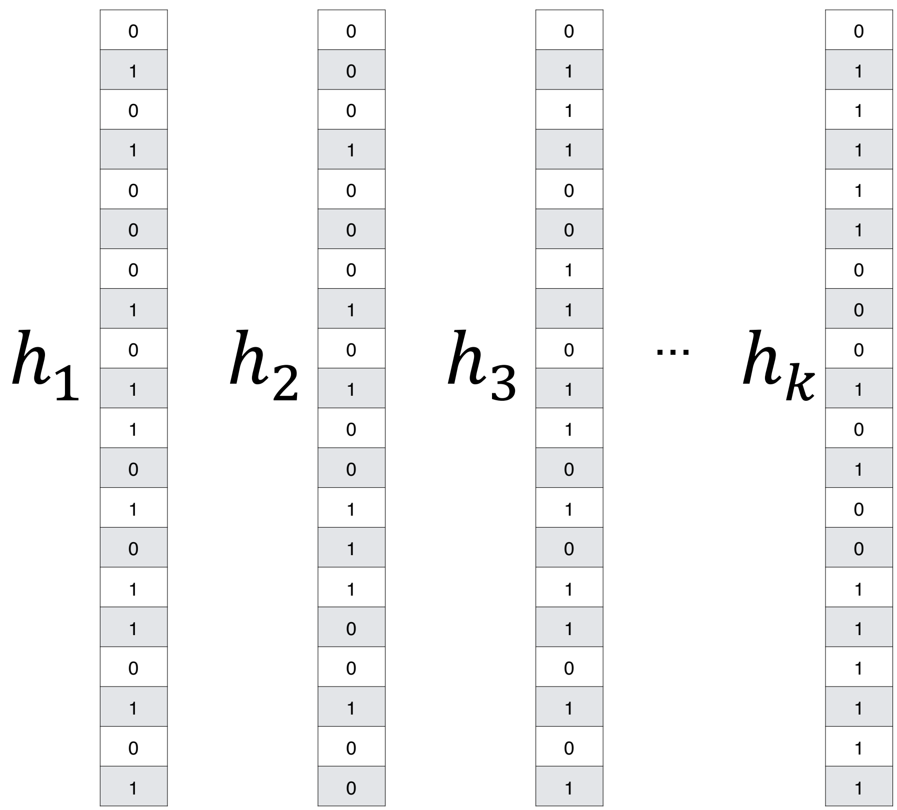
Bloom Filter: Repeated Trials

Using repeated trials, even a very bad filter can still have a very low FPR!

If we have k bloom filter, each with a FPR p, what is the likelihood that ***all*** filters return the value '1' for an item we didn't insert?

Bloom Filter: Repeated Trials

But doesn't this hurt our storage costs by storing k separate filters?



Bloom Filter: Repeated Trials

Rather than use a new filter for each hash, one filter can use k hashes

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

$S = \{ 6, 8, 4 \}$

$h_1(x) = x \% 10$ **$h_2(x) = 2x \% 10$** **$h_3(x) = (5+3x) \% 10$**

Bloom Filter: Repeated Trials

Rather than use a new filter for each hash, one filter can use k hashes

0	0
1	0
2	1
3	1
4	1
5	0
6	1
7	1
8	1
9	1

$h_1(x)$ = x % 10 **$h_2(x)$ = 2x % 10** **$h_3(x)$ = (5+3x) % 10**

```
_find(1)
```

```
_find(16)
```

Bloom Filter

A probabilistic data structure storing a set of values

Built from a bit vector of length m and k hash functions

Insert / Find runs in: ________________

Delete is not possible (yet)!

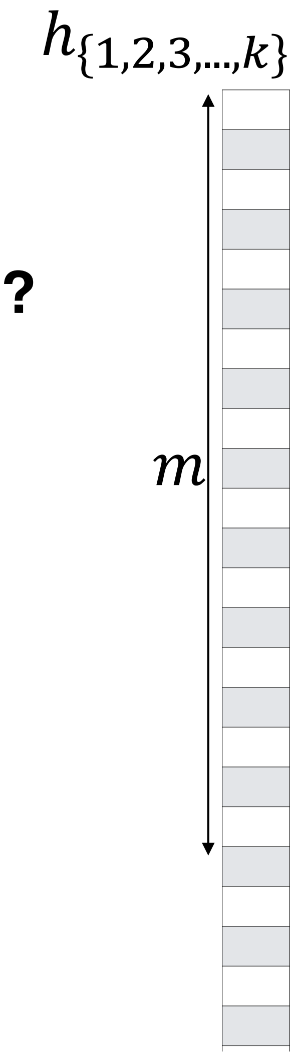
$$H = \{h_1, h_2, \ldots, h_k\}$$

0
0
1
0
0
1
0
1
0
0

Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

What is our expected FPR after n objects are inserted?

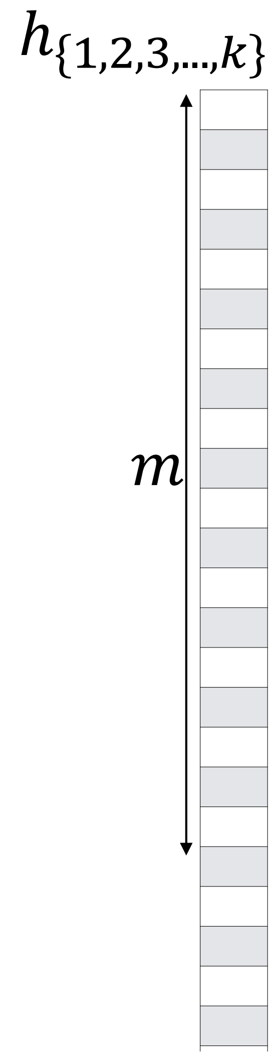


Bloom Filter: Error Rate

Given bit vector of size m and 1 SUHA hash function

What's the probability a specific bucket is 1 after one object is inserted?

Same probability given k SUHA hash function?

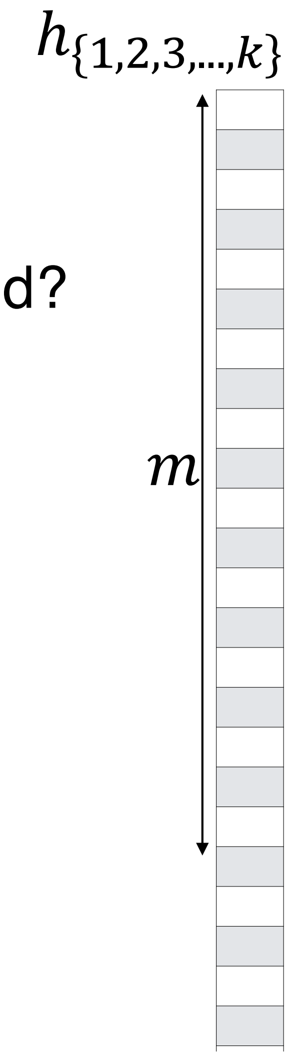


Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

Probability a specific bucket is 0 after one object is inserted?

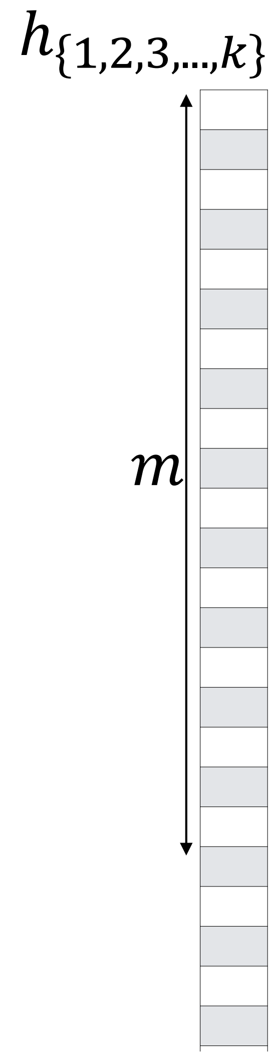
After n objects are inserted?



Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

What's the probability a specific bucket is 1 after n objects are inserted?



Bloom Filter: Error Rate

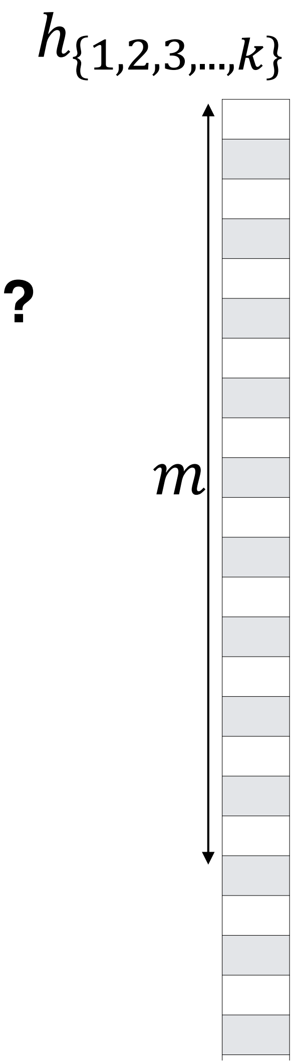
Given bit vector of size m and k SUHA hash function

What is our expected FPR after n objects are inserted?

The probability my bit is 1 after n objects inserted

$$\left(1 - \left(1 - \frac{1}{m}\right)^{nk}\right)^{k}$$

The number of [assumed independent] trials



Bloom Filter: Error Rate

Vector of size m, k SUHA hash function, and n objects

To minimize the FPR, do we prefer…

(A) large k **(B) small k**

$$\left(1 - \left(1 - \frac{1}{m}\right)^{nk}\right)^{k}$$