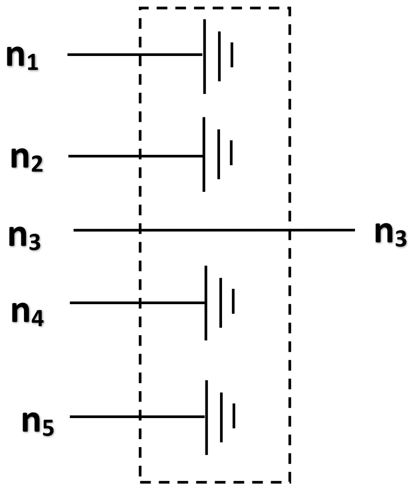
(1) Question

First we can use the the projection Π_i^k : $N^k \to N$ :: $(n_1, ..., n_k) \mapsto n_i$ for each k and every i with $1 \leqslant i \leqslant k$ function to rewrite Π_3^5:

So we can get that Π_3^5 : $N^5 \to N$ :: $(n_1, n_2, n_3, n_4, n_5) \mapsto n_3$

Then we can use the box-and-wire notation to show that:



Then the definition of primitive recursive: A function is "primitive recursive" if and only if first it is one of the zero, successor function, discarding, duplication, identity, swap, or second it can be obtained from other primitive recursive functions by (serial and parallel) composition and primitive recursion. So according to this diagram, We can see that this is just one of five inputs selected for n3 output, while the others are discarded. This is consistent with the basic primitive recursive functions such as the identity function and discard function mentioned in the definition. So this satisfies the definition. Therefore, according to the definition of primitive recursive, we can find that the projection Π_3^5 is primitive recursive.

(2) . Question:

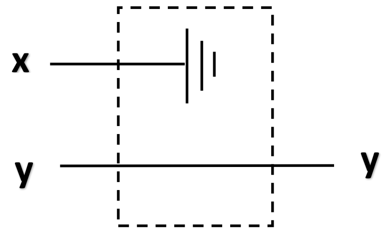
(a) Question:

First, we need to find the base clause and the inductive clause. So first about base clause because from the content of the conditional function given in the question, we can find that when n = 0, f(x, y) will output y. Therefore, we can get cond(x, y, 0) := f(x, y) = y.

Then second about inductive clause, according to the conditions given in the title, we can find that when n > 0, it always returns x, no matter what the value of n is.

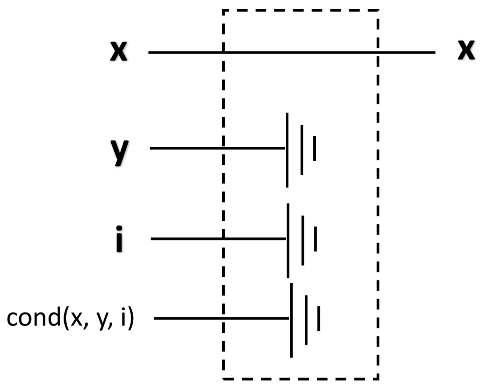
So we can get that cont(x, y, i + 1) := g(x, y, i, cond(x, y, i)) = x.

(b) <u>Question:</u>
First, we analyze whether f is primitive recursive. I want to use box-and-wire notation to show show that F(x, y) = y:



So according to this diagram, we can see that this function is a projection function. It contains a discard function and an identity function. So according to the definition of primitive recursive, the function f is primitive recursive.

Second, we analyze whether f is primitive recursive. I also want to use box-and-wire notation to show show that g(x, y, i, cond(x, y, i)) = x:



So according to this diagram, we can see that this function is also a projection function. It contains a discard function and an identity function. So according to the definition of primitive recursive, the function g is primitive recursive.

Therefore, We can prove that two function f and g are both primitive recursive by using box-and-wire notation.

(3) Question:

First according to the definition of β -reduction: ((λx.M)N) $\xrightarrow{\beta}$ M[N/x]), we can use it to

prove that (($\overline{\boldsymbol{True}}$ **M)N)** $\xrightarrow{\beta}$ .... $\xrightarrow{\beta}$ M.

Since the question mention that $\overline{\boldsymbol{True}}$ $:=$ (λx. (λy. x)), So we have (($\overline{true}$ M)N) = ((λx. (λy. x))M)N. By definition of β -reduction, we have ((λx. (λy. x))M)N→ (λy. M)N. Then we need to use β -reduction again, and then we can have (λy. M)N → M. Therefore we can

prove that (($\overline{True}$ M)N) $\xrightarrow{\beta}$ ((λx. (λy. x))M)N $\xrightarrow{\beta}$ (λy. M)N $\xrightarrow{\beta}$ M.

First according to the definition of β -reduction: ((λx.M)N) $\xrightarrow{\beta}$ M[N/x]), we can use it to

prove that (($\overline{\boldsymbol{False}}$ **M)N)** $\xrightarrow{\beta}$ ... $\xrightarrow{\beta}$ N.

Since the question mention that $\overline{\boldsymbol{False}}$ $:=$ (λx. (λy. y)), So we have (($\overline{true}$ M)N) = ((λx. (λy. y))M)N. By definition of β -reduction, we have ((λx. (λy. y))M)N→ (λy. y)N. Then we need to use β -reduction again, and then we can have (λy. y)N→ N. Therefore we can prove

That (($\overline{\boldsymbol{False}}$ **M)N)** $\xrightarrow{\beta}$ ((λx. (λy. y))M)N $\xrightarrow{\beta}$ (λy. y)N $\xrightarrow{\beta}$ N.