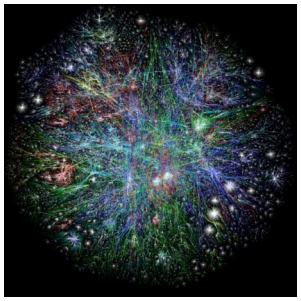
CS 225

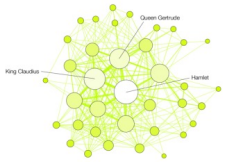
Data Structures

March 28 – Graph Implementations and Traversals

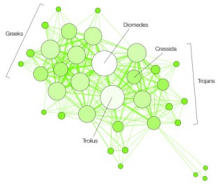
G Carl Evans

Graphs

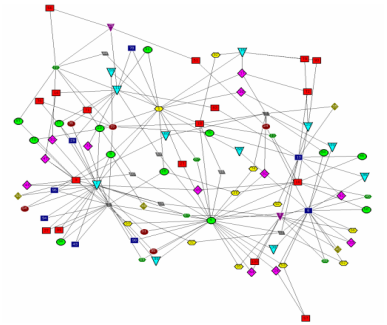




HAMLET

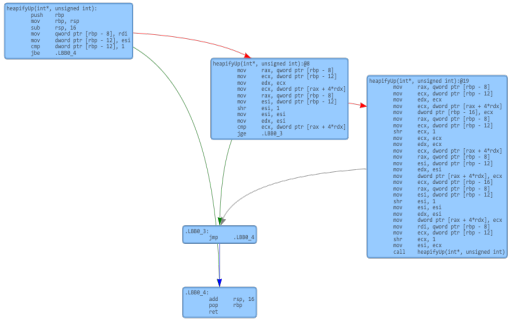


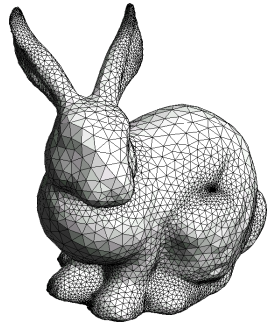
TROILUS AND CRESSIDA

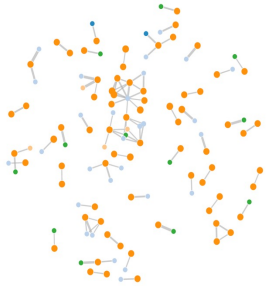


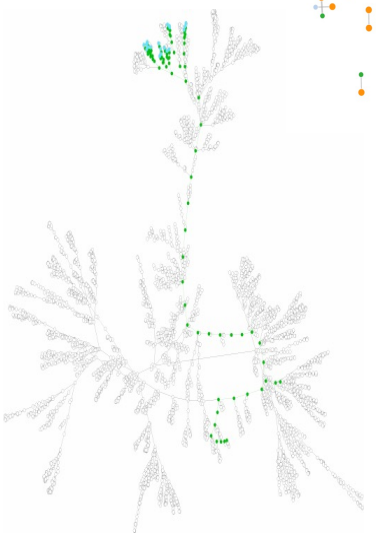
To study all of these structures:

1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms

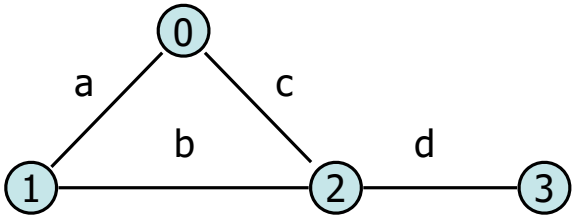








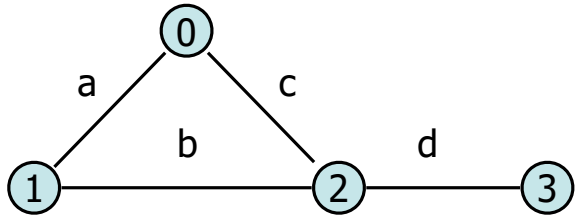
Graph Implementation: Edge List

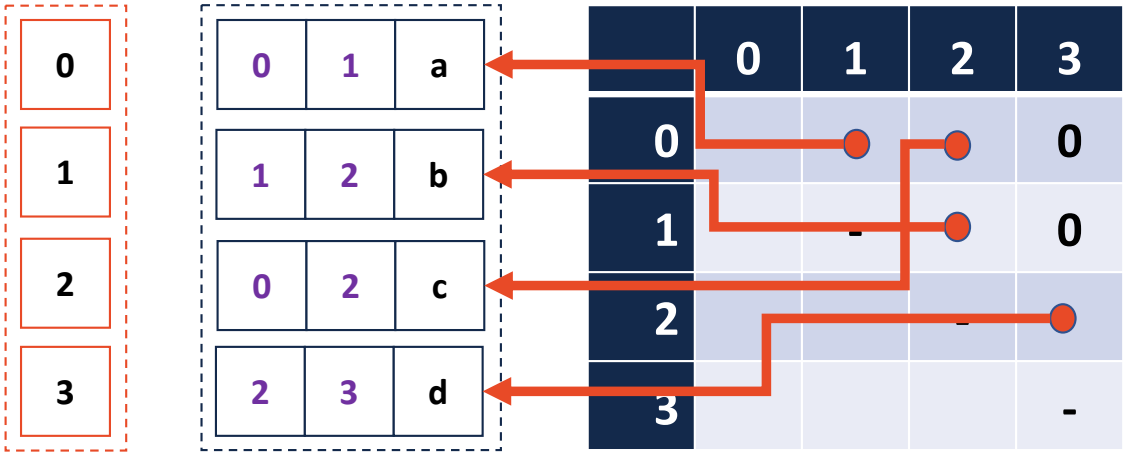


0		
1		
2		
3		

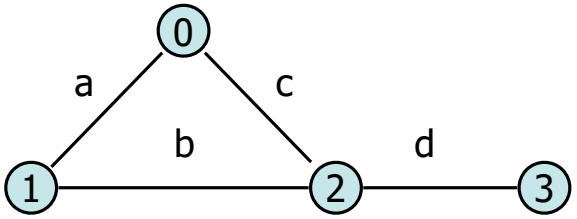
0	1	a
1	2	b
0	2	c
2	3	d

Graph Implementation: Adjacency Matrix





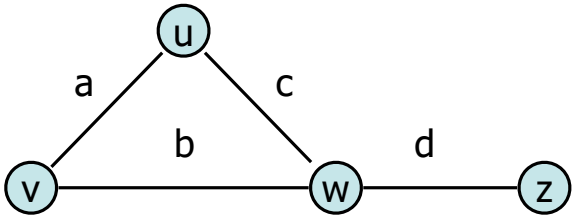
Graph Implementation: Edge List + ?

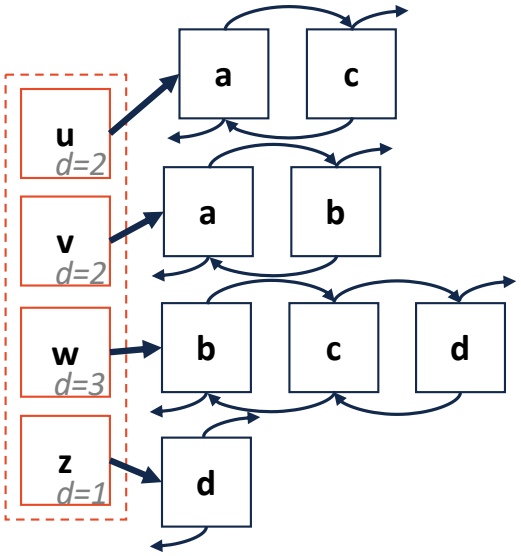


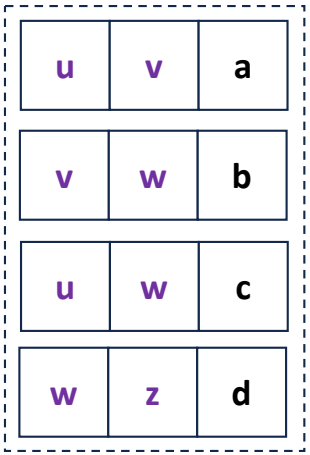
0
1
2
3

0	1	a
1	2	b
0	2	c
2	3	d

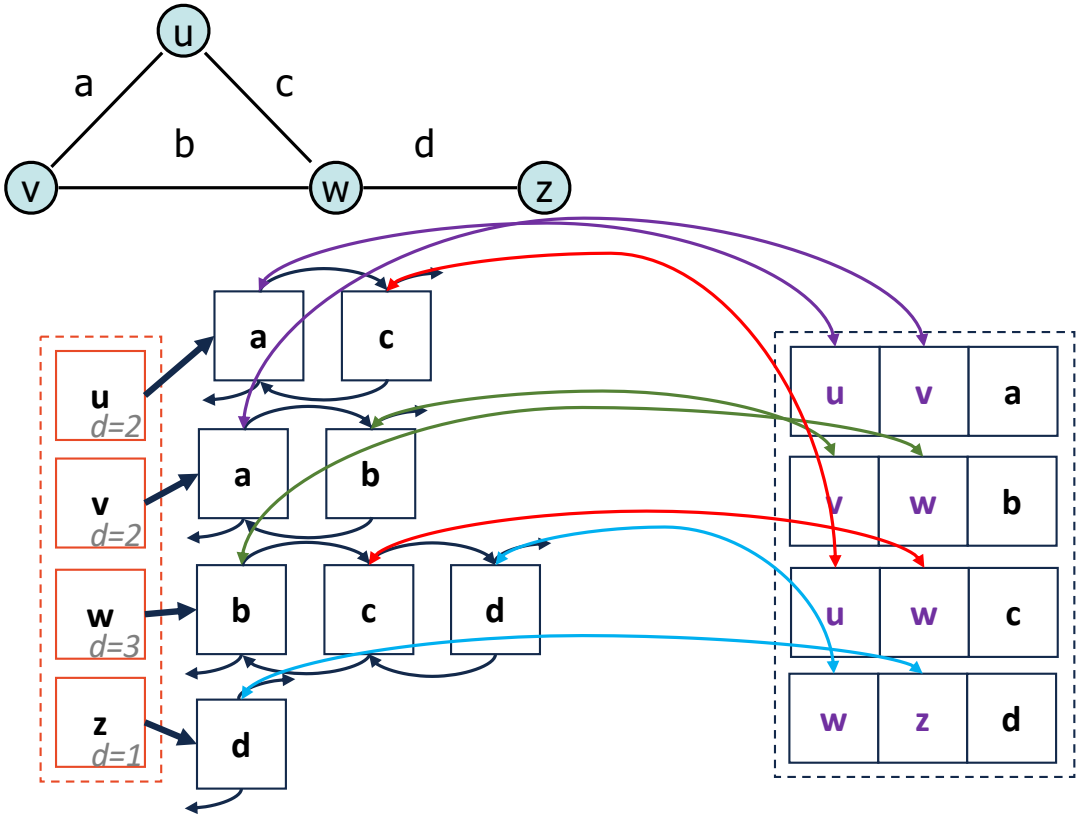
Graph Implementation: Adjacency List





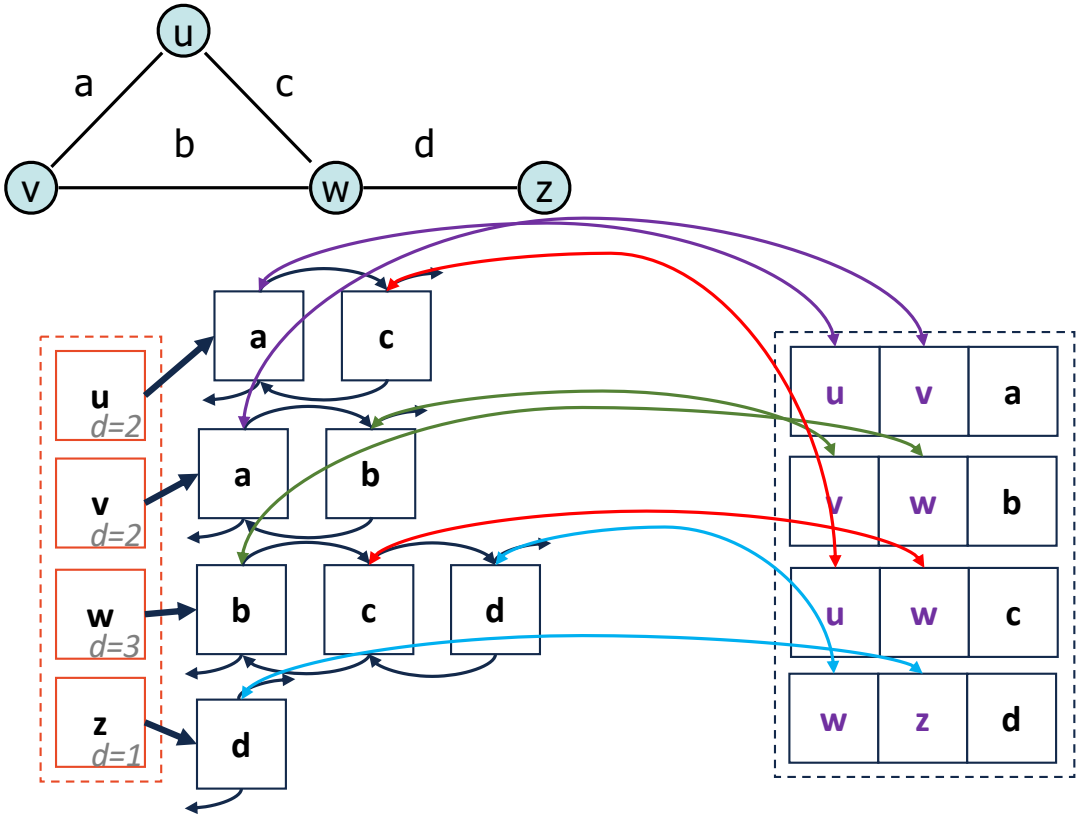


Graph Implementation: Adjacency List



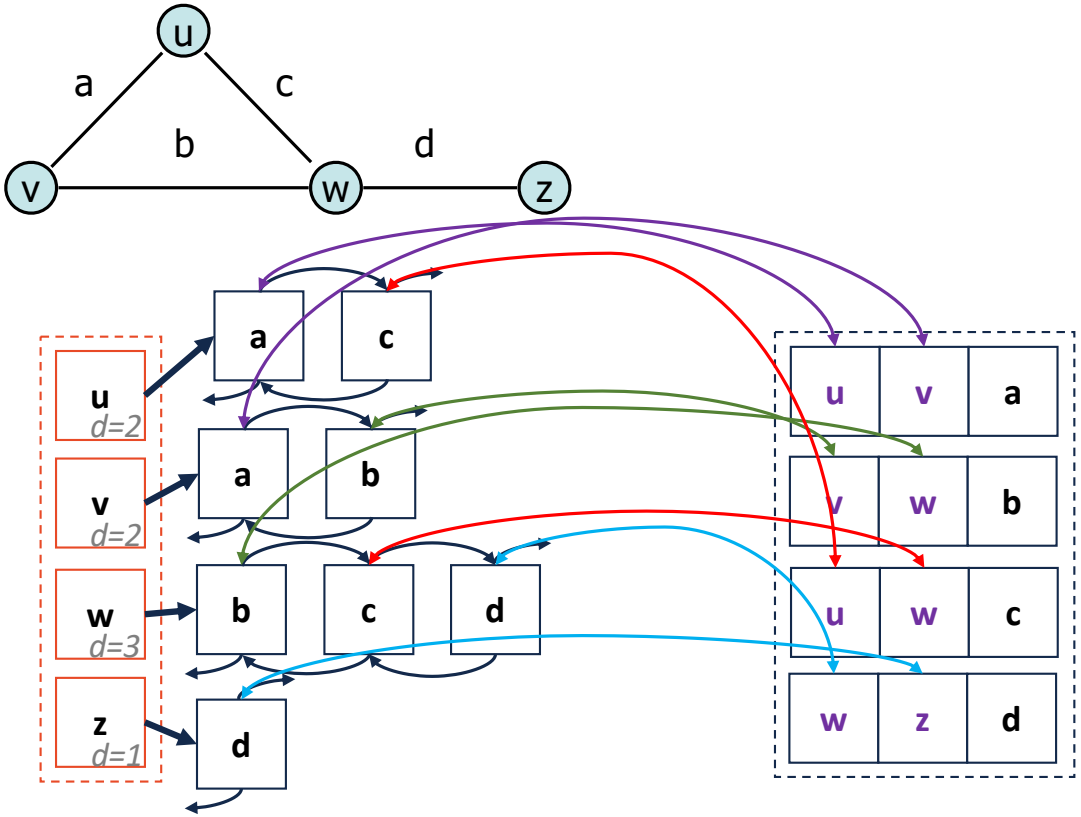
Adjacency List

insertVertex(K key):



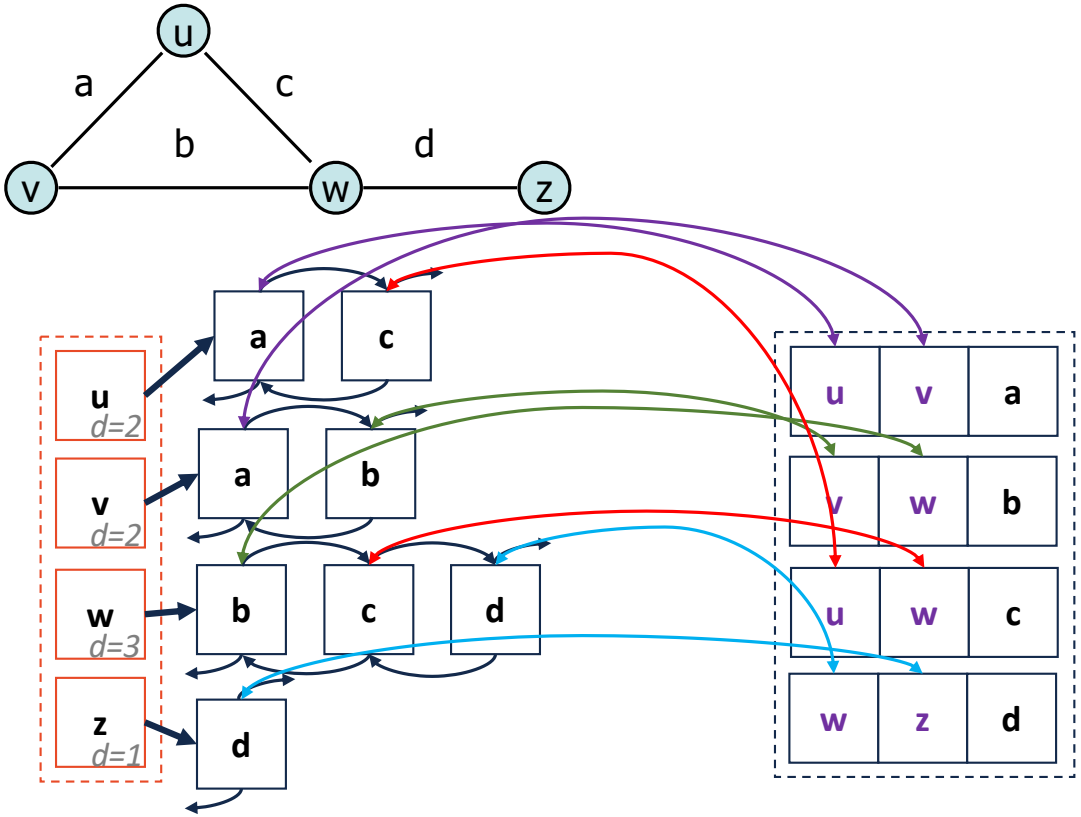
Adjacency List

removeVertex(Vertex v):



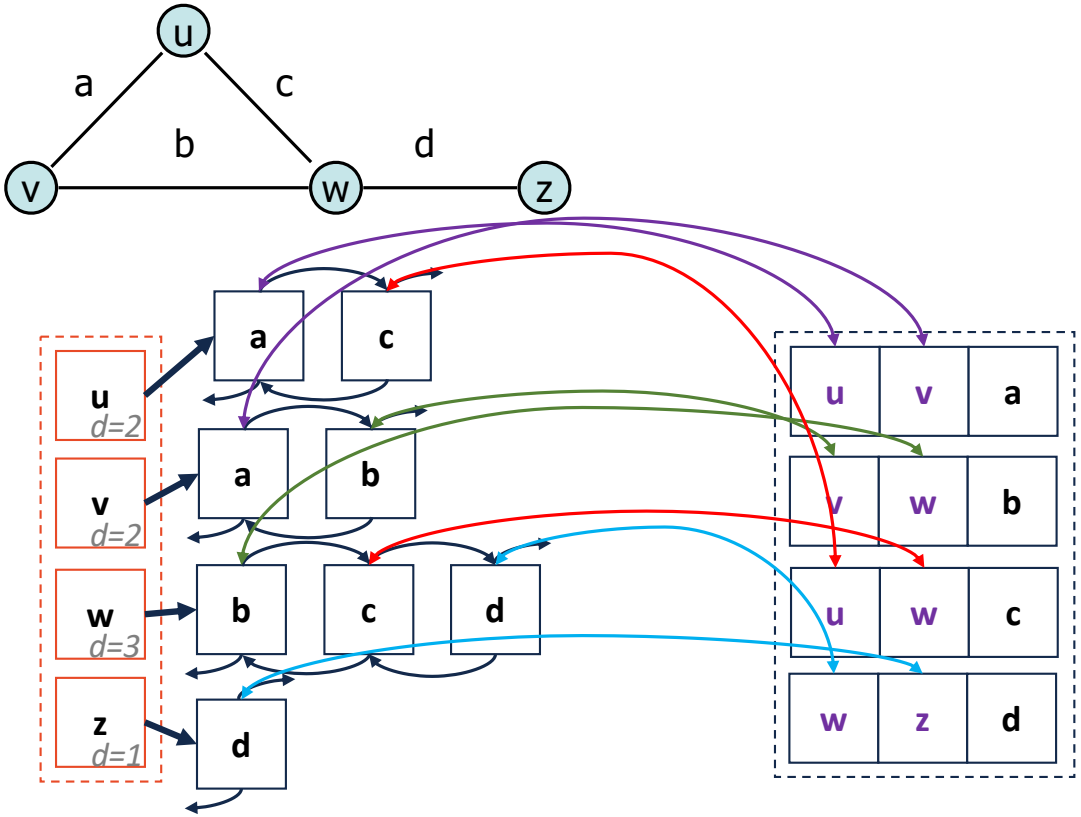
Adjacency List

incidentEdges(Vertex v):



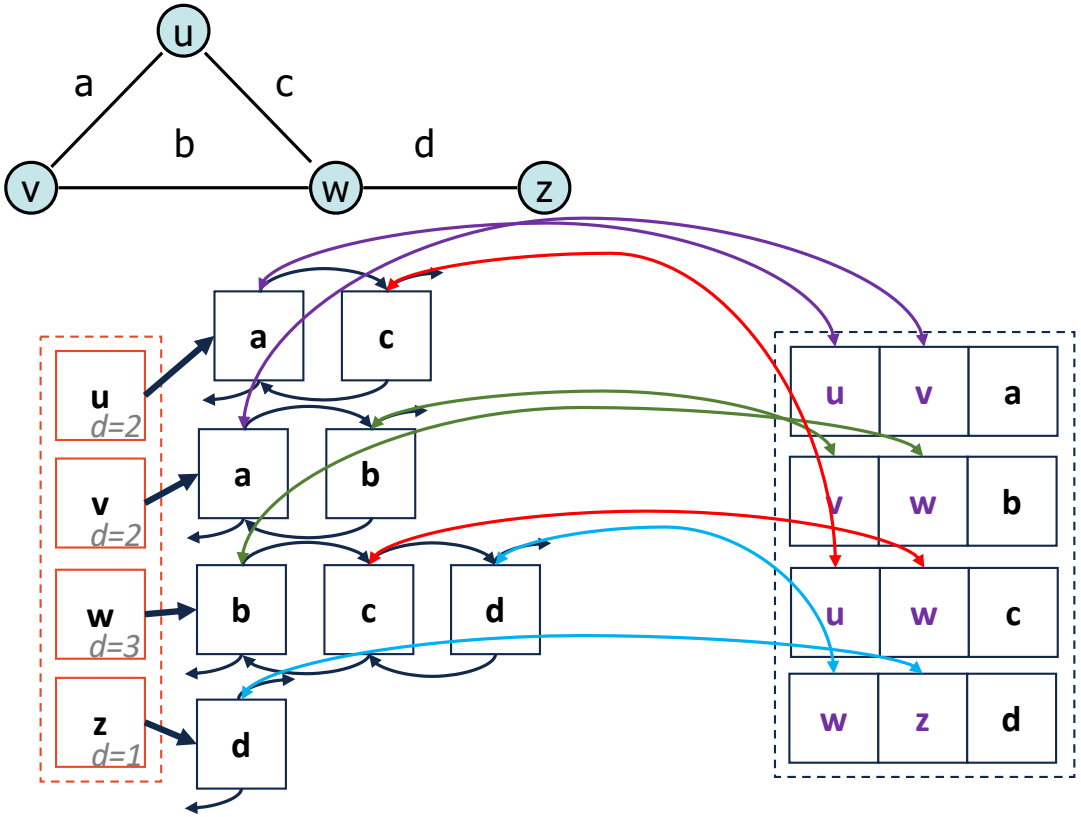
Adjacency List

areAdjacent(Vertex v1, Vertex v2):



Adjacency List

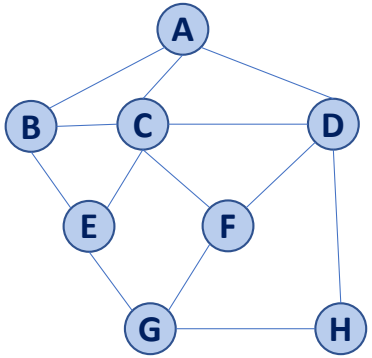
insertEdge(Vertex v1, Vertex v2, K key):



```
 1  BFS(G):
 2    Input: Graph, G
 3    Output: A labeling of the edges on
 4        G as discovery and cross edges
 5
 6    foreach (Vertex v : G.vertices()):
 7      setLabel(v, UNEXPLORED)
 8    foreach (Edge e : G.edges()):
 9      setLabel(e, UNEXPLORED)
10    foreach (Vertex v : G.vertices()):
11      if getLabel(v) == UNEXPLORED:
12         BFS(G, v)
```

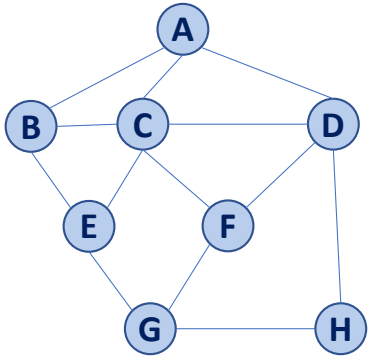
```
14  BFS(G, v):
15    Queue q
16    setLabel(v, VISITED)
17    q.enqueue(v)
18
19    while !q.empty():
20      v = q.dequeue()
21      foreach (Vertex w : G.adjacent(v)):
22        if getLabel(w) == UNEXPLORED:
23           setLabel(v, w, DISCOVERY)
24           setLabel(w, VISITED)
25           q.enqueue(w)
26        elseif getLabel(v, w) == UNEXPLORED:
27           setLabel(v, w, CROSS)
```

Traversal: BFS



v	d	P	Adjacent Edges
A			
B			
C			
D			
E			
F			
G			
H			

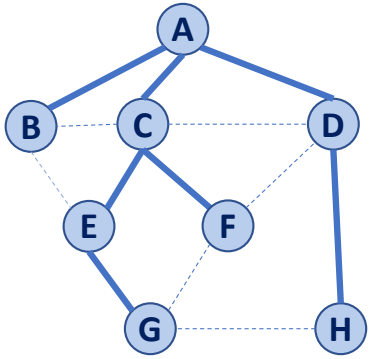
Traversal: BFS



v	d	P	Adjacent Edges
A	0	-	C B D
B			A C E
C			B A D E F
D			A C F H
E			B C G
F			C D G
G			E F H
H			D G

A

Traversal: BFS



v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	B A D E F
D	1	A	A C F H
E	2	C	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G

BFS Analysis

Q: Does our implementation handle disjoint graphs? If so, what code handles this?

- ***How do we use this to count components?***

Q: Does our implementation detect a cycle?

- ***How do we update our code to detect a cycle?***

Q: What is the running time?

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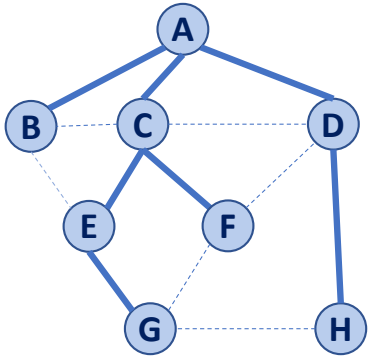
```

Running time of BFS



While-loop at **:19**?

For-loop at **:21**?

v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	B A D E F
D	1	A	A C F H
E	2	C	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G

G H F E D B C A

BFS Observations

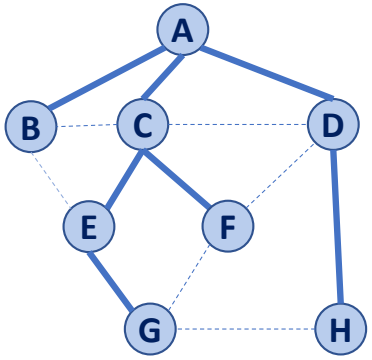
Q: What is a shortest path from **A** to **H**?

Q: What is a shortest path from **E** to **H**?

Q: How does a cross edge relate to **d**?

Q: What structure is made from discovery edges?

v	d	P	Adjacent Edges
A	0	-	**C B D**
B	1	**A**	**A C E**
C	1	**A**	**B A D E F**
D	1	**A**	**A C F H**
E	2	**C**	**B C G**
F	2	**C**	**C D G**
G	3	**E**	**E F H**
H	2	**D**	**D G**



BFS Observations

Obs. 1: BFS can be used to count components.

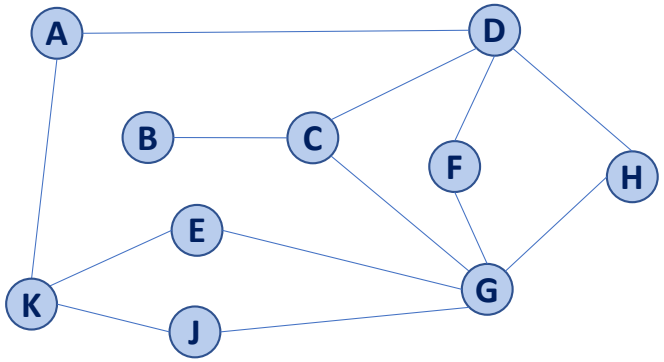
Obs. 2: BFS can be used to detect cycles.

Obs. 3: In BFS, **d** provides the shortest distance to every vertex.

Obs. 4: In BFS, the endpoints of a cross edge never differ in distance, **d**, by more than 1:

$$|d(u) - d(v)| = 1$$

Traversal: DFS



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12         BFS(G, v)
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24           setLabel(w, VISITED)
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27           setLabel(v, w, CROSS)
```

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 1  DFS(G):
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 3    Output: A labeling of the edges on
 4        G as discovery and back edges
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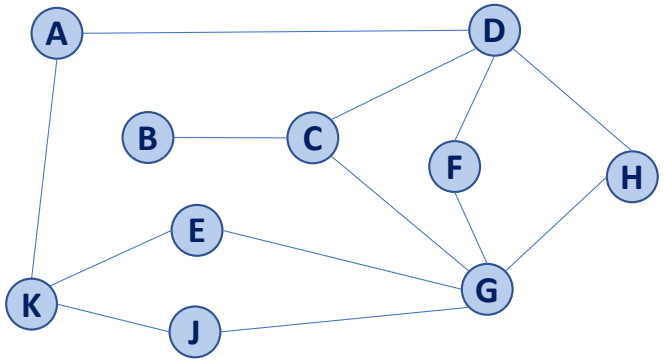
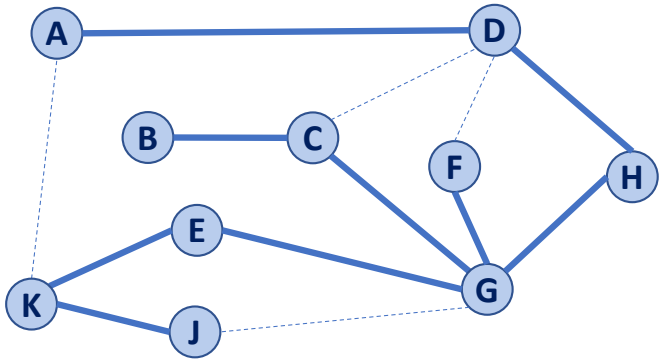
27           setLabel(v, w, BACK)
```

Traversal: DFS



Traversal: DFS



Discovery Edge

Back Edge

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26        elseif getLabel(v, w) == UNEXPLORED:
27           setLabel(v, w, BACK)
```

Running time of DFS

Labeling:

- Vertex:

- Edge:

Queries:

- Vertex:

- Edge: