



CS 225

Data Structures

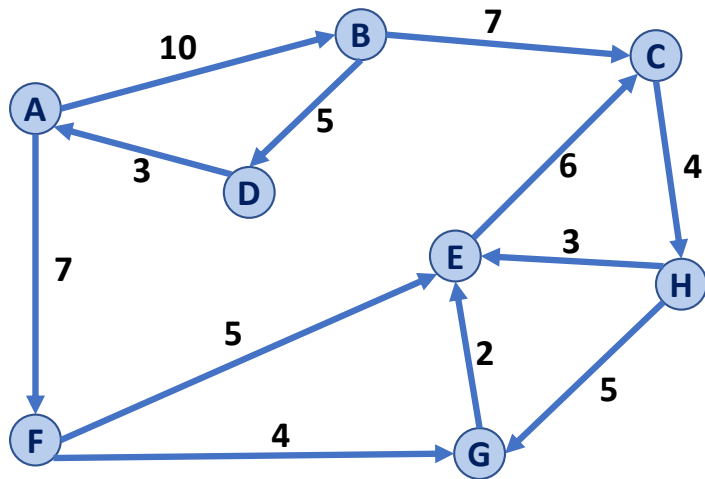
April 7 – Dijkstra's Algorithm

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Shortest Path

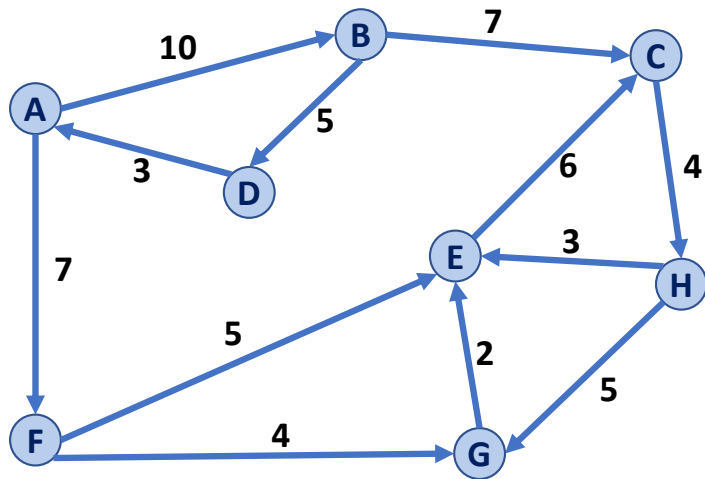


Dijkstra's Algorithm (SSSP)



```
PrimMST(G, s):
6  foreach (Vertex v : G):
7      d[v] = +inf
8      p[v] = NULL
9  d[s] = 0
10
11  PriorityQueue Q // min distance, defined by d[v]
12  Q.buildHeap(G.vertices())
13
14  repeat n times:
15      Vertex u = Q.removeMin()
16      foreach (Vertex v : neighbors of u not in T):
17          if cost(u, v) < d[v]:
18              d[v] = cost(u, v)
19              p[v] = u
20
21
```

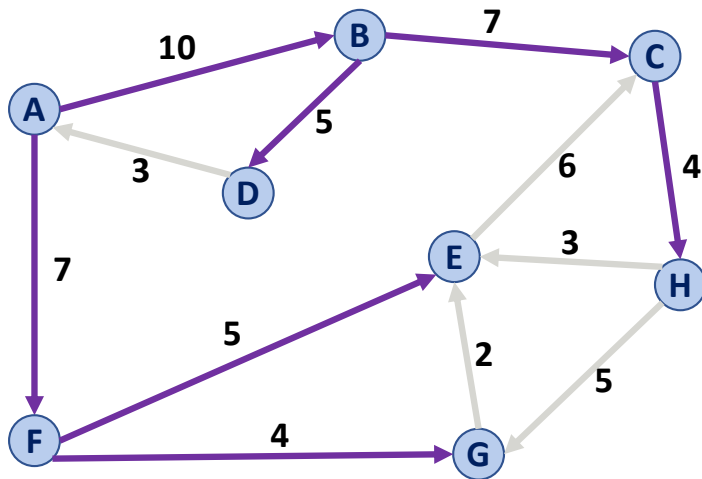
Dijkstra's Algorithm (SSSP)



```
DijkstraSSSP(G, s):
```

```
6  foreach (Vertex v : G):
7      d[v] = +inf
8      p[v] = NULL
9  d[s] = 0
10
11  PriorityQueue Q // min distance, defined by d[v]
12  Q.buildHeap(G.vertices())
13  Graph T          // "labeled set"
14
15  repeat n times:
16      Vertex u = Q.removeMin()
17      T.add(u)
18      foreach (Vertex v : neighbors of u not in T):
19          if _____ < d[v]:
20              d[v] = _____
21              p[v] = u
```

Dijkstra's Algorithm (SSSP)

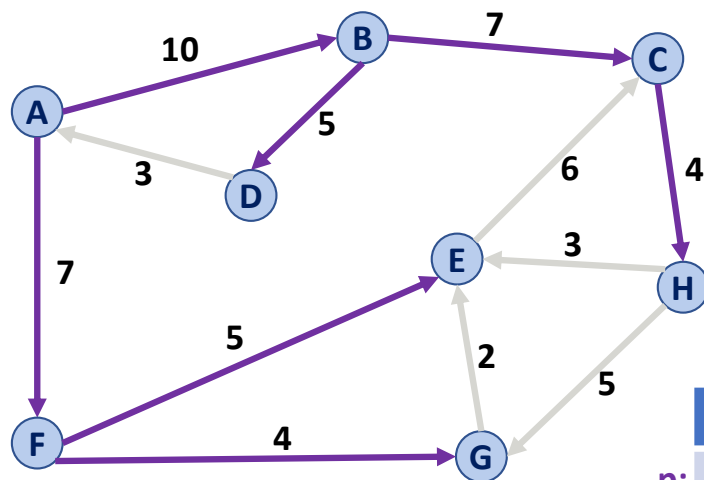


```
DijkstraSSSP(G, s):
```

```
6   foreach (Vertex v : G):
7       d[v] = +inf
8       p[v] = NULL
9   d[s] = 0
10
11   PriorityQueue Q // min distance, defined by d[v]
12   Q.buildHeap(G.vertices())
13
14   repeat n times:
15       Vertex u = Q.removeMin()
16       foreach (Vertex v : neighbors of u not in T):
17           if cost(u, v) + d[u] < d[v]:
18               d[v] = cost(u, v) + d[u]
19               p[v] = u
20
21
```

Dijkstra's Algorithm (SSSP)

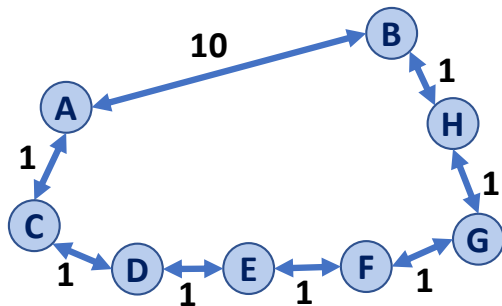
Dijkstra gives us the shortest path from our path (single source) to **every** connected vertex!



	A	B	C	D	E	F	G	H
p:	--	A	B	B	F	A	F	C
d:	0	10	17	15	12	7	11	21

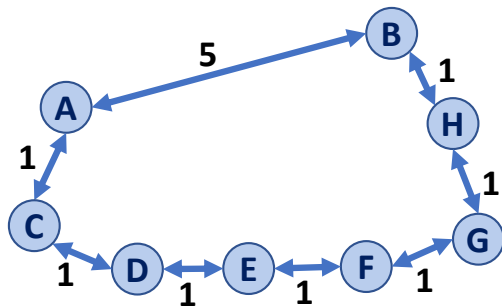
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



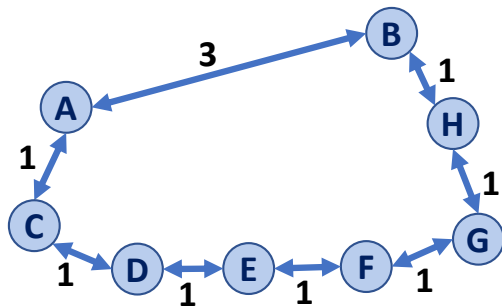
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle undirected graphs?



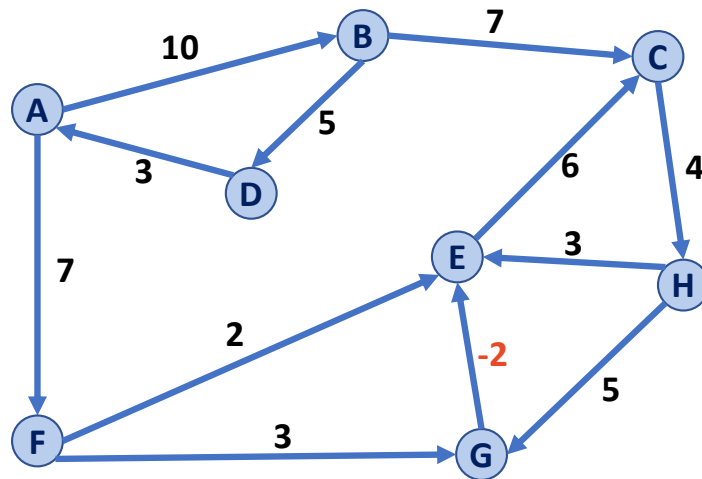
Dijkstra's Algorithm (SSSP)

What is Dijkstra's running time?

```
DijkstraSSSP(G, s):
6   foreach (Vertex v : G):
7       d[v] = +inf
8       p[v] = NULL
9   d[s] = 0
10
11   PriorityQueue Q // min distance, defined by d[v]
12   Q.buildHeap(G.vertices())
13
14   repeat n times:
15       Vertex u = Q.removeMin()
16       foreach (Vertex v : neighbors of u not in T):
17           if cost(u, v) + d[u] < d[v]:
18               d[v] = cost(u, v) + d[u]
19               p[v] = u
20
21   return T
22
23
```

Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight edges?

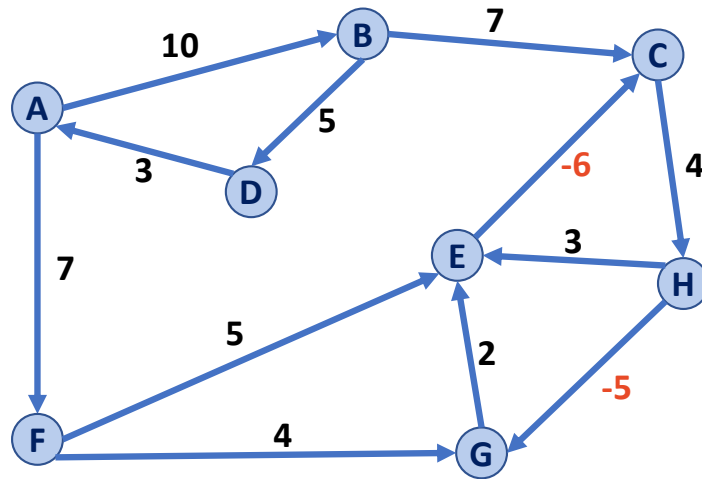


Modified Dijkstra's Algorithm (SSSP)

```
DijkstraSSSP(G, s):
6   foreach (Vertex v : G):
7       d[v] = +inf
8       p[v] = NULL
9   d[s] = 0
10
11   PriorityQueue Q // min distance, defined by d[v]
12   Q.buildHeap(G.vertices())
13   Graph T          // "labeled set"
14
15   repeat until Q.empty() times:
16       Vertex u = Q.removeMin()
17       foreach (Vertex v : neighbors of u not in T):
18           if cost(u, v) + d[u] < d[v]:
19               d[v] = cost(u, v) + d[u]
20               p[v] = u
21               Q.push(v)
22
23   return T
```

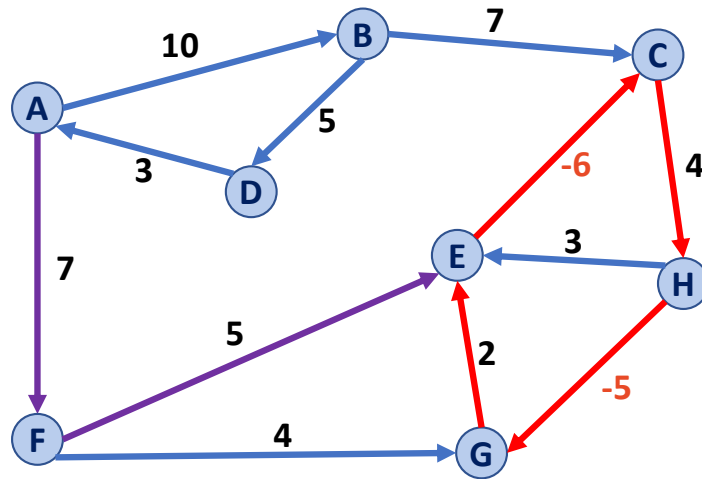
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



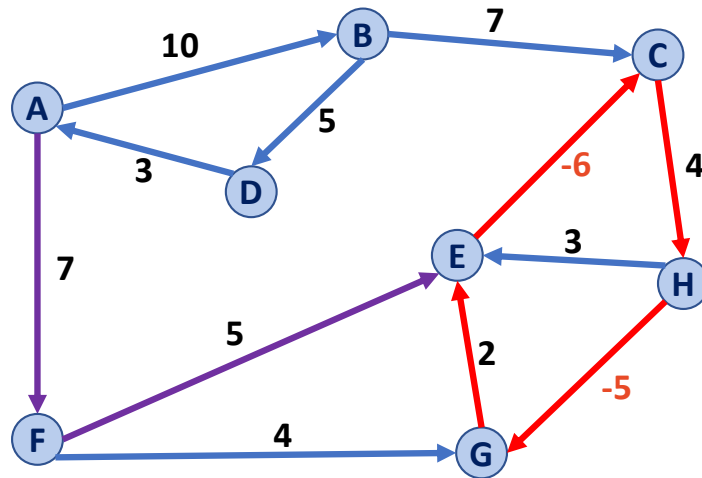
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



Shortest Path ($A \rightarrow E$): $A \rightarrow F \rightarrow E$ $\rightarrow (C \rightarrow H \rightarrow G \rightarrow E)^*$
Length: 12 Length: -5 (repeatable)