MAT1841 Continuous Mathematics for Computer Science

Assignment 3

*The assignment is to be submitted via MOODLE via **11:55 pm AEST Friday 11 October 2024**.*

See the instructions under the assessment tab on MOODLE. Be sure to press the "submit assignment" button to complete the submission. You must submit a single PDF document no larger than 100MB in size. It's the student's responsibility to ensure that the file is not corrupted.

Assignment 3 is worth 7.5% of the final mark. There are four questions.

The standard penalty of 5% of the total mark per day will apply for late work.

***Show your working.** You are required to clearly explain your steps in both English and mathematical expressions. Most of the marks will be allocated for clear working and explanations. A mathematical writing guide is available on Moodle.*

Generative AI tools cannot be used in assessment. In this assessment, you must not use generative artificial intelligence (AI) to generate any materials or content in relation to the assessment task.

1. Compute the first four non-zero terms in the Taylor series for the following functions centred about the specified point a. **[5 + 5 = 10 marks]**

   a. $f(x) = \ln(x^3)$, $a = 1$

   b. $f(x) = \cos^{-1}(e^x - 1)$, $a = 0$

2. Calculate the four cubic splines that piecewise pass through the five points

x	-1	1	2	3	5
$f(x)$	1	0	0	1	3

   with the second derivatives set equal to zero at the two endpoints (i.e., 'natural cubic spline'). Use software to graph these four splines over the domain $x \in [-1, 5]$. **[16 + 2 = 18 marks]**

3. Use integration by parts to calculate the following integrals. **[4 + 4 = 8 marks]**

   a. $I = \int x^2 \ln(2x)\, dx$

   b. $I = \int e^{2x} \cos(x)\, dx$

4. Consider the area bounded by the two functions $y = (x^3 - 4x^2 - 12x)/12$ and $y = (-x^2 + 4x + 12)/4$ over the domain $-2 \le x \le 6$. **[2 + 4 + 2 + 2 + 4 = 14 marks]**

   a. Sketch/graph the two curves.

   b. Use the Fundamental Theorem of Calculus to calculate the area between the curves.

   c. Approximate the area between the curves using Riemann sums with n = 4.

   d. Approximate the area between the curves using the Trapezoidal rule with n = 4.

   e. Approximate the area between the curves using the Trapezoidal rule with n = 8.