Midterm 1Feb. 24

YOUR NAME:

Student ID:

Instructions: This exam is closed-book. You are allowed to use a 1-page cheat sheet. Please turn off and put away electronic devices such as cell phones, laptops, etc.

You have a total of 80 minutes. The exam has 5 questions that are worth 30+10+15+15+10 = 80 points. In Q1, there's an additional 5 extra credit points. The questions may vary in difficulty, so if you get stuck on any question, it might help to leave it and try another one. In general, aim to spend no more than X minutes per X points.

Answer each question in the space provided below the question. If you need more space, you can use the reverse side of that page (and clearly indicate that you've done so). You can use without proof any result proved in class, in Sipser's book, or during discussion sections, but clearly state the result you are using.

Please indicate your name and Student ID on the top of every page.

Do not turn this page until the instructor tells you to do so!

Good luck!

Q 1. (30 pts + 5 pts extra credit = 5 pts per item) Are the following statements always **true** or sometimes **false**? Provide a brief justification (1-3 sentences) for each answer. If true explain why, if false give a counterexample.

1. Let L_1 be a non-regular language. Then $\overline{L_1}$ is non-regular.
   Answer:

2. Let L_1 and L_2 be non-regular languages. Then $L_1 \cup L_2$ is non-regular.
   Answer:

3. Let L_1 and L_2 be non-regular languages. Then $L_1 \cap L_2$ is non-regular.
   Answer:

4. Let L_1 be a non-regular language and L_2 be a regular language. Then $L_1 \cdot L_2$ is non-regular.
   Answer:

5. Let L_1 be a non-regular language. Then $L = \{0w : w \in L_1\}$ is non-regular.
   Answer:

6. Let $L_0, L_1, L_2, \cdots$ be an infinite sequence of regular languages, one for each $i \in \mathbb{N}$. Then $L = \bigcup_{i=0}^{\infty} L_i$ is regular.

   (Note that a word $w \in L$ if and only if there is some $i \in \mathbb{N}$ such that $w \in L_i$.)
   Answer:

7. **(Extra Credit)** Let L_1 be a non-regular language and L_2 be a regular language. Then $L_1 \oplus L_2$ is non-regular. [1]

   Answer:

[1] Recall that for two languages L_1 and L_2 over alphabet Σ, their XOR is defined as

$$L_1 \oplus L_2 = \{w : \ (w \in L_1 \text{ and } w \notin L_2) \text{ or } (w \notin L_1 \text{ and } w \in L_2)\ \},$$

i.e., words which belong to exactly one of L_1, L_2.

Q 2. (5 + 5 = 10 pts) Recall that for a regex R, $L(R)$ denotes the regular language of all strings matched by this regular expression. For example, $L(1(0+1)^*) = \{w \in \{0,1\}^* : w \text{ starts with } 1\}$.

1. Let $L_1 = L((0+1)^*10)$. Find a regular expression for the complement language $\overline{L_1}$.

   Answer:

2. Let $L_1 = L((0+1)^*010)$ and $L_2 = L(00(0+1)^*)$. Find a regular expression for their intersection $L_1 \cap L_2$.

   Answer:

Q 3. (10 + 5 = 15 pts) Recall that for two languages L_1 and L_2 over alphabet Σ, their XOR is defined as

$$L_1 \oplus L_2 = \{w : \ (w \in L_1 \text{ and } w \notin L_2) \text{ or } (w \notin L_1 \text{ and } w \in L_2) \ \},$$

i.e., words which belong to exactly one of L_1, L_2.

Consider two deterministic automata $M_1 = (Q_1, \Sigma, \delta_1, q_0^{(1)}, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_0^{(2)}, F_2)$ with n_1 and n_2 states respectively. Let L_1 be the language of M_1 and L_2 be the language of M_2. Let $L = L_1 \oplus L_2$.

1. Describe a DFA for L with at most $n_1 n_2$ states (it suffices to specify the construction - no further proof is needed)
   Answer:

2. Show that if $L \neq \phi$ then there is a word $w \in L$ such that $|w| \leq n_1 n_2$.
   Answer:

Q 4. (15 pts) Show that the following language over alphabet $\{0, 1, 2\}$ is not regular:

$$L_{eq} = \{0^m 1^n 2^r \mid (m = n) \text{ or } (n = r)\}.$$

Answer:

Q 5. (10 pts) Find a context-free grammar for the following language over alphabet $\Sigma = \{0, 1\}$

$$L = \{0^n w w^R 1^n \mid w \in \{0,1\}^*, n \in \mathbb{N}\}.$$

(Providing the grammar is sufficient, no justification is required.)
Answer: