

5. When  $m = 1$   $\varphi$  only have 1 clause:  $C$   
because  $\varphi$  is formula.

So there is a pure variable  $x$  in  $C$

if  $x$  only positively set  $x = \text{True}$

if  $x$  only negatively set  $x = \text{False}$ .

Then  $C$  is satisfied

$\varphi$  is satisfiable

Set: For all  $k \leq m$ , any formula CNF containing  $k$  clauses is satisfiable.

when  $m \geq 2$

take any one of the clause  $C$ .

because  $\varphi$  formula.

also a pure variable  $x$  in  $C$ .

if  $x$  only positively  $x = \text{True}$

if  $x$  only negatively  $x = \text{False}$ .

every clause with  $x$  is satisfied now.

delete all these clauses.

so get the remaining formula  $\varphi'$

$\varphi'$  also CNF and clauses' number is less than  $n$

take any clause  $D$  of  $\varphi'$

in  $\varphi$  before,  $D$  had a witness variable  $y$

because  $\varphi$  is formulas.

so  $y$  only appears in one polarity

just delete the clause with  $x$  do not change  $y$ 's polarity.

if  $D$  have  $x$ , it has already been deleted.

so  $D$  also have one polarity variable  $y$ .

$\varphi'$  formulas.

$\varphi'$  is satisfiable, all deleted clauses are satisfied by  $x$   
the clauses in  $\varphi'$  are satisfied by  $y$ . so  $\varphi$  is satisfiable.