Data Structures and Algorithms
Hash Table Finale and Bloom Filters

CS 225
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Department of Computer Science

From slides by Brad Solomon

A Hash Table based Dictionary

Client Code:

```
1  Dictionary<KeyType, ValueType> d;

2  d[k] = v;
```

A **Hash Table** consists of three things:

1. A hash function

2. A data storage structure

3. A method of addressing *hash collisions*

Running Times *(Don't memorize these equations, no nee*

The expected number of probes for find(key) under SUHA

Linear Probing:

- Successful: **½(1 + 1/(1-α))**
- Unsuccessful: **½(1 + 1/(1-α))²**

Double Hashing:

- Successful: **1/α * ln(1/(1-α))**
- Unsuccessful: **1/(1-α)**

Separate Chaining:

- Successful: **1 + α/2**
- Unsuccessful: **1 + α**

Instead, observe:

- As α increases:

- If α is constant:

Running Times

The expected number of probes for find(key) under SUHA

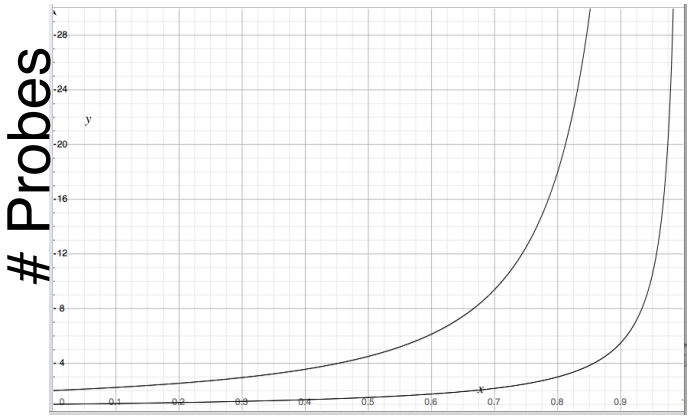
Linear Probing:

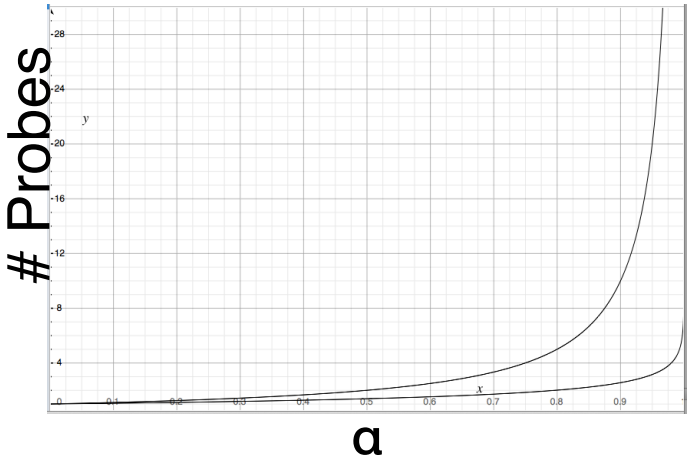
- Successful: **½(1 + 1/(1-α))**
- Unsuccessful: **½(1 + 1/(1-α))²**

Double Hashing:

- Successful: **1/α * ln(1/(1-α))**
- Unsuccessful: **1/(1-α)**

When do we resize?





Which collision resolution strategy is better?

- Big Records:

- Structure Speed:

What structure do hash tables implement?

What constraint exists on hashing that doesn't exist with BSTs?

Why talk about BSTs at all?

Running Times

	Hash Table	AVL	Linked List
Find	Expectation*: Worst Case:		
Insert	Expectation*:Worst Case:		
Storage Space			

std data structures

std::map

::operator[]

::insert

::erase

::lower_bound(key) ➔ Iterator to first element ≤ key

::upper_bound(key) ➔ Iterator to first element > key

std data structures

std::unordered_map

::operator[]

::insert

::erase

~~::lower_bound(key) ➔ Iterator to first element ≤ key~~

~~::upper_bound(key) ➔ Iterator to first element > key~~

::load_factor()

::max_load_factor(ml) ➔ Sets the max load factor

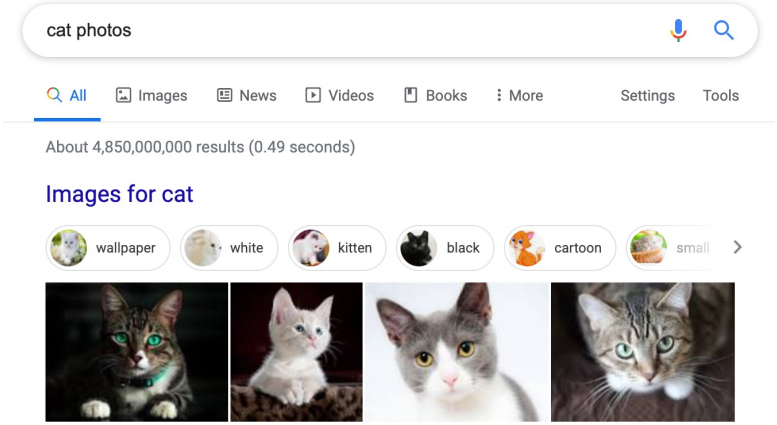
Data Structures Review

What method would you use to build a search index on a collection of objects?

Memory-Constrained Data Structures

What method would you use to build a search index on a collection of objects *in a memory-constrained environment*?

Constrained by Big Data (Large N)



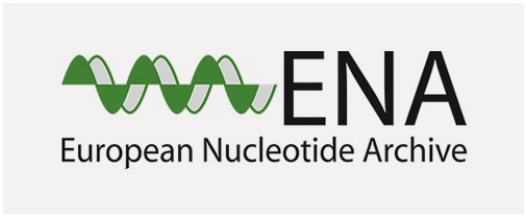
Google Index Estimate: >60 billion webpages

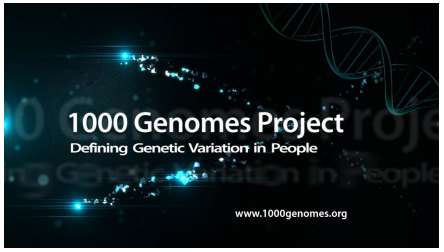
Google Universe Estimate (2013): >130 trillion webpages

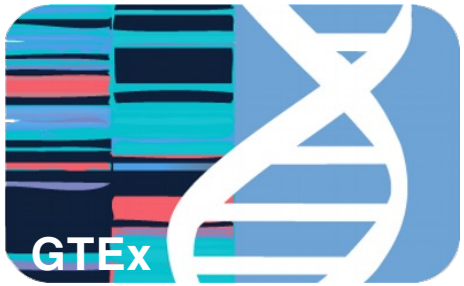
Memory-Constrained Data Structures

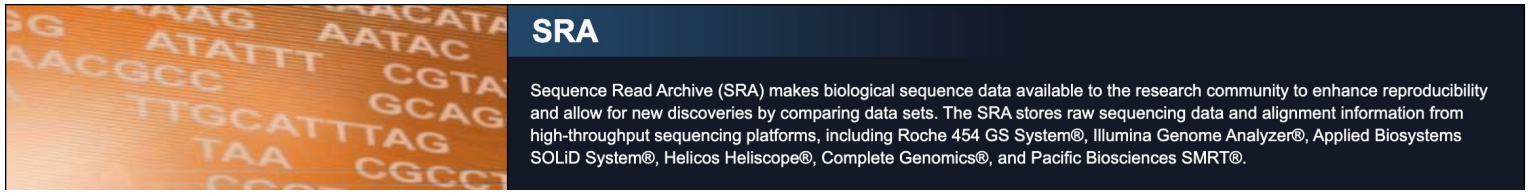
What method would you use to build a search index on a collection of objects *in a memory-constrained environment*?

Constrained by Big Data (Large N)









Sequence Read Archive Size: >60 petabases (10^{15})

Memory-Constrained Data Structures

What method would you use to build a search index on a collection of objects *in a memory-constrained environment*?

Constrained by Big Data (Large N)

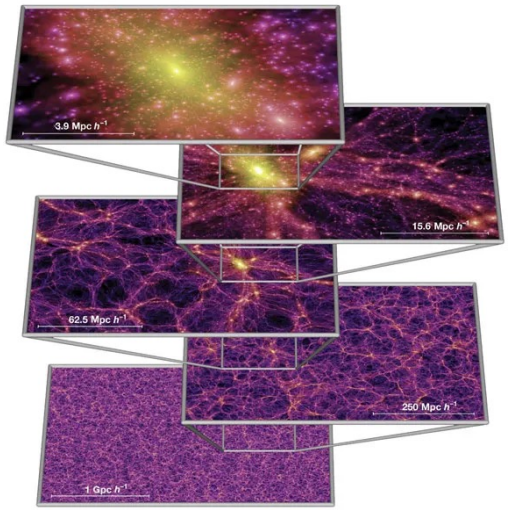


Image: https://doi.org/10.1038/nature03597

Sky Survey Projects	Data Volume
DPOSS (The Palomar Digital Sky Survey)	3 TB
2MASS (The Two Micron All-Sky Survey)	10 TB
GBT (Green Bank Telescope)	20 PB
GALEX (The Galaxy Evolution Explorer)	30 TB
SDSS (The Sloan Digital Sky Survey)	40 TB
SkyMapper Southern Sky Survey	500 TB
PanSTARRS (The Panoramic Survey Telescope and Rapid Response System)	~ 40 PB expected
LSST (The Large Synoptic Survey Telescope)	~ 200 PB expected
SKA (The Square Kilometer Array)	~ 4.6 EB expected

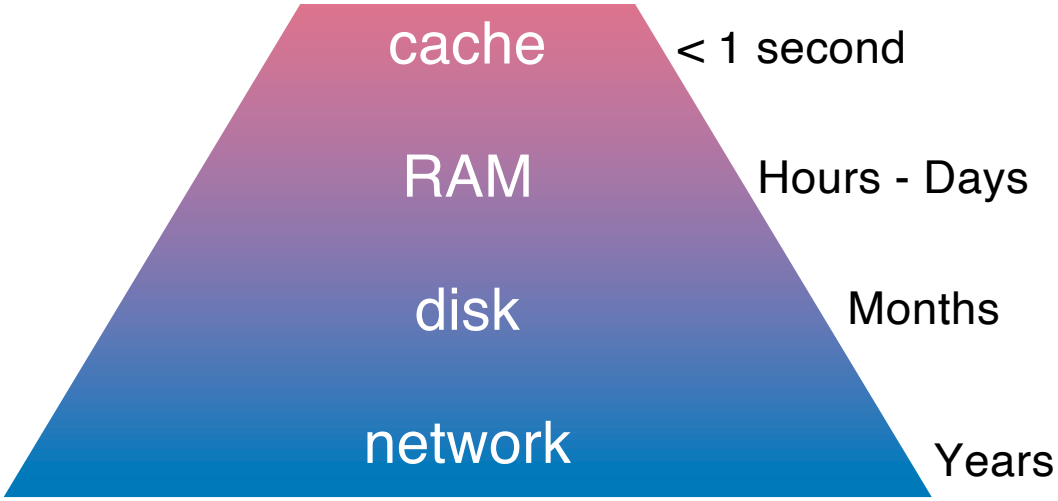
Table: http://doi.org/10.5334/dsj-2015-011

Estimated total volume of one array: 4.6 EB

Memory-Constrained Data Structures

What method would you use to build a search index on a collection of objects *in a memory-constrained environment*?

Constrained by resource limitations



(Estimates are Time x 1 billion courtesy of https://gist.github.com/hellerbarde/2843375)

Memory-Constrained Data Structures

What method would you use to build a search index on a collection of objects *in a memory-constrained environment*?

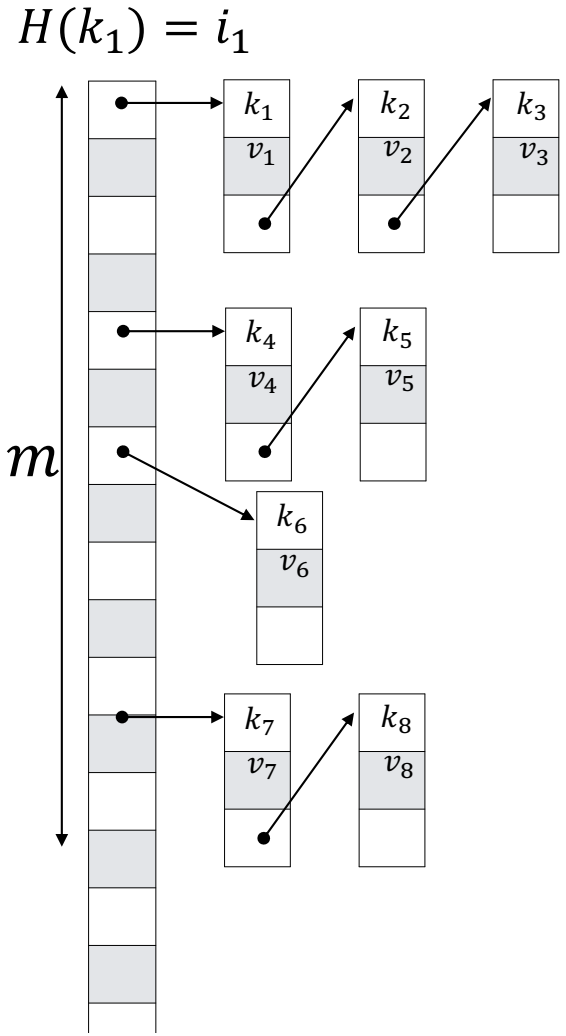
Reducing storage costs

1) Throw out information that isn't needed

2) Compress the dataset

Reducing a hash table

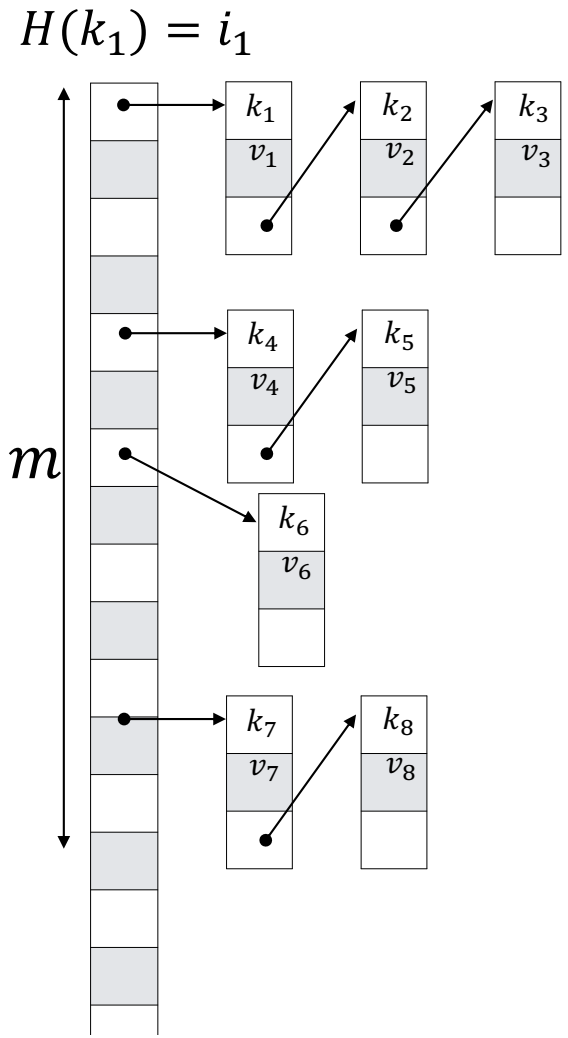
What can we remove from a hash table?



Reducing a hash table

What can we remove from a hash table?

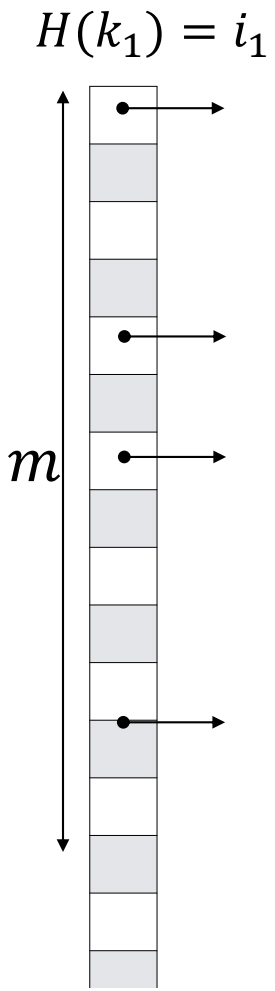
Take away values



Reducing a hash table

What can we remove from a hash table?

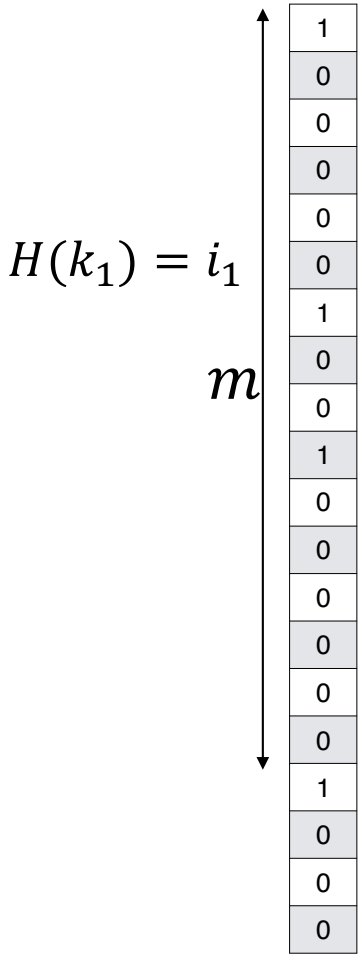
Take away values and keys



Reducing a hash table

What can we remove from a hash table?

Take away values and keys



Bloom Filter: Insertion

S = { 16, 8, 4, 13, 29, 11, 22 }

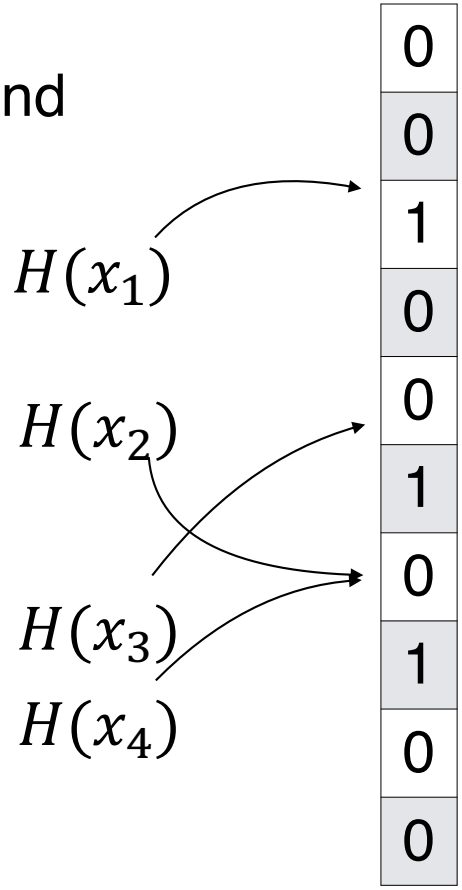
h(k) = k % 7

0	0
1	0
2	0
3	0
4	0
5	0
6	0

Bloom Filter: Insertion

An item is inserted into a bloom filter by hashing and then setting the hash-valued bit to 1

If the bit was already one, it stays 1



Bloom Filter: Deletion

S = { 16, 8, 4, 13, 29, 11, 22 }

h(k) = k % 7

0	0
1	1
2	1
3	0
4	1
5	0
6	1

```
_delete(13)
```

```
_delete(29)
```

Bloom Filter: Search

S = { 16, 8, 4, 13, 29, 11, 22 }

h(k) = k % 7

0	0
1	1
2	1
3	0
4	1
5	0
6	1

`_find(16)`

`_find(20)`

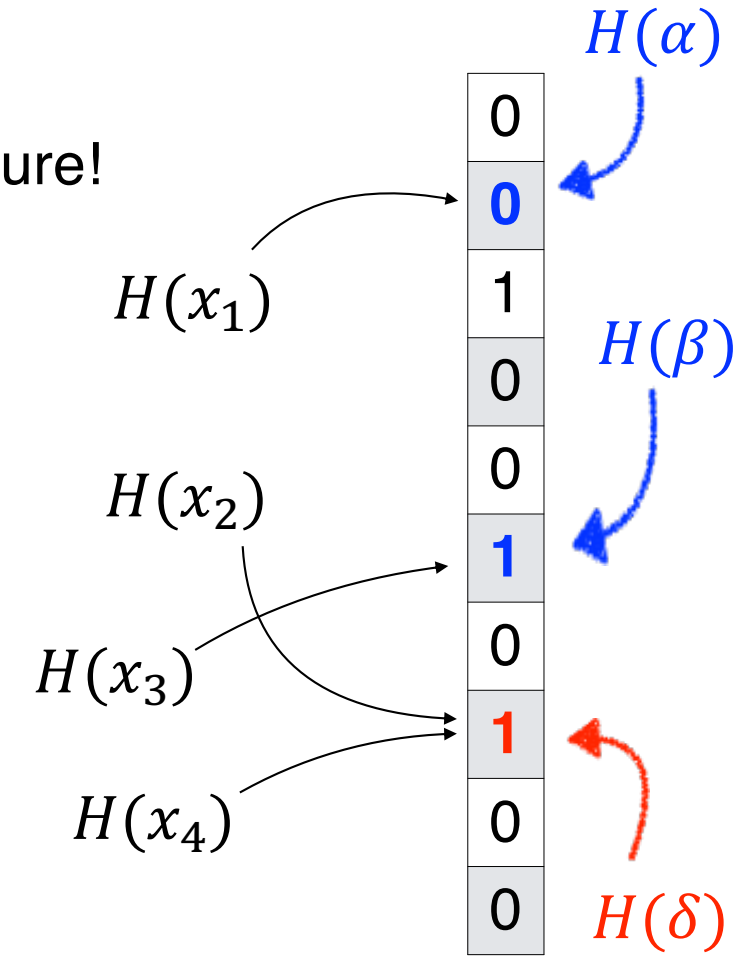
`_find(3)`

Bloom Filter: Search

The bloom filter is a *probabilistic* data structure!

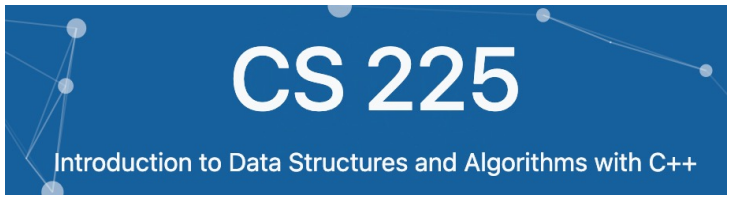
If the value in the BF is 0:

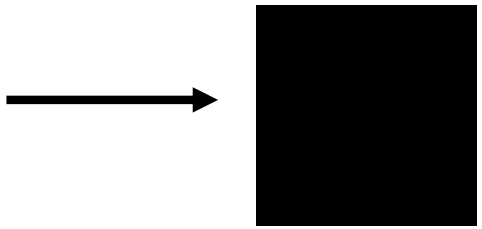
If the value in the BF is 1:



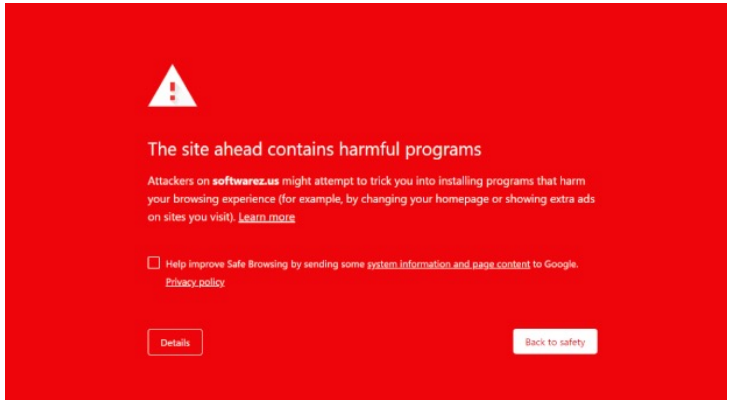
Probabilistic Accuracy: Malicious Websites

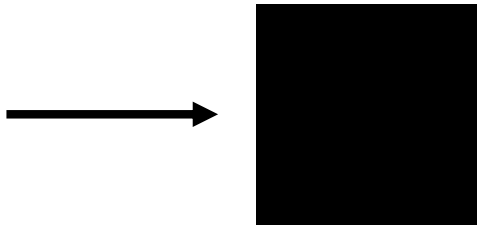
Imagine we have a detection oracle that identifies if a site is malicious





“Not malicious”





“Malicious”

Probabilistic Accuracy: Malicious Websites

Imagine we have a detection oracle that identifies if a site is malicious

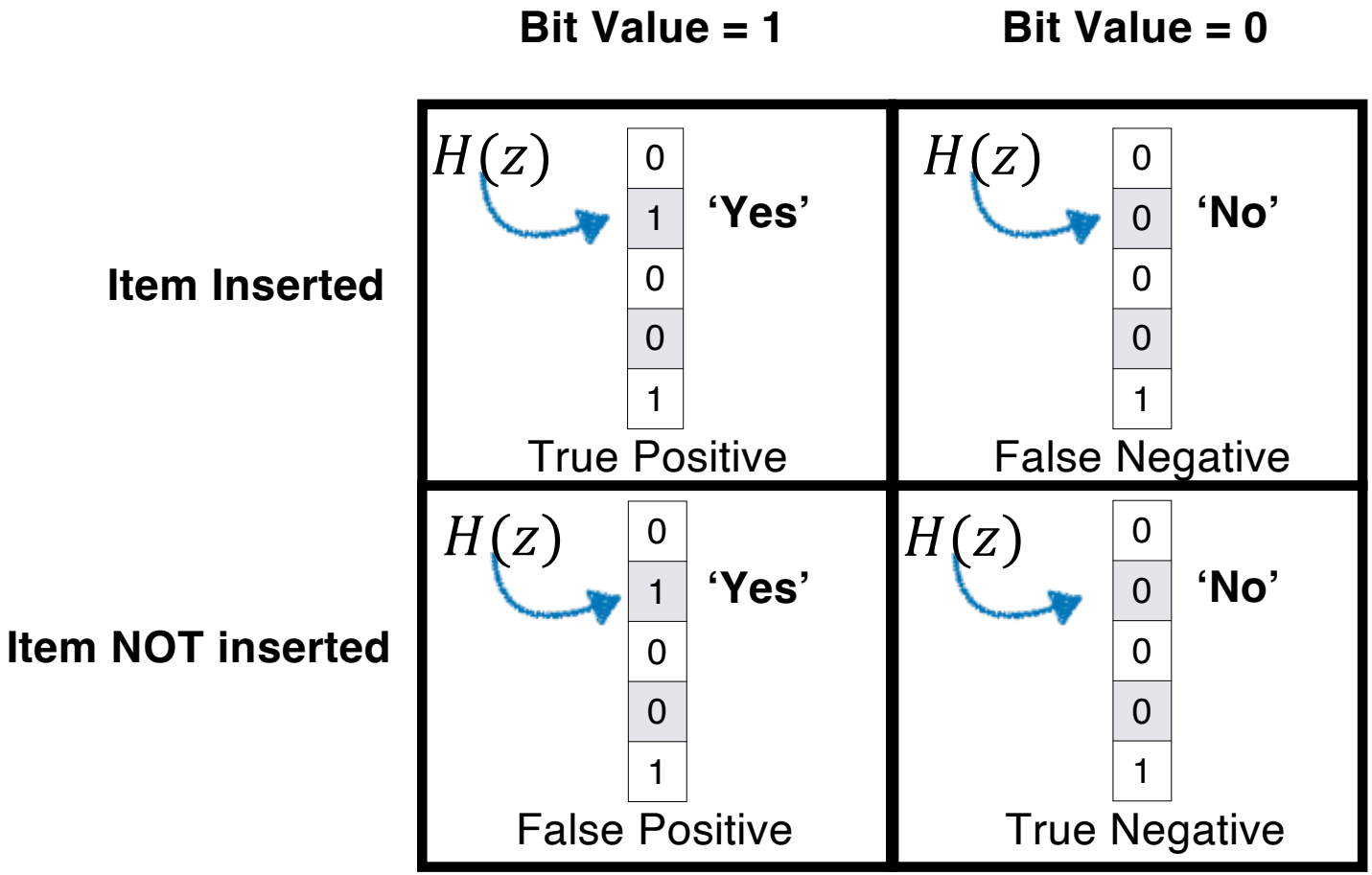
True Positive:

False Positive:

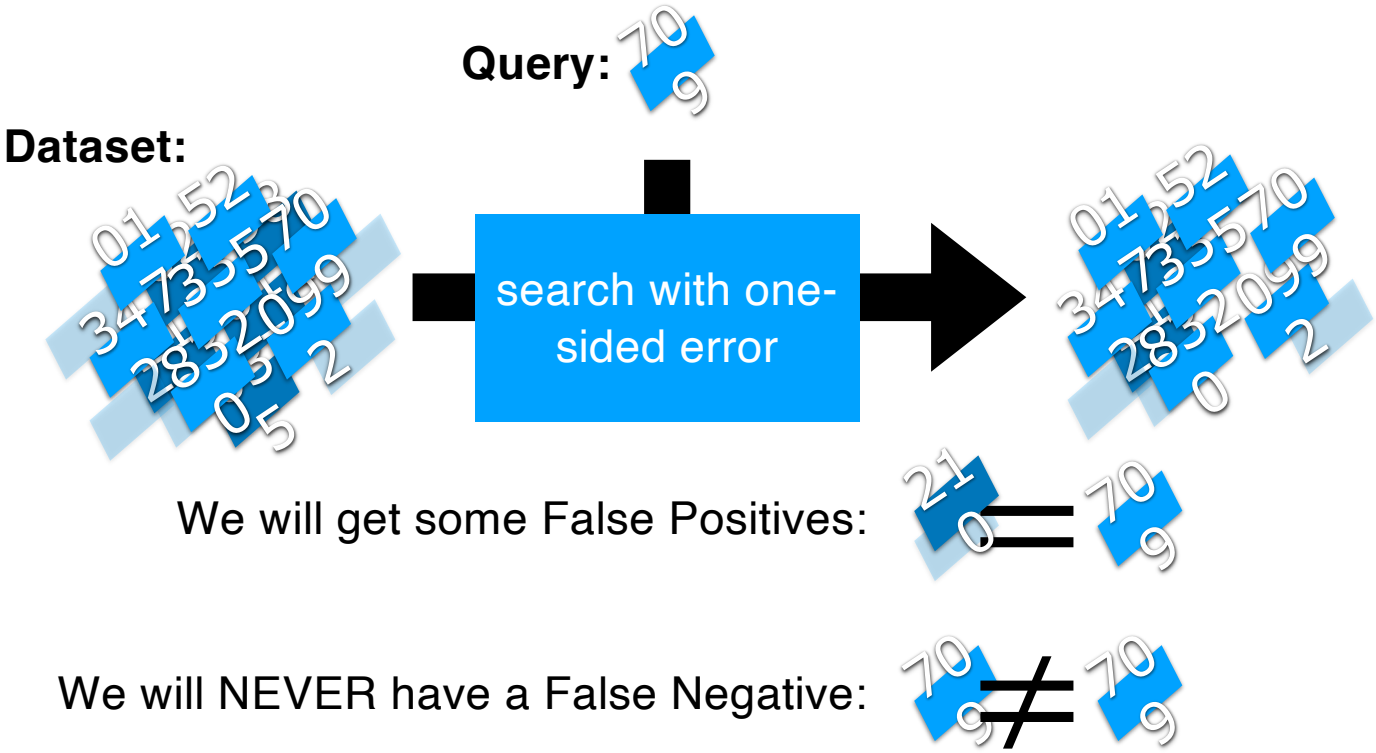
False Negative:

True Negative:

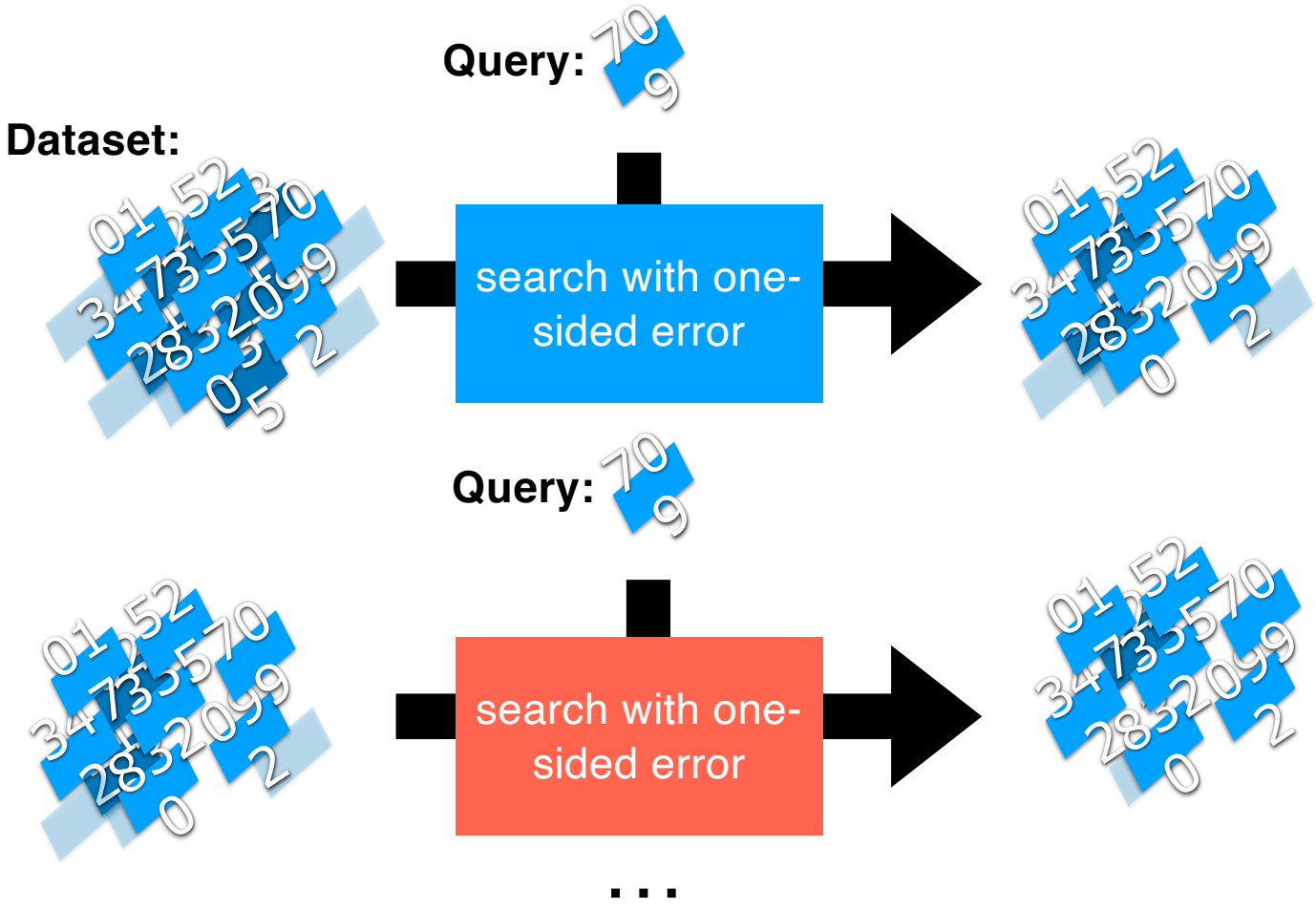
Imagine we have a **bloom filter** that **stores malicious sites…**



Probabilistic Accuracy: One-sided error



Probabilistic Accuracy: One-sided error

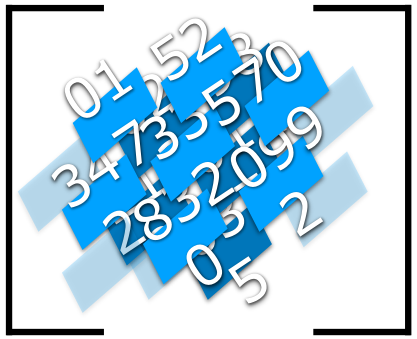


Bloom Filter: Repeated Trials

Use many hashes/filters; add each item to each filter

h_1

0
1
0
1
0
0
0
1
0
1
1
0
1
0
1
1
0
1
0
1



Bloom Filter: Repeated Trials

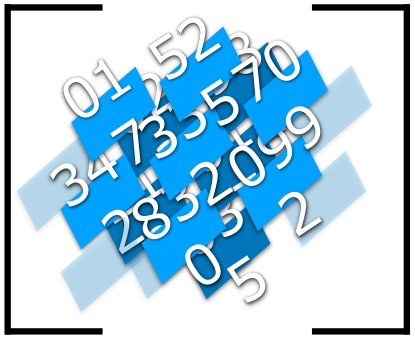
Use many hashes/filters; add each item to each filter

h_1

0
1
0
1
0
0
0
1
0
1
1
0
1
0
1
1
0
1
0
1

h_2

0
0
0
1
0
0
0
1
0
1
0
0
1
1
1
0
0
1
0
0



Bloom Filter: Repeated Trials

Use many hashes/filters; add each item to each filter

h_1

0
1
0
1
0
0
0
1
0
1
1
0
1
0
1
1
0
1
0
1

h_2

0
0
0
1
0
0
0
1
0
1
0
0
1
1
1
0
0
1
0
0

h_3

0
1
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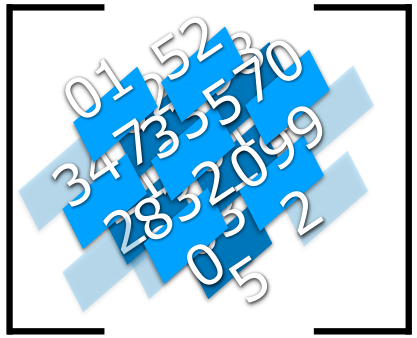
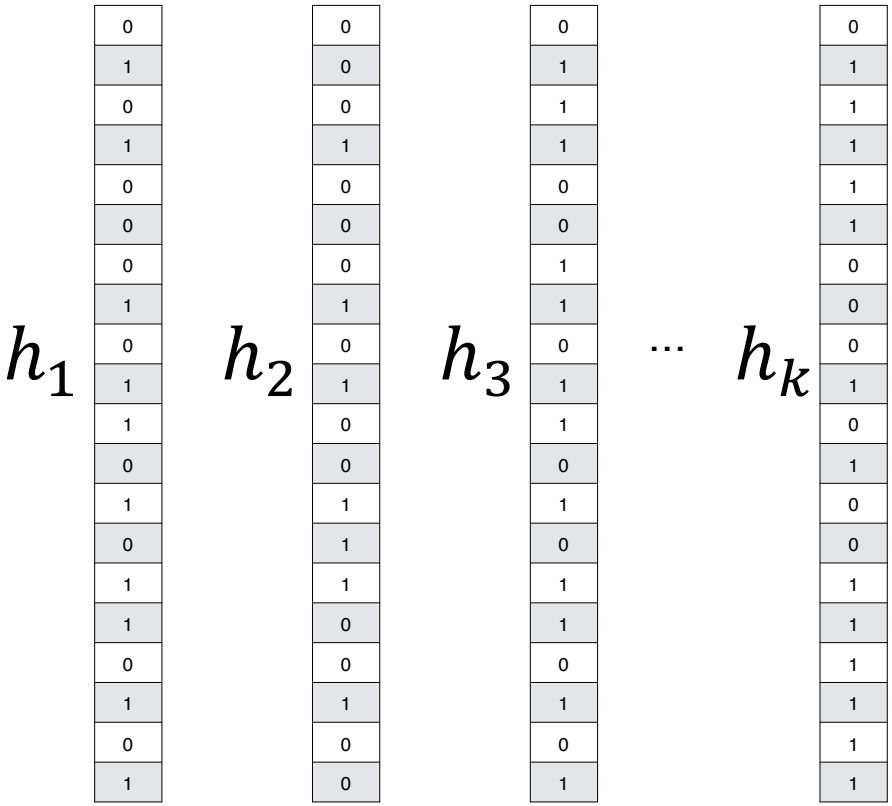
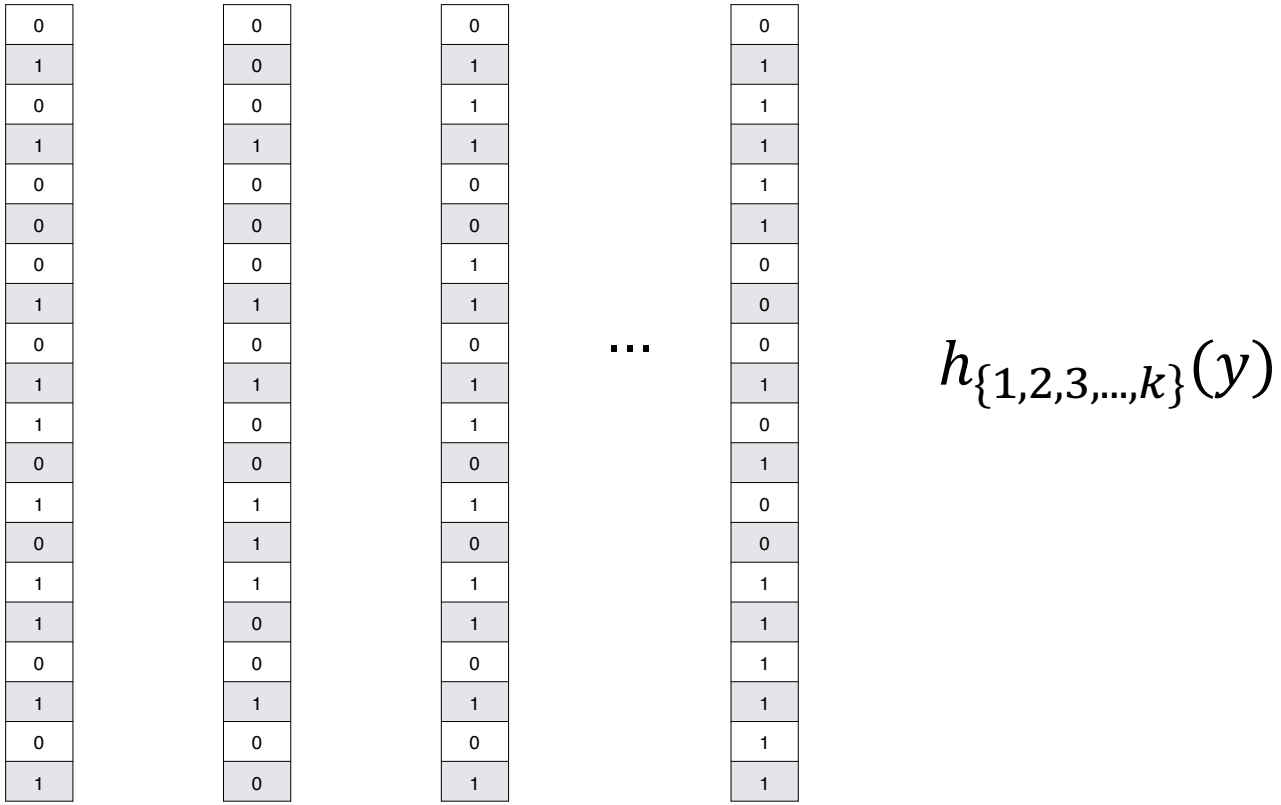


Bloom Filter: Repeated Trials

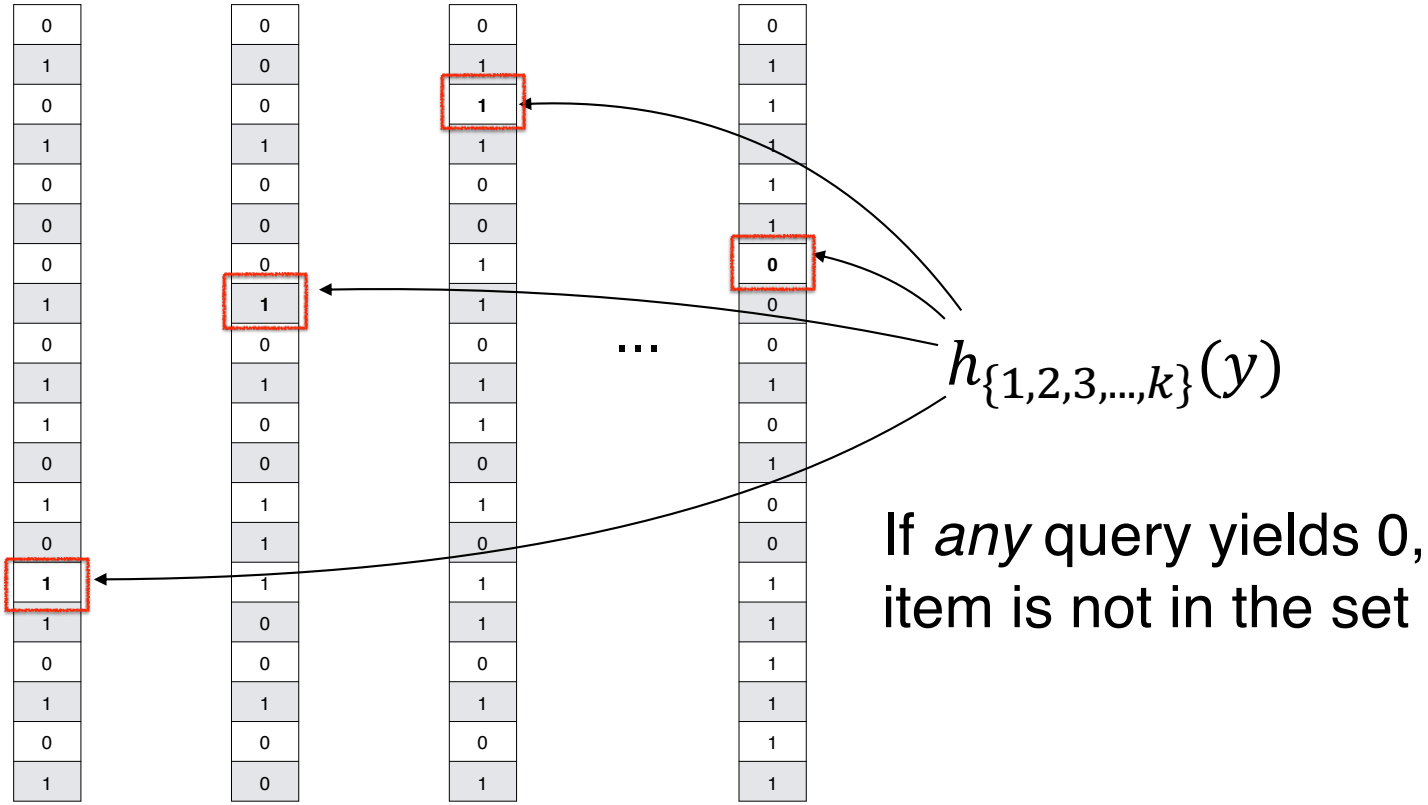
Use many hashes/filters; add each item to each filter



Bloom Filter: Repeated Trials

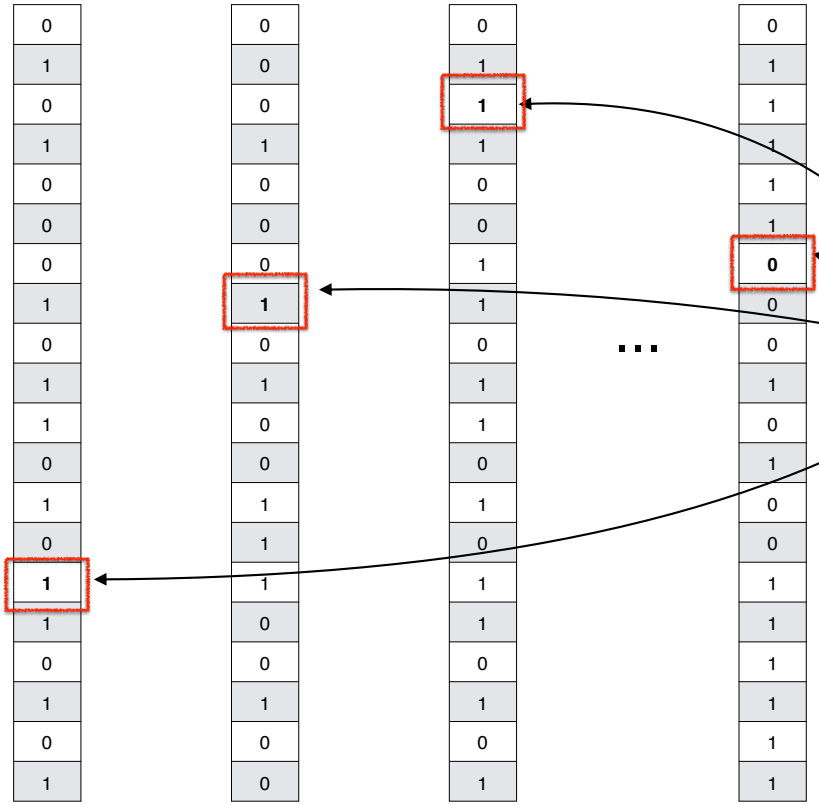


Bloom Filter: Repeated Trials



If *any* query yields 0, item is not in the set

Bloom Filter: Repeated Trials



If *all* queries yield 1, item *may* be in the set; or we might have collided *k* times

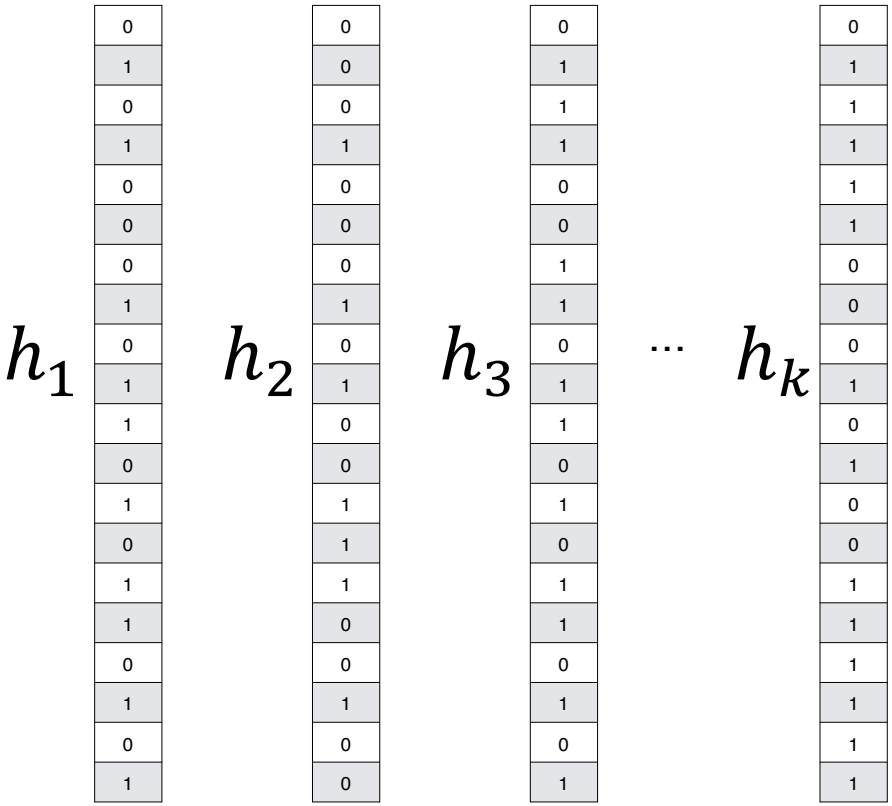
Bloom Filter: Repeated Trials

Using repeated trials, even a very bad filter can still have a very low FPR!

If we have k bloom filter, each with a FPR p, what is the likelihood that ***all*** filters return the value ‘1’ for an item we didn’t insert?

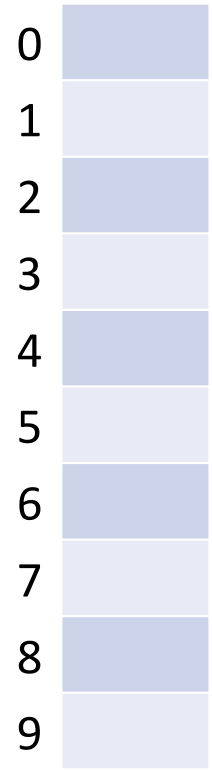
Bloom Filter: Repeated Trials

But doesn't this hurt our storage costs by storing k separate filters?



Bloom Filter: Repeated Trials

Rather than use a new filter for each hash, one filter can use k hashes



S = { 6, 8, 4 }

$h_1(x) = x \% 10$ **$h_2(x) = 2x \% 10$** **$h_3(x) = (5+3x) \% 10$**

Bloom Filter: Repeated Trials

Rather than use a new filter for each hash, one filter can use k hashes

0	0
1	0
2	1
3	1
4	1
5	0
6	1
7	1
8	1
9	1

$h_1(x) = x \% 10$ **$h_2(x) = 2x \% 10$** **$h_3(x) = (5+3x) \% 10$**

```
_find(1)
```

```
_find(16)
```

Bloom Filter

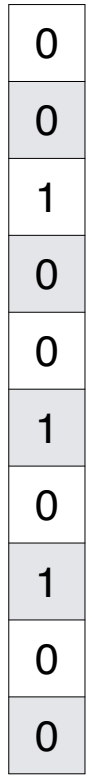
A probabilistic data structure storing a set of values

$H = \{h_1, h_2, \ldots, h_k\}$

Built from a bit vector of length m and k hash functions

Insert / Find runs in: _____

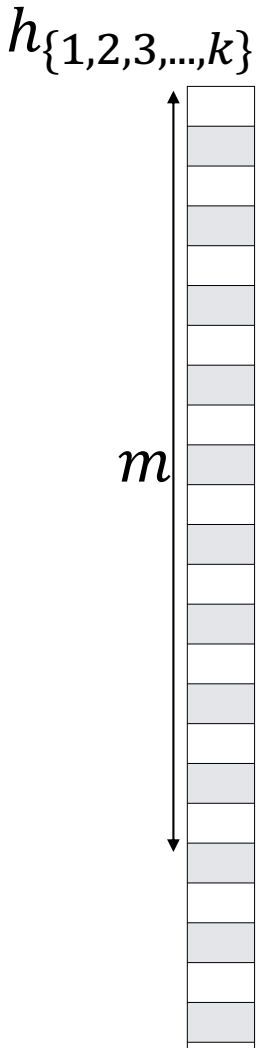
Delete is not possible (yet)!



Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

What is our expected FPR after n objects are inserted?

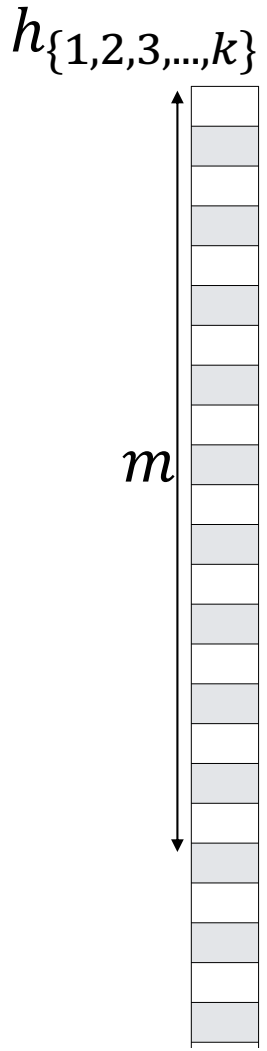


Bloom Filter: Error Rate

Given bit vector of size m and 1 SUHA hash function

What's the probability a specific bucket is 1 after one object is inserted?

Same probability given k SUHA hash function?

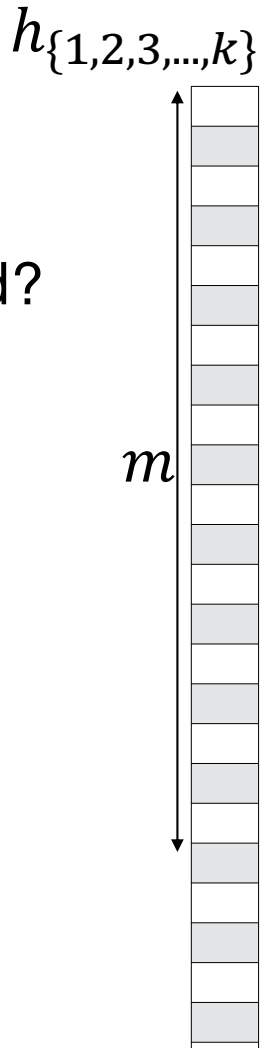


Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

Probability a specific bucket is 0 after one object is inserted?

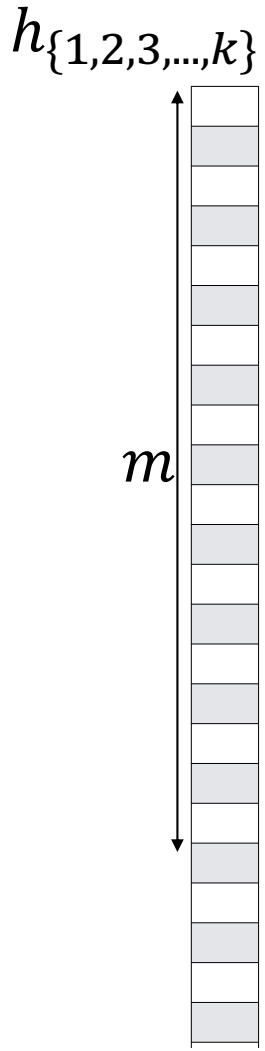
After n objects are inserted?



Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

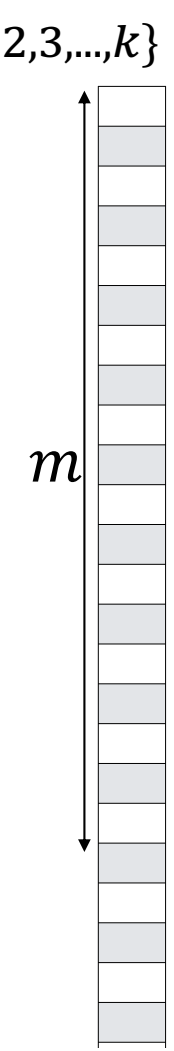
What's the probability a specific bucket is 1 after n objects are inserted?



Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

What is our expected FPR after n objects are inserted?



The probability my bit is 1 after n objects inserted

$$\left(1 - \left(1 - \frac{1}{m}\right)^{nk}\right)^{k}$$

The number of [assumed independent] trials

Bloom Filter: Error Rate

Vector of size m, k SUHA hash function, and n objects

To minimize the FPR, do we prefer…

(A) large k (**B) small k**

$$\left(1 - \left(1 - \frac{1}{m}\right)^{nk}\right)^{k}$$