Math 20C, Final Exam
Winter 2024

Name:

PID:

1. You have **180 minutes** to complete this exam.
2. Write your name on every page.
3. There are **9 questions** in this exam.
4. You may use one page (both sides) of **handwritten** notes.
5. No books or calculators are allowed.
6. No cellular phones or any other electronic devices are allowed.
7. All work must be your individual efforts.
8. **Always justify your answers and show all your work.** Write your answers and all accompanying work *neatly*

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1. (5 pts) Find an equation of the plane that passes through the points $(3, 0, -1)$ and $(1, -1, 2)$ and that is parallel to the y-axis. (Give your answer in the form $ax + by + cz = d$.)

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2. (5 pts) Let $u = \sqrt{r^2 + s^2 + 4}$ where $r = y(x - 2z)^3$ and $s = x + \sin(yz)$. **Use the chain rule** to find

$$\frac{\partial u}{\partial y} \qquad \text{and} \qquad \frac{\partial u}{\partial z}$$

when $x = 1$, $y = 2$, and $z = 0$. (Chain rule must be used in order to receive credit.)

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3. Find the limit if it exists or show that the limit does not exist.

   (a) (4 pts) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\cos(xy)-1}{x^2y^2}$

   (b) (4 pts) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2\tan(x^2)}{x^2+y^2}$

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4. (5 pts) Find parametric equations for the line through the point $P = (0, 1, 2)$ that intersects and is perpendicular to the line $\mathbf{r}(t) = \langle 2 + t, 3 + 2t, 2 - t \rangle$.

(Hint : First find the point Q on the line $\mathbf{r}(t)$ so that $\overrightarrow{PQ}$ is perpendicular to the line $\mathbf{r}(t)$.)

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5. (5 pts) Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors in $\mathbb{R}^3$. Assume $\|\mathbf{u}\| = 3$ and that the angle between $\mathbf{u}$ and $\mathbf{v}$ is $\frac{\pi}{3}$. Suppose $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-3\mathbf{v}$ are perpendicular. Find $\|\mathbf{v}\|$.

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6. Let $f(x, y, z) = \dfrac{x^2}{4} + y^2 + \dfrac{z^2}{16}$.

(a) (4 pts) Find the directional derivative of f at $(-2, 1, 4)$ in the direction of $\mathbf{v} = \langle 1, 2, -2 \rangle$.

(b) (5 pts) Find all the points on the surface $f(x, y, z) = 3$ at which the tangent plane is normal to the vector $\mathbf{n} = \langle 1, 0, \frac{1}{2} \rangle$.

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7. (7 pts) Find the global minimum and maximum values of the function

$$f(x, y) = x^2 - y^2 - 4x + 2y$$

on the domain $D = \{(x, y) \,|\, 0 \leq x,\, 0 \leq y,\, y \leq 4 - x\}$.

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8. (7 pts) **Use the method of Lagrange multipliers** to find the maximum and minimum values of

$$f(x, y, z) = x^2 + 2y + 4z$$

subject to the constraint $x^2 + y^2 + 2z^2 = 12$. (Lagrange multipliers must be used in order to receive credit.)

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9. Compute the following integrals:

(a) (5 pts) $\displaystyle\iint_R x^2 \cos(xy)\, dA \quad \text{where } R = [1,2] \times [0,\pi].$

(b) (5 pts) $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{2x}{y^5+1}\, dy\, dx$