Homework 3

Release date: Sep. 13, Due date: Sep. 20.

Guidelines: Submit your solutions in pdf format on Gradescope by **10pm on Sep. 20**. Solutions may be either typed in LaTeX or Word (with either machine-drawn or hand-drawn diagrams) or written **legibly** by hand. Please be sure to begin the solution for each problem on a new page, and to tag each of your solutions to the correct problem. Take time to write **clear** and **concise** answers. You are encouraged to form small groups to work through the homework, but you **must** write up all your solutions on your own. Depending on grading resources, we reserve the right to grade a random subset of the problems and check off the rest; so you are advised to attempt all the problems.

Q 1 (30 points - 5 points per item)**.** For each of the following languages, determine whether they are regular or non-regular.

If you claim the language is regular, please provide evidence for that statement, i.e., a description of a DFA or an NFA or a regular expression recognizing the language (there's no need to formally prove that the DFA/NFA/regex recognizes the language, just to give a brief description of it).

If you claim the language is non-regular, please provide proof using either the pumping lemma, the Myhill-Nerode theorem, or closure properties.

(a) $L_{\text{OR}} = \{a^i b^j \mid \text{either } i \text{ is odd or } i \geq j\}$

(b) $L_{\text{NEQ}} = \{a^i b^j \mid i \neq j\}$

(c) $L_{\text{PROD}} = \{w \in \{a,b\}^* \mid \#_a(w) \cdot \#_b(w) \text{ is } \textit{even}\}$.
(Here $\#_a(w), \#_b(w)$ denote number of occurrences of a, b in w respectively.)

(d) $L_{01=10} = \{w \in \{0,1,2\}^* \mid w \text{ has the same number of 01's as 10's}\}$
(Note that this is over the ternary alphabet $\{0,1,2\}$).

(e) $L_{sts} = \{s \cdot t \cdot s \mid s, t \in \{0,1\}^+\}$.

(f) $L_{\text{palindrome}} = \{w \in \{0,1\}^* \mid w = w^R\}$.

Q 2. (10 points - 5 points per item) Each of the following languages is regular, and hence has a pumping length. Determine the minimum pumping length (the smallest p that satisfies the condition of the pumping lemma).

(a) $L = L(1011)$

(b) $L = L((01)^*)$

Q 3 (8 + 8 + 4 = 20 points)**.** Consider the following language over $\Sigma = \{a, b, c\}$:

$$F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}.$$

(a) Show that F is not regular.

(b) Show that F acts like a regular language in the pumping lemma. In other words, give a pumping length p and demonstrate that F satisfies the three conditions of the pumping lemma for this value of p.

(c) Explain why parts (a) and (b) do not contradict the pumping lemma.

Q 4 (5 + 2 + 8 = 15 points)**.** In this problem, we will classify all regular languages over the unary alphabet $\Sigma = \{1\}$.

(a) Let $a, b \in \mathbb{Z}_{\geq 0}$. Show that the following language is regular:

$$L = \{1^{ax+b} : x \in \mathbb{Z}_{\geq 0}\}.$$

(b) Let $a_1, b_1, \ldots, a_k, b_k \in \mathbb{Z}_{\geq 0}$. Show that the following language is regular:

$$L = \{1^{a_i x+b_i} : x \in \mathbb{Z}_{\geq 0}, i \in \{1, \ldots, k\}\}.$$

(c) Show that every regular language L over the unary alphabet is of the form

$$L = \{1^{a_i x+b_i} : x \in \mathbb{Z}_{\geq 0}, i \in \{1, \ldots, k\}\}$$

for some $a_1, b_1, \ldots, a_k, b_k \in \mathbb{Z}_{\geq 0}$.

Q 5 (Extra credit = 8 points)**.** Let L be an infinite regular language. Prove that L can be partitioned into two infinite disjoint regular languages. That is, there are two infinite regular languages A and B such that $A \cup B = L$ and $A \cap B = \emptyset$.