

Math 20C : Final exam

Instructions. You are allowed to consult your textbook, your notes, and the lecture videos. Show all of your work. **No credit will be given for unsupported answers, even if correct. You are not allowed to collaborate or communicate with any other humans while working on this exam.**

This exam is worth 60 points.

1. (14 points) Answer the following questions:

(a) (6 points) Find an equation for the line that passes through the point $(1, 4, -2)$ and is parallel to the line $\langle 5 + t, 3 - t, -2 + 2t \rangle$.

(b) (8 points) Find an equation for the plane that passes through the point $(-2, 3, -1)$ and is orthogonal to the line of intersection of the planes $2x + y - 3z = 0$ and $x - y + 2z = 3$.

2. (8 points) Find the point on the graph of $z = (x^2 + y)^2$ at which the tangent plane is parallel to the plane $2x + 2y - z = 0$.

3. (11 points)

(a) (6 points) Let $f(x, y, z) = \frac{xy}{z^2 + 1}$.

Find the maximum rate of increase of f at $(2, 2, 1)$.

(b) (5 points) Let $g(t)$ be a differentiable function.

Show that $u(x, y) = g(xy)$ satisfies $x \frac{\partial u}{\partial x} = y \frac{\partial u}{\partial y}$.

4. (12 points) Use Lagrange multipliers to find the absolute maximum and minimum values of $f(x, y) = x^2 + xy$ subject to the constraint $x^2 + xy + y^2 \leq 3$.

5. (15 points) Compute the following integrals:

(a) (5 points) $\iint_R e^{x-y} dA$, where $R = [1, 2] \times [0, 1]$.

(b) (6 points) $\int_0^1 \int_{y^2}^1 y \sin(x^2) dx dy$.

(c) (4 points) $\iint_R (\sin^2(\sqrt{x}) + \cos^2(\sqrt{y})) dA$, where $R = [0, 2] \times [0, 2]$.