Midterm 1 Exam

CS 172 Fall 2022
Date: 29 Sep 2022

YOUR NAME:

Student ID:

Instructions: This exam is closed-book. You are allowed to use a 1-page cheat sheet. Please turn off and put away electronic devices such as cell phones, laptops, etc.
You have a total of 90 minutes. The exam has 5 questions that are worth 30+18+12+18+12 = 90 points. The questions may vary in difficulty, so if you get stuck on any question, it might help to leave it and try another one. In general, aim to spend no more than X minutes per X points.

Answer each question in the space provided below the question in **legible handwriting**. If you need more space, you can use the reverse side of that page (and clearly indicate that you've done so). You can use without proof any result proved in class, in Sipser's book, or during discussion sections, but **clearly state** the result you are using.

In some questions we offered hints, which we believe are helpful. Nonetheless, any correct solution, even one that doesn't follow the hint, would be acceptable.

Please indicate your name and Student ID on the top of every page.

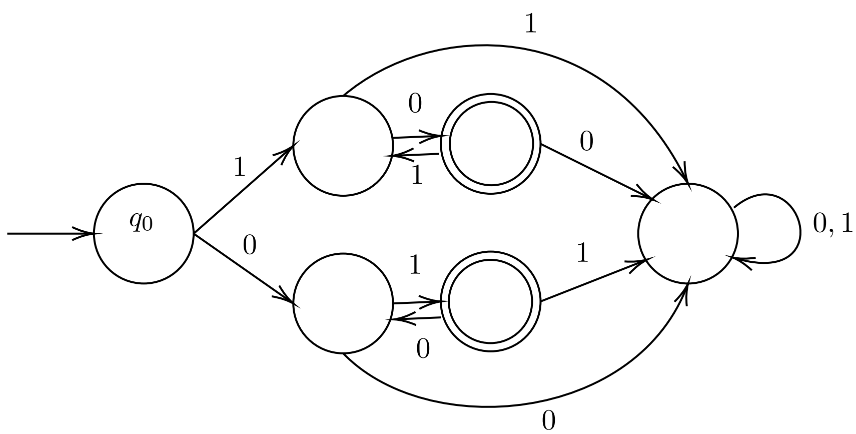
Do not turn this page until the instructor tells you to do so!

Good luck!

Q 1. (30 pts = 5 pts per item) Are the following statements **true** or **false**? Provide a brief justification (1-3 sentences) for each answer (e.g., a brief explanation or a counterexample).

(a) Let L_1, L_2 be arbitrary languages, then $(L_1 \cap L_2)^R = L_1^R \cap L_2^R$.
(Recall that L^R stands for L-reverse.)
Answer:

(b) Consider the language $L_\cup = L((10)^+ \cup (01)^+)$. The DFA below is the minimum DFA for $L_\cup$.



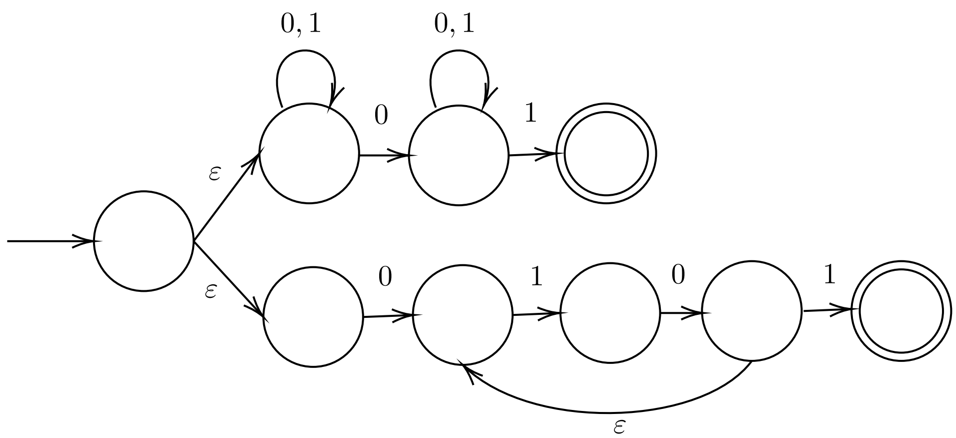
Answer:

(c) The minimum pumping length of $L_\cup$ is 6.
Answer:

(d) Consider the language $L = L((10)^*)$. The strings 0 and 11 are indistinguishable with respect to L.
Answer:

(e) $L(1(01)^*0) = L(10(10)^*)$.
Answer:

(f) The following NFA recognizes the intersection language $L\left((0 \cup 1)^*0(0 \cup 1)^*1\right) \cap L\left(0(10)^*1\right)$.



Answer:

Q 2. (18 pts = 10 + 8 pts) Let M be a 3-state DFA. Recall that R_M denotes the equivalence relation containing all pairs of strings (x, y) such that x and y reach the same state in M. Suppose the strings $\varepsilon, 1, 11$ are all in different equivalent classes, and among them only the string 1 is accepted. Additionally we know that $\varepsilon \sim_M 0$, $1 \sim_M 10 \sim_M 111$ and $11 \sim_M 110$.

(a) Draw the DFA diagram of M.

Answer:

(b) Is the DFA minimal?
If so, briefly justify your answer.
If not, draw the minimum DFA that is equivalent to M.
Answer:

Q 3. (12 pts) Consider the language over the alphabet $\{0, 1, 2\}$

$$L = \left\{ 0^i \cdot s \;\middle|\; i \geq 0, s \in \{1, 2\}^* \text{ and if } i = 2, s = s^R \right\}.$$

Show that any streaming algorithm for this language requires linear memory (i.e., the algorithm must have memory $\Omega(n)$ on strings of length n).

Answer:

Q 4. (18 pts = 6 pts per item) We say that a language L is *pumpable* if there exists a constant p such that any string $w \in L$ of length at least p can be partitioned to $xyz = w$ with: (i) $|xy| \leq p$, (ii) $|y| \geq 1$, and (iii) $\forall k \geq 0 : \ xy^k z \in L$.

Recall that the pumping lemma states that every regular language is pumpable. In addition, in Homework 4, you showed that the following *non-regular* language is also pumpable:

$$F = \{a^i b^j c^\ell \mid i, j, \ell \geq 0 \text{ and if } i = 1 \text{ then } j = \ell\}.$$

(a) Show that the pumpable languages are closed under union.
Answer:

(b) Show that the pumpable languages are **not** closed under intersection.
(**Hint:** take the intersection of F with a regular language to make it non-pumpable.)
Answer:

(c) Show that the pumpable languages are closed under the star operator.

Answer:

Q 5. (12 pts) Recall that for any binary strings x, y of the same length, $x \oplus y$ denotes the bitwise XOR of the two strings. Concretely, $x \oplus y$ is a binary string of the same length as x such that $(x \oplus y)_i = x_i \oplus y_i$ for all $i = 1, \ldots, |x|$.

Let $L_1, L_2 \subseteq \{0,1\}^*$ be two regular languages, computed by the **deterministic** finite automata $M_1 = (Q_1, \Sigma, \delta_1, q_0^{(1)}, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_0^{(2)}, F_2)$ with n_1 and n_2 states respectively.

Describe an NFA with at most $n_1 n_2$ states that recognizes the language

$$L = \{x \oplus y \mid |x| = |y|,\ x \in L_1,\ y \in L_2\}$$

over the alphabet $\{0,1\}$.

(**Hint:** Note that we are asking you to describe an NFA, not a DFA, with $n_1 n_2$ many states.)
Answer: