make history.

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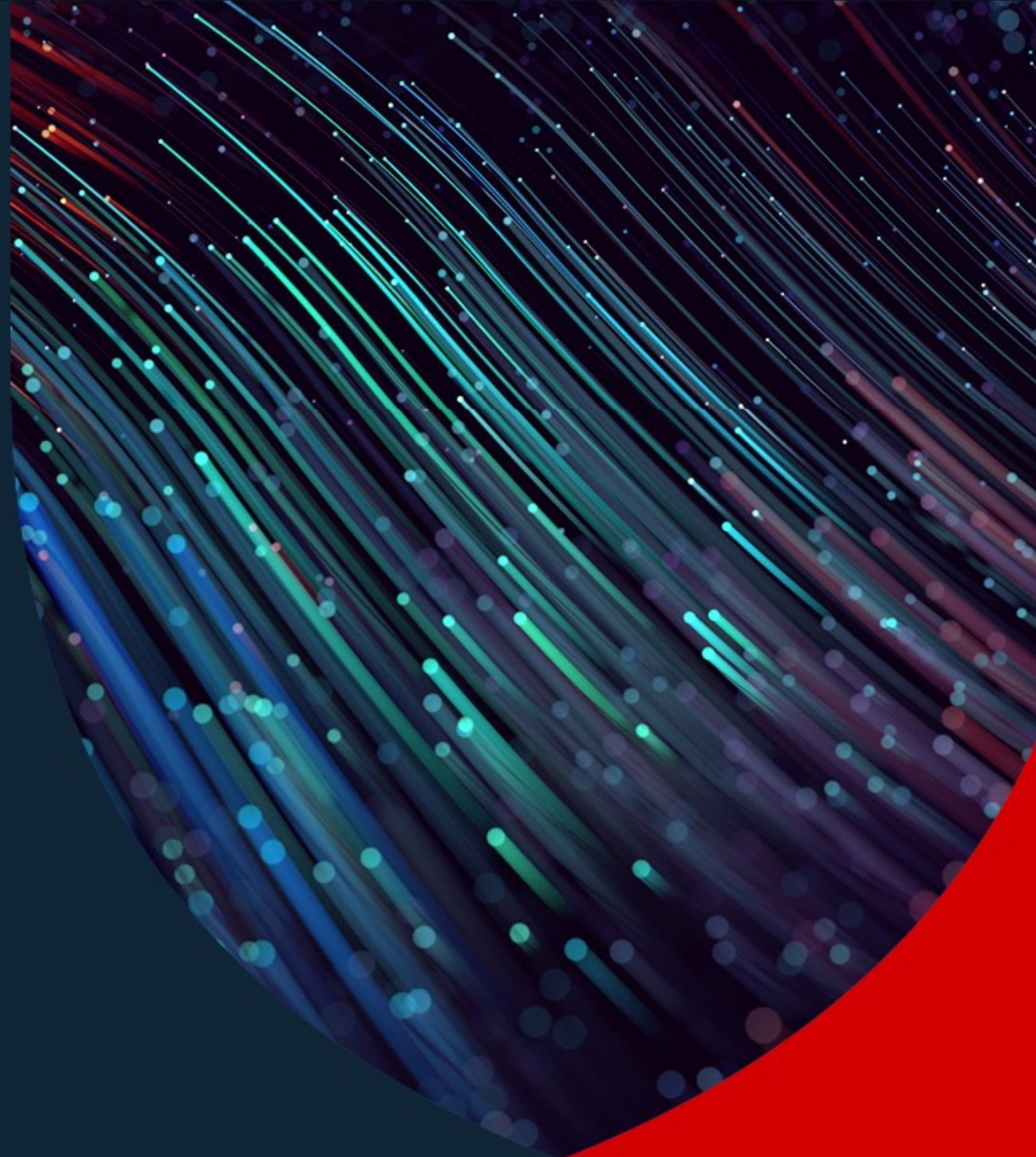
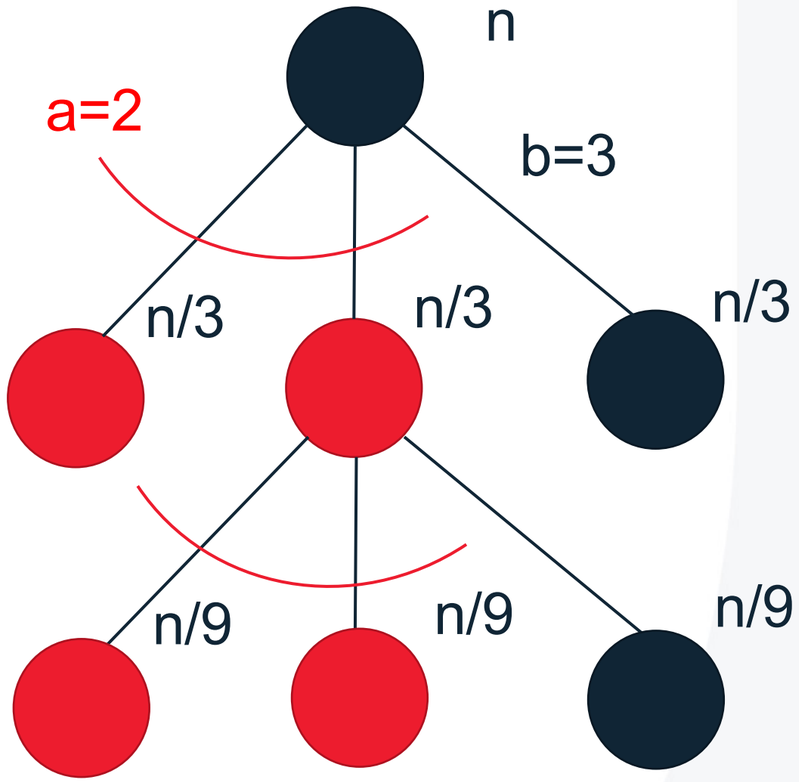
Master theorem

Dr. Anna Kalenkova



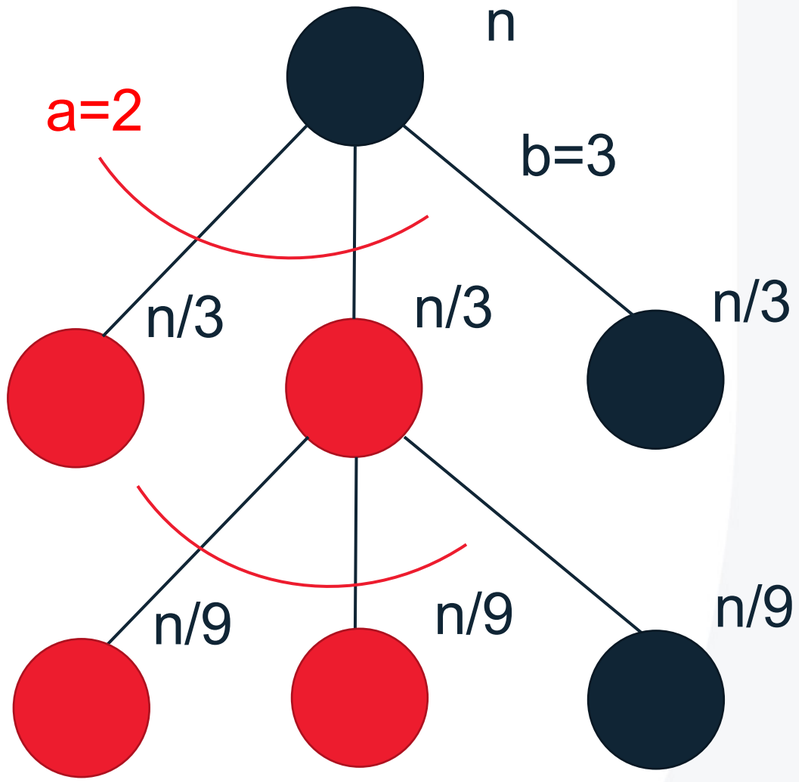
Divide and conquer

- Create *a* subproblems, each having size *n*/*b*
- Call the procedure recursively on each subproblem
- Combine the results from the subproblems



Master theorem

$$T(n) = a\,T\left(\frac{n}{b}\right) + f(n)$$



T(n) – computational complexity to solve problem of size n;

f(n) – computational complexity to combine results from subproblems.

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Master theorem

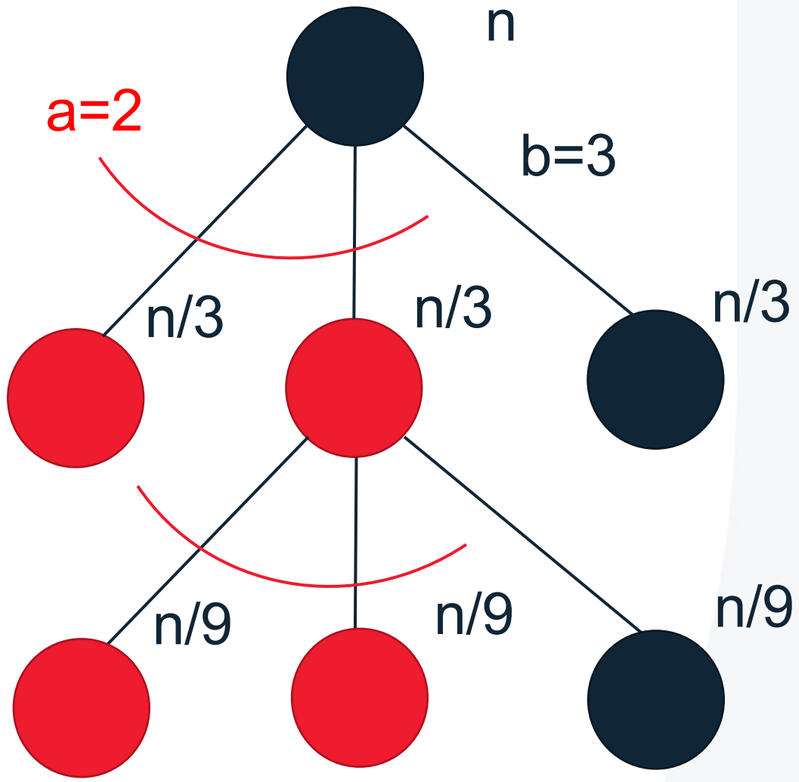
For constants $a \geq 1, b \geq 2, d \geq 0$ and $f(n) \in \Theta(n^d)$,

consider the recurrence:

$$T(n) = a\,T\left(\frac{n}{b}\right) + f(n)$$

then

$$\mathrm{T}(n) \in \begin{cases} \Theta(n^d), & \text{if } a < b^d \\ \Theta(n^d \log n), & \text{if } a = b^d \\ \Theta\left(n^{\log_b a}\right), & \text{if } a > b^d \end{cases}$$



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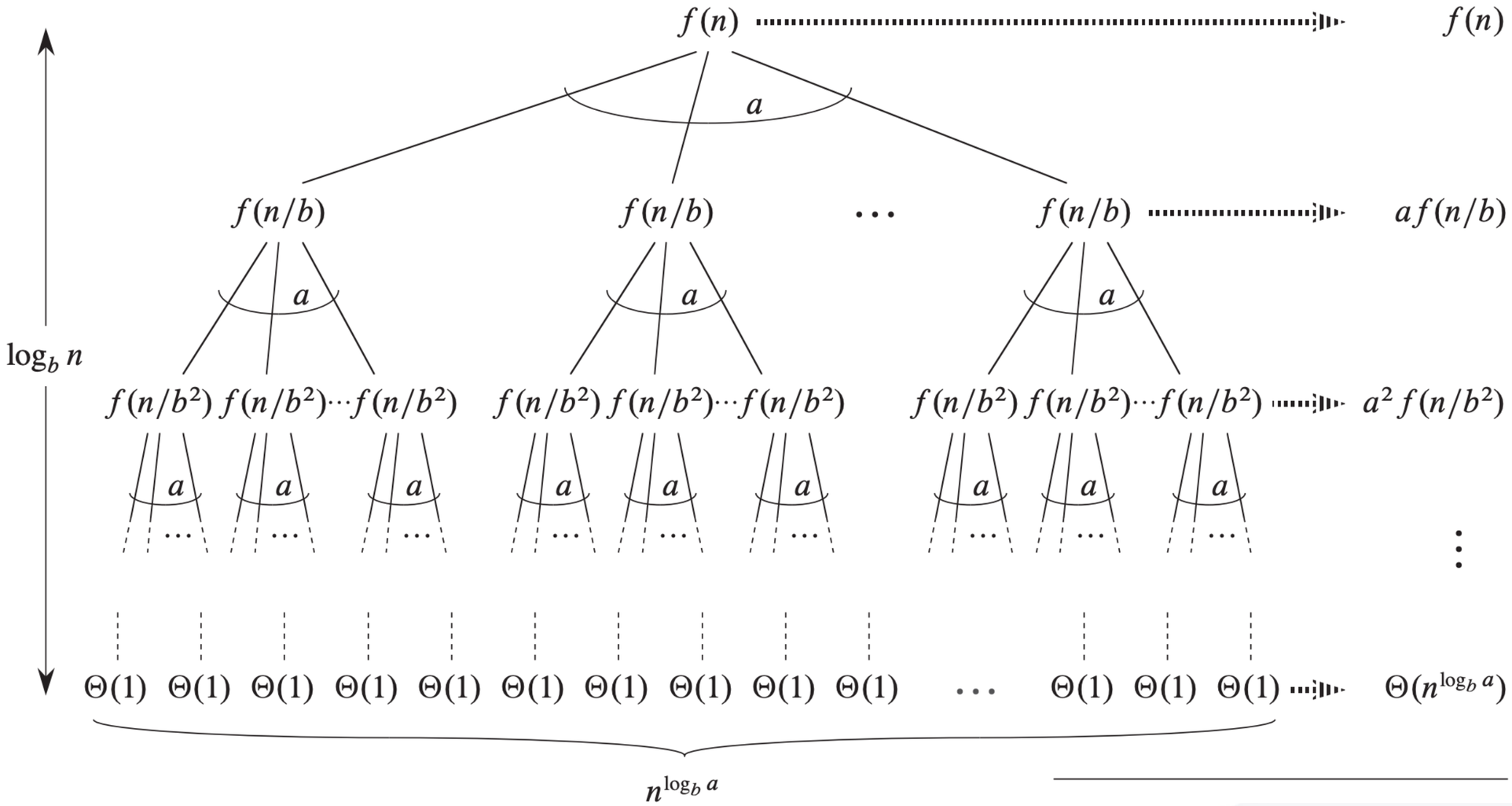

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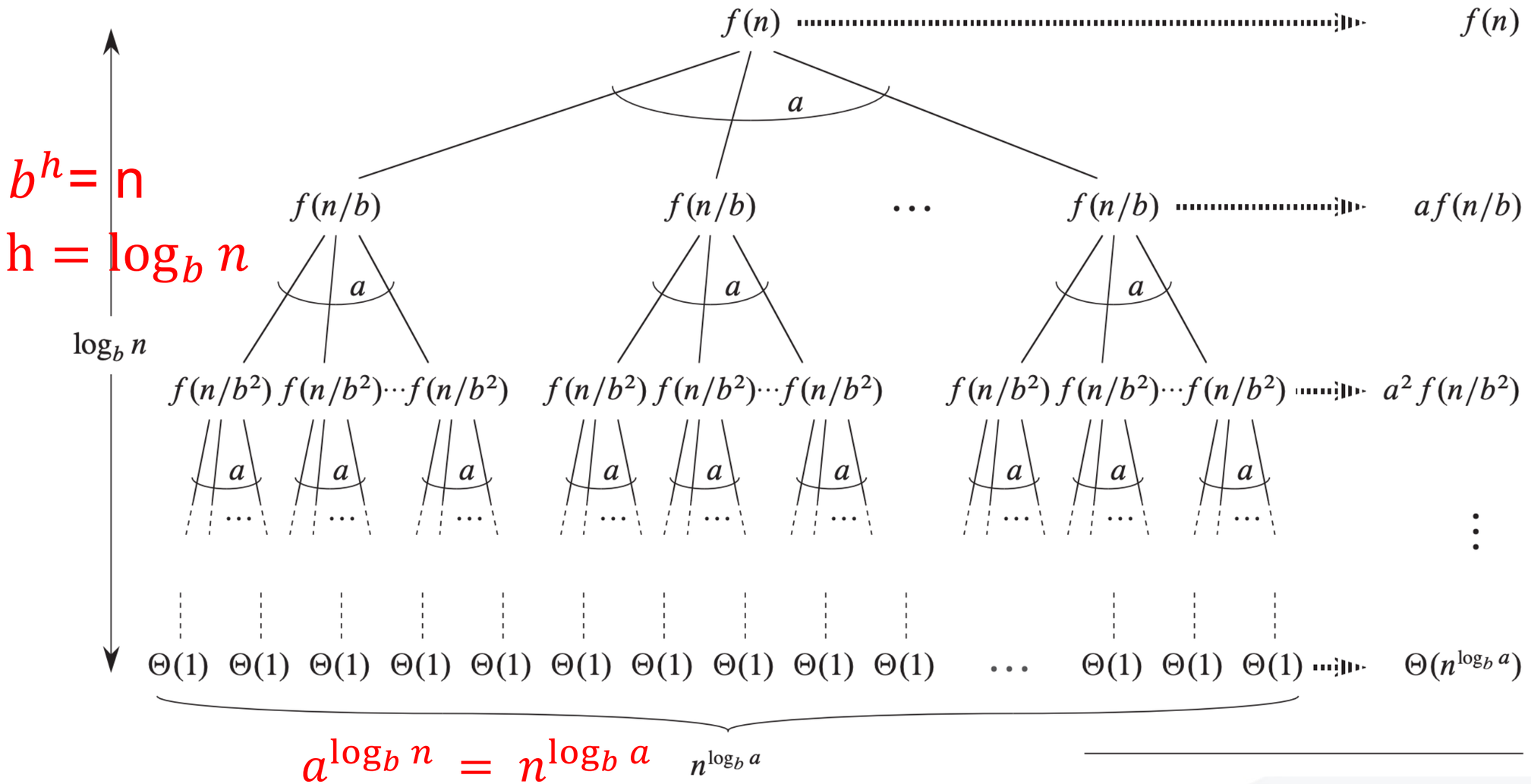
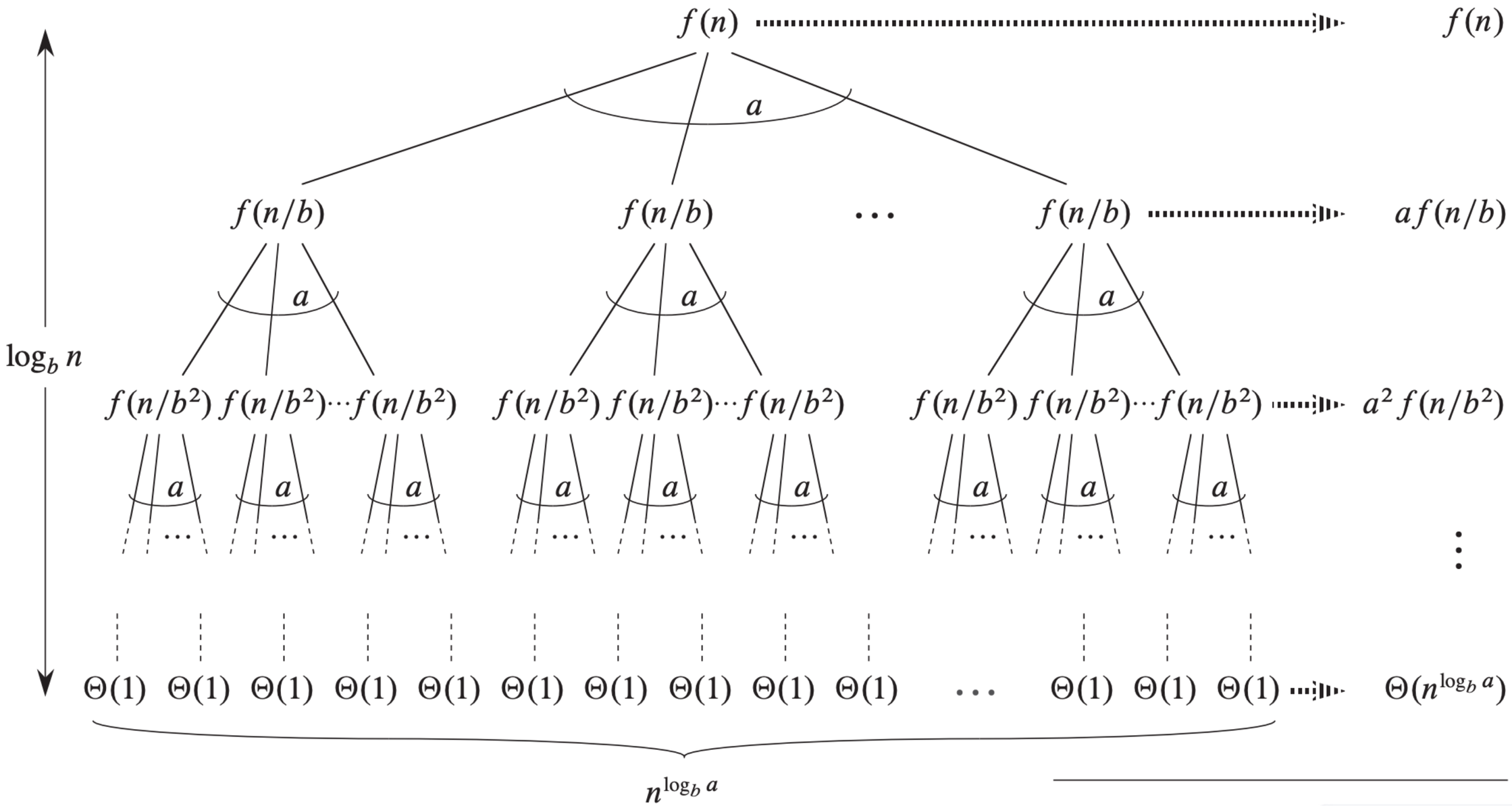
Master theorem





$$\Theta(n^{\log_b a}) + \sum_{j=0}^{\log_b n - 1} a^j \left(\frac{n^d}{b^{dj}}\right) = \Theta(n^{\log_b a}) + n^d \sum_{j=0}^{\log_b n - 1} \left(\frac{a^j}{b^{dj}}\right)$$

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