Trigonometry

Reciprocal & Quotient Identities:

$\tan x = \frac{\sin x}{\cos x}$
$\cos x = \frac{1}{\sin x}$
$\sec x = \frac{1}{\cos x}$
$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

Pythagorean Identities:

$\sin^2 x + \cos^2 x = 1$
$\cos^2 x - \cot^2 x = 1$
$\sec^2 x - \tan^2 x = 1$

Angle Sum & Difference:

$\sin(ab) = \sin a \cos b \cos a \sin b$
$\sin(ab) = \sin a \cos b \cos a \sin b$
$\cos(ab) = \cos a \cos b \sin a \sin b$
$\cos(ab) = \cos a \cos b \sin a \sin b$
$\tan(ab) = \frac{\tan a \tan b}{1 \tan a \tan b}$
$\tan(ab) = \frac{\tan a \tan b}{1 \tan a \tan b}$

Product Identities:

$\sin a \cos b = \frac{1}{2}\{\sin(a+b) + \sin(a-b)\}$
$\cos a \sin b = \frac{1}{2}\{\sin(a+b) - \sin(a-b)\}$
$\cos a \cos b = \frac{1}{2}\{\cos(a-b) + \cos(a+b)\}$
$\sin a \sin b = \frac{1}{2}\{\cos(a-b) - \cos(a+b)\}$

Sum & Difference Identities:

$\sin a + \sin b = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
$\sin a - \sin b = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$
$\cos a + \cos b = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
$\cos a - \cos b = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$

Double Angle Identities:

$\sin 2a = 2 \sin a \cos b$
$\sin 2a = \frac{2 \tan a}{1 \tan^2 a}$
$\cos 2a = \frac{1-tan^2 a}{1+tan^2 a}$
$\cos 2a = cos^2 a - sin^2 a$
$\cos 2a = 1 - 2sin^2 a$
$\cos 2a = 2cos^2 a - 1$
$\tan 2a = \frac{2tana}{1-tan^2 a}$

Triple Angle Identities:

$\sin 3a = 3 \sin a - 4 \sin^3 a$
$\cos 3a = 4 \cos^3 a - 3 \cos a$
$\tan 3a = \frac{(3 \tan a - \tan^3 a)}{1 - 3 \tan^2 a}$

Quadruple Angle Identities:

$\sin 4a = 4 \sin a \cos^3 a - 4 \sin^3 a \cos a$
$\cos 4a = \cos^4 a + \sin^4 a - 6 \sin^2 a \cos^2 a$
$\tan 4a = \frac{(4 \tan a - 4 \tan^3 a)}{1 + \tan^4 a - 6 \tan^2 a}$

Half Angle Identities:

$\sin\left(\frac{a}{2}\right) = \pm\sqrt{\frac{1-\cos a}{2}}$
$\cos\left(\frac{a}{2}\right) = \pm\sqrt{\frac{1+\cos a}{2}}$
$\tan\left(\frac{a}{2}\right) = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$
$\sin a = \frac{2\tan\left(\frac{a}{2}\right)}{1+\tan^2\left(\frac{a}{2}\right)}$
$\cos a = \frac{1-\tan^2\left(\frac{a}{2}\right)}{1+\tan^2\left(\frac{a}{2}\right)}$
$\tan a = \frac{2\tan\left(\frac{a}{2}\right)}{1-\tan^2 a}$

Trigonometric Identities for Complementary & Supplementary Angles:

$\angle(x)$	$\sin x$	$\cos x$	$\tan x$	$\cot x$	$\sec x$	$\csc x$
$-x$	$-\sin x$	$\cos x$	$-\tan x$	$-\cot x$	$\sec x$	$-\csc x$
$90° - x$	$\cos x$	$\sin x$	$\cot x$	$\tan x$	$\csc x$	$\sec x$
$90° + x$	$\cos x$	$-\sin x$	$-\cot x$	$-\tan x$	$-\csc x$	$\sec x$
$180° - x$	$\sin x$	$-\cos x$	$-\tan x$	$-\cot x$	$-\sec x$	$\csc x$
$180° + x$	$-\sin x$	$-\cos x$	$\tan x$	$\cot x$	$-\sec x$	$-\csc x$
$360° - x$	$-\sin x$	$\cos x$	$-\tan x$	$-\cot x$	$\sec x$	$-\csc x$
$360° + x$	$\sin x$	$\cos x$	$\tan x$	$\cot x$	$\sec x$	$\csc x$

Differentiation of Products & Quotients:

Product Rule:

$$y = u * v$$

$$\frac{dy}{dx} = u * \frac{dv}{dx} + v * \frac{du}{dx}$$

Quotient Rule:

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v * \frac{du}{dx} - u * \frac{dv}{dx}}{v^2}$$

Derivatives of Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

Trigonometric Ratio Table

$\angle$(Deg)	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
$\angle$(Rad)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–	0	–	1
cot	–	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	–	0	–
csc	–	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	–	-1	–
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	–	-1	–	1

Differentiation:

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(n) = 0$$

$$\frac{d}{dx}(e^{mx}) = me^{mx}$$

$$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(\ln(x \pm a)) = \frac{1}{x \pm a}$$

$$\frac{d}{dx}(\ln(ax + b)) = \frac{1}{ax + b}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\sin(mx)) = m\cos(mx)$$

$$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\cos(mx)) = -m\sin(mx)$$

$$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin(f(x))$$

$$\frac{d}{dx}(\sin^2 x) = 2\sin x\cos x = \sin 2x$$

$$\frac{d}{dx}(\sec x) = \sec x\tan x$$

$$\frac{d}{dx}(\sec(mx)) = m\sec(mx)\tan(mx)$$

$$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$$

$$\frac{d}{dx}(\csc x) = -\csc x\cot x$$

$$\frac{d}{dx}(\csc(mx)) = -m\csc(mx)\cot(mx)$$

$$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\tan(mx)) = m\sec^2(mx)$$

$$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2(f(x))$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\cot(mx)) = -m\csc^2(mx)$$

$$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a}$$

$$\frac{d}{dx}(\log_a f(x)) = \frac{f'(x)}{f(x)\ln a}$$

Integration:

$$\int (x^n)\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int e^{mx}\,dx = \frac{e^{mx}}{m} + c$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int \sin(mx)\,dx = -\frac{1}{m}\cos(mx) + c$$

$$\int \sin(mx+n)\,dx = -\frac{1}{m}\cos(mx+n) + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \cos(mx)\,dx = \frac{1}{m}\sin(mx) + c$$

$$\int \cos(mx+n)\,dx = \frac{1}{m}\sin(mx+n) + c$$

$$\int \sec x \tan x\,dx = \sec x + c$$

$$\int \sec(mx)\tan(mx)\,dx = \frac{1}{m}\sec(mx) + c$$

$$\int \sec^2 x\,dx = \tan x + c$$

$$\int \sec^2(mx)\,dx = \frac{1}{m}\tan(mx) + c$$

$$\int \sec^2(mx+n)\,dx = \frac{1}{m}\tan(mx+n) + c$$

$$\int \csc x \cot x\,dx = -\csc x + c$$

$$\int \csc(mx)\cot(mx)\,dx = -\frac{1}{m}\cot(mx) + c$$

$$\int \csc^2 x\,dx = -\cot x + c$$

$$\int \csc^2(mx)\,dx = -\frac{1}{m}\cot(mx) + c$$

$$\int \csc x\,dx = |\ln|\csc x - \cot x|| + c$$

$$\int \tan x\,dx = \ln|\sec x| + c$$

$$\int \cot x\,dx = \ln|\sin x| + c$$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx$$

$$\int \sin^2 x\,dx = \frac{1}{2}\int (1 - \cos 2x)\,dx$$

$$\int \sin^3 x\,dx = \frac{1}{4}\int (3\sin x - \sin^3 x)\,dx$$

$$\int \cos^2 x\,dx = \frac{1}{2}\int (1 + \cos 2x)\,dx$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}(x) + c$$

$$\int \frac{1}{ax \pm b}\,dx = \frac{1}{a}\ln|ax \pm b| + c$$

$$\int \frac{1}{x \pm a}\,dx = \ln|x \pm a| + c$$

$$\int \frac{1}{a - x}\,dx = -\ln|a - x| + c$$

$$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c$$

$$\int \frac{1}{x}\,dx = \ln x + c$$

Further Integration

$$\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\cdot\int v\,dx\right)dx$$

$$\int \frac{1}{a-bx}\,dx = -\frac{1}{b}\ln|a-bx| + c$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + c$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \ln\left(x+\sqrt{a^2+x^2}\right) + c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right) + c$$

$$\int \csc x\,dx = \ln|\csc x - \cot x| + c$$

Rules of Logarithm

$$\ln(xy) = \ln x + \ln y$$

$$\ln\left(\tfrac{x}{y}\right) = \ln x - \ln y$$

$$\ln\left(\tfrac{1}{x}\right) = -\ln x$$

$$\ln(a-b) = \tfrac{\ln a}{\ln b}$$

$$\ln(a+b) = \ln ab$$

$$\ln(e) = 1$$

$$\ln(1) = 0$$

$$e^{\ln f(x)} = f(x)$$