Problem A:

Solution:

Given, $f(x) = x + x^2 - x^3$ for $x \geq 0$

So,

$f'(x) = 1 + 2x - 3x^2$

$f''(x) = 0 + 2 - 6x$

$= 2 - 6x$

For maximum or minimum

$f'(x) = 0$

$\Rightarrow 1 + 2x - 3x^2 = 0$

$\Rightarrow 3x^2 - 2x - 1 = 0$ [Divided by (-1)]

$\Rightarrow 3x^2 - 3x + x - 1 = 0$

$\Rightarrow 3x(x-1) + 1(x-1) = 0$

$\Rightarrow (x-1)(3x+1) = 0$

So, $x = 1$ or $x = -\frac{1}{3}$

For checking max or min

$f''(1) = 2 - (6 \times 1) = 2 - 6 = -4 < 0$

$f''(-\frac{1}{3}) = 2 - 6 \times (-\frac{1}{3}) = 4 > 0$

So, maximum value $= f(1) = 1$

Answer: (1)

problem B

solution:

Given,

$$1n^3 + 2n + 3n^2$$

$$= n(n^2 + 3n + 2)$$

$$= n\{n(n) + 2n + n + 2\}$$

$$= n\{n(n+2) + 1(n+2)\}$$

$$= n(n+1)(n+2)$$

Here, It's a number of product of 3 consecutive number.

So, It is divisible by 2 and 3 [showed]

problem - c

Solution:

Given,

$$\sin\left(x+\frac{\pi^3+2\sqrt{\pi^6}}{\pi^2+\pi^2}+\pi^{\pi^0}\right)=\cos\left(x+\frac{(-1)^{16}}{2}-\frac{\log_2\sqrt{8}}{3}\right)$$

$$\Rightarrow \sin\left(x+\frac{3\pi^3}{2\pi^2}+\pi\right)=\cos\left(x+\frac{1}{2}-\frac{3}{2}\cdot\frac{1}{3}\right)$$

$$\Rightarrow \sin\left(x+\frac{5\pi}{2}\right)=\cos(x) \quad \text{------ (i)}$$

(i) $\Rightarrow$

$$\sin\left(x+\frac{5\pi}{2}\right)=\sin\left(\frac{\pi}{2}-x\right)$$

$$\Rightarrow x+\frac{5\pi}{2}=\frac{\pi}{2}-x$$

$$\Rightarrow 2x=\frac{\pi}{2}-\frac{5\pi}{2}=-2\pi$$

$$\therefore x=-\pi$$

Again, (i) $\Rightarrow$

$$\sin^2\left(x+\frac{5\pi}{2}\right)=\cos^2 x$$

$$\Rightarrow \frac{1}{2}\left(1-\cos(2x+5\pi)\right)=\frac{1}{2}(1+\cos 2x)$$

$$\Rightarrow \cos(2x+5\pi)+\cos 2x=0$$

$$\Rightarrow 2\cos\frac{4x+5\pi}{2}\cos\frac{5\pi}{2}=0$$

$$\Rightarrow \cos\frac{4x+5\pi}{2}=0=\cos\frac{\pi}{2}$$

$$\Rightarrow \frac{4x+5\pi}{2}=\frac{\pi}{2}$$

$$\Rightarrow 4x=\pi-5\pi=-4\pi$$

$$\therefore x=-\pi$$

Again, $x=-3\pi, -5\pi$

Answer, $x=(-\pi, -3\pi, -5\pi)$

problem - D

solution:

Given,

$$\alpha+\beta+\gamma=1 \quad \text{-----(i)}$$

$$\beta+\gamma+\beta=1$$

$$\Rightarrow \gamma+2\beta=1 \quad \text{—(ii)}$$

$$\gamma+\beta+\gamma=1 \quad \text{—(iii)}$$

$$\Rightarrow \beta+2\gamma=1 \quad \text{---(iii)}$$

(ii) $\Rightarrow$

$$\gamma=1-2\beta \quad \text{—(iv)}$$

(iii) & (iv) $\Rightarrow$

$$\beta+2\gamma=1$$

$$\Rightarrow \beta+2(1-2\beta)=1$$

$$\Rightarrow \beta+2-4\beta=1$$

$$\Rightarrow -3\beta=-1$$

$$\therefore \beta=\frac{1}{3}$$

$$\therefore \gamma=1-2\cdot\frac{1}{3}=\frac{1}{3}$$

(i) $\Rightarrow$

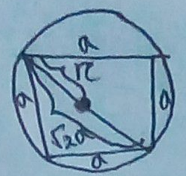
$$\therefore \alpha=1-(\beta+\gamma)=1-\left(\frac{1}{3}+\frac{1}{3}\right)=\frac{1}{3}$$

$$\therefore \alpha=\frac{1}{3}$$

Answer: $\left(\frac{1}{3}\right)$

Problem E

Solution:



Given,

$$A_1 = 1 \, m^2$$

we know,

$$A_1 = \pi r^2 = 1 \quad [r = \text{the Radius of circle}]$$

$$\Rightarrow r^2 = \frac{1}{\pi}$$

$$\Rightarrow r = \sqrt{1/\pi}$$

$$\Rightarrow 2r = 2\sqrt{1/\pi}$$

Let,

The length of the square's side is a

So,

$$\sqrt{2}a = 2r$$

$$\Rightarrow a = \frac{2r}{\sqrt{2}} = \frac{2\sqrt{1/\pi}}{\sqrt{2}}$$

$$\therefore a = \sqrt{2}\sqrt{1/\pi}$$

So, Area of square, $A_2 = a^2 m^2 = \left(\sqrt{2}\sqrt{\frac{1}{\pi}}\right)^2 m^2$

$$= 2 \cdot \frac{1}{\pi} \, m^2$$

$$\therefore A_2 = 0.6366 \, m^2$$

Answer: $0.6366 \, m^2$.