

AdaBoost: From Weak Learners to Strong Classifiers

Minimizing Exponential Error via Sequential Learning

Nikhil, Jannen

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The Power of the Committee

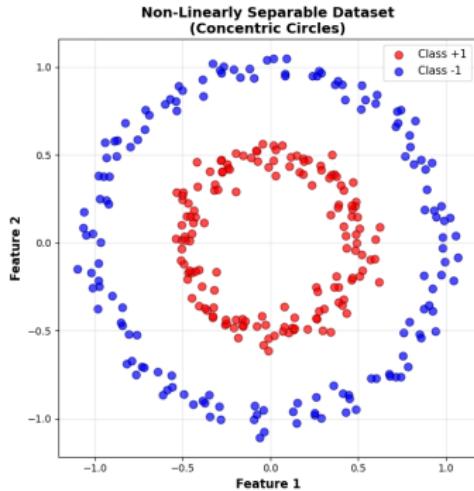


Figure: Linear Separability Challenge

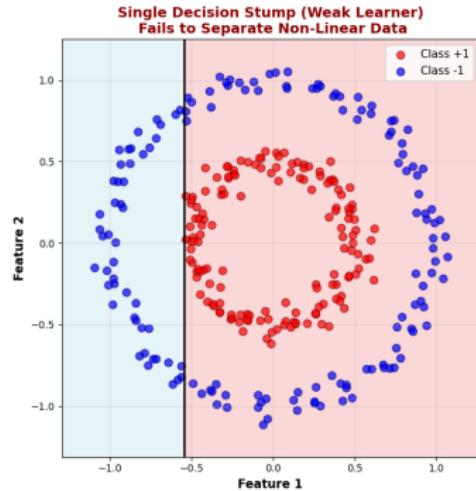


Figure: Decision Stump fails

- **The Problem:** Complex data is rarely linearly separable.
- **The Naive Solution:** Build a massive, complex model.
- **The AdaBoost Solution:** Combine simple decision boundaries.

$$H(x) = \text{sign} \left(\sum \alpha_k h_k(x) \right)$$

The Dataset: Breast Cancer Wisconsin (Diagnostic)

What are we classifying?

- **Source:** FNA biopsy images
- **Samples:** 569 patients
- **Task:** Binary classification
- Malignant (M): ~ 37%
- Benign (B): ~ 63%

Why it matters:

- Early cancer detection
- Non-invasive procedure
- Real-world ML

Feature Engineering:

- 10 cell nucleus characteristics
 - 3 measurements each
 - ⇒ 30 total features
- 1 Radius
 - 2 Texture
 - 3 Perimeter
 - 4 Area
 - 5 Smoothness
 - 6 Compactness
 - 7 Concavity
 - 8 Concave Points
 - 9 Symmetry
 - 10 Fractal Dimension

Visualization Strategy: 30D to 2D Projection

- **Challenge:** 30 features impossible to visualize
- **Solution:** Principal Component Analysis (PCA)
 - Projects 30D to 2D while preserving variance
 - Classes remain **non-linearly separable**
 - Perfect for demo
- **Key Insight:** Single stump (line) still fails in 2D

Why Not Squared Error?

- **Regression:** Minimizes Squared Error (MSE).
- **Classification:** MSE is unreliable.
 - Penalizes “too correct” predictions.
 - Sensitive to outliers in the wrong way.
- **AdaBoost:** Minimizes Exponential Error.

$$E = \sum_{n=1}^N \exp(-t_n f(x_n))$$

Focuses heavily on misclassified points ($t_n \neq \text{sign}(f(x_n))$).

Sequential Minimization

$$E = \sum_{n=1}^N \exp(-t_n [H_{k-1}(x_n) + \alpha_k h_k(x_n)])$$

- Cannot optimize all α and h at once.
- **Greedy Approach:** Freeze past, optimize current step.
- **The Weight Trick:**

$$w_n^{(k)} = \exp(-t_n H_{k-1}(x_n))$$

- Previous errors become current weights!
- Misclassified points get higher weights.

Finding the Optimal Weight (α_k)

- **Goal:** Find α_k to minimize E .
- Taking the derivative: $\frac{\partial E}{\partial \alpha_k} = 0$
- **Result:**

$$\alpha_k = \frac{1}{2} \ln \left(\frac{1 - \epsilon_k}{\epsilon_k} \right)$$

- Low Error ($\epsilon_k \rightarrow 0$) \Rightarrow High Alpha
- Random Guess ($\epsilon_k = 0.5$) \Rightarrow Zero Alpha
- Worse than random ($\epsilon_k > 0.5$) \Rightarrow Negative Alpha

Updating Sample Weights

- After computing α_k and training h_k , update weights:

$$w_n^{(k+1)} = w_n^{(k)} \cdot \exp(-\alpha_k \cdot t_n \cdot h_k(x_n))$$

- If correct:** $t_n \cdot h_k(x_n) = 1 \Rightarrow$ weight decreases
- If wrong:** $t_n \cdot h_k(x_n) = -1 \Rightarrow$ weight increases
- Normalize:** $w_n^{(k+1)} \leftarrow \frac{w_n^{(k+1)}}{\sum_i w_i^{(k+1)}}$

Hard examples get harder weights: forces next learner to focus.

The AdaBoost Algorithm (Summary)

① **Initialize:** $w_n^{(1)} = \frac{1}{N}$

② **For** $k = 1, 2, \dots, K$:

 ① Train weak learner h_k on weighted data

 ② Calculate error: $\epsilon_k = \sum_n w_n^{(k)} \cdot I(h_k(x_n) \neq t_n)$

 ③ Compute weight: $\alpha_k = \frac{1}{2} \ln \left(\frac{1-\epsilon_k}{\epsilon_k} \right)$

 ④ Update: $w_n^{(k+1)} = w_n^{(k)} \cdot \exp(-\alpha_k \cdot t_n \cdot h_k(x_n))$

 ⑤ Normalize: $w_n^{(k+1)} \leftarrow \frac{w_n^{(k+1)}}{\sum_i w_i^{(k+1)}}$

③ **Output:** $H(x) = \text{sign} \left(\sum_{k=1}^K \alpha_k h_k(x) \right)$

Live Demo: AdaBoost on Breast Cancer (2D PCA)

- Switch to Python console
- Run: `python visualize_adaboost_pca.py`
- Evolution of decision boundary:
 - Panel 1: Single stump (fails)
 - Panels 2-5: Ensemble grows
 - Right: Error curves decreasing
- Compare to data (red = malignant, blue = benign)

Key: Exponential error decreases each iteration!

Why AdaBoost Works: The Exponential Error

Figure: Decision boundary evolution: 1, 2, 3, 5 weak learners

- Single stump: Misclassifies $\sim 50\%$
- 2 stumps: Better separation
- 5 stumps: $> 95\%$ accuracy

Summary

- **AdaBoost minimizes Exponential Error**
- Turns complex problem into simple sequence (greedy)
- **Weight update:** Focuses on misclassified examples
- **Key Takeaway:** Smart combination of simple models
- **Rigorous:** Every formula from exponential loss

AdaBoost: Mathematically principled and empirically powerful

References & Tools

- **Literature:**

- Bishop, C. M. (2006). *Pattern Recognition* (Ch. 14).
- Friedman, Hastie, Tibshirani (2000). *Additive Logistic Regression*.
- Viola, Jones (2004). *Robust Real-Time Face Detection*.

- **Dataset:**

- Wolberg et al. (1995). *Breast Cancer Wisconsin*. UCI ML. DOI: 10.24432/C5DW2B

- **Tools:**

- scikit-learn, ucimlrepo, matplotlib