Sensitivity analysis for agent based models in the philosophy of science

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Abstract

Agent Based Models (ABMs) have become an increasingly common tool amongst philosophers of science. However, sensitivity analyses, or assessments of how changes in model elements and parameters affect model outputs, remain uncommon in this field. Here I present a simple case study in which I re-implement Kevin Zollman's Science Cliques model [15] and perform a very simple sensitivity analysis. I present information gleaned from this sensitivity analysis that was not apparent in the original publication.

1 Introduction

Philosophers of science use Agent Based Models (ABMs) to study how collaboration and communication choices made by individuals affect communities [5, 6, 8, 9, 10, 13, 14, 16]. Models in this field have allowed researchers to generate new hypotheses relating individuals' behaviors to emergent network structures and community beliefs. However, few philosophy of science ABM publications include discussion of the robustness of model results to changes in (parametric and non parametric) model elements nor characterization of which elements have the greatest impact on emergent behavior. These analysis are known as sensitivity analyses. Researchers would benefit from sensitivity analyses as they both help identify potential weaknesses (or unexpected outcomes that result from implementation choices) of a model and allow researchers to extract more information about how parameters affect model behavior. As ABMs become increasingly common in philosophy, modelers must adopt strategies to assess model robustness and the relative impact different aspects of the model have on emergent model behavior.

The sensitivity analysis presented in this paper is very simple. It is meant to be easily replicable and to highlight some of the advantages of performing sensitivity analyses. Though informative, this sensitivity analysis should not be considered comprehensive.

1.1 Agent based models in the philosophy of science

ABMs are computational models that characterize the dynamics that result from the collective actions of autonomous agents. Agents exist in an environment and follow rules that dictate their behavior. Agents can interact with each other and with their environment. ABMs usually progress step-wise through time with agents taking actions at each step that depend on their internal and external conditions.

ABMs have been used for years by social scientists to study how individual-level social choices can result in the emergence of social phenomenon like cooperation[1], social network structures[3], and housing segregation[11]. More recently ABMs have been used by philosophers of science to study how social collaboration networks emerge between scientists given the incentive landscapes modern scientists must navigate and the biases individuals may hold [5, 6, 9, 10, 13]. In these social ABMs, agents typically represent people and the rules guiding their behavior are defined to reflect hypotheses about how certain methods of interaction might affect emergent model properties and network structures. After simulating interactions between these individuals, researchers analyze model properties to assess their initial hypotheses.

1.2 Sensitivity analysis

Sensitivity analysis is defined as "The study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input"[12]. In the context of philosophy AMBs, model input comprises both input parameters and the guiding assumptions researchers used to design the model. These assumptions include both input parameters and non-parametric model elements like rulesets and orders of actions taken by individuals at each step. Model output usually comprises the properties of the model and social network structures present in the model at the end of each simulation. Thus sensitivity analysis is a method for quantifying how changes in model elements affect the emergent behavior of the model.

Identifying model elements that have the greatest impact on emergent model behavior can be valuable for research prioritization, model simplification, and assessment of model robustness [2, 4]. Knowing which model elements most impact model behavior can help researchers prioritize where to focus additional research efforts (for example, if a certain model output depends on a certain parameter, researchers may want to focus on assessing why that parameter is so important and which ranges of that parameter yield which output ranges). If model output is universally unaffected by changes in a certain model element, it is possible that element is extraneous and can be removed thus simplifying the model. Because agent based models are stochastic, analyzing a model's robustness to changes is important when determining whether or not an observed model behavior is common.

1.3 Science Cliques Model

For this case study I have focused on conducting a simple sensitivity analysis on Kevin Zollman's Science Cliques model [15]. This model was designed to study how different methods for choosing what individuals one should believe when developing new opinions can yield different ratios of true to false opinions and different raw numbers of true opinions throughout the community. This model was chosen because its source code was freely available and the publication explored a small subset of the model's possible parameter space.

In the original implementation of the model, there are 100 individuals. Each individual has a reliability which is drawn from a beta distribution and averages .6 across all individuals. There are 1500 facts and each individual has a correct belief, incorrect belief, or no belief about each fact. Individuals begin the simulation with beliefs about 15 facts. The accuracy of these initial beliefs that are correct is proportional to each individual's reliability (individuals with a reliability of .6 will have about 60% correct initial beliefs and 40% incorrect initial beliefs.

The model is advanced 500 time steps per simulation. At each time step, each individual progresses through three phases:

1. **Investigate**: With a probability of 10%, independently generate a new belief about a fact the individual previously had no opinion about. The probability of this new belief being true is the individual's reliability.

2. **Select Teachers**: Select a set of unique individuals from whom to solicit testimony. Each teacher presents a random fact and opinion they hold about that fact. Teachers only present facts about which they have a correct or incorrect opinion.

3. **Learn**: Adopt the opinions of teachers if they offer an opinion about a fact this individual does not currently have an opinion about, otherwise ignore new teacher and keep previous opinion.

All individuals in a model follow one of four philosophies; skeptical, reid, direct, and indirect. The individuals' philosophy determines each individual's behavior in each phase. Skeptical individuals do not select teachers nor learn from teachers. They only learn new facts through investigation. Reid individuals select teachers randomly. Direct individuals select the individuals who have the highest percentage of true beliefs as their teachers, calculated using equation 1.

$$\frac{\text{true beliefs}}{(\text{true beliefs} + \text{false beliefs})} \tag{1}$$

Indirect individuals select individuals who have the most beliefs in common with themselves as their teachers. All ties when direct or indirect individuals are selecting teachers are broken randomly.

1.3.1 Possible parameter values

The original model was implemented in Netlogo and allowed users to modify parameters. The range of parameters allowed were as described in Table 1.

Though these ranges of parameters were available to users who wished to run the original model, almost all model parameters were kept constant throughout all model runs discussed in the paper (parameter values used in the original paper were as described in Table 1. The notable exception is the number of neighbors. In the paper, the model was run with 2, 4, 6, and 8 neighbors (all possible neighbor values). The original paper ran the model 100 times with each number of neighbors (and the constant parameters described in Table 1.

Table 1: Possible parameter values and tested parameter values in original model

Parameter	**Description**	**Possible Range of Values**	**Values Tested**
number of individuals	the number of individuals in the model	1 - 100	100
number of neighbors	the number of individuals each individual will solicit for opinions	2, 4, 6, 8	2, 4, 6, 8
number of facts	the number of facts in the model	1 - 1500	1500
investigation probability	the probability that an individual will independently generate a belief on any time step	0 - 1	0, 1
starting knowledge	the number of facts each individual has an opinion about at the start of the simulation	0 - 1500	1500
reliability alpha	the alpha of the beta distribution each individual's reliability is drawn from	1 - 100	1.5
reliability beta	the beta of the beta distribution each individual's reliability is drawn from	.001 - 1	1

1.3.2 Original Paper Results

Zollman assessed each philosophy's ability to minimize false beliefs and maximize true beliefs. Figure 1 (taken from Zollman 2015 [15]) depicts the proportion of beliefs that are true across all individuals in the model (calculated using equation 1). Across all community sizes, models using the direct philosophy yielded the greatest proportion of true beliefs across all individuals. There was no significant difference between the proportion of true beliefs across individuals in any of the three other philosophies. Note that the Y axis of this bar graph does not start at 0. All three other philosophies yielded proportions of beliefs that are true roughly equivalent to the average reliability of the agents in the model. Figure 3 (taken from Zollman 2015 [15]) depicts the average number of beliefs each individual has that are true. The overall trend in these data is the direct (or creudulist) communities typically had the greatest number of true beliefs overall (only tied by the direct communities when the model is run with a number of neighbors of 2). Zollman concludes that when one can not objectively assess the reliability of peers (or when the direct option is unavailable, as is the case in real life) there is no reason to prefer an indirect philosophy to a reid philosophy.

2 Model Reimplementation

My code is publicly available at `https://github.com/Jannetty/science_cliques`. I reimplemented the model using the python agent-based-modeling package Mesa [7]. First I ran the model with the parameters specified in the original paper repeating each set of parameters for 10 model runs. I decreased the number of replicate model runs from 100 to 10 due to limitations in my computational capacity (all model runs were run locally on my personal computer). I recreated the data summary figures from the paper using data from my model.

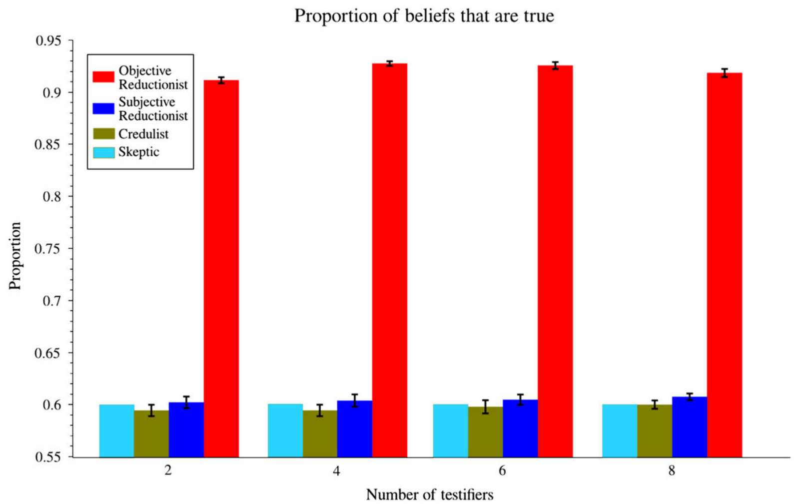

Figure 1: Zollman's Figure 1. From Zollman 2015 [15]: "The average proportions of an individual's non-abstention beliefs that are true for each of several testimonial strategies... The bars represents the means for the individuals from 100 runs of the simulation. The error bars represent the 95% confidence interval for the average performance of a community of 100 individuals". Skeptic models used the skeptical philosophy, creudulist models used the reid philosophy, subjective reductionist models used the indirect philosophy, and objective reductionist models used the direct philosophy.

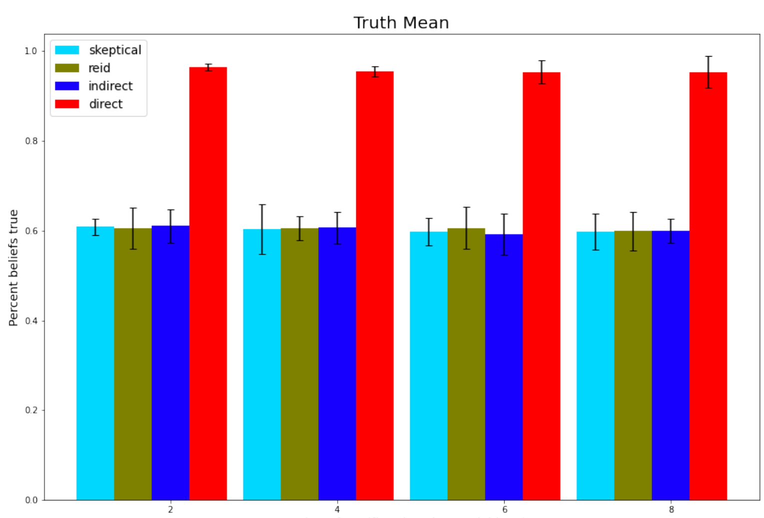

Figure 2: My average proportion of non-abstention beliefs that are true for each of several testimonial strategies. Bars represent the means for the individuals from 10 runs of the simulation. Error bars represent 95% confidence interval for average performance of a community of 100 individuals.

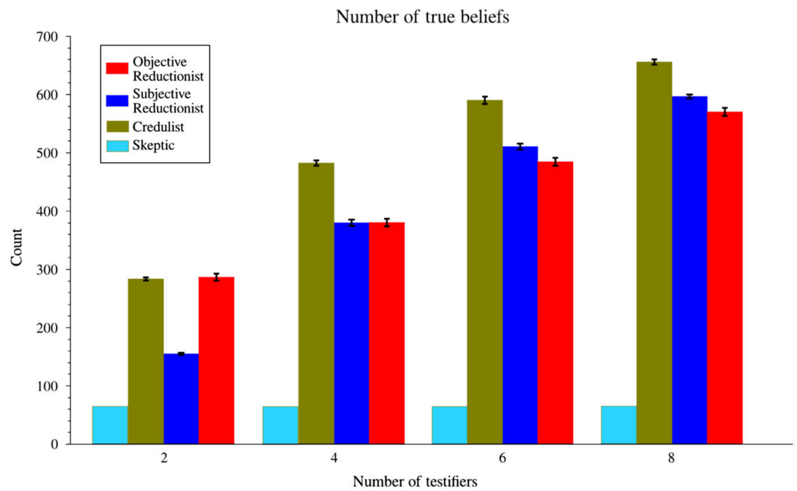

Figure 3: Zollman's Figure 3. From Zollman 2015 [15]: Average Total number of true things that an individual believes. Bars are means for the individuals from 100 runs of the simulation. Error bars represent 95% confidence interval for average performance of a community of 100 individuals. Skeptic models used the skeptical philosophy, creudulist models used the reid philosophy, subjective reductionist models used the indirect philosophy, and objective reductionist models used the direct philosophy.

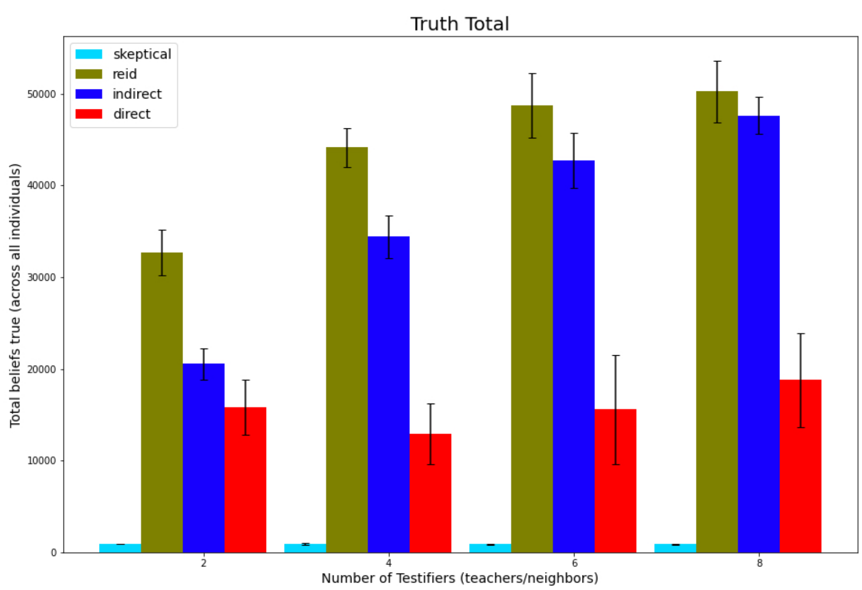

Figure 4: My total number of true beliefs across all individuals in the simulation. The bars represents the means for 10 runs of the simulation. The error bars represent the 95% confidence interval for the average performance of a community of 100 individuals.

Figure 2 depicts the average truth mean (calculated using equation 1) among all individuals in my model for each philosophy and for each number of neighbors. My data for this point are very similar to the data from Zollman 2015. Two differences may strike viewers. The Zollman figure (1) appears to have a more drastic difference between the truth mean of the direct individuals and the other three philosophies. This is only because the minimum value on the y axis of the Zollman figure is set to .55 instead of 0. The other key difference is the 95% confidence intervals (depicted in the error bars in each figure) are wider in my data vs the Zollman figure. This is because my figure depicts the average performance over 10 runs of the model whereas the Zollman figure depicts the average performance over 100 runs of the model. The increased number of replicates performed in the original paper resulted in smaller 95% confidence intervals.

Figure 4 depicts the total number of true beliefs held by all individuals in each population in my model. Zollman's similar figure can be seen in Figure 3. A key distinction between these figures is that Zollman's figure depicts the average number of true beliefs in each population whereas my figure depicts the total number of true beliefs in each population. To recreate Zollman's figure, I would have to divide each truth-total value in my figure by 100. I did not do this in Figure 4 because the 95% confidence intervals of my data became so large that they extended into the negatives. I have however included this version of my figure without confidence intervals in Figure 5. The trends that Zollman observed of skeptical communities having the lowest rate of true beliefs and reid communities having the highest rate of true beliefs remained consistent in my simulations, and the total average number of truths held by each indirect community is similar between our simulations.

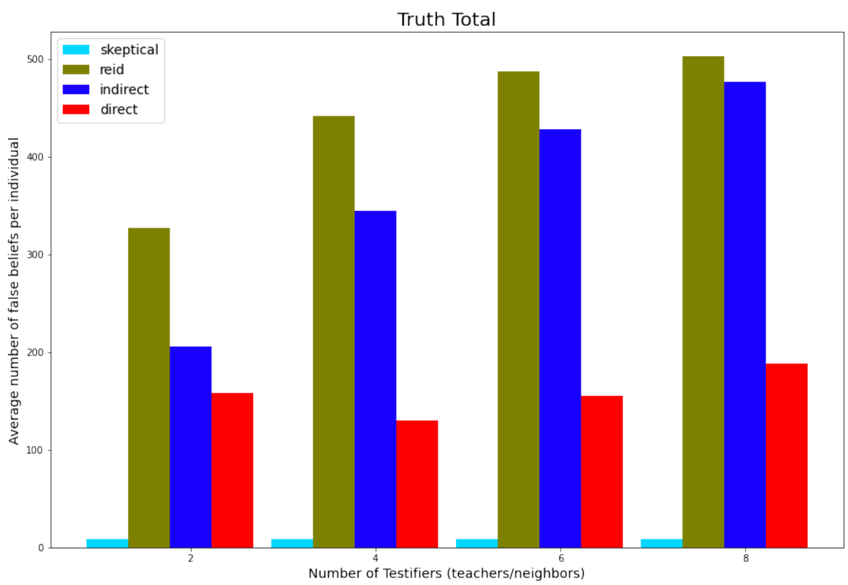

Figure 5: My average number of true beliefs across all individuals in the simulation. Bars represents means for 10 runs of the simulation.

There are a few notable differences between Zollman's and my truth total data that I cannot definitively explain. Both my skeptical and my direct communities ended with far fewer total truths overall than Zollman observed. I suspect these differences may not have persisted had I run my model 100 times per replicate instead of 10 times per replicate. I also suspect that I may have implemented my direct philosophy slightly differently from Zollman which my explain some of the difference between our community's performances. In my simulation, a sorted list is maintained tracking each individual and the percentage of each individual's beliefs that are true. In my direct implementation, individuals select the top-ranking individuals on this list (the individuals with the highest percentage of true beliefs) and break ties randomly. Zollman's implementation may not break ties randomly, resulting in all individuals in the community adopting the beliefs of a small number of teachers. Honestly I would expect that this would result in Zollman's implementation having a lower number of true direct beliefs compared to mine. I also suspect there may be an error in the way I implemented my investigation probability (a point I recognized during my sensitivity analysis, which I will discuss later).

On the whole, my model was able to capture the overall trends seen in the Zollman paper even when run with far fewer replicates. Though there were some differences observed in the truth total data, the width of the 95% confidence interval of my data suggests that more replicates would be needed to recapture the exact ranges seen in in Zollman. Unfortunately limits in my computational resources kept me from running additional replicates. I took this initial data as validation that my implementation was at least similar to Zollman's implementation and moved forward to conducting a very simple sensitivity analysis.

3 Sensitivity Analysis

The sensitivity analysis used here is a simple one-at-a-time method. The parameter values used during this analysis are described in Table 2. For each philosophy, the model was run ten times with each permutation of parameters. The outputs measured after each model runs were the truth means (the percent of facts agents' have opinions about that are true, see 1), truth totals (the total number of true opinions held by all agents in the model), false means ($1-$ the truth mean), and false totals (the total number of false opinions held by all agents in the model).

The parameters selected for sensitivity analysis were number of individuals, number of facts, and investigation probability. The starting knowledge, reliability alpha, and reliability beta were kept consistent with the values used in Zollman's original model [15]. The number of neighbors was fixed to eight. Varying all parameters would have yielded a more complete sensitivity analysis. However, the number of model runs I could perform was limited by my computational resources. I selected parameters I hypothesized might have different effects on the model outputs under different philosophies.

The parameter values selected were as evenly spaced between the minimum and maximum values allowed in the original Zollman implementation as possible. In retrospect, I regret not centering the distribution of parameters sampled on the values used in the Zollman paper. This analysis would be strengthened by additional model runs with parameter values above those used in the Zollman paper in addition to the values (below those used in the Zollman paper) used here.

Table 2: Parameter values used during sensitivity analysis

Parameter	**Description**	**Values Tested**
number of individuals	the number of individuals in the model	8, 20, 40, 60, 80, 100
number of neighbors	the number of individuals each individual will solicit for opinions	8
number of facts	the number of facts in the model	16, 300, 600, 900, 1200, 1500
investigation probability	the probability that an individual will independently generate a belief on any time step	.01, .2, .4, .6, .8, 1
starting knowledge	the number of facts each individual has an opinion about at the start of the simulation	1500
reliability alpha	the alpha of the beta distribution each individual's reliability is drawn from	1.5
reliability beta	the beta of the beta distribution each individual's reliability is drawn from	1

Once all the model runs concluded, I calculated the correlations between each parameter varied and output measured. I will discuss observations that can be drawn from these correlation calculations in the following sections. Note that correlation coefficients are shown between the set of all inputs and outputs and the set of all outputs, so each output has at least one correlation of 1 (with itself). I also calculated the R squared values between parameters and outputs. The R squared values can be seen in Figure 10 in Appendix A.

3.1 Skeptical

The correlation coefficients between the skeptical model inputs and outputs are shown in Figure 6. Both the false total and the truth total outputs are strongly correlated with the number of individuals in the simulation. This makes sense because skeptical communities did not solicit testimony from their neighbors, thus new beliefs were formed exclusively through independent discovery. The more individuals there are in the simulation, the more instances of independent discovery there will be, and thus the more true and false opinions held. Because the false total and the truth total are both sensitive to the same input (the number of individuals), they are also correlated with each other. A good experiment to decouple these outputs would have been to collect data from the model while setting the reliability of all individuals to 0 or 1. This would make all new observations either true or false. In the case that reliability is 1 and all new observations are true, we would expect to still see a correlation between the truth total and the number of individuals with no correlation between false total and number of individuals. With reliability set to 0, we would expect to see correlation between false total and number of individuals with no correlation between truth total and number of individuals.

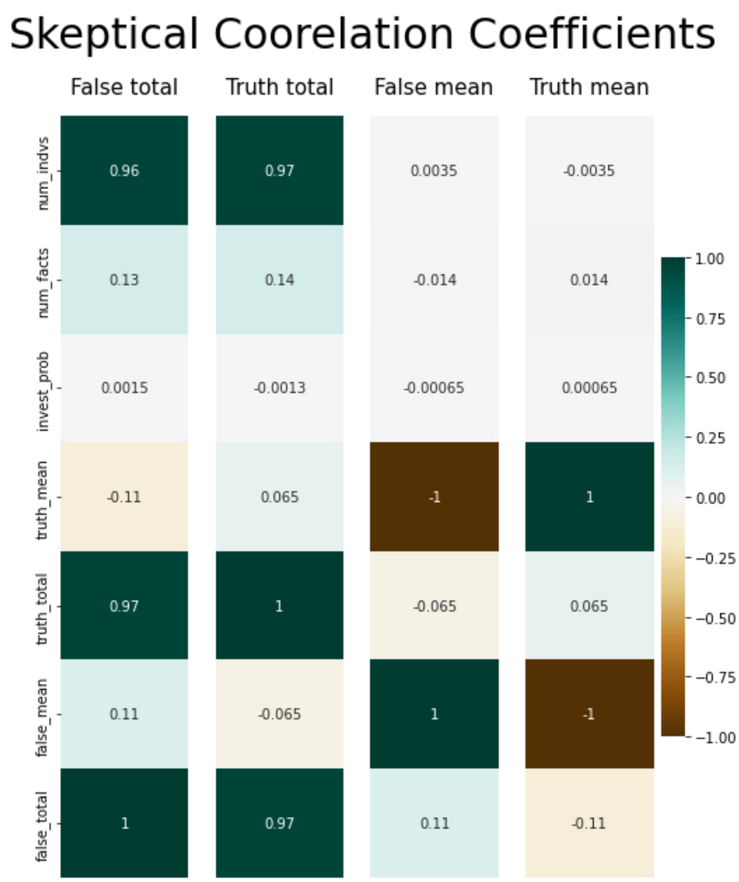

Figure 6: Correlations between all parameters and outputs assessed in sensitivity analysis for skeptical communities.

I expected both the false total and the truth total to be positively correlated to the investigation probability as an increased investigation probability would increase the rate at which new beliefs were discovered. I have no explanation for why this was not the case and suspect it may be an implementation error.

There were no notable correlations between inputs and the truth mean and false mean outputs. This indicates that none of the varied parameters greatly affected the truth or false means. This is expected amongst skeptical communities because, due to the lack of information exchange between individuals, we expect the proportion of true beliefs in the community to be purely a function of the reliability of the individuals. All new beliefs are discovered by individuals independently, so we expect the proportion of true beliefs to be roughly equal to the average reliability in the population. A good experiment to strengthen this justification would have been to run the model with various average reliabilities. We would expect to see a strong positive correlation between the average reliability in the community and the True and False means.

3.2 Reid

The correlation coefficients between the reid model inputs and outputs are shown in Figure 7.

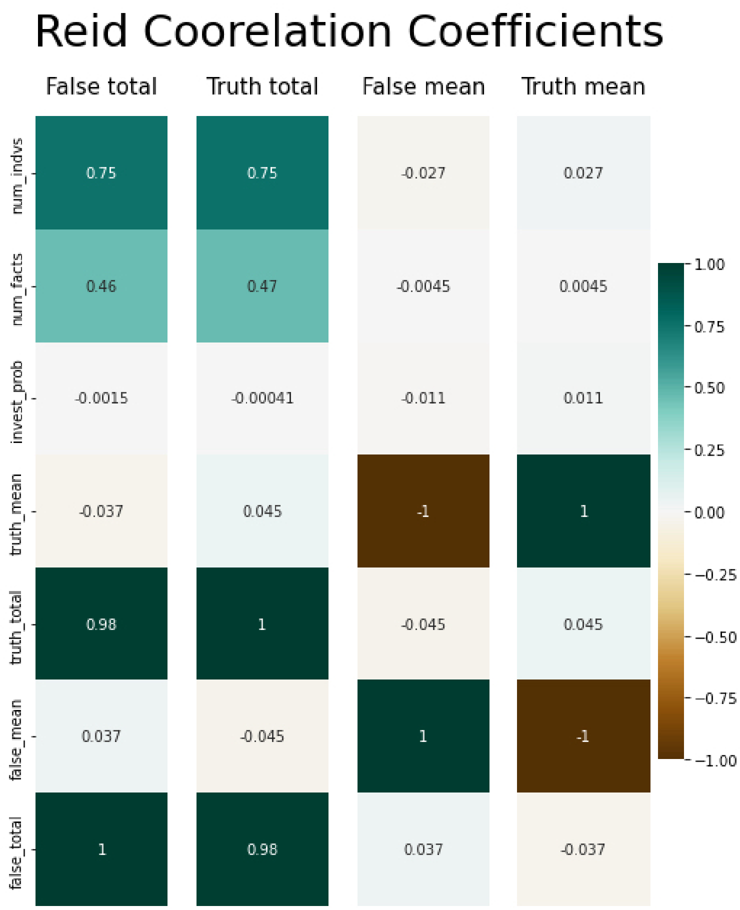

Figure 7: Correlations between all parameters and outputs assessed in sensitivity analysis for reid communities.

As in the skeptical community, the false total and the truth total outputs are correlated with the number of individuals in the simulation, though the correlation here is not as strong. Similar to skeptical communities, the number of true and false beliefs uncovered through discovery increases as these populations get larger. However in addition to discovering new beliefs independently, reid communities solicit testimony randomly and adopt any new beliefs they encounter. This means the total number of true and false beliefs in the community is a function both of the discovery rate of new beliefs and the rate at which individuals encounter new beliefs through testimony. Logically we would expect the number of neighbors (or individuals from whom each individual solicits testimony) to correlate with the number of true and false beliefs. Though the number of neighbors was not varied in the sensitivity analysis, it was varied in the runs recreating the data from the original paper. A positive correlation (of .895) between the number of neighbors and the truth total for reid communities can be seen in Figure 4. There was also a positive correlation (.866) between the number of neighbors and the false total for reid communities (data not shown). The truth total and the false total were strongly correlated with each other, as was seen in the skeptical communities. This implies that conditions that yield high truth totals also yield high false totals and vice versa.

There were again no notable correlations between inputs and the truth mean and false mean outputs. These means were also not correlated with the number of neighbors in the data shown in Figure 2 (the correlation between the number of neighbors and the truth mean was .079 and the correlation between the number of neighbors and the false mean -.079). This is an interesting finding as it suggests that none of the parameters assessed have an influence on the proportion of true beliefs in a reid population.

3.3 Direct

The correlation coefficients between the direct model inputs and outputs are shown in Figure 8.

Figure 8: Correlations between all parameters and outputs assessed in sensitivity analysis for direct communities.

As in the skeptical and reid communities, increasing the number of individuals in the simulation increased both the false and the truth totals. The false total and the truth total are correlated with each other, meaning circumstances that increase one also tended to increase the other. However unlike the other communities, increasing the number of individuals had a substantially greater affect on the truth total than the false total. This can be seen both in comparing the correlation between the number of individuals and the false total (.36) to the correlation between the number of individuals and the truth total (.75), and when considering the correlations between the number of individuals and the true and false means. Increasing the number of individuals correlated with an increase of the truth mean (.68) and a decrease of the false mean (-.68). This is reasonable as members of direct communities solicit testimony from the individuals in the population who have the highest ratio of true beliefs to false beliefs. Thus it makes sense that a higher proportion of beliefs learned from teachers in direct communities will be true compared to communities that solicit testimony randomly, and therefore that the number of true beliefs in the population will grow faster than the number of false beliefs.

3.4 Indirect

The correlation coefficients between the direct model inputs and outputs are shown in Figure 9.

Figure 9: Correlations between all parameters and outputs assessed in sensitivity analysis for indirect communities.

These correlations are very similar to the correlations seen in the reid communities seen in Figure 7. As in the all other communities, the false total and the truth total outputs are correlated with the number of individuals in the simulation. The strengths of the correlations here (.75 for false total and .76 for truth total) are very similar to the strengths of the correlations seen in the reid communities (.75 and .75). In communities of both philosophies, the number of true and false beliefs uncovered through discovery increases as these populations get larger. In addition to discovering new beliefs independently, both communities solicit testimony and adopt any new beliefs they encounter. The equivalency of the correlation between the number of individuals and the false total and the number of individuals and the truth total suggests that, as was seen in the reid communities, increasing the number of individuals increases the truth total and the false total roughly equally. However, unlike reid communities, indirect communities solicit testimony from individuals who have the most beliefs in common with themselves. This method of choosing teachers maximises the likelihood that a teacher will present a belief about a fact that the solicitor already has a belief about. Because teachers randomly select and present a belief they currently have about a fact, and teacher are matched to students in a way that maximises facts that students and teachers have in common, indirect teachers are more likely to present a belief the solicitor already holds than teachers in the reid communities. Individuals in this model ignore any beliefs about facts they already hold a belief about, so by selecting teachers this way indirect communities maximize the likelihood they will not learn anything from their teachers. This could explain the difference in magnitudes between the reid truth totals and the indirect truth totals seen in Figures 3, 4, and 5.

There were again no notable correlations between inputs and the truth mean and false mean outputs. This again suggests that none of the parameters assessed have an influence on the proportion of true beliefs in an indirect population (as was seen in the reid population).

4 Conclusion

The simple sensitivity analysis presented here both strengthens elements of Zollman's conclusions and opens new questions. Zollman's ultimate conclusion was that there is no reason to prefer an indirect philosophy to a reid philosophy. In some ways the sensitivity analysis supports this conclusion. The truth and false totals of both philosophies are sensitive to changes in the same model parameters, and the truth and false means were insensitive to all varied model parameters. The number of of true and false beliefs in both types of communities positively correlated with the number of individuals in those communities. These findings support Zollman's conclusions. However, when considered alongside the data in Zollman's paper showing that reid communities have the most true beliefs amongst these communities (Figure 3), the sensitivity analysis raises an interesting question. The correlation between the number of individuals and the number of true beliefs was the same as the correlation between the number of individuals and the number of false beliefs for both reid and indirect communities. In response to an increase in individuals in the simulation, the number of false beliefs in the community increases at the same rate as the number of true beliefs. Thus even though reid communities have the highest number of true beliefs, they also have the highest number of false beliefs (for further confirmation, false total data for the original simulation runs can be found in Appendix A in Figure 11). With this in mind, the question of whether to prefer an indirect philosophy to a reid philosophy translates to whether one prefers maximizing true beliefs in a community at the expense of maximizing false beliefs versus minimizing false beliefs at the expense of minimizing true beliefs. This question is not discussed in the original publication.

I hope the case study presented here highlights the advantages of conducting sensitivity analyses on agent based models in philosophy of science research. Sensitivity analysis highlights key interactions between model inputs and model outputs, which can both raise interesting questions and serve as a sanity check for a model's implementation. When a correlation between a change in a parameter and a model output is identified, it is a useful exercise to consider whether this correlation makes sense. Each of the strong correlations identified in this sensitivity analysis was considered in the context of the rules agents were following under the philosophy being assessed. Most of the correlations (and lack of correlations) identified here had intuitive explanations, but this may not always be the case. Correlations without obvious explanations could either be exciting findings resulting from complex interactions in the model or unintended consequences of model implementation choices (or mistakes, as I suspect is the case with the lack of correlation between the investigation probability and the true/false totals of the skeptical community). These are important aspects of the model for a researcher to be aware of.

The sensitivity analysis presented here is flawed. I am unsatisfied with the differences in the truth total of direct communities between my model and Zollman's model (Figure 3 vs Figure 4). Furthermore, I am unsatisfied with my selection of parameter ranges for the sensitivity analysis (Table 2) as the parameter values do not range higher than the values used in Zollman's implementation. I also excluded certain parameters from assessment. A complete sensitivity analysis should have explored parameter values higher and lower than those used in the original publication and should have varied all parameters (including the number of neighbors and the average reliability of all individuals in the simulation). I also did no analysis of differences in neighborhood structures between communities of different philosophies (a point discussed in the original Zollman publication), and I never explored the potential mistake yielding the lack of correlation between the investigation probability and the True/False totals of the skeptical community.

Even with these shortcomings, this sensitivity analysis highlighted the key interactions driving outcomes for each philosophy and enabled a deeper discussion of the differences and similarities between these philosophies. Though imperfect, I hope this case study highlights the benefits of conducting sensitivity analysis on philosophy of science agent based models.

A Supplemental Figures

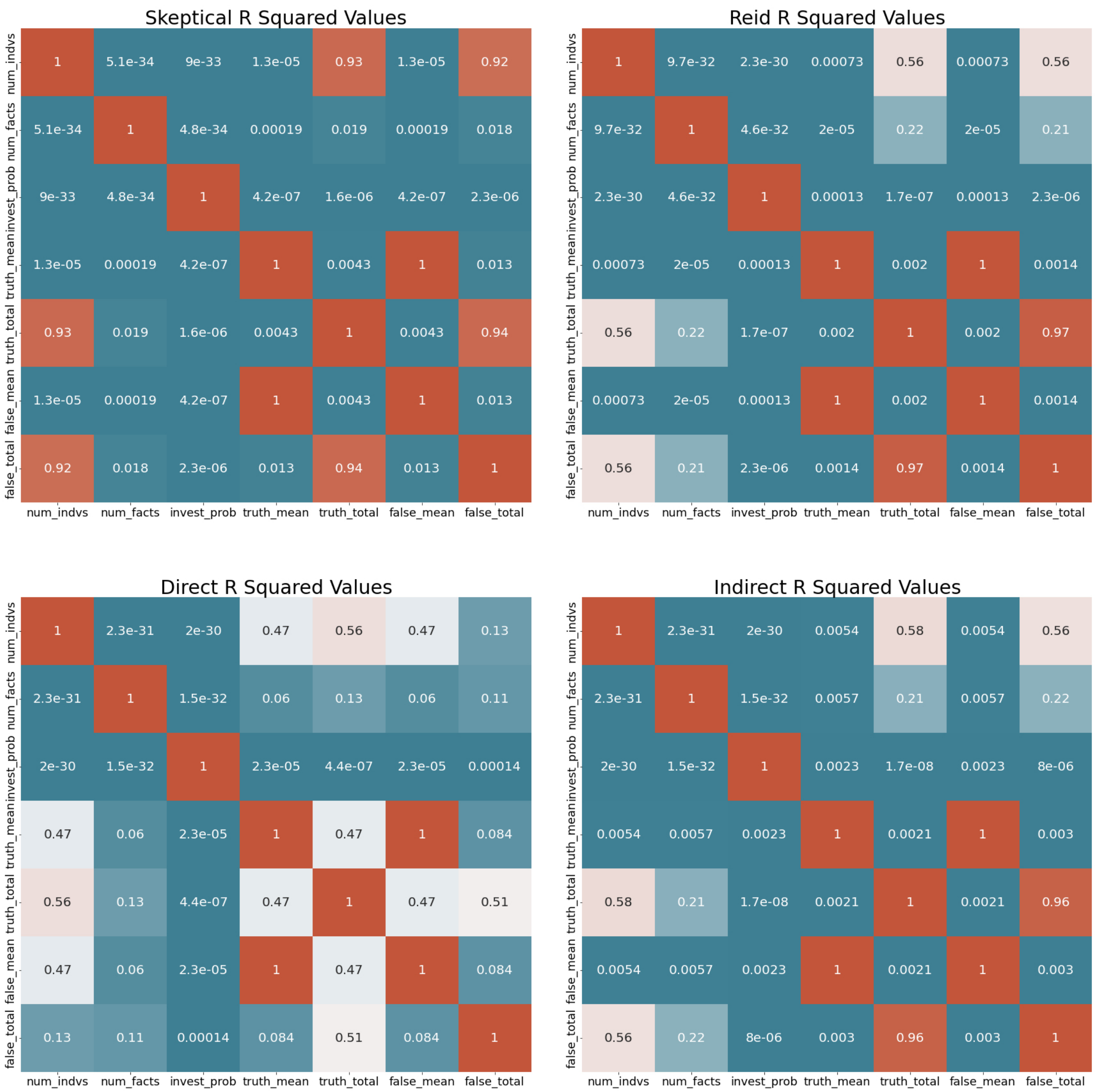

Figure 10: R^2 values between all parameters and outputs assessed in sensitivity analysis.

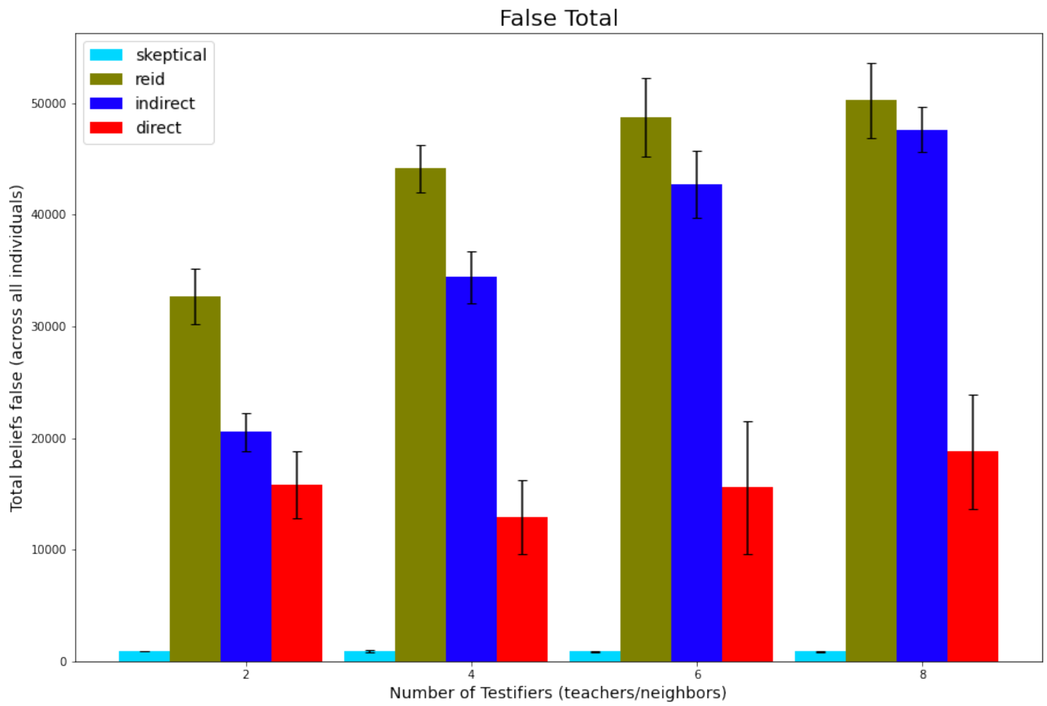

Figure 11: Total number of false beliefs across all individuals in the original simulation runs (the false total equivalent of figure 4

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