A1

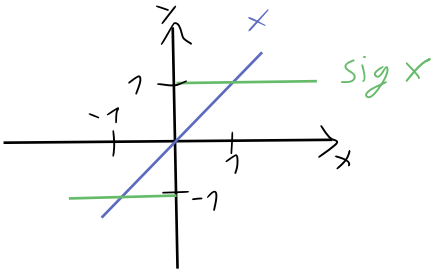
(a) $\mathbb{D} = (-\infty; +\infty)$ b) $\mathbb{D} = [0; +\infty)$ c) $\mathbb{D} = (-2; \infty)$

d) $\mathbb{D} = (0; \infty)$ e) $\mathbb{D} = \mathbb{R} \setminus \{0\}$ f) $\mathbb{D} = (-8; 4]$

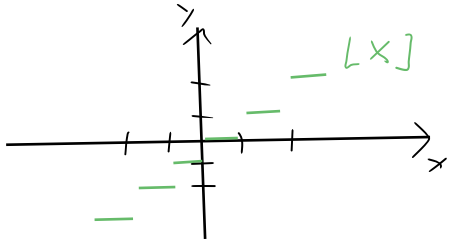
g) $\mathbb{D} = \mathbb{R} / n \cdot \frac{\pi}{2}$ $n \in \mathbb{Z}$ h) $\mathbb{D} = \mathbb{R} / x = (k+1)\pi$

A2

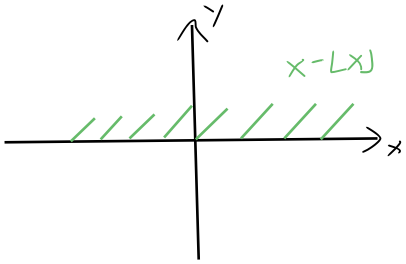
a)



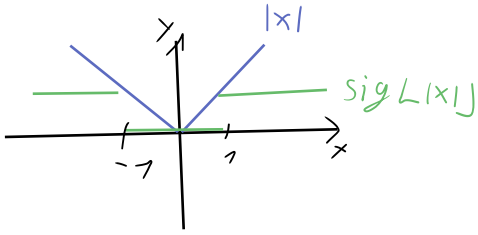
b)



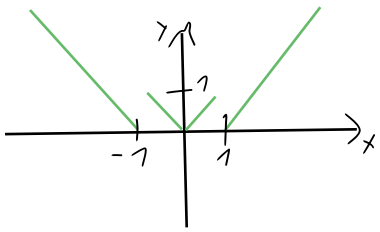
c)



d)



e)



A3

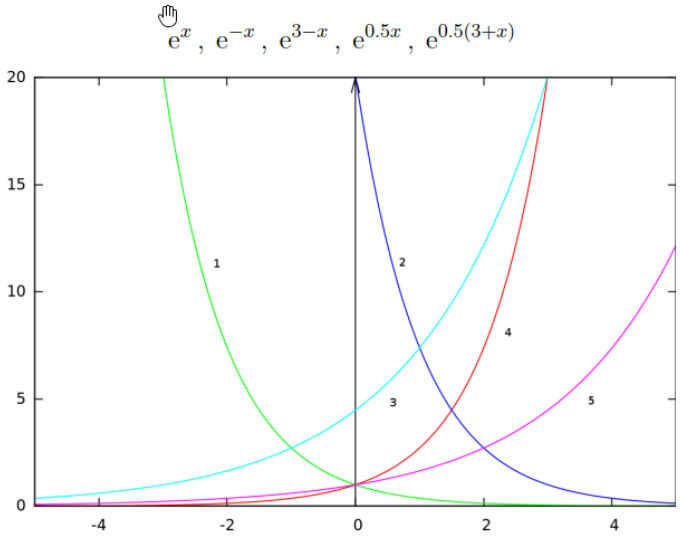
1: e^{-x}

2: e^{3-x}

3: $e^{0,5(3+x)}$

4: e^{x}

5: $e^{0,5x}$



A4

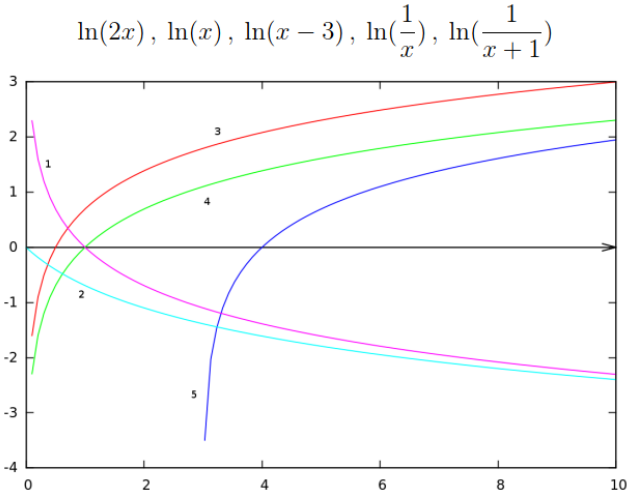
1: $\ln(\frac{1}{x})$

2: $\ln(\frac{1}{x+1})$

3: $\ln(2x)$

4: $\ln(x)$

5: $\ln(x-3)$



A5

a) $f(x) \to \infty$

b) $f(x) \to 0$

c) $f(x) \to \infty$

d) $f(x) \to 2$

e) $f(x) \to -\infty$

f) $f(x) \to -2$

g) $= \frac{2x^2-4}{3x^2+8x-3}$

$f(x) \to \frac{2}{3}$

h) $= \frac{x^3-4x^2}{2x-2x^3}$ $\quad f(x) \to -\frac{1}{2}$

A7

a) $: f(x) = \frac{1-x}{1-\sqrt{x}}$ $\quad \mathbb{D} = \mathbb{R}_0^+ \setminus \{1\}$

$\lim_{x \to 1} = \frac{0}{0}$

$$\frac{1-x}{1-\sqrt{x}} = \frac{1-(\sqrt{x})^2}{1-\sqrt{x}} = \frac{(1-\sqrt{x})(1+\sqrt{x})}{1-\sqrt{x}} = 1+\sqrt{x}$$

$\lim_{x \to 1} 1+\sqrt{x} = 2$

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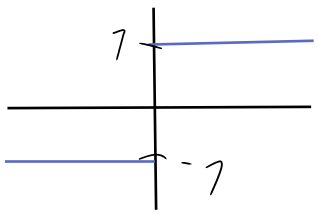
b) $g(t) = \frac{t}{|t|}$ $\quad \mathbb{D}_g = \mathbb{R} \setminus \{0\}$



$\lim_{-1 > 0} \frac{t}{|t|} = -1$ $\quad \lim_{1 \to 0} \frac{t}{|t|} = 1$

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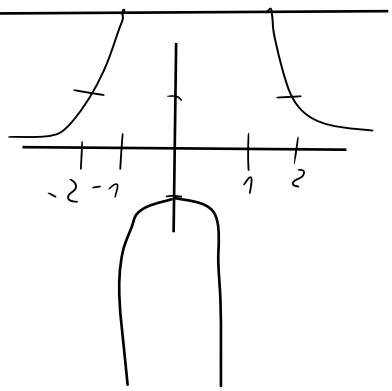
c) $h(s) = \frac{1}{|s|-1}$ $\quad \mathbb{D}_h = \mathbb{R} \setminus \{1,-1\}$

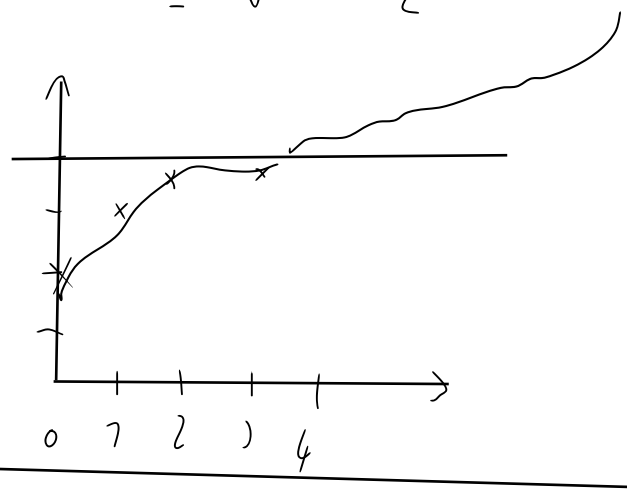


$\lim_{2 \to 1} \frac{1}{|s|-1} = \frac{1}{0,00\ldots 1} = \infty$ $\quad \lim_{0 \to 1} \frac{1}{|s|-1} = \frac{1}{-0,0\ldots 1} = -\infty$

d) $V(a) = \frac{a-4}{\sqrt{a}-2}$ $\quad D_V = \mathbb{R}^+ \setminus \{4\}$

$= \frac{(\sqrt{a})^2 - 4}{\sqrt{a}-2} = \frac{(\sqrt{a}-2)\cdot(\sqrt{a}+2)}{\sqrt{a}-2} = \sqrt{a}+2$

$\lim_{x \to 4} \sqrt{a}+2 = 2+2 = \underline{4}$



A 9

I $f(x) = 2x - 2$ $\quad D_f = \mathbb{R}$

$y = 2x - 2 \quad |+2$

$y+2 = 2x \quad |:2$

$\frac{y}{2} + 1 = x$ $\quad f^{-1}(x) = \frac{x}{2} + 1$ $\quad D_{f^{-1}} = \mathbb{R}$

II $f(x) = x^2 + 12x - 4$ $\quad D_f = \mathbb{R}$?

III $f(x) = \frac{1}{x}$ $\quad D_f = \mathbb{R} \setminus \{0\}$

$y = \frac{1}{x} \quad |\cdot x$

$x \cdot y = 1 \quad |:y$

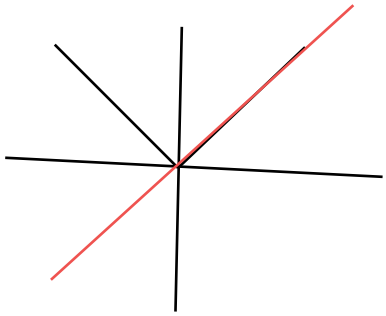
$x = \frac{1}{y}$ $\quad f^{-1}(x) = \frac{1}{x}$ $\quad D_{f^{-1}} = \mathbb{R} \setminus \{0\}$

IV $f(x) = \frac{x+3}{2x}$ $\quad D_f = \mathbb{R} \setminus \{0\}$

$f^{-1}(x) = \frac{3}{2x-1}$ $\quad D_{f^{-1}} = \mathbb{R} \setminus \{\frac{1}{2}\}$

V $f(x) = |x|$ $\quad D_f = \mathbb{R}$

$f^{-1}(x) = x$ $\quad D_{f^{-1}} = \mathbb{R}_0^+$



VI $f(x) = \sqrt{x}$ $\quad D_f = \mathbb{R}_0^+$

$f^{-1}(x) = x^2$ $\quad D_{f^{-1}} = \mathbb{R}$

A 10

a) $f(x) = a^x$ $\quad y = a^x$ $\quad f^{-1}(x) = \log_a(x)$

$\log_a(y) = x$

$f^{-1}(f) = f^{-1}(a^x) = \log_a(a^x) = x$

b) $f(x) = 1 - a^x$

$a^x = 1 - y$

$x = \log_a(1-y)$

$f^{-1}(x) = \log_a(1-x)$

$D_{f^{-1}} = \mathbb{R} < 1$

$f^{-1}(f) = f^{-1}(1 - a^x)$

$= \log_a(1 - 1 - a^x)$

$= \log_a(-a^x)$

c) $f(x) = a^{x-2}$

$y = a^{x-2}$

$\log_a(y) = x - 2$

$f^{-1}(x) = 2 + \log_a(x)$

$f^{-1}(f) = f^{-1}(a^{x-2})$

$= 2 + \log_a(a^{x-2})$

$= 2 + x - 2$

$= x$

d) $f(x) = 2a^{1-x}$

$f^{-1} = -\log_a\left(\frac{y}{2}\right) + 1$

$-\log_a\left(\frac{a^{x-2}}{2}\right) + 1$