1)

$$\begin{pmatrix} 1 & 3 & 4 & 5 \\ 2 & 4 & 5 & 6 \\ 3 & 6 & 9 & 7 \\ 4 & 8 & 12 & 16 \end{pmatrix}$$

2)

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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9)

$S, B, W = $ Anzahl Taschen $\neq 0$

$$S + B + W = 100$$

$$10\,S + 0{,}5\,B + 3W = 150$$

$$-9{,}5\,B - 7W = -950$$

$$-7W = -950 + 9{,}5\,B$$

$$\boxed{W = \frac{950}{7} - \frac{9{,}5\,B}{7}}$$

$$10\,S + 0{,}5\,B + \frac{3 \times 950}{7} - \frac{3 \times 9{,}5\,B}{7} = 150$$

$$10\,S = \frac{100}{7} + \frac{6}{7}\,B$$

$$\boxed{S = -\frac{1800}{70} + \frac{25\,B}{70}}$$

$$\frac{1800}{70} + \frac{25B}{70} + \frac{950}{7} - \frac{9{,}5\,B}{7} = 100 \qquad | \cdot 70$$

$$1800 + 25\,B + 9500 - 95\,B = 7000$$

$$11300 \quad - 70\,B = 7000 \qquad B =$$

2)

$A\ 2\times1 \qquad B\ 3\times2$

$C\ 2\times4 \qquad D\ 1\times4$

$F\ 4\times3$

a) I $\quad A\cdot B = 2\times1 \cdot 3\times2 \qquad \text{\Lightning}$

b) 3×3 II $\quad B\cdot A = 3\times2 \cdot 2\times3 \qquad \checkmark$

III $\quad A\cdot C = 2\times1 \cdot 2\times4 \qquad \text{\Lightning}$

IV $\quad D\cdot C = 1\times4 \cdot 2\times4 \qquad \text{\Lightning}$

V $\quad C\cdot D^T \approx 2\times4 \cdot 4\times1 \qquad \checkmark$

VI $\quad C\cdot D \approx 2\times4 \cdot 1\times4 \qquad \text{\Lightning}$

2×3 VII $\quad C\cdot F = 2\times4 \cdot 4\times3 \qquad \checkmark$

4×2 VIII $\quad F\cdot B = 4\times3 \cdot 3\times2 \qquad \checkmark$

IX $\quad B\cdot F = 3\times2 \cdot 4\times3 \qquad \text{\Lightning}$

c) $\quad B\times A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 1\cdot 1 + 2\cdot 2 \\ 2\cdot 1 + 3\cdot 2 \\ 3\cdot 1 + 4\cdot 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 11 \end{pmatrix}$

2) c)

$$C \cdot D^T = \begin{pmatrix} 1 & 0 & 3 & 1 \\ 2 & 1 & 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \cdot 1 + 0 \cdot 2 + 3 \cdot 3 + 1 \cdot 4 \\ 2 \cdot 1 + 1 \cdot 2 + 4 \cdot 3 + 5 \cdot 4 \end{pmatrix} = \begin{pmatrix} 1 + 0 + 9 + 4 \\ 2 + 2 + 12 + 20 \end{pmatrix}$$

$$= \begin{pmatrix} 14 \\ 36 \end{pmatrix}$$

$$C \cdot F = \begin{pmatrix} 1 & 0 & 3 & 1 \\ 2 & 1 & 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \cdot 1 + 0 \cdot 2 + 3 \cdot 3 + 1 \cdot 4 & \cdots & | & \cdots \\ 2 \cdot 1 + 1 \cdot 2 + 4 \cdot 3 + 5 \cdot 4 & \cdots & | & \cdots \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 34 & 54 \\ 36 & 84 & 132 \end{pmatrix}$$

$$F \cdot B = \begin{pmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \cdot 1 + 5 \cdot 2 + 9 \cdot 3 & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{pmatrix}$$

$$= \begin{pmatrix} 38 & 53 \\ 44 & 64 \\ 50 & 71 \\ 56 & 80 \end{pmatrix}$$

3)

$$V^T \cdot W = (1\ 2\ 3) \cdot \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = (1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6)$$

$$= (4 + 10 + 18) = (32)$$

$$V \cdot W^T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot (4\ 5\ 6) = \begin{pmatrix} 1 \cdot 4 \\ 2 \cdot 5 \\ 3 \cdot 6 \end{pmatrix} = \begin{pmatrix} 4 \\ 10 \\ 18 \end{pmatrix}$$

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4)

$$A \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \qquad B \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 1 \cdot 2 + 2 \cdot 1 & 1 \cdot 1 + 2 \cdot 1 \\ 1 \cdot 2 + 1 \cdot 2 & 1 \cdot 1 + 2 \cdot 1 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 4 & 3 \end{pmatrix}$$

$$A \cdot C$$

$$1 \cdot x_1 + 2 \cdot x_2 = 4 \qquad\qquad x_1 \neq 2$$

$$1 \cdot x_3 + 2 \cdot x_3 = 3 \qquad\qquad x_2 \neq 1$$

$$3 + 2 \cdot x_2 = 4 \qquad\qquad x_3 \neq 2$$

$$3 + 2 \cdot x_3 = 3 \qquad\qquad x_4 \neq 1$$

$$C \begin{pmatrix} 0,5 & 0 \\ 0,5 & 0 \end{pmatrix}$$

$$2 x_2 = 1 \qquad\qquad 0,5$$

$$2 x_3 = 0 \qquad\qquad 0$$

5)

Ordnen Sie passende Ausdrücke einander zu:

$A\,x$	$\displaystyle\sum_{i,j} a_{ji} x_i x_j$
$x^T A$	$\left(\displaystyle\sum_i a_{ij} x_i\right)_j$
$A^T x$	$\displaystyle\sum_{i,j} a_{ij} x_i x_j$
$x^T A x$	$\left(\displaystyle\sum_j a_{ij} x_j\right)_i$
$\left(x^T A^T x\right)^T$	$\left(\displaystyle\sum_i a_{ij} x_i\right)_j$
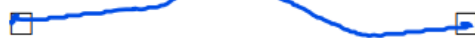
6) $_B \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

7) 1. $R3 \qquad x_4 = 3$

$x_2 - x_3 = 1 \implies x_3 = x_2 - 1$

$x_2 = 1 + x_3$

$x_1 + 2x_3 = -2 \implies x_1 = -2 - 2x_3$

$L\{\infty\} \quad Dim \, L = 1$

2. $r = 2 \qquad Dim \, L = 3 \qquad L = \emptyset$ $\qquad$ 8. $r = 3 \qquad D = 2$

∞

3. $r = 2 \quad Dim \, L = 1 \qquad \infty$ $\qquad$ 9. $r = 4 \quad D = 1$

∞ $\qquad\qquad\qquad\qquad\qquad\qquad$ ∞

4. $r = 3 \quad Dim \, L = 1 \qquad \infty$

5. $r = 3 \quad Dim \, L = 0 \qquad L\begin{pmatrix} -4 \\ 5 \\ 7 \end{pmatrix}$

6. $V = 4 \quad Dim \, L = 1 \qquad \infty$

7. $r = 2 \quad Dim \, L = 2 \qquad \infty$

8)

x	Y	Z	
1	1	1	60
1	-3	2	-4
2	5	-5	68

$24 \quad 20 \quad 16$

$II - I$
$III - 2 \cdot I$

x	Y	Z	
1	1	1	60
0	-4	1	-56
0	3	-7	-52

x	y	z	
1	1	1	60
0	-4	1	-56
0	0	$\frac{25}{4}$	10

$III + \frac{3}{4} II$

$z = \frac{40}{25} = \frac{8}{5}$ $\qquad$ $y = -56 + 4 - \frac{8}{5} = -\frac{268}{5}$

$x = 60 - \frac{8}{5} + \frac{268}{5} = 41,8$

b) 3,4,16