

$$1) a) \quad t \cdot \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + s \cdot \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} + v \cdot \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -9 \\ -7 \\ -15 \end{pmatrix}$$

$$\begin{array}{ccc|c} t & s & v & \\ \hline 2 & 1 & 3 & -9 \\ 1 & -1 & 2 & -7 \\ \hline 4 & 3 & 5 & -15 \end{array} \quad \begin{array}{l} 2 \cdot \text{II} - \text{I} \\ \text{III} - 2 \cdot \text{I} \end{array}$$

$$\begin{array}{ccc|c} t & s & v & \\ \hline 2 & 1 & 3 & -9 \\ \hline 0 & -3 & 1 & -5 \\ \hline 0 & 1 & -1 & 3 \end{array}$$

$$\text{III} = 3 \cdot \text{III} + \text{II} = \begin{array}{ccc|c} t & s & v & \\ \hline 0 & 0 & -2 & 4 \end{array} \rightarrow \underline{v = -2}$$

$$\begin{aligned} \rightarrow \text{II} - 3s + (-2) &= -5 \\ -3s &= -3 \\ \underline{s} &= \underline{1} \end{aligned}$$

$$\begin{aligned} \rightarrow \text{I} \quad 2t + 1 + 3 \cdot (-2) &= -9 \\ 2t + 1 - 6 &= -9 \\ 2t &= -4 \\ \underline{t} &= \underline{-2} \end{aligned}$$

$$x = -2 \cdot \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} - 2 \cdot \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$$

$$(ii) \quad \dots = \begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix}$$

$$\begin{array}{ccc|c} s & t & v & \\ \hline 2 & 1 & 3 & 6 \\ \hline 0 & -3 & 1 & 16 \\ \hline 0 & 1 & -1 & -6 \end{array}$$

$$\begin{array}{ccc|c} s & t & v & \\ \hline 2 & 1 & 3 & 6 \\ \hline 1 & -1 & 2 & -1 \\ \hline 4 & 3 & 5 & 6 \end{array} \quad \begin{array}{l} 2 \cdot \text{II} - 1 \\ \text{III} - 2 \cdot \text{I} \end{array}$$

$$\text{III} = 3 \cdot \text{III} + \text{II} = \begin{array}{ccc|c} s & t & v & \\ \hline 0 & 0 & -2 & -2 \end{array} \rightarrow \underline{v = 1}$$

$$\text{II} - 3t + 1 = 16 \Rightarrow \underline{t = -5}$$

$$\text{I} \quad 2s - 3 \cdot (-5) + 3 \cdot (1) = 6$$

$$2s + 15 + 3 = 6 \Rightarrow \underline{s = -6}$$

$$\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} - 5 \cdot \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} - 6 \cdot \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix} \quad \sqrt{4 \mid -5 \mid 1}$$

$$b) \quad a = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad v = \left\{ \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \end{pmatrix} \right\} \quad w = \left\{ \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$$

$$s \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{array}{cc|c} s & t & \\ \hline -1 & 0 & 1 \\ 3 & 4 & 2 \end{array} \quad \longrightarrow \quad \begin{array}{l} s = -1 \\ t = \frac{5}{4} \end{array}$$

$$\begin{pmatrix} 1 \\ -3 \end{pmatrix} + \begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$2) \quad P(x) = 3x^3 - 10x + 2$$

$$P = \{3, 2x, -5x^2, 4x^3\}$$

$$(3, 2x, -5, 4x^3) \cdot \begin{pmatrix} \frac{2}{3} \\ 3 \\ -5 \\ 0 \\ 3 \\ 4 \end{pmatrix}$$

3) 2.

4) a) Ja, Null an unterschiedl. Positionen

b) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, nicht eindeutig $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$
 aber nicht $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

c) nein, weil nur Matrix A_{ij}
 mit $I_2 = J_2$ möglich

d) 2.

