1 General Topology

Definition 1.1 (Topology)**.** A topological space is a set X with a collection of subsets $\mathcal{U}$, called open sets, such that

1. $\emptyset, X \in \mathcal{U}$.
2. The arbitrary union of open sets is open.
3. The finite union of open sets is open.

The complement $X - U$ of an open set U is called closed.

Definition 1.2. Let $(X,\mathcal{U})$, $(X,\mathcal{V})$ be topologies. $\mathcal{U}$ is called stronger(finer) than $\mathcal{V}$ if $\mathcal{V} \in \mathcal{U}$, and weaker(coarser) if $\mathcal{V} \in \mathcal{U}$.

Definition 1.3. A basis $\mathcal{B}$ of a topology for X is a collection of subsets of X such that

1. For each $x \in X$ there is at least one $B \in \mathcal{B}$ with $x \in B$.
2. If $x \in B_1 \cap B_2$ then there exists a $B_3 \subset B_1 \cap B_2$ with $x \in B$.

We say that $\mathcal{B}$ generates the topology $\mathcal{U}$ if U is open iff for every $x \in U$ there exits $B \in \mathcal{B}$ with $x \in B \subset U$.

Lemma 1.4. bla