1 General Topology

Definition 1.1 (Topology)**.** A topological space is a set X with a collection of subsets $\mathcal{U}$, called open sets, such that

1. $\emptyset, X \in \mathcal{U}$.
2. The arbitrary union of open sets is open.
3. The finite union of open sets is open.

The complement $X - U$ of an open set U is called closed.

Definition 1.2. Let $(X,\mathcal{U})$, $(X,\mathcal{V})$ be topologies. $\mathcal{U}$ is called stronger(finer) than $\mathcal{V}$ if $\mathcal{V} \subset \mathcal{U}$, and weaker(coarser) if $\mathcal{U} \subset \mathcal{V}$.

Definition 1.3. A basis $\mathcal{B}$ of a topology for X is a collection of subsets of X such that

1. For each $x \in X$ there is at least one $B \in \mathcal{B}$ with $x \in B$.
2. If $x \in B_1 \cap B_2$ then there exists a $B_3 \subset B_1 \cap B_2$ with $x \in B_3$.

We say that $\mathcal{B}$ generates the topology $\mathcal{U}$ if U is open iff for every $x \in U$ there exits $B \in \mathcal{B}$ with $x \in B \subset U$.

Lemma 1.4. Let $\mathcal{B}$ be a basis for a topology $\mathcal{U}$ on X. Then $\mathcal{U}$ is equal to the collection of all unions of elements of $\mathcal{B}$.

Lemma 1.5. If $\mathcal{C}$ is a collection of open sets, such that for each $U \subset X$ open, $x \in U$ there is $C \in \mathcal{C}$ such that $x \in C \subset U$, then $\mathcal{C}$ is a basis for X.

Lemma 1.6. Let $\mathcal{B}$, $\mathcal{B}'$ be bases for topologies $\mathcal{U}$, $\mathcal{U}'$ respectively on X. Then the following are equivalent:

1. $\mathcal{U}'$ is finer than $\mathcal{U}$.
2. For each $x \in B \in \mathcal{B}$, there is a $B' \in \mathcal{B}'$ with $x \in B' \subset B$.

Definition 1.7. A subbasis $\mathcal{S}$ for a topology on X is a collection of subsets whose union equals X. The topology generated by the subbasis is defined to be the collection of all unions of finite intersections of elements of $\mathcal{S}$.