

## Chapter 2

# Coq and SSReflect

We decided to employ the Small Scale Reflection Extension (**SSReflect**<sup>1</sup>) for the **Coq**<sup>2</sup> proof assistant. The most important factors in this decision were SSREFLECT's excellent support for finite types, list operations and graphs. SSREFLECT also introduces an alternative scripting language that can often be used to shorten the bookkeeping overhead of proofs considerably.

### 2.1 Coq

Description,  
citation

### 2.2 SSReflect

SSREFLECT is a set of extensions to the proof scripting language of the COQ proof assistant. They were originally developed to support small-scale reflection. However, most of them are of quite general nature and improve the functionality of COQ in most basic areas such as script layout and structuring, proof context management and rewriting [4].

#### 2.2.1 Finite Types and Ordinals

The most important feature of SSREFLECT for our purpose are finite types. Finite types are based on lists. Every element of a finite type is contained in the associated (de-duplicated) list and vice versa. SSREFLECT's support for finite types is based on canonical structures, instances of which come predefined for basic finite types and type constructors. This allows us to easily combine basic finite types such as `bool` with type constructors such as `option` and `sum`. We can also create finite types directly from lists.

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<sup>1</sup><http://www.msr-inria.inria.fr/Projects/math-components>

<sup>2</sup><http://coq.inria.fr/>

SSREFLECT provides boolean versions of the universal and existential quantifiers on finite types, **forallb** and **existsb**. We can compute the number of elements in a finite type  $F$  with  $\#|F|$ . `enum` gives a list of all items of a finite type. Finite types also come with enumeration functions which provide a consistent ordering. The corresponding functions are `enum_rank` and `enum_val`. The input of `enum_val` and the result of `enum_rank` are ordinals, i.e. values in  $[0, \#|F|-1]$ . The corresponding type can be written as  $1_{\#|F|}$ .

### 2.2.2 Boolean Reflection

SSREFLECT offers boolean reflections for decidable propositions. This allows us to switch back and forth between equivalent boolean and propositional predicates.

### 2.2.3 Boolean Predicates

We make use of SSREFLECT's syntax to specify boolean predicates. This allows us to specify predicates in a way that resembles set-theoretic notation, e.g. `[pred x | <boolean expression in x>]`. Furthermore, we can use the functions `pred1` and `pred0` to specify the singleton predicate and the empty predicate, respectively. The complement of a predicate can be written as `[predC p]`. The syntax for combining predicates is `[pred? p1 & p2]`, with `?` being replaced with one of `U` (union), `I` (intersection) or `D` (difference). There is also syntax for the preimage of a predicate under a function which can be written as `[preim f of p]`.

There are also applicative (functional) versions of `predC`, `predU`, `predI`, `predD` which are functions that take predicates and return predicates.