

## Assignment 5 DESIGN.pdf

Ning Jiang

**Description of Program:** For this assignment, we mainly need to Encrypt and decrypt files using RSA. The keygen program will be in charge of key generation, producing RSA public and private key pairs. The encrypt program will encrypt files using a public key, and the decrypt program will decrypt the encrypted files using the corresponding private key.

### Basic rules of RSA:

The image shows handwritten notes detailing the RSA process. On the left, under 'Encryption. public key', it shows the encryption of text 'A' (represented by 5) using a public key (11, 14). The calculation is  $2^5 \pmod{14} = 32 \pmod{14} = 4$ . The ciphertext is 'D' (represented by 4). On the right, under 'Decryption. Private key', it shows the decryption of ciphertext 'D' (4) using a private key (11, 14). The calculation is  $4^{11} \pmod{14} = 4$ , which corresponds to the original text 'A'. In the center, under 'How does it work??', it lists the steps for key generation: 1. pick 2 random numbers (prime)  $p=2, q=7$ ; 2.  $N=pq=14$ ; 3.  $\phi(N)=(p-1)(q-1)=1 \times 6=6$ ; 4. choose  $e$  such that  $1 < e < \phi(N)$  and coprime with  $N, \phi(N)$ . It shows  $e=5$  is chosen. On the far right, a list of numbers 1-14 is shown, with 5 and 11 circled as coprime with 14.

**Encryption. public key**

text:  $(5, 14) \rightarrow 2$   
number

$2^5 \pmod{14} = 32 \pmod{14}$   
 $= (32 + 14) \% 14$   
 $= 4 \pmod{14}$

Ciphertext:  $(11, 14)$   
D.  $\rightarrow 4$   
number

**Decryption. Private key**

$4^{11} \pmod{14} = 4^{11} \pmod{14}$   
 $= 2 \pmod{14}$

How to know 11 would be the key that fits (5, 14)?

**How does it work??**

- pick 2 random numbers (prime)  
 $p=2, q=7$
- $N=pq=14$
- $\phi(N)=(p-1)(q-1)=1 \times 6=6$   
coprime number
- choose  $e$ 
  - $1 < e < \phi(N)$
  - coprime with  $N, \phi(N)$

$e=5$  Lock (5, 14)

**choose d:**  $d \cdot e \pmod{\phi(N)} = 1$   
 $d \cdot 5 \pmod{6} = 1$

1, 10, 15, 20, 25, 30, ...  
5, 4, 3, 2, 1, 0  
 $d=5$  ✓

X common factor of 14

### Files to be included in directory "asn5":

1. decrypt.c: This contains the implementation and main() function for the decrypt program.
2. encrypt.c: This contains the implementation and main() function for the encrypt program.
3. keygen.c: This contains the implementation and main() function for the keygen program.
4. numtheory.c: This contains the implementations of the number theory functions.
5. numtheory.h: This specifies the interface for the number theory

functions.

6. `randstate.c`: This contains the implementation of the random state interface for the RSA library and number theory functions.
7. `randstate.h`: This specifies the interface for initializing and clearing the random state.
8. `rsa.c`: This contains the implementation of the RSA library.
9. `rsa.h`: This specifies the interface for the RSA library.
10. `Makefile`
11. `README.md`
12. `DESIGN.pdf`

## Structure & explanation / Pseudocode:

We first implement two libraries and a random state module that will be used in keygen, encrypt and decrypt. The pseudocode will be provided in the Pseudocode part.

Explanations for some of the equations are cited from `asgn5.pdf`.

### ***randstate.c:***

*Global variable state*

#### **void randstate\_init(uint64\_t seed):**

`gmp_randinit_mt()` and `gmp_randseed_ui()`.

Initializes the global random state named `state`.

#### **void randstate\_clear(void):**

Clears and frees all memory used by the initialized global random state named `state`.

### ***numtheory.c:***

#### **void pow\_mod(mpz\_t out, mpz\_t base, mpz\_t exponent, mpz\_t modulus):**

*POWER-MOD(a, d, n)*

```
1  v ← 1
2  p ← a
3  while d > 0
4      if ODD(d)
5          v ← (v × p) mod n
6          p ← (p × p) mod n
7          d ← ⌊d/2⌋
8  return v
```

Performs fast modular exponentiation, computing base raised to the exponent power modulo modulus, and storing the computed result in out.

**bool is\_prime(mpz\_t n, uint64\_t iters):**

```

MILLER-RABIN(n,k)
1  write  $n-1 = 2^s r$  such that  $r$  is odd
2  for  $i \leftarrow 1$  to  $k$ 
3      choose random  $a \in \{2, 3, \dots, n-2\}$ 
4       $y = \text{POWER-MOD}(a, r, n)$ 
5      if  $y \neq 1$  and  $y \neq n-1$ 
6           $j \leftarrow 1$ 
7          while  $j \leq s-1$  and  $y \neq n-1$ 
8               $y \leftarrow \text{POWER-MOD}(y, 2, n)$ 
9              if  $y == 1$ 
10                 return FALSE
11              $j \leftarrow j+1$ 
12         if  $y \neq n-1$ 
13             return FALSE
14  return TRUE

```

indicate whether or not  $n$  is prime.

**void make\_prime(mpz\_t p, uint64\_t bits, uint64\_t iters):**

Generates a new prime number stored in  $p$ . The generated prime should be at least  $\text{bits}$  number of bits long.

1. pick a random number which is at least  $\text{nbits}$  long.
2. Use `is_prime` to check whether it is prime or not.
3. If yes, return. Else, pick another random number.

**void gcd(mpz\_t d, mpz\_t a, mpz\_t b):**

```

GCD(a,b)
1  while  $b \neq 0$ 
2       $t \leftarrow b$ 
3       $b \leftarrow a \bmod b$ 
4       $a \leftarrow t$ 
5  return  $a$ 

```

Computes the greatest common divisor of  $a$  and  $b$ , storing the value of the computed divisor in  $d$ .

**void mod\_inverse(mpz\_t i, mpz\_t a, mpz\_t n):**

```

MOD-INVERSE(a,n)
1  (r,r') ← (n,a)
2  (t,t') ← (0,1)
3  while r' ≠ 0
4      q ← ⌊r/r'⌋
5      (r,r') ← (r', r - q × r')
6      (t,t') ← (t', t - q × t')
7  if r > 1
8      return no inverse
9  if t < 0
10     t ← t + n
11  return t

```

Computes the inverse  $i$  of  $a$  modulo  $n$ .

### ***rsa.c***

```

void rsa_make_pub(mpz_t p, mpz_t q, mpz_t n, mpz_t e, uint64_t nbits,
uint64_t iters):

```

Create a random bits

make\_prime(p)

make\_primeq(q)

$\lambda(n) = \text{lcm}(p-1, q-1) = \text{abs}((p-1)(q-1)) / \text{gcd}(p-1, q-1)$

While true:

    Create a random number

    Check whether it is coprime with  $\lambda(n)$

    If true, return.

Creates parts of a new RSA public key: two large primes  $p$  and  $q$ , their product  $n$ , and the public exponent

```

void rsa_write_pub(mpz_t n, mpz_t e, mpz_t s, char username[], FILE
*pbfile):

```

If the file is valid:

    Print  $n$ ,  $e$ ,  $s$ ,  $username$  to the pbfile.

Writes a public RSA key to pbfile.

```

void rsa_read_pub(mpz_t n, mpz_t e, mpz_t s, char username[], FILE
*pbfile):

```

If the file is valid:

    Read  $n$ ,  $e$ ,  $s$ ,  $username$  from the pbfile.

Reads a public RSA key from pbfile.

```

void rsa_make_priv(mpz_t d, mpz_t e, mpz_t p, mpz_t q):

```

$\lambda(n) = \text{lcm}(p-1, q-1) = \text{abs}((p-1)(q-1)) / \text{gcd}(p-1, q-1)$   
`mod_inverse(e, l)`

Creates a new RSA private key `d` given primes `p` and `q` and public exponent `e`.

**`void rsa_write_priv(mpz_t n, mpz_t d, FILE *pvfile):`**

If the file is valid:

Print `n`, `d`, username to the `pvfile`.

Writes a private RSA key to `pvfile`.

**`void rsa_read_priv(mpz_t n, mpz_t d, FILE *pvfile):`**

If the file is valid:

Read `n`, `d`, username from the `pvfile`.

Reads a private RSA key from `pvfile`.

**`void rsa_encrypt(mpz_t c, mpz_t m, mpz_t e, mpz_t n):`**

`pow_mod(m, e, n)`

computing ciphertext `c` by encrypting message `m` using public exponent `e` and modulus `n`.

**`void rsa_encrypt_file(FILE *infile, FILE *outfile, mpz_t n, mpz_t e):`**

block size `k = floor((log2(n)-1)/8)`

Dynamically allocate an array: (`arr`) that can hold `k` bytes.

`Arr[0] = 0xFF`

While there are still unprocessed bytes in `infile` // use `fread()`

`mpz_import()`, convert the read bytes, including the prepended `0xFF` into an `mpz_t m`. 1 for most significant word first, 1 for the endian parameter, and 0 for the nails parameter.

Encrypt `m` with `rsa_encrypt()`,

encrypted number to `outfile` as a hexstring

Encrypts the contents of `infile`, writing the encrypted contents to `outfile`.

**`void rsa_decrypt(mpz_t m, mpz_t c, mpz_t d, mpz_t n):`**

$M = c^d \pmod n$ .

computing message `m` by decrypting ciphertext `c` using private key `d` and public modulus `n`.

```
void rsa_decrypt_file(FILE *infile, FILE *outfile, mpz_t n, mpz_t d):
```

```
block size k= floor((log2(n)-1)/8)
```

Dynamically allocate an array: (arr) that can hold kbytes.

While there are still unprocessed bytes in infile // gmp\_fscanf()

Using mpz\_export(), convert c back into bytes, storing them in the allocated block. 1 for most significant word first, 1 for the endian parameter, and 0 for the nails parameter.

Write out j-1 bytes starting from index 1 of the block to outfile.

Decrypts the contents of infile, writing the decrypted contents to outfile.

```
void rsa_sign(mpz_t s, mpz_t m, mpz_t d, mpz_t n):
```

```
s= m^d (mod n)
```

producing signature s by signing message m using private key d and public modulus n.

```
bool rsa_verify(mpz_t m, mpz_t s, mpz_t e, mpz_t n):
```

```
t= V(s) = s^e (mod n).
```

```
If t == m:
```

```
Return true
```

```
Else:
```

```
return false
```

returning true if signature s is verified and false otherwise.

### ***keygen.c***

This contains the implementation and main() function for the keygen program including make the public and private keys, get the current user's name, write the computed public and private key to their respective files.

The program should follow these steps:

1. Parse command-line options using getopt() and handle them accordingly.
2. Open the public and private key files using fopen(). Print a helpful error and exit the program in the event of failure.
3. Using fchmod() and fileno(), make sure that the private key file permissions are set to 0600, indicating read and write permissions for the user, and no

permissions for anyone else.

4. Initialize the random state using `randstate_init()`, using the set seed.

5. Make the public and private keys using `rsa_make_pub()` and `rsa_make_priv()`, respectively.

6. Get the current user's name as a string. You will want to use `getenv()`.

7. Convert the username into an `mpz_t` with `mpz_set_str()`, specifying the base as 62. Then, use `rsa_sign()` to compute the signature of the username.

8. Write the computed public and private key to their respective files.

If verbose output is enabled print the following, each with a trailing newline, in order:

(a) username

(b) the signature `s`

(c) the first large prime `p`

(d) the second large prime `q`

(e) the public modulus `n`

(f ) the public exponent `e`

(g) the private key `d`

All of the `mpz_t` values should be printed with information about the number of bits that constitute them, along with their respective values in decimal. See the reference key generator program for an example.

10. Close the public and private key files, clear the random state with `randstate_clear()`, and clear any `mpz_t` variables you may have used.

### ***encrypt.c***

This contains the implementation and `main()` function for the encrypt program including read the public key from the opened public key file and encrypt the file.

The program should follow these steps:

1. Parse command-line options using `getopt()` and handle them accordingly.

2. Open the public key file using `fopen()`. Print a helpful error and exit the program in the event of failure.

3. Read the public key from the opened public key file.

4. If verbose output is enabled print the following, each with a trailing newline, in order:

- (a) username
- (b) the signature `s`
- (c) the public modulus `n`
- (d) the public exponent `e`

All of the `mpz_t` values should be printed with information about the number of bits that constitute them, along with their respective values in decimal. See the reference encryptor program for an example.

5. Convert the username that was read in to an `mpz_t`. This will be the expected value of the verified signature. Verify the signature using `rsa_verify()`, reporting an error and exiting the program if the signature couldn't be verified.
6. Encrypt the file using `rsa_encrypt_file()`.
7. Close the public key file and clear any `mpz_t` variables you have used.

### ***decrypt.c***

This contains the implementation and `main()` function for the decrypt program including read the private key from the opened private key file, decrypt the file.

The program should follow these steps:

1. Parse command-line options using `getopt()` and handle them accordingly.
2. Open the private key file using `fopen()`. Print a helpful error and exit the program in the event of failure.
3. Read the private key from the opened private key file.
4. If verbose output is enabled print the following, each with a trailing newline, in order:
  - (a) the public modulus `n`
  - (b) the private key `e`

Both these values should be printed with information about the number of bits that constitute them, along with their respective values in decimal. See the reference decryptor program for an example.

5. Decrypt the file using `rsa_decrypt_file()`.
6. Close the private key file and clear any `mpz_t` variables you have used.



**Error Handling:**

Credit: