

# RankPL: A Qualitative Probabilistic Programming Language

Tjitze Rienstra

University of Luxembourg  
tjitze@gmail.com

**Abstract.** In this paper we introduce *RankPL*, a modeling language that can be thought of as a qualitative variant of a probabilistic programming language. It has a semantics based on Spohn’s ranking theory and can be used, broadly speaking, to represent and reason about processes that exhibit uncertainty expressible by distinguishing “normal” from “surprising” events. It supports various types of reasoning, including causal inference, abduction and (iterated) belief revision. We present syntax and semantics, demonstrate the language with practical examples, and discuss an implementation that we made available for download.

## 1 Introduction

*Probabilistic programming languages* (PPLs) are regular programming languages extended with statements to (1) draw values at random from a given probability distribution, and (2) condition values of variables due to observation. Rather than a deterministic outcome, a probabilistic program represents a probability distribution over different possible outcomes. PPLs greatly simplify the problem of representing and reasoning with rich probabilistic models and have received increasing attention in recent years, mainly in the context of Bayesian machine learning. Examples of PPLs include Church [?], Venture [?] and Figaro [?]. Early work on PPLs goes back to Kozen [?].

*Ranking theory* is a qualitative abstraction of probability theory in which events receive discrete degrees of surprise called *ranks* [?]. That is, event can be ranked 0 (not surprising), 1 (surprising), 2 (very surprising), and so on, or  $\infty$  if impossible. Ranking theory is computationally simpler than probability theory and does not require precise probabilities to perform meaningful inference. Still, it provides analogues to many powerful probabilistic notions, including conditioning and independence. Ranking theory has been applied in logic-based AI approaches such as belief revision and non-monotonic reasoning [?, ?] as well as formal epistemology [?].

In this paper we develop a language called RankPL. Semantically, the language draws a parallel with probabilistic programming in terms of ranking theory. We start with a minimal imperative programming language (**if-then-else**, **while**, etc.) and extended it with statements to (1) draw choices at random from a given ranking function and (2) perform ranking-theoretic conditioning

on values of variables due to observation. Analogous to probabilistic programs, RankPL programs represent ranking functions over possible outcomes.

Broadly speaking, RankPL can be used to represent and reason about processes whose input or behavior exhibits uncertainty expressible by distinguishing normal (rank 0) from surprising (rank  $> 0$ ) events. Reasoning tasks that are supported include causal inference, abduction and different types of (iterated) belief revision. The latter are consistent with the AGM postulates for belief revision and DP postulates for iterated belief revision [?,?]. Just as PPLs provide a high degree of flexibility in reasoning about rich probabilistic models, RankPL does so for ranking-theoretic models.

The overview of this paper is as follows. Section ?? deals with basics of ranking theory. In section ?? we introduce RankPL and present its syntax and semantics. This is a *denotational* semantics, which defines the meaning of a statement in terms of a ranking transformation function. In section ?? we discuss two generalized conditioning schemes (L-conditioning and J-conditioning) and show how they can be implemented in RankPL. All the above will be demonstrated by practical examples of RankPL programs. In section ?? we discuss our RankPL implementation and conclude in section ??.

## 2 Ranking Theory

In this section we present the necessary basics of ranking theory, all of which is due to Spohn [?]. The definition of a *ranking function* presupposes a finite set  $\Omega$  of possibilities and a boolean algebra  $\mathcal{A}$  over the subsets of  $\Omega$ , called *propositions*.

**Definition 1.** *A ranking function is a function  $\kappa : \Omega \rightarrow \mathbb{N} \cup \{\infty\}$  that associates events with ranks.  $\kappa$  is extended to a function over propositions by defining  $\kappa(\emptyset) = \infty$  and  $\kappa(A) = \min(\{\kappa(w) \mid w \in A\})$  for each  $A \in \mathcal{A} \setminus \emptyset$ . A ranking function must satisfy  $\kappa(\Omega) = 0$ .*

As mentioned in the introduction, ranks can be understood as degrees of surprise or, alternatively, as inverse degrees of plausibility. The requirement that  $\kappa(\Omega) = 0$  is equivalent to the condition that at least one  $w \in \Omega$  receives a rank of 0. We sometimes work with functions  $\lambda : \Omega \rightarrow \mathbb{N} \cup \{\infty\}$  that violate this condition. The *normalization* of  $\lambda$  is a ranking function denoted by  $\|\lambda\|$  and defined by  $\|\lambda\|(w) = \lambda(w) - \lambda(\Omega)$ . Conditional ranks are defined as follows.

**Definition 2.** *Given a ranking function  $\kappa$ , the rank of  $A$  conditional on  $B$  (denoted  $\kappa(A \mid B)$ ) is defined by*

$$\kappa(A \mid B) = \begin{cases} \kappa(A \cap B) - \kappa(B) & \text{if } \kappa(B) \neq \infty, \\ \infty & \text{otherwise.} \end{cases}$$

We denote by  $\kappa_B$  the ranking function defined by  $\kappa_B(A) = \kappa(A \mid B)$ .

In words, the effect of conditioning on  $B$  is that the rank of  $B$  is shifted down to zero (keeping the relative ranks of the possibilities in  $B$  constant) while the rank of its complement is shifted up to  $\infty$ .

How do ranks compare to probabilities? An important difference is that ranks of events do not add up as probabilities do. That is, if  $A$  and  $B$  are disjoint, then  $\kappa(A \cup B) = \min(\kappa(A), \kappa(B))$ , while  $P(A \cup B) = P(A) + P(B)$ . This is, however, consistent with the interpretation of ranks as degrees of surprise (i.e.,  $A \cup B$  is no less surprising than  $A$  or  $B$ ). Furthermore, ranks provide deductively closed beliefs, whereas probabilities do not. More precisely, if we say that  $A$  is believed with firmness  $x$  (for some  $x > 0$ ) with respect to  $\kappa$  iff  $\kappa(\overline{A}) > x$ , then if  $A$  and  $B$  are believed with firmness  $x$  then so is  $A \cap B$ . A similar notion of belief does not exist for probabilities, as is demonstrated by the Lottery paradox [?].

Finally, note that  $\infty$  and 0 in ranking theory can be thought of as playing the role of 0 and 1 in probability, while  $\min$ , subtraction and addition play the role, respectively, of addition, division and multiplication. Recall, for example, the definition of conditional probability, and compare it with definition ???. This correspondence also underlies notions such as (conditional) independence and ranking nets (the ranking-based counterpart of Bayesian networks) that have been defined in terms of rankings [?].

### 3 RankPL

Let start with a brief overview of the features of RankPL. The basis is a minimal imperative language consisting of integer-typed variables, an **if-then-else** statement and a **while-do** construct. We extend it with the two special statements mentioned in the introduction. We call the first one *ranked choice*. It has the form  $\{s_1\} \langle e \rangle \{s_2\}$ . Intuitively, it states that either  $s_1$  or  $s_2$  is executed, where the former is a normal (rank 0) event and the latter a typically surprising event whose rank is the value of the expression  $e$ . Put differently, it represents a draw of a statement to be executed, at random, from a ranking function over two choices. Note that we can set  $e$  to zero to represent a draw from two equally likely choices, and that larger sets of choices can be represented through nesting.

The second special statement is called the *observe* statement **observe**  $b$ . It states that the condition  $b$  is observed to hold. Its semantics corresponds to ranking-theoretic conditioning.

To illustrate these statements, consider the program

$$x := 10; \{y := 1\} \langle 1 \rangle \{\{y := 2\} \langle 1 \rangle \{y := 3\}\}; x := x \times y;$$

This program has three possible outcomes:  $x = 10$ ,  $x = 20$  and  $x = 30$ , ranked 0, 1 and 2, respectively. Now suppose we extend the program as follows:

$$x := 10; \{y := 1\} \langle 1 \rangle \{\{y := 2\} \langle 1 \rangle \{y := 3\}\}; \textbf{observe } y > 1; x := x \times y;$$

Here, the observation rules out the possibility  $y = 1$ , and the ranks of the remaining possibilities are shifted down, resulting in two outcomes  $x = 20$  and  $x = 30$ , ranked 0 and 1, respectively.

Finally, the language also includes the *rank expression* **rank**  $b$ . This is an integer expression that evaluates to the rank of the boolean expression  $b$ . Its use will be demonstrated later.

### 3.1 Syntax

We fix a set  $Vars$  of variables (ranged over by  $x$ ) and denote by  $Val$  the set of integers including  $\infty$  (ranged over by  $n$ ). We include  $\infty$  to be able to express ranks. We use  $e$ ,  $b$  and  $s$  to range over the numerical expressions, boolean expressions, and statements. They are defined by the following BNF rules:

$$\begin{aligned} e &:= n \mid x \mid \mathbf{rank} \ b \mid (e_1 \diamond e_2) \text{ (for } \diamond \in \{-, +, \times, \div\}) \\ b &:= \neg b \mid (b_1 \vee b_2) \mid (e_1 \blacklozenge e_2) \text{ (for } \blacklozenge \in \{=, <\}) \\ s &:= \{s\} \mid s_0; s_1 \mid x := e \mid \mathbf{if} \ b \ \mathbf{then} \ s_1 \ \mathbf{else} \ s_2 \mid \\ &\quad \mathbf{while} \ b \ \mathbf{do} \ s \mid s_1 \ \langle e \rangle \ s_2 \mid \mathbf{observe} \ b \mid \mathbf{skip} \end{aligned}$$

Some syntactical shortcuts: Brackets and parentheses will be omitted unless this leads to ambiguity. We define  $\wedge$  in terms of  $\vee$  and  $\neg$ . We write  $\mathbf{if} \ b \ \mathbf{then} \ s$  instead of  $\mathbf{if} \ b \ \mathbf{then} \ s \ \mathbf{else} \ \mathbf{skip}$ , and abbreviate statements of the form  $\{x := e_1\} \langle e \rangle \{x := e_2\}$  to  $x := e_1 \langle e \rangle e_2$ . Note that the **skip** statement does nothing and is added for technical convenience.

### 3.2 Semantics

The denotational semantics of RankPL defines the meaning of a statement  $s$  as a function  $D[s]$  that maps prior rankings into posterior rankings. The objects of these rankings are *valuations*, i.e., functions denoted by  $\sigma$  that assign values to all variables. The *initial valuation* sets all variables to 0 and is denoted by  $\sigma_0$ . The *initial ranking* assigns 0 to  $\sigma_0$  and  $\infty$  to all others, and is denoted by  $\kappa_0$ . We denote by  $\sigma[x \rightarrow n]$  the valuation equivalent to  $\sigma$  except for assigning  $n$  to  $x$ .

From now on we associate  $\Omega$  with the set of valuations and denote the set of rankings over  $\Omega$  by  $K$ . Intuitively, if  $\kappa$  yields the degree of surprise that a given valuation is the actual valuation *before* executing  $s$ , then  $D[s](\kappa)$  yields the degree of surprise that a given valuation is the actual valuation *after* executing  $s$ . If we refer to the result of running the *program*  $s$ , we refer to the ranking  $D[s](\kappa_0)$ .

Because a statement  $s$  might not execute successfully,  $D[s]$  is not a total function over  $K$ . There are two issues to deal with. First of all, non-termination of a loop leads to an undefined outcome. Therefore  $D[s]$  is a partial function whose value  $D[s](\kappa)$  is defined only if  $s$  terminates given  $\kappa$ . Secondly, observe statements may rule out all possibilities. A program whose outcome is empty because of this is said to *fail*. We denote failure with a special ranking  $\kappa_\infty$  that assigns  $\infty$  to all valuations. Since  $\kappa_\infty \notin K$ , we define the range of  $D[s]$  by  $K^* = K \cup \{\kappa_\infty\}$ . Thus, the semantics will be defined by a partial function

$$D : S \rightarrow (K^* \rightarrow K^*).$$

But first, we define the semantics of expressions. A numerical expression is evaluated with respect to both a ranking function (to determine values of **rank** expressions) and a valuation (to determine values of variables). Boolean expressions may also contain **rank** expressions and therefore also depend on a ranking function. Given a valuation  $\sigma$  and ranking  $\kappa$ , we denote by  $\sigma_\kappa(e)$  the value of the

numerical expression  $e$  w.r.t.  $\sigma$  and  $\kappa$ , and by  $[b]_\kappa$  the set of valuations satisfying the boolean expression  $b$  w.r.t.  $\kappa$ . These functions are defined as follows.<sup>1</sup>

$$\begin{aligned} \sigma_\kappa(n) &= n & [\neg b]_\kappa &= \Omega \setminus [b]_\kappa \\ \sigma_\kappa(x) &= \sigma(x) & [b_1 \vee b_2]_\kappa &= [b_1]_\kappa \cup [b_2]_\kappa \\ \sigma_\kappa(\mathbf{rank} \ b) &= \kappa([b]_\kappa) & [a_1 \blacklozenge a_2]_\kappa &= \{\sigma \in \Omega \mid \sigma_\kappa(a_1) \blacklozenge \sigma_\kappa(a_2)\} \\ \sigma_\kappa(a_1 \blacklozenge a_2) &= \sigma_\kappa(a_1) \blacklozenge \sigma_\kappa(a_2) \end{aligned}$$

Given a boolean expression  $b$  we will write  $\kappa(b)$  as shorthand for  $\kappa([b]_\kappa)$  and  $\kappa_b$  as shorthand for  $\kappa_{[b]_\kappa}$ . We are now ready to define the semantics of statements, which is captured by seven rules, numbered **D1** to **D7**. The first deals with the **skip** statement, which does nothing and therefore maps to the identity function.

$$D[\mathbf{skip}](\kappa) = \kappa. \quad (\mathbf{D1})$$

The meaning of  $s_1; s_2$  is the composition of  $D[s_1]$  and  $D[s_2]$ .

$$D[s_1; s_2](\kappa) = D[s_2](D[s_1](\kappa)) \quad (\mathbf{D2})$$

The rank of a valuation  $\sigma$  after executing an assignment  $x := e$  is the minimum of all ranks of valuations that equal  $\sigma$  after assigning the value of  $e$  to  $x$ .

$$D[x := e](\kappa)(\sigma) = \kappa(\{\sigma' \in \Omega \mid \sigma = \sigma'[x \rightarrow \sigma'_\kappa(e)]\}) \quad (\mathbf{D3})$$

To execute the **if**  $b$  **then**  $s_1$  **else**  $s_2$  statement we first execute  $s_1$  and  $s_2$  conditional on  $b$  and  $\neg b$ , respectively, yielding the rankings  $D[s_1](\kappa_b)$  and  $D[s_2](\kappa_{\neg b})$ . These are then adjusted by adding the prior ranks of  $b$  and  $\neg b$ , and combined by taking the minimum of the two. The result is normalized to account for the case where one branch fails.

$$\begin{aligned} D[\mathbf{if} \ e \ \mathbf{then} \ s_1 \ \mathbf{else} \ s_2](\kappa) &= ||\lambda||, \\ \text{where } \lambda(\sigma) &= \min \left( \frac{D[s_1](\kappa_b)(\sigma) + \kappa(b)}{D[s_2](\kappa_{\neg b})(\sigma) + \kappa(\neg b)} \right) \end{aligned} \quad (\mathbf{D4})$$

Given a prior  $\kappa$ , the rank of a valuation after executing  $s_1 \langle e \rangle s_2$  is the minimum of the ranks assigned by  $D[s_1](\kappa)$  and  $D[s_2](\kappa)$ , where the latter is increased by  $e$ . The result is normalized to account for the case where one branch fails.

$$D[s_1 \langle e \rangle s_2](\kappa) = ||\lambda||, \text{ where } \lambda(\sigma) = \min \left( \frac{D[s_1](\kappa)(\sigma)}{D[s_2](\kappa)(\sigma) + \sigma_\kappa(e)} \right) \quad (\mathbf{D5})$$

The semantics of the observe statement **observe**  $b$  corresponds to conditioning on the valuations satisfying  $b$ , unless either the rank of this set is  $\infty$  or the prior ranking equals  $\kappa_\infty$  (i.e., represents failure).

$$D[\mathbf{observe} \ b](\kappa) = \begin{cases} \kappa_\infty & \text{if } \kappa = \kappa_\infty \text{ or } \kappa(b) = \infty, \text{ or} \\ \kappa_b & \text{otherwise.} \end{cases} \quad (\mathbf{D6})$$

<sup>1</sup> We omit explicit treatment of undefined operations (i.e. division by zero and some operations involving  $\infty$ ). They lead to program termination.

We define the semantics of **while**  $b$  **do**  $s$  as the iterative execution of **if**  $b$  **then**  $s$  **else skip** until the rank of  $b$  is  $\infty$  (the loop terminates normally) or the result is undefined ( $s$  does not terminate). If neither of these conditions is ever met (i.e., if the **while** statement loops endlessly) then the result is undefined.

$$D[\text{while } b \text{ do } s](\kappa) = \begin{cases} F_{b,s}^n(\kappa) & \text{for the first } n \text{ s.t. } F_{b,s}^n(\kappa)(b) = \infty, \text{ or} \\ \text{undefined} & \text{if there is no such } n. \end{cases} \quad (\mathbf{D7})$$

where  $F_{b,s} : K_{\perp} \rightarrow K_{\perp}$  is defined by

$$f(b, s)(\kappa) = D[\text{if } b \text{ then } s \text{ else skip}](\kappa).$$

Some remarks. First of all, the semantics described here can be thought of as a ranking-based variation of the semantics of probabilistic programs due to Kozen [?]. That is, if we replace  $+$  with  $\times$  and  $\min$  with  $+$ , then our definitions are very similar to their probabilistic counterparts. Secondly, it is not necessary to compute complete rankings to execute a program. Our implementation (discussed in section ??) follows a *most-plausible-first* execution strategy. That is, the possible outcomes of a RankPL program can be iterated over in ascending order with respect to rank, and higher-ranked alternatives do not need to be explored if knowing only the lowest-ranked outcomes is sufficient, which is often the case.

**Example** Consider the *two-bit full adder* circuit shown in figure ?? . It contains two *XOR* gates  $X_1, X_2$ , two *AND* gates  $A_1, A_2$  and an *OR* gate  $O_1$ . The 2-bit output  $b_1 b_2$  is a binary representation of the number of inputs  $a_1 a_2 a_3$  that are high. The *circuit diagnosis problem* is about explaining observed incorrect behavior by finding minimal sets of gates that, if faulty, cause this behavior.<sup>2</sup>

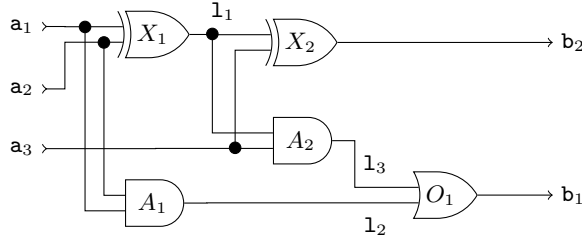


Fig. 1: A two-bit full adder

The listing below shows a RankPL solution. Line 1 encodes the space of possible inputs (0 or 1, equally likely). The failure variables  $\mathbf{fa}_1$ ,  $\mathbf{fa}_2$ ,  $\mathbf{fo}_1$ ,  $\mathbf{fx}_2$  and  $\mathbf{fx}_1$  represent the events of individual gates failing and are set on line 2 (0 = OK, 1 = failing). Here, we assume that failure is surprising to degree 1.

<sup>2</sup> See Halpern [?] for a similar treatment of this example.

The circuit's logic is encoded on lines 3-7, where the output of a failing gate is arbitrarily set to 0 or 1. Note that  $\oplus$  stands for XOR. At the end we observe  $\phi$ .

---

```

1  $\mathbf{a}_1 := (0 \langle 0 \rangle 1); \mathbf{a}_2 := (0 \langle 0 \rangle 1); \mathbf{a}_3 := (0 \langle 0 \rangle 1);$ 
2  $\mathbf{fx}_1 := (0 \langle 1 \rangle 1); \mathbf{fx}_2 := (0 \langle 1 \rangle 1); \mathbf{fa}_1 := (0 \langle 1 \rangle 1); \mathbf{fa}_2 := (0 \langle 1 \rangle 1); \mathbf{fo}_1 := (0 \langle 1 \rangle 1);$ 
3 if  $\mathbf{fx}_1 = 0$  then  $\mathbf{l}_1 := \mathbf{a}_1 \oplus \mathbf{a}_2$  else  $\mathbf{l}_1 := (0 \langle 0 \rangle 1);$ 
4 if  $\mathbf{fa}_1 = 0$  then  $\mathbf{l}_2 := \mathbf{a}_1 \wedge \mathbf{a}_2$  else  $\mathbf{l}_2 := (0 \langle 0 \rangle 1);$ 
5 if  $\mathbf{fa}_2 = 0$  then  $\mathbf{l}_3 := \mathbf{l}_1 \wedge \mathbf{a}_3$  else  $\mathbf{l}_3 := (0 \langle 0 \rangle 1);$ 
6 if  $\mathbf{fx}_2 = 0$  then  $\mathbf{b}_2 := \mathbf{l}_1 \oplus \mathbf{a}_3$  else  $\mathbf{b}_2 := (0 \langle 0 \rangle 1);$ 
7 if  $\mathbf{fo}_1 = 0$  then  $\mathbf{b}_1 := \mathbf{l}_3 \vee \mathbf{l}_2$  else  $\mathbf{b}_1 := (0 \langle 0 \rangle 1);$ 
8 observe  $\phi;$ 

```

---

The different valuations of the failure variables produced by this program represent explanations for the observation  $\phi$ , ranked according to plausibility. Suppose we observe that the input  $\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3$  is valued 001 while the output  $\mathbf{b}_1\mathbf{b}_2$  is incorrectly valued 10 instead of 01. Thus, we set  $\phi$  to  $\mathbf{a}_1 = 0 \wedge \mathbf{a}_2 = 0 \wedge \mathbf{a}_3 = 1 \wedge \mathbf{b}_1 = 1 \wedge \mathbf{b}_2 = 0$ . This results in one outcome ranked 0, namely  $\mathbf{fa}_1\mathbf{fa}_2\mathbf{fo}_1\mathbf{fx}_2\mathbf{fx}_2 = 00010$ . That is,  $\phi$  is most plausibly explained by failure of  $X_1$ . Other explanations are ranked higher than 0 and involve more than one faulty gate.

## 4 Noisy Observation and Iterated Revision

Conditioning by  $A$  means that  $A$  is afterwards believed with absolute certainty. This is undesirable if we have to deal with iterated belief change or noisy or untrustworthy observations, since we cannot afterwards condition on something inconsistent with  $A$ . There are a number of generalized forms of ranking-based conditioning that address this problem. In this section we discuss two of them, and show how they can be expressed in RankPL.

The first is *J-conditioning* [?] (also called *result-oriented conditioning* [?]). It is parametrized by a rank  $x$  that indicates the firmness with which the evidence must be believed. It satisfies the DP postulates of iterated belief revision [?].

**Definition 3.** Let  $A \in \Omega$ ,  $\kappa$  a ranking function over  $\Omega$  such that  $\kappa(A), \kappa(\bar{A}) < \infty$ , and  $x$  a rank. The J-conditioning of  $\kappa$  by  $A$  with strength  $x$ , denoted by  $\kappa_{A \rightarrow x}$ , is defined by  $\kappa_{A \rightarrow x}(B) = \min(\kappa(B|A), \kappa(B|\bar{A}) + x)$ .

In words, the effect of J-conditioning by  $A$  with strength  $x$  is that  $A$  becomes believed with firmness  $x$ . This permits iterated belief change, because the rank of  $\bar{A}$  is shifted up only by a finite number of ranks and hence can be shifted down again afterwards. Instead of introducing a special statement representing J-conditioning, we show that we can already express it in RankPL. Below we write  $\kappa_{b \rightarrow x}$  as shorthand for  $\kappa_{\beta_{\kappa}(b) \rightarrow x}$ . Proofs are omitted due to space constraints.

**Theorem 1.** Let  $b$  be a boolean expression and  $\kappa$  a ranking function s.t.  $\kappa(b) < \infty$  and  $\kappa(\neg b) < \infty$ . We then have  $\kappa_{b \rightarrow x} = S[\mathbf{observe} \ b \ \langle x \rangle \ \mathbf{observe} \ \neg b](\kappa)$ .

Next, we consider *L-conditioning* [?] (also called *evidence-oriented conditioning* [?]). Here, the parameter  $x$  characterizes the ‘impact’ of the evidence. Again, this operation satisfies the DP postulates of iterated belief revision.

**Definition 4.** Let  $A \in \Omega$ ,  $\kappa$  a ranking function over  $\Omega$  such that  $\kappa(A), \kappa(\bar{A}) < \infty$ , and  $x$  a rank. The L-conditioning of  $\kappa$  is denoted by  $\kappa_{A \uparrow x}$  and is defined by  $\kappa_{A \uparrow x}(B) = \min(\kappa(A \cap B) - y, \kappa(\neg A \cap B) + x - y)$ , where  $y = \min(\kappa(A), x)$ .

The effect of L-conditioning by  $A$  with strength  $x$  is that the rank of  $A$  improves by  $x$  units with respect to the rank of  $\neg A$ . L-conditioning satisfies two intuitively desirable properties not satisfied by J-conditioning, making it suitable to model noisy observation: it is *reversible* ( $(\kappa_{A \uparrow x})_{\bar{A} \uparrow x} = \kappa$ ) and *commutative* ( $(\kappa_{A \uparrow x})_{B \uparrow x} = (\kappa_{B \uparrow x})_{A \uparrow x}$ ) [?]. Again, we will show how to express it in RankPL. Like before, we will write  $\kappa_{b \uparrow x}$  as shorthand for  $\kappa_{\beta_\kappa(b) \uparrow x}$ .

**Theorem 2.** Let  $b$  be a boolean expression,  $\kappa$  a ranking function over  $\Omega$  such that  $\kappa(b), \kappa(\neg b) < \infty$ , and  $x$  a rank. We then have:

$$\kappa_{b \uparrow x} = S \left[ \begin{array}{c} \text{if rank}(b) \leq x \text{ then} \\ \text{observe } b \langle x - \text{rank}(b) + \text{rank}(\neg b) \rangle \text{ observe } \neg b \\ \text{else} \\ \text{observe } \neg b \langle \text{rank}(b) - x \rangle \text{ observe } b \end{array} \right] (\kappa)$$

In what follows we use the statement **observe<sub>L</sub>**( $b, x$ ) as shorthand for the statement that represents L-conditioning as defined in theorem ??.

**Example** The following problem involves both iterated revision and noisy observation. A robot navigates a grid world and has to determine its location using a map and two distance sensors. The grids in figure ?? depict the map, where a gray cell represents a wall and all other cells are empty (ignoring, for now, the red cells and dots). The sensors (oriented north and south) measure the distance to the nearest wall or obstacle. To complicate matters, the sensor readings might not match the map. For example, the  $X$  in figure ?? marks an obstacle that affects sensor readings, but as far as the robot knows, this cell is empty.

The listing below shows a RankPL solution. The program takes as input: (1) A map, held by an array<sup>3</sup> **map** of size  $n \times m$ , storing the content of each cell (0 = empty, 1 = wall); (2) An array **mv** (length  $k$ ) of movements (N/E/S/W for north/east/south/west) at given time points; and (3) Two arrays **ns** and **ss** (length  $k$ ) with distances read by the north and south sensor at given time points.

---

```

1 t := 0; x := 0 {0} {1 {0} {2 {0} ... m}}; y := 0 {0} {1 {0} {2 {0} ... n}};
2 while (t ≤ k) do {
3   if (mv[t] = N) then y := y + 1
4   else if (mv[t] = S) then y := y - 1
5   else if (mv[t] = W) then x := x - 1
6   else if (mv[t] = E) then x := x + 1 else skip;
7   n := 0; while map[x][y + n + 1] = 0 do n := n + 1;
8   observeL(1, n = ns[t]);
9   s := 0; while map[x][y - s - 1] = 0 do s := s + 1;
10  observeL(1, s = ss[t]);
11  t := t + 1;
12 }
```

---

<sup>3</sup> Semantically, arrays are just indexed variables.



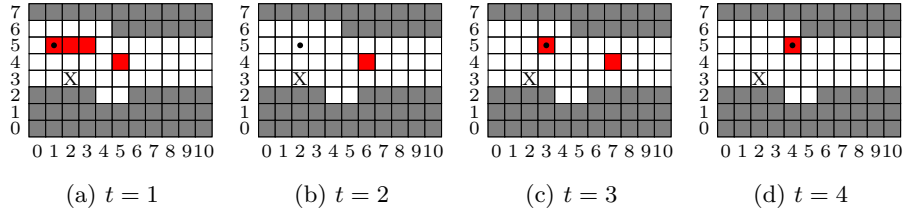


Fig. 2: Most plausible inferred locations during four iterations

The program works as follows. On line 1 the robot's location  $\mathbf{x}, \mathbf{y}$  is initialized to range over all coordinates using nested ranked choice statements. The **while** loop iterates over the time point  $\mathbf{t}$  and first processes the movement  $\mathbf{mv}[\mathbf{t}]$  (lines 3-6). On lines 7-8 the north sensor reading is processed by counting the empty cells between the robot and nearest wall, the result of which is observed to equal  $\mathbf{ns}[\mathbf{t}]$ —and likewise for the south sensor (lines 9-10). Here, we use L-conditioning with strength 1, to account for possible incorrect observations. We then update  $\mathbf{t}$ .

The different values for  $\mathbf{x}, \mathbf{y}$  generated by this program encode possible locations of the robot, ranked according to plausibility. Suppose the robot moves from (0,5) to (4,5). Thus, we set  $\mathbf{mv} = \{E, E, E, E\}$ ,  $\mathbf{ns} = \{1, 1, 1, 1\}$  and  $\mathbf{ss} = \{2, 1, 2, 3\}$ . The dots in figure ?? show robot's position after each step, while the red cells represent the inferred most plausible (i.e., rank zero) locations. If we terminate at  $t = 1$  (i.e., we set  $k$  to 0) we get four locations, all of which are consistent with the observed distances 1 (north) and 2 (south). After two iterations, the robot wrongly believes to be at (6, 4), due to having observed the obstacle. However, after two more iterations, the robot found its actual location.

Note that using L-conditioning here is essential. Regular conditioning would cause failure after the third iteration. We could also have used J-conditioning, which gives different rankings of intermediate results.

## 5 Implementation

A RankPL interpreter written in Java can be found at <http://github.com/tjitze/RankPL>. It runs programs written using the syntax described in this paper, or constructed using Java classes that map to this syntax. The latter makes it possible to embed RankPL programs inside Java code and to make it interact with and use classes and methods written Java. The interpreter is faithful to the semantics described in section ?? and implements the *most-plausible-first* execution strategy discussed in section ?. All examples discussed in this paper are included, as well as a number of additional examples.

## 6 Conclusion and Future Work

We have introduced RankPL, a language semantically similar to probabilistic programming, but based on Spohn's ranking theory, and demonstrated its utility

using examples involving abduction and iterated revision. We believe that the approach has great potential for applications where PPLs are too demanding due to their computational complexity and dependence on precise probability values. Moreover, we hope that our approach will generate a broader and more practical scope for the topics of ranking theory and belief revision which, in the past, have been studied mostly from purely theoretical perspectives.

A number of aspects were not touched upon and will be addressed in future work. This includes a more fine grained characterization of termination and a discussion of the relationship with nondeterministic programming, which is a special case of RankPL. Furthermore, we have omitted examples to show that RankPL subsumes ranking networks and can be used to reason about causal rules and actions [?]. We also did not contrast our approach with default reasoning formalisms that use ranking theory as a semantic foundation (see, e.g., [?]).

Even though we demonstrated that RankPL is expressive enough to solve fairly complex tasks in a compact manner, it is a very basic language that is best regarded as proof of concept. In principle, the approach can be applied to any programming language, whether object-oriented, functional, or LISP-like. Doing so would make it possible to reason about ranking-based models expressed using, for example, recursion and complex data structures. These features are also supported by PPLs such as Church [?], Venture [?] and Figaro [?].

## References

1. Darwiche, A., Pearl, J.: On the logic of iterated belief revision. *Artificial intelligence* 89(1-2), 1–29 (1996)
2. Gärdenfors, P., Rott, H.: Handbook of logic in artificial intelligence and logic programming (vol. 4). chap. Belief Revision, pp. 35–132. Oxford University Press, Oxford, UK (1995)
3. Goldszmidt, M., Pearl, J.: Qualitative probabilities for default reasoning, belief revision, and causal modeling. *Artificial Intelligence* 84(1), 57–112 (1996)
4. Goodman, N.D., Mansinghka, V.K., Roy, D.M., Bonawitz, K., Tenenbaum, J.B.: Church: a language for generative models. In: McAllester, D.A., Myllymäki, P. (eds.) *UAI 2008, Proceedings of the 24th Conference in Uncertainty in Artificial Intelligence*, Helsinki, Finland, July 9-12, 2008. pp. 220–229. AUA Press (2008)
5. Halpern, J.Y.: Reasoning about uncertainty. MIT Press (2005)
6. Kozen, D.: Semantics of probabilistic programs. *J. Comput. Syst. Sci.* 22(3), 328–350 (1981)
7. Kyburg Jr, H.E.: Probability and the logic of rational belief (1961)
8. Mansinghka, V.K., Selsam, D., Perov, Y.N.: Venture: a higher-order probabilistic programming platform with programmable inference. CoRR abs/1404.0099 (2014)
9. Pearl, J.: System Z: A natural ordering of defaults with tractable applications to nonmonotonic reasoning. In: Parikh, R. (ed.) *Proceedings of the 3rd Conference on Theoretical Aspects of Reasoning about Knowledge*, Pacific Grove, CA, March 1990. pp. 121–135. Morgan Kaufmann (1990)
10. Pfeffer, A.: Figaro: An object-oriented probabilistic programming language. Charles River Analytics Technical Report 137 (2009)
11. Spohn, W.: The Laws of Belief - Ranking Theory and Its Philosophical Applications. Oxford University Press (2014)