Homework 3

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Problem 3.1

a) $f(n) = 9n$; $g(n) = 5n^3$

$f \in \Theta(g)$ $\Rightarrow$ false since $g(n)$ is of higher order than $f(n)$ and therefore there is no constant c_1 which would make $\lim_{n\to\infty} c_1 g(n) \leq f(n)$

$f \in O(g)$ $\Rightarrow$ $f(n) = O(n)$; $g(n) = O(n^3)$ Therefore $0 \leq f(n) \leq g(n)$, $\forall n \geq n_0$ which means $f \in O(g)$

$f \in o(g)$ $\Rightarrow$ $\lim_{n\to\infty} \frac{9n}{5n^3} = \lim_{n\to\infty} \frac{9}{5n^2} \Rightarrow \lim_{n\to\infty} \frac{9}{5n^2} = \frac{9}{\infty} = 0$
Therefore $f \in o(g)$

$f \in \Omega(g)$ $\Rightarrow$ false since $f = O(n)$ and $g = O(n^3)$. This means that there is no constant for which $0 \leq c\, g(n) \leq f(n)$, $\forall n \geq n_0$ holds true

$f \in \omega(g)$ $\Rightarrow$ we already proved that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$. Since the requirement for $f \in \omega(g)$ is $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$ this relation is not true

$g \in \Theta(f)$ $\Rightarrow$ Also not possible since $g(n)$ is of higher order than $f(n)$ and therefore there is no constant to which $\lim_{n\to\infty} c_2 f(n) \geq g(n)$ with $n > n_0$

$g \in O(f)$ $\Rightarrow$ can't be true since we already proved that $f \in \omega(g)$ can't be true

$g \in o(f)$ $\Rightarrow$ also can't be true since we already proved that $f \in \omega(g)$ can't be true

$g \in \Omega(f)$ $\Rightarrow$ $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{5n^3}{9n} = \lim_{n\to\infty} \frac{5n^2}{9} = \infty \Rightarrow \lim_{n\to\infty} \frac{g(n)}{f(n)} > 0$
Therefore the relation is true

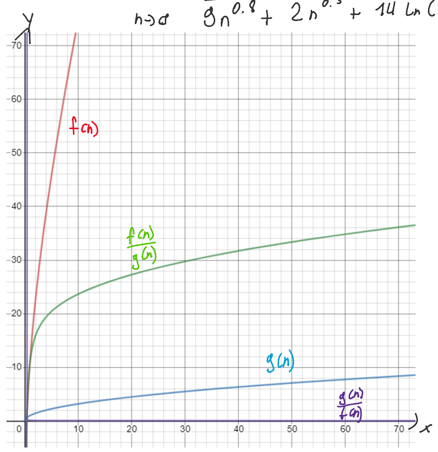
$g \in \omega(f)$ $\Rightarrow$ $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \infty \Rightarrow g \in \omega(f)$

b)

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{9n^{0.8} + 2n^{0.3} + 14\ln(n)}{\sqrt{n}} = \infty$$

Can be seen by plotting both functions and the devision.

$$\lim_{n\to\infty} \frac{\sqrt{n}}{9n^{0.8} + 2n^{0.3} + 14\ln(n)} = 0$$



Therefore the following relations are correct:

$f \in \Omega(g)$, $f \in \omega(g)$, $g \in O(f)$, $g \in o(f)$

And these are not correct:

$f \in \Theta(g)$, $f \in O(g)$, $f \in o(g)$, $g \in \Theta(f)$, $g \in \Omega(f)$, $g \in \omega(f)$

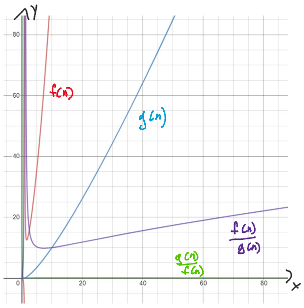
c)

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\frac{n^2}{\log n}}{n \log n} = \lim_{n\to\infty} \frac{n}{\log(n)^2} = \infty$$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{n \log n}{\frac{n^2}{\log n}} = \lim_{n\to\infty} \frac{\log(n)^2}{n} = 0$$

$\Rightarrow$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{n \log n}{\frac{n^2}{\log n}} = \lim_{n\to\infty} \frac{\log(n)^2}{n} = 0$$



true:

$f \in \Omega(g)$, $f \in \omega(g)$, $g \in O(f)$, $g \in o(f)$

false:

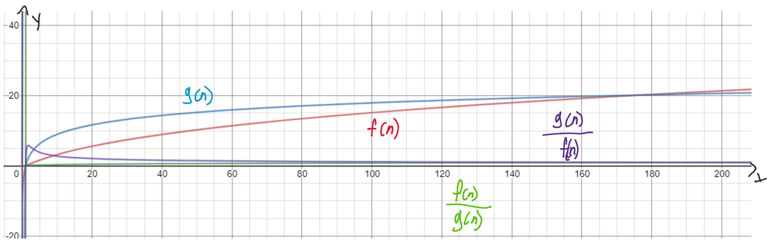
$f \in \Theta(g)$, $f \in O(g)$, $f \in o(g)$, $g \in \Theta(f)$, $g \in \Omega(f)$, $g \in \omega(f)$

d)

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\log(3n)^3}{9\log(n)} = \lim_{n\to\infty} \frac{\frac{\log(3n)^3}{\log(n)}}{9}$$

$$= \lim_{n\to\infty} \frac{3\log(3n)^2}{9} = \infty$$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{9\log(n)}{\log(3n)^3} = \lim_{n\to\infty} \frac{9}{\frac{\log(n)}{\log(3n)^3}} = \lim_{n\to\infty} \frac{9}{3\log(3n)^2} = 0$$



true:

$f \in \Omega(g)$, $f \in \omega(g)$, $g \in O(f)$, $g \in o(f)$

false:

$f \in \Theta(g)$, $f \in O(g)$, $f \in o(g)$, $g \in \Theta(f)$, $g \in \Omega(f)$, $g \in \omega(f)$

problem 3.2:

b) Loop invarient:

At each iteration of the i loop the subarray arr[0... i-1] contains the i-1 smallest elements in sorted order. Therefore with each iteration we add another object to the sorted array which decreases the number of unsorted elements by at least 1.

c) The case B is when the array is already sorted. This is trivial since no numbers need to be sorted (1 2 3 4 5)

The case A is when the biggest number is in the front and the array is sorted after that. The algorithm therefore has to swap n-1 times which is the maximum. (5 1 2 3 4)

d) I tried to use chrono to get the computation time but I couldn't get it to work in time.

e) couldn't try it yet.