



A corridor-centric approach to planning electric vehicle charging infrastructure



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ABSTRACT

The transition to electric vehicles (EV) faces two major barriers. On one hand, EV batteries are still expensive and limited by range, owing to the lack of technology breakthrough. On the other hand, the underdeveloped supporting infrastructure, particularly the lack of fast refueling facilities, makes EVs unsuitable for medium and long distance travel. The primary purpose of this study is to better understand these hurdles and to develop strategies to overcome them. To this end, a conceptual optimization model is proposed to analyze travel by EVs along a long corridor. The objective of the model is to select the battery size and charging capacity (in terms of both the charging power at each station and the number of stations needed along the corridor) to meet a given level of service in such a way that the total social cost is minimized. Two extensions of the base model are also considered. The first relaxes the assumption that the charging power at the stations is a continuous variable. The second variant considers battery swapping as an alternative to charging. Our analysis suggests that (1) the current paradigm of charging facility development that focuses on level 2 charging delivers poor level of service for long distance travel; (2) the level 3 charging method is necessary not only to achieve a reasonable level of service, but also to minimize the social cost; (3) investing on battery technology to reduce battery cost is likely to have larger impacts on reducing the charging cost; and (4) battery swapping promises high level of service, but it may not be socially optimal for a modest level of service, especially when the costs of constructing swapping and charging stations are close.

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1. Introduction

1.1. Background

Transportation accounts for over 60% of all petroleum consumed in the US, of which 60% must be imported (NRS, 2010). While the world has witnessed a remarkably stable increase in oil production and consumption for a long period of time, the trend is clearly unsustainable (Energy Information Administration, 2007). Without alternative sources of energy, the price of petroleum is likely to rise at an ever-increasing pace and with greater volatility, to which those countries heavily dependent on imported oil are especially vulnerable (NRS, 2010). An oil-dependent transportation industry is not only a concern for energy security, it also creates major environmental problems (United Nations Climate Change Secretariat, 2006). In

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2005, transportation is responsible for roughly 23% of the world's carbon dioxide (CO₂) emissions, a major greenhouse gas linked to global climate change (Ohnishi, 2008). Data also suggest that between 1990 and 2006 the growth in transportation GHG emissions represented almost 50% of the total growth (Cambridge Systematics, 2009). Transition to alternative fuel vehicles can effectively reduce oil use, and is widely considered an important ingredient in the solution to these grand challenges of our time, namely energy security, climate change and sustainable development (Ohnishi, 2008; NRS, 2010). Of particular interest to this study is plug-in electric vehicles (PEV) or battery electric vehicles (BEV), which will be simply referred to as electric vehicles hereafter.

Electric vehicles have a few notable advantages compared to conventional internal combustion engine (ICE) automobiles and other alternative fuel vehicles (e.g. hybrid vehicle). For one thing, electric vehicles are more energy efficient (Romm, 2006). Eberhard and Tarpenning (2006) show that the well-to-wheel efficiency of electric cars is around 1.15 kilo meter per million Joule (km/mJ). The same study estimates an efficiency of 0.56 km/mJ for the celebrated hybrid model, Toyota Prius, and much lower rates for conventional ICE cars (Toyota Camry, for example, is rated at 0.28 km/mJ). Second, because electric cars have zero emission at the point of operation, they contribute significantly to the reduction of local air pollution. Electric cars also help reduce greenhouse emissions (Samaras and Meisterling, 2008). A recent study (Crist, 2012) estimates a four-door electric Sedan could save up to 17 tone of CO₂ in its lifetime compared to an ICE vehicle equipped with improved diesel engine technology.

PHEVs and EVs sales are appraised to be approximately 50,000 vehicle by 2015, which accounts for 0.3% of all cars sales (Newman, 2010). President Obama has promised to make the US “become the first country to have one million electric vehicles on the road by 2015”.¹ Accordingly, the US government has pledged \$2.4 billion in federal grants to further development of EVs (Canis, 2011). Still, the great promises of electric cars are crippled by the lack of enabling infrastructure and technology breakthrough. Electric vehicles are still considerably more expensive than ICE vehicles, largely due to the cost of battery packs that is still over €10,000 on average according to recent estimates (Crist, 2012). Another major hurdle is the so-called range anxiety, which has to do with the drivers' fear of batteries running out power en-route. Range anxiety is closely related drivers' refueling behavior in general, as shown by Kitamura and Sperling (1987). For EV drivers, the range anxiety is much more of a concern, mainly due to the limited range of the current batteries and the lack of public and private charging infrastructure.

1.2. Literature review

Overcoming the above barriers demands an EV charging network that is optimally configured to meet the needs of current and future EV fleets. A widely adopted approach to this design problem aims at locating charging facilities near the urban activity centers of EV owners (e.g. home, shopping malls and workplaces) so as to maximize the overall accessibility. Typically underlying this approach are various set covering or P-median facility location models (Dashora et al., 2010, e.g.; Frade et al., 2011; Chen et al., 2013; Sweda and Klabjan, 2011). Variants of this approach also address the interactions with power grids and route/destination choices (He et al., 2013) and the location of battery swapping stations (Pan et al., 2010). Most existing studies along this direction do not focus on long distance trips (more than 100 miles round trip) traditionally made using passenger cars, even though charging seems much more important for these trips than those near home (see Acknowledgements).

Flow capturing facility location models (FCLM) are better suited to tackle long-distance travel. Unlike the traditional facility location models which assume point demands (Daskin, 1952), flow capturing models assume that the demands are given in the form of origin–destination (O–D) flows. Hodgson (1990) proposes the first FCLM that seeks to locate a given number of facilities so as to intercept as much O–D flows as possible. Kuby and Lim (2005) applies the FCLM in the context of refueling problem for range-limited vehicles. In their flow refueling location model (FRLM), an O–D flow is “captured” only if vehicles never run out of fuel along their designated travel path. The objective of FRLM is to locate p refueling facilities to maximize the total vehicle flows refueled. Kuby and Lim (2007) extend FRLM so that facilities can be located along the arcs of the network, and later, Lim and Kuby (2010) proposes a few efficient heuristic algorithms for solving the FRLM problems. The refueling station location problem studied in Wang and Lin (2009) also considers O–D demands. Yet, instead of trying to maximize flow being captured, their model minimizes the total facility cost while ensuring all flows are properly served according to a “refueling logic”. Later, Wang and Wang (2010) proposes a hybrid version of the refueling station location problem, along the line of Hodgson and Rosing (1992), which considers both point and O–D demands. Recently, Mak et al. (2013) studies a robust location problem of battery swapping stations. Similar to Wang and Lin (2009), their model attempts to minimize the total cost (or maximize the chance of meeting an investment-to-return goal) and embeds a recharging logic to ensure that all demands are served by swapping stations. Moreover, Mak et al. (2013) consider the capacity limits at the swapping stations and demand uncertainty.

The modeling approach adopted in this paper employs an embedded refueling logic. Yet, the main tradeoff concerned herein arises from the interdependence between the cost of building recharging infrastructure and manufacturing batteries, as elaborated below. We also explicitly consider the level of services experienced by EV users in the form of extra time spent on recharging batteries in their journey.

¹ Energy Speech Fact Sheet. http://www.barackobama.com/pdf/factsheet_energy_speech_080308.pdf.

1.3. Overview of the research approach proposed

Clearly, rapid adoption of EVs can benefit from the improvements on both or either: the availability of charging facilities and/or cheaper batteries with greater capacity. On one hand, if a dense fast charging network is built, smaller and less expensive batteries would probably meet the needs of most consumers. On the other hand, should batteries with much larger capacities become available at a reasonable price, significant investment on charging facilities may be avoided. From the societal point of view, therefore, questions that can be asked now are (1) If the society can freely decide the capacities of charging facilities and batteries, how the decision can be made in a way that minimizes cost while providing satisfactory level of service; (2) Which factors should have important influences on the decision; and (3) What policies may be implemented to facilitate the optimal allocation of resources for expanding these capacities? The purpose of this study is to explore answers to these questions through simplified analyses. We note that these analyses are not meant to be used a design tool for the manufacturing of next-generation EVs and their supporting infrastructure. Rather, the goal is to merely recognize the tradeoff between the overall charging capacity and the battery size, and to show that it is useful for the society to examine this tradeoff before committing to certain level of resources on either investment. The hope is that the results could stimulate policies/regulations that would promote batteries and charging facilities of "socially optimal" sizes.

To this end, a simple optimization model is built that focuses on travel along corridors long enough to trigger range anxiety. We believe that these medium range low frequency trips traditionally served by passenger cars are likely one of the main reason why single-car families have to say no to the current generation of EVs. Our model aims to minimize the total cost of providing recharging facilities and manufacturing batteries, while ensuring all EV users can complete their trips with a desired level of service.

In the next section, we shall review the relevant facts about the EVs and their charging facilities. Sections 3 and 4 present and analyze the single corridor base model. Section 5 presents two special cases: one considers a discrete power function and the other addresses the battery swapping problem. Section 6 reports results of numerical experiments and Section 7 concludes the study.

2. Characteristics of batteries and charging stations

In this section, we briefly review the type of charging facilities and batteries and how their properties affect travel.

There are many types of batteries currently available for EVs in the market. These batteries have different energy capacities, which affect not only the price but also the maximum range of the EV. The average battery cost is found to be around 650 (\$/kW h) per battery pack for mid size cars in 2013 (Cluzed and Douglas, 2012). It is expected that this cost would decline as the manufacturing technology advances (Rio, 2012).

An important performance measure of an EV is the distance that it runs on each unit of battery energy consumed, denoted using the symbol β in this paper. A study conducted by the US Department of Energy (Electric Vehicle Operation Program, 1999) examined six different types of vehicles in urban versus highway driving under various conditions (e.g. headlight setting, auxiliary loads, and A/C). It found on average an EV can travel 2.5 miles for each kW h (kilo Watt hour) of energy. Another factor critical to EV performance is the charging time, which depends on not only the type of the battery but also the power of the charging facilities (NECA413, 2011). Simply speaking, let P denote the charging power, E the energy capacity of the battery, then the charging time can be estimated as

$$t_r = \alpha E / P \quad (1)$$

where $\alpha \geq 1$ is a unitless parameter that measures the battery's charging efficiency. In reality, α varies across batteries and charging stations.

Table 1 reports the charging times of seven EV models, using data obtained from Garthwaite (2012) to calculate the recharging efficiency. In this table charging station power was calculated as

$$P = v \times I \quad (2)$$

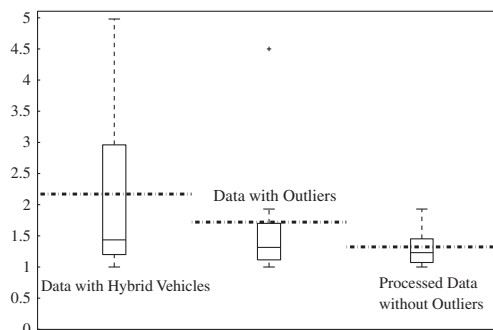
where I is the electric current, v is electric potential and P is the power. $\frac{E}{P}$ produces a theoretical charging time based solely on battery energy and charging station power. Note that the seventh column in the table is the reported charging time, and the last column gives the estimated α in each case. We first exclude α values smaller than 1 from consideration, because they seem to indicate problems in the reported charging time. As Toyota Prius is the only plug-in hybrid electric vehicle in this data set, it is also excluded. The box plot for all the acceptable α values is plotted in Fig. 1(a), which reveals an outlier. After removing the outlier, we draw the data again in a power vs. α plot, as shown Fig. 1(b). As the plot reveals no obvious relationship between α and the power, we simply take the average of all α for our study, which equals 1.3.

At present there are three types of charging facilities available for EVs in the US, each associated with a different level of voltage and current (Morrow et al., 2008). The Level 1 charging method uses a standard 120 VAC, 15amp (12 amp useable) or 20 amp (16 amp useable) branch circuit, and has a maximum of 1.44 kW charging power. Because of its low power, the level 1 charging typically requires long charging time and was only intended to be used at home or places where the vehicle can stay for an extended period of time. Level 2 is an improved charging option that comes with 240 VAC and about 40 amps of current, which delivers a maximum charging power close to 10 kW. Level 2 charging is often considered the "preferred"

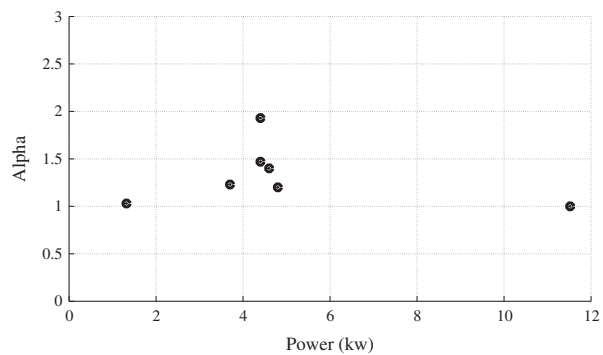
Table 1

EVs charging performance at different types of charging facilities (Data source: Garthwaite (2012)).

Vehicle type	E (kW h)	I (amp)	v (V)	P (kW)	E/P (h)	t_r (h)	α
BMW Mini E	35	12	110	1.32	27	26.00	0.98
		32	240	7.68	5	4.50	0.99
		48	240	11.52	3	3.00	1.00
Chevy Volt	16	12	120	1.44	11	10.00	0.90
		20	240	4.8	3	4.00	1.20
		20	230	4.6	5	7.00	1.40
Ford Focus EV	23	12	110	1.32	12	12.50	1.03
Mitsubishi iMiEV	16	20	220	4.4	4	7.00	1.93
		60	480	28.8	0.6	2.50	4.50
		20	220	4.4	5	8.00	1.47
Nissan LEAF	24	60	480	28.8	0.8	0.60	0.72
		16	230	3.7	7	8.00	1.23
		12	110	1.32	1.0	3.00	2.96
Volvo C30	24	20	200	4	0.3	1.67	4.98
Toyota PRIUS	1.34						

 I – electric current; E – battery energy; V – electric potential; P – power.

(a) Box plots for determining battery charging efficiency



(b) Recharging efficiency changes with power

Fig. 1. Estimation of battery recharging efficiency.

option for EVs (Morrow et al., 2008). The last option, Level 3 charging, operates with 480 VAC and can deliver a power ranging between 60 kW and 150 kW. Level 3 charging usually brings the car to 50% of its charge level in 10–15 min (Morrow et al., 2008). Currently, EVs may be recharged at home, in public areas and at some work places (Pound, 2012). According to US Department of Energy,² the US now has between 6000 and 7000 electric charging stations, of which the majority (more than 5000) are privately owned. Moreover, nearly 80% of all existing charging stations are level 2. In Illinois, for example, there are currently about 250 charging stations, of which 90% are level 2. Naturally, different charging facilities come at different prices, which mostly depend on the cost of purchasing and installing the associated power outlets. Fig. 2 shows power and cost of different levels and brands of single connector charging facilities. For simplicity, this paper assumes that the cost of installing each charging slot increases linearly with the charging power P . Accordingly, a simple linear regression based on the above data suggests that the unit construction cost per charging outlet is approximately 500 (\$/kW).

Finally, building charging facilities may also involve actual construction of the structures that house the chargers, if they are not readily available. As these infrastructure costs are potentially running high, we shall consider two scenarios in this study: one that makes use of the existing gas stations/public parking lots to avoid extensive construction, and the other that builds stations dedicated to EV charging.

3. Design model

To enable the use of EVs, the society may need to trade off two long-term investments: manufacturing batteries that hold energy for travel and providing facilities to charge the batteries in long journeys.

To model this decision, we assume that the society has to choose the energy capacity of each EV's primary battery (denoted as E), and the power of the charging facilities (denoted as P). Cost of building a charging station is considered as a function of P and the number of charging outlets n_0 , denoted as $R(P, n_0)$. We shall call this function charging cost function.

² <http://www.afdc.energy.gov/locator/stations>, last visited 12.15.12.

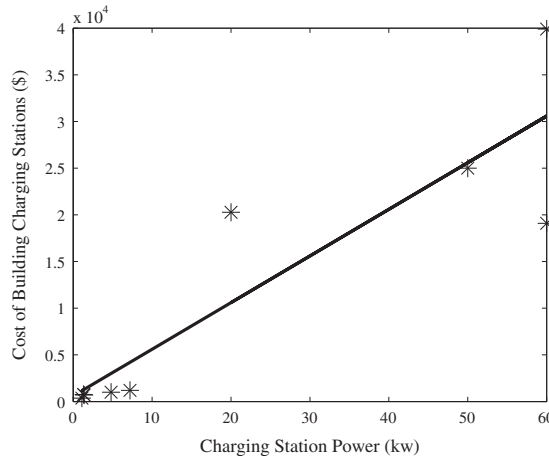


Fig. 2. Power-cost relationship of existing charging stations.

Similarly, the cost of each battery is a function of its energy capacity E , denote as $B(E)$, which will be referred to as battery cost function. The cost of purchasing electricity is assumed to be identical regardless of the availability or location of the charging facilities, and hence is not included in the total cost for this particular design decision. The objective of the model in this study is to minimize the total cost of charging stations and batteries while maintaining the level of service, distinguished by the time required to regain the battery energy.

The model considers a long corridor with a maximum length of l , serving EV drivers traveling along one direction. Let λ denote the density of the EVs (measured in vehicle per unit distance), and f be the average frequency of the trips made by each EV for a given analysis period (typically a day).

The total number of EV drivers is given as λl , and the total number of trips in the analysis period can be computed as $\lambda l f$. For simplicity, a conservative design is adopted, which assumes that (1) all trips are concentrated at the two ends of the corridor and that (2) each station must have enough charging units to accommodate all trips. These assumptions lead to the following requirement:

$$n_o \geq \min(\lambda l, \lambda l f) = \lambda l \min(1, f) \quad (3)$$

Note that the number of charging units at each station should be an integer number. Thus the number given by the right hand side of the inequality (3) should be rounded up to the next integer. Moreover, let m be the number of charging stations constructed along the corridor. For simplicity, we assume that these stations are uniformly spaced, that is, the distance between any pair of the stations and the distance between a station and the origin/destination is identical, and equals $l/(m+1)$. The design model for manufacturing and charging EV batteries can now be formulated as follows:

$$\min \quad z(P, E) = mR(P, n_o) + \lambda l B(E) \quad (4)$$

s. t. :

$$m \left(\frac{\alpha \theta E}{P} + t_s \right) \leq T_0 \quad (5)$$

$$\beta \theta E \geq \frac{l}{m+1} \quad (6)$$

The objective function value is the total cost of manufacturing λl units of battery pack at the capacity E and constructing m charging facilities with power P and n_o spots. Constraint (5) deals with the corridor's level of service. Specifically, it requires the total wasted time due to mandatory battery charging be smaller than a predetermined threshold T_0 ³. We assume that, due to the range anxiety, traveler would like to recharge the battery when it uses 100% of its capacity. $\theta \in [0, 1]$ is referred to as the range tolerance. Thus, the total wasted time consists of the time required to fully recharge the battery and the lost time due to deceleration and acceleration (denoted as t_s). It seems reasonable to think T_0 as a function of the total trip length. In this study, we assume that T_0 is an affine function of the longest free flow travel time, i.e. the time it takes to travel through the corridor without any delay. Specifically, we rewrite T_0 as

$$T_0 = \delta \frac{l}{v_0} \quad (7)$$

³ Alternatively, the charging time may be directly included in the objective function. Yet, such a modeling approach would require estimating the value of charging time, which is a difficult task in its own right. Specifically, it may lead to optimal solutions with unacceptably low level of service, if the charging time is not properly valued compared to the infrastructure costs.

where v_0 is the free flow speed and $\delta > 0$ is called delay tolerance, which measures travelers' tolerance to travel delay. The larger δ is, the less sensitive the drivers is to travel delay, and lower the level of service is. Finally, note that α in (5) is the unitless parameter that converts energy/power ratio to charging time, as explained in Section 2.

Constraint (6) dictates that the anxiety-free range of the EV has to exceed the distance between any two charging facilities. Recall that β converts the total battery energy to travel distance. Note that Constraint (6) implicitly assumes that all EV users start from the origin with a fully charged battery and fully charge it whenever the battery is depleted exactly to $(1 - \theta)100\%$ of the full capacity.

We now need to specify the charging and battery cost functions in order to analyze this model. As an initial approximation, we assume these functions are linear with respect to their arguments. Specifically,

$$R(P, n_o) = (C_p + P n_o C_s), \quad B(E) = C_e E \quad (8)$$

where C_p is the construction cost for building/remodeling a charging station, C_s is the installation cost per unit charging power, and C_e the unit manufacturing cost for battery. We assume that

$$C_p = (A_0 + n_o a_o) C_a \quad (9)$$

where A_0 is the fixed construction area for each charging station, a_o is the variable construction area required for each charging slot, and C_a is the unit construction cost (\$ per square feet). If the existing public/private facilities can be utilized to host the chargers, the fixed construction cost is waived (i.e. $A_0 = 0$), and the unit variable construction cost is assumed to be much lower (they may be related to remodeling the existing structure to fit the chargers).

Clearly, drivers would always charge their battery to full capacity at a charging facility.⁴ In order to minimize the cost, the society needs to build as few charging stations as possible. Therefore, we can replace m in the formulation with its lower bound given by (6), i.e.

$$m \geq \frac{l}{\beta \theta E} - 1 \quad (10)$$

Another worthwhile simplification has to do with t_s . The lost time due to deceleration and acceleration are at most in the range of a couple of minutes. Because it is relatively insignificant compared to the typical charging time, we can safely exclude them from the model. Finally, we note that for cost minimization, n_o should always take its lower bound in (3), i.e. $\lambda \min(1, f)$. These considerations lead to the following simplified model

$$\min \quad z(P, E) = (C_p + P \lambda \min(1, f) C_s) \left(\frac{l}{\beta \theta E} - 1 \right) + \lambda C_e E \quad (11)$$

s. t. :

$$\left(\frac{l}{\beta \theta E} - 1 \right) \frac{\alpha \theta E}{P} \leq T_0 \quad (12)$$

where Constraint (12) is obtained by combining (5) and (6) in the original model. We note that Constraint (12) can be further simplified as follows:

$$T_0 P + \alpha \theta E \geq \frac{l \alpha}{\beta} \quad (13)$$

Before analyzing the optimization problem presented in Section 3, let us further simplify the notation by introducing the following constants

$$c_1 \equiv \lambda \min(1, f) C_s; \quad c_2 \equiv \lambda C_e; \quad c_3 \equiv \frac{l}{\beta \theta} \quad (14)$$

where c_1 is the variable cost of charging facility, c_2 is the total cost to manufacture all batteries to meet the demand, and c_3 is the energy a battery has to hold in order to travel through corridor without being depleted below the range tolerance. If $c_2 \leq C_p$, i.e., the cost of manufacturing all needed batteries large enough for traversing the corridor without having to stop is no greater than the fixed construction cost of a single charging station, then it is clear that no charging facility should ever be built. Hence only the case where $c_2 c_3 > C_p$ needs to concern us in what follows.

4. Solution and analysis

4.1. Two analytical solutions

With the new notation, the Lagrangian of our design problem can be written as

$$L = c_1 c_3 \frac{P}{E} + C_p \left(\frac{c_3}{E} - 1 \right) - c_1 P + c_2 E - \mu (T_0 P + \alpha \theta E - c_3 \alpha \theta) \quad (15)$$

⁴ This may not be the case when the remaining journey is shorter than the full range, which likely occurs at the "last" charging facility.

and the Karush–Kuhn–Tucker (KKT) conditions can be stated as

$$\frac{\partial L}{\partial P} = \frac{c_1 c_3}{E} - c_1 - \mu T_0 = 0 \quad (16)$$

$$\frac{\partial L}{\partial E} = -\frac{c_1 c_3 P}{E^2} - \frac{C_p c_3}{E^2} + c_2 - \mu \alpha \theta = 0 \quad (17)$$

$$\mu(T_0 P + \alpha \theta E - c_3 \alpha \theta) = 0 \quad (18)$$

$$\mu \geq 0; T_0 P + \alpha \theta E - c_3 \alpha \theta \geq 0 \quad (19)$$

Note that the above conditions discards the corner solutions where either $P = 0$ or $E = 0$ because these solutions are of little practical utility. The objective function (11) is not convex, because its Hessian

$$H = \begin{bmatrix} 0 & -\frac{c_1 c_2}{E^2} \\ -\frac{c_1 c_2}{E^2} & -2\frac{c_3(c_1 P + C_p)}{E^3} \end{bmatrix}$$

is evidently not positive semi-definite. Accordingly, satisfying the KKT conditions is necessary but not sufficient for optimality.

To solve the KKT system, we consider the following two cases: $\mu = 0$ and $\mu > 0$. When $\mu = 0$, we can solve from (16) and (17)

$$E_0^* = c_3; \quad P_0^* = \frac{c_2 c_3 - C_p}{c_1} \quad (20)$$

Replacing E and P in Constraint (13) with this solution we find

$$\frac{T_0 c_2 c_3}{c_1} + c_3 \alpha \theta > c_3 \alpha \theta$$

for any positive T_0 . Thus, (E_0^*, P_0^*) is indeed a solution to the KKT conditions, which means it is at least a local solution for the design problem. Note that $c_3 = l/(\theta \beta)$ can be interpreted as the battery energy required to travel up to a distance l without range anxiety. Thus, the above solution essentially advises completely giving up charging stations, and building a battery with enough capacity to traverse the corridor without having to stop.

Let's now consider the other case, i.e. $\mu > 0$. In this case, $E^* = \frac{c_1 c_3}{c_1 + \mu T_0}$; $P^* = \frac{c_2 - \mu \alpha \theta}{c_1 c_3} E^2 - \frac{C_p}{c_1}$. From the complementarity condition (18) we can solve μ^* as

$$\mu^* = c_1(\eta - 1)/T_0; \quad \eta \equiv \sqrt{\frac{T_0(c_2 c_3 - C_p)}{\alpha \theta c_1 c_3 + T_0 C_p}} + 1 \equiv \sqrt{\frac{c_3(T_0 c_2 + \alpha \theta c_1)}{\alpha \theta c_1 c_3 + T_0 C_p}} \quad (21)$$

Also, the following identity always holds

$$\frac{c_2 c_3 - C_p \eta^2}{c_1} = \frac{(\eta^2 - 1) \alpha \theta c_3}{T_0} \quad (22)$$

Accordingly, the optimal E and P can be obtained as follows:

$$E_1^* = \frac{c_3}{\eta} \quad (23)$$

$$\begin{aligned} P_1^* &= \frac{c_2 c_3}{c_1} \frac{1}{\eta^2} - \frac{c_3 \alpha \theta}{T_0} \frac{1}{\eta} \left(1 - \frac{1}{\eta}\right) - \frac{C_p}{c_1} = \frac{1}{\eta^2} \left(\frac{c_2 c_3}{c_1} - \frac{C_p \eta^2}{c_1} + \frac{c_3 \alpha \theta}{T_0} \right) - \frac{c_3 \alpha \theta}{T_0} \frac{1}{\eta} = \frac{1}{\eta^2} \left(\frac{c_3 \alpha \theta}{T_0} (\eta^2 - 1) + \frac{c_3 \alpha \theta}{T_0} \right) - \frac{c_3 \alpha \theta}{T_0} \frac{1}{\eta} \\ &= \frac{c_3 \alpha \theta}{T_0} \left(1 - \frac{1}{\eta}\right) \end{aligned} \quad (24)$$

Note that the third equality above uses the identity (22). To better understand the second optimal solution, we focus on the newly emerged parameter η . It is easy to verify that η is always well defined because $\frac{T_0(c_2 c_3 - C_p)}{\alpha \theta c_1 c_3 + T_0 C_p} > -1$. To guarantee the multiplier $\mu > 0$, and accordingly, the level of service constraint to be binding, the inequality $c_2 c_3 > C_p$ should hold which leads to $\eta > 1$. If $\eta < 1$ then $c_2 c_3 < C_p$ (Eq. 21) implying relatively smaller battery cost and also $P_1^* < 0$ (from Eq. 24, which means due to the battery manufacturing cost it is optimal to build a large enough battery and avoid building recharging facilities. The following observations can be made by inspecting the above analytical relationship.

- A higher battery construction cost (i.e. larger C_c) leads to larger η , hence smaller optimal battery size and larger charging capacity. Conversely, a higher fixed (C_p) or variable (c_1) construction cost results in smaller η , hence larger optimal battery size and smaller charging capacity.
- A lower level of service requirement (i.e. larger T_0) increases η , which in turn reduces the optimal battery size. The impacts on the optimal charging capacity is likely parameter specific and not always monotone.

- The growth in the EV population (i.e. larger λ) will proportionally increase both c_1 and c_2 , and hence increase η . This makes it more desirable to have a smaller battery size and larger charging capacity.
- For long-distance travel such as considered in this study, it is mostly likely $f \ll 1$. If this assumption holds, higher trip frequency will lead to higher c_1 and lower η . Consequently, the optimal battery size will increase, whereas the capacity of charging facility should decrease.

An important question that may be asked now is which of the two solutions is the global optima. It seems intuitive that the answer may well be “it depends” (on the input parameters). Yet the following proposition provides a somewhat unexpected answer.

Proposition 1. Assume all other parameters in the design model (11) and (12) are positive. If $c_2c_3 > C_p$, then $z(P_0^*, E_0^*) > z(P_1^*, E_1^*)$.

Proof. The proof is straightforward. First, since for the corner solution, no charging facility is needed, $z(P_0^*, E_0^*)$ is simply the cost of manufacturing all the batteries, which equals c_2c_3 . For the second solution, the objective function can be evaluated as

$$Z(P_1^*, E_1^*) = \frac{c_1c_3\alpha\theta}{T_0} \left(\eta + \frac{1}{\eta} - 2 \right) + \frac{c_2c_3}{\eta}$$

Note that

$$\begin{aligned} \frac{c_1c_3\alpha\theta}{T_0} \left(\eta + \frac{1}{\eta} - 2 \right) + \frac{c_2c_3}{\eta} < c_2c_3 &\Leftrightarrow c_1\alpha\theta(\eta - 1)^2 - c_2T_0(\eta - 1) < 0 \Leftrightarrow c_1\alpha\theta(\eta - 1) - c_2T_0 < 0 \Leftrightarrow \eta < \frac{c_2T_0}{c_1\alpha\theta} + 1 \\ &\Leftrightarrow \eta < \eta^2 \end{aligned} \quad (25)$$

It is evident that the last inequality holds for any $\eta > 1$, which is true if and only if $c_2c_3 > C_p$. \square

The above result indicates that manufacturing batteries with excessive capacity is unlikely to be a socially optimal solution when $c_2c_3 > C_p$. Assuming the trip frequency $f < 1$,

$$c_2c_3 > C_p \rightarrow \lambda > \frac{C_p\beta\theta}{C_e l^2} = \frac{(A_0 + n_0 a_0) C_a \beta \theta}{C_e l^2} = \frac{(A_0 + \lambda f a_0) C_a \beta \theta}{C_e l^2}$$

For $C_a = 104$ \$/sqf, $f = 0.13$, $A_0 = 2000$ sqf, $a_0 = 300$ sqf, $C_e = 600$ \$/kW h, $\beta = 2.5$ mile/kW h, $\theta = 0.8$, $\lambda = 100$ mile) (see Table 2 for the choice of the parameter values), the above relationship becomes

$$\lambda > 0.0993 \text{ (veh/mile)}$$

That is, as long as the density of EV demand exceeds certain threshold (which is relatively low for the given parameters), it is always beneficial to provide recharging facilities, which promises the scale of economy because they are shared by all EVs. This observation makes a strong case for the public investment in building recharging facilities.

4.2. Integer solutions

The analytical solution obtained in the previous section could lead to non-integer number of charging facilities, which is obtained as

$$m^* = \frac{l}{\beta\theta E^*} - 1$$

Table 2

Parameters definitions and default values.

Parameter	Description	Unit	Value
l	Corridor length	mile	150
f	Average trip frequency	trip/day	0.13
λ	EV fleet density	vehicle/mile	0.5
α	Energy efficiency (converting energy/power ratio to charging time)	–	1.3
β	Battery performance	mile/kW h	2.5
δ	Delay tolerance	–	15%
A_0	Minimum construction area	sqf	2000
a_0	Per spot construction area	sqf	300
C_a	Unit construction cost for new stations (charging or swapping)	\$/sqf	104
	Unit construction cost for existing charging stations	\$/sqf	20
C_e	Unit manufacturing cost of battery	\$/kW h	650
C_s	Per spot construction cost of charging outlet	\$/kW	500
θ	Range tolerance (confident range)	–	0.8

once the optimal E is given. To determine the optimal integer solution, we need to compare the solution associated with $[m^*]^+$ and $[m^*]^-$ where $[a]^+$ and $[a]^-$ are the first integers obtained by rounding the real number a up and down respectively. Using the constraints of the design problem, $[E^*]^\#$ and $[P^*]^\#$ ($\# = +, -$) can be computed from

$$[E^*]^\# = \frac{l}{\beta\theta([m^*]^\# + 1)}; \quad [P^*]^\# = \frac{\alpha\theta[E^*]^\#[m^*]^\#}{T_0}, \# = +, -;$$

Then, the optimal integer solution would be the one that minimizes $z([E^*]^\#, [P^*]^\#)$, $\forall \# = +, -$.

5. Special cases

In this section we consider two variants of the design problem presented in the previous section. In the first, we relax the assumption that the power of charging station (P) is a continuous variable. As mentioned in Section 2, only three levels of charging facilities are available, each associated with a specific range of power. The second variant deals with an alternative charging solution that allows an EV driver to swap his/her depleted battery for a fully charged one (Pan et al., 2010).

5.1. Discrete capacity for charging facility

Allowing the discrete capacity for charging facility is relatively straightforward. Note that we can simply solve the problem for each discrete level of P and compare the solutions. As P is known for each level, the design problem is reduced to a single-variable optimization problem that has a box constraint, formulated as follows:

$$\min \quad z(E) = (C_p + Pc_1)\left(\frac{C_3}{E} - 1\right) + c_2E \quad (26)$$

s. t. :

$$c_3 - \frac{T_0P}{\alpha\theta} \leq E \leq c_3 \quad (27)$$

The lower bound in (27) is derived from Constraint (12). As to the upper bound, recall that c_3 is the energy that allows the vehicle to travel through corridor without being depleted below the range tolerance. Hence, holding more energy than that would be a waste.

The optimal solution of the above optimization problem is obtained from the first-order optimality condition as

$$E_r^* = \begin{cases} \sqrt{\frac{(C_p + Pc_1)c_3}{c_2}} & c_3 - \frac{T_0P}{\alpha\theta} \leq \sqrt{\frac{(C_p + Pc_1)c_3}{c_2}} \leq c_3 \\ c_3 - \frac{T_0P}{\alpha\theta} & \sqrt{\frac{(C_p + Pc_1)c_3}{c_2}} < c_3 - \frac{T_0P}{\alpha\theta} \\ c_3 & \sqrt{\frac{(C_p + Pc_1)c_3}{c_2}} > c_3 \end{cases} \quad (28)$$

Fig. 3 shows the relationship between a given P and optimal battery energy E^* . We note that the intersection of the two lines in the figure, denoted as $P0$, can be found as

$$P0 = \frac{c_3\alpha\theta}{T_0} \left(1 + 0.5\sigma - \sqrt{(1 + 0.5\sigma)^2 + \left(\frac{C_p}{c_2c_3} - 1\right)} \right), \quad \sigma \equiv \frac{c_1\alpha\theta}{c_2T_0}$$

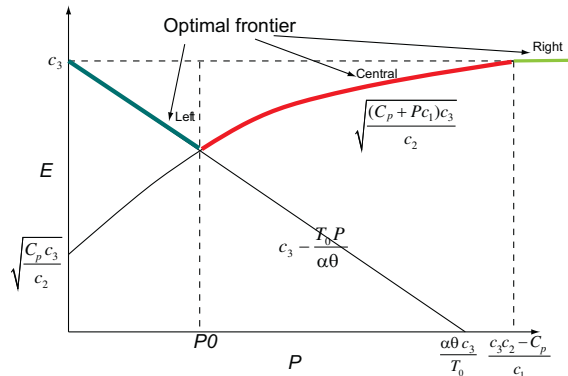


Fig. 3. Relationship between P and optimal E .

It is easy to see (both from the plot and the basic algebra) that $0 < P_0 < \alpha \theta c_3 / T_0$ because $C_p < c_2 c_3$. Therefore, for the value of P that is very large (the right part of the optimal frontier), no station would ever be built because constructing cost of a single station would exceed that of the cost of building a battery large enough to traverse through the corridor without charging.

For small P (the left part of the frontier), the optimal battery capacity is given by $c_3 - \frac{T_0 P}{\alpha \theta}$. In this case, the number of charging stations is

$$m^* = \frac{c_3}{c_3 - \frac{T_0 P}{\alpha \theta}} - 1$$

Again, in reality we will have to round m^* to an integer as explained in Section 4.3. To get a rough qualitative assessment, note that if m^* is closer to zero than to 1 (i.e. $m^* < 0.5$), it is likely that the optimal integer solution would be $m^* = 0$, i.e., no charging facility is needed. That is

$$m^* > 0.5 \Rightarrow \frac{T_0 P}{\alpha} > \frac{l}{3\beta} \Rightarrow P > \frac{v_0 \alpha}{3\delta \beta}$$

where v_0 (free flow speed) and δ (level of service measured as the percentage of the free flow travel time) are used to define T_0 in Eq. (7). For $v_0 = 55$ mile/h, $\alpha = 1.3$, $\beta = 2.5$ mile/kW h and $\delta = 50\%$, the charging power that might warrant building the first charging facility can be estimated as 19.1 kW, which is almost twice as much as the highest power offered by level 2 charging option. In other words, if we only have level 2 chargers, chances are that they will not meet the level of service requirement specified here- it will require the user tolerate almost 100% of the free flow travel time instead. We emphasize that the bound derived above will be invalid if $P > P_0$, because in that case the optimal E is given by a different formula (the middle section of the optimal frontier).

Plots in Fig. 4 provides a conceptual illustration of the objective function, lower and upper bounds associated with the three charging levels.

5.2. Battery swapping problem

The battery swapping problem introduces three changes into the basic model. First of all, the system now needs more batteries, depending on the number of EVs and how often they travel along the corridor. Second, the EV users no longer need to wait for charging at the station since charging now takes place at other times and possibly even at other locations. This leads to the third change: a charging space is no longer needed for each charger.

For simplicity, we shall assume that (1) each battery swapping station will store extra batteries and recharge the depleted ones that the EV users leave behind, (2) the station only uses level 3 charging method for higher battery reuse rate, and (3) the battery models are standardized so that they can be shared across all EVs. We denote the time it takes to exchange the battery at each station as t_e . Thus, the level of service constraint dictates that

$$\left(\frac{l}{\beta \theta E} - 1 \right) t_e \leq T_0 \rightarrow E \geq \frac{l}{\beta \theta (T_0 / t_e + 1)} \quad (29)$$

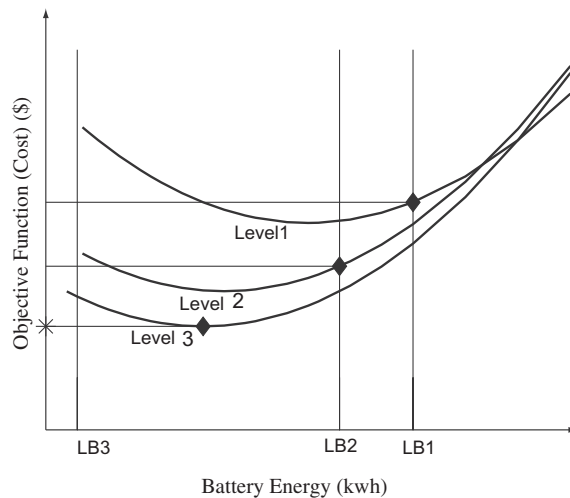


Fig. 4. Illustration of objective functions corresponding to three discrete levels of charging power; LB1–LB3 in the plot represent the lower bounds associated with the lower bound (right hand side) in Eq. (27) for each level of charging power.

Let us now figure out how many batteries the system needs in order to operate. As each car has a battery already installed on it, there would be at least λl batteries in the system. In the worst case, each swapping station would need to exchange the battery $\lambda l f$ times per day (where f is the trip frequency), assuming every EV trip stops at the station for exchange at the same time. In reality, the stations probably would not need $\lambda l f$ batteries in their inventory because they could recharge and reuse the batteries. However, for simplicity and for consistency with the charging problem (where the number of chargers installed equals $\lambda l f$), the above worst case assumption is adopted. Moreover, we assume that each station will need to install $r\lambda l f$ level 3 chargers to ensure full operation, where $r \in (0, 1]$ is a parameter. In summary, the total number of batteries is

$$n_b = \lambda l + \left(\frac{l}{\beta \theta E} - 1 \right) \lambda l f \quad (30)$$

The cost of building a new battery swapping station consists of the construction cost C'_p and the cost of purchasing and installing level 3 chargers. The charging cost function becomes

$$R = C'_p + r\lambda l f C_s P_3 = C'_p + rP_3 c_1, \quad C'_p = A_0 C_a \quad (31)$$

where P_3 is the power of the level 3 charger, C_s is the cost of acquiring a unit charging power, and C_a is the unit construction cost. Note that, unlike in the base model, C'_p here does not include the variable construction area for each charging slot. That is $C'_p < C_p$.

The battery swapping model can now be formulated as follows.

$$\min z(E) = (C'_p + rP_3 c_1) \left(\frac{l}{\beta \theta E} - 1 \right) + n_b C_e E \quad (32)$$

s. t. :

$$\frac{c_3}{(T_0/t_e + 1)} \leq E \leq c_3 \quad (33)$$

Recalling the definitions of c_2 and c_3 in (14), we can rewrite the objective function (32) as follows:

$$z(E) = (C'_p + rP_3 c_1) \left(\frac{c_3}{E} - 1 \right) + c_2(1-f)E + c_2 c_3 f \quad (34)$$

It is clear from the objective function that if $f \geq 1$, then the optimal solution would not build any exchange station because it would be too expensive to hold so many batteries at the stations compared to simply using batteries large enough to support the entire trip. As mentioned before, because we are mainly concerned with infrequent long-distance travel here, it is safe to assume that $f \ll 1$.

The optimal solution to the problem can be easily obtained from the first order optimality condition as

$$E_e^* = \begin{cases} \sqrt{\frac{(C'_p + rP_3 c_1) c_3}{c_2(1-f)}} & \frac{c_3}{(T_0/t_e + 1)} \leq \sqrt{\frac{(C'_p + rP_3 c_1) c_3}{c_2(1-f)}} \leq c_3 \\ \frac{c_3}{(T_0/t_e + 1)} & \sqrt{\frac{(C'_p + rP_3 c_1) c_3}{c_2(1-f)}} < \frac{c_3}{(T_0/t_e + 1)} \\ c_3 & \sqrt{\frac{(C'_p + rP_3 c_1) c_3}{c_2(1-f)}} > c_3 \end{cases} \quad (35)$$

The solution clearly indicates that the maximum number of charging stations (hence the minimum battery size) is determined only by T_0 and t_e , i.e., the level of service requirement and how fast the battery swapping can take place. Ignoring the corner solutions, let us now compare side by side the optimal battery sizes obtained in the charging and exchange scenarios.

$$E_r^* = \sqrt{\frac{(C_p + P_3 c_1) c_3}{c_2}}, \quad E_e^* = \sqrt{\frac{(C'_p + rP_3 c_1) c_3}{c_2(1-f)}}$$

Note that here we assume the level 3 charging for the charging option so P in Eq. (28) is replaced with P_3 .

It can be observed from these equations that E_e^* is likely to be smaller than E_r^* (since $C'_p < C_p$ and $r < 1$). This is especially true for low trip frequency (i.e., f is close to 0). Although the exchange option seems to require smaller battery sizes than the charging option at optimum, it is difficult to analyze which option would be better in terms of the overall cost, which is subject to conflicting influences of numerous parameters. The fact that we have to find the best integer solutions makes it even harder to compare these solutions. It seems appropriate to explore these effects using numerical experiments based on reasonable estimates of the parameters. To these efforts we now turn.

6. Case study

The purpose of the case study is twofold. First, it is designed to establish the sensitivity of the optimal integer solutions to various parameters. We are especially interested in two types of parameters: those related to the demand, such as the line density of EVs and the level of service requirements, and those related to the technology, such as the unit manufacture cost of batteries and the installation cost of the chargers. The second focus of the case study is to examine the relative performance of the charging and swapping options under different conditions.

Our case study is based on a model corridor that connects Chicago, IL to Madison, WI (cf. Acknowledgements to see why this corridor is chosen). The length of the corridor is about 150 miles. We assume the density of EVs is 0.5 veh/mile, which is translated to about 75 EVs in total along the corridor. We assume that on average the EV users will use the corridor for long-distance travel once a week, or 0.13 trip per day. Turrentine et al. (1992) suggested that EV users usually leave 20 miles as the spare range for a 100 mile range battery. Therefore the range tolerance is selected as 80% in this study. Description and default values for other parameters used in the model can be found in Table 2. We have explained in Section 2 how the default values of α , β , C_e , and C_s are estimated. The fixed construction cost of building a charging station is estimated based on the cost for building a gas station, including construction, contract and architectural fees. According to Reed Construction Data (2008), the construction of a gas station has an overall unit cost of about 104 (\$/sqf). Also, the average construction area of a gas station is about 4000 (sqf).⁵ Accordingly, this study assumes a 2000 (sqf) fixed area for each charging station, and 300 (sqf) area for each charging spot. Since building charging stations in existing parking lots or gas stations only requires obtaining charging spots, the fixed area for charging stations is set to zero. The per spot cost of building a charging station excluding the acquisition cost of the charger varies widely depending on installation area, electric circuit, etc. This cost is assumed to be \$6000 in this study as per the data given by (NREL, 2012). Assuming 300 (sqf) is needed for each charging spot, the per unit area cost is $20 \left(\frac{\$}{\text{sqf}} \right)$ as reported in the table.

In what follows, we shall first consider the case where the existing gas stations/public parking lots can be used to house the chargers, which implies $A_0 = 0$ and $C_a = 20 \left(\frac{\$}{\text{sqf}} \right)$ in Eq. (9). Results on the dedicated EV charging facilities will be reported and compared in Section 6.5.

6.1. Baseline model

Table 3 reports the optimal solutions of the base model (11) and (12) corresponding to various levels of service, measured by the tolerance to travel delays. Note that the parameters take the default values in Table 2, and the number of charging stations m is forced to be an integer. As expected, both the battery size and charging power decrease as δ increases (corresponding to a degrading level of service). For a reasonable level of service ($\delta \leq 0.25$, or travelers do not waste 25% extra time waiting for charging), up to two charging stations of relatively high power (level 3 facility) are needed. The range requirement is relatively low in this case, even compared to the current-generation EVs (Nissan Leaf, for example, has a range of about 100 mile). Yet, even if the travelers are willing to double their travel time ($\delta = 1$) by using an EV, the required optimal charging power is still more than 20 kW, which well exceeds the limit of level 2 charging facilities.

Since all optimal solutions require level 3 facilities, the current EV supporting infrastructure that predominately rely on level 2 charging seems a sub-optimal solution. It either mandates the use of expensive high-capacity battery or forces the EV users to tolerate extremely lower level of service.

6.2. Sensitivity analysis of the base scenario

This section focuses on exploring how the optimum number of charging stations m , optimum battery energy E^* , optimum charging station power P^* and optimal cost z^* are affected by input parameters. Two types of parameters are considered separately. The first includes those related to users, specifically the level of service δ and the density λ . The second type of parameters have to do with technology, including the unit battery manufacturing cost C_e and the unit charging power acquisition cost C_s . We focus on C_e and C_s because they are likely to change significantly as technologies advance. Parameters such as the unit construction cost C_p , trip frequency f and range tolerance are considered relatively stable so the sensitivity to them is less of a concern. In addition, there seems no much room to improve α and β unless major technology breakthrough takes place. Accordingly, they are also excluded from the analysis.

Fig. 5a–d plots how P^* , E^* , z^* and m^* change with δ and λ , respectively. As expected, the total system cost rises with density (as more batteries have to be built), and decreases with the delay tolerance. The demand density does not seem to affect the three other outputs much, except when it is small. Taking Fig. 5d as an example. When $\lambda > 0.4$, the number of charging stations primarily depend on the level of service requirement δ . However, when λ is at a very low level (e.g. < 0.1), m^* is still sensitive to its change for some values of δ . It is worth noting the similarity and difference between Fig. 6b and d. This is also expected because the battery range is inversely related to the number of charging stations.

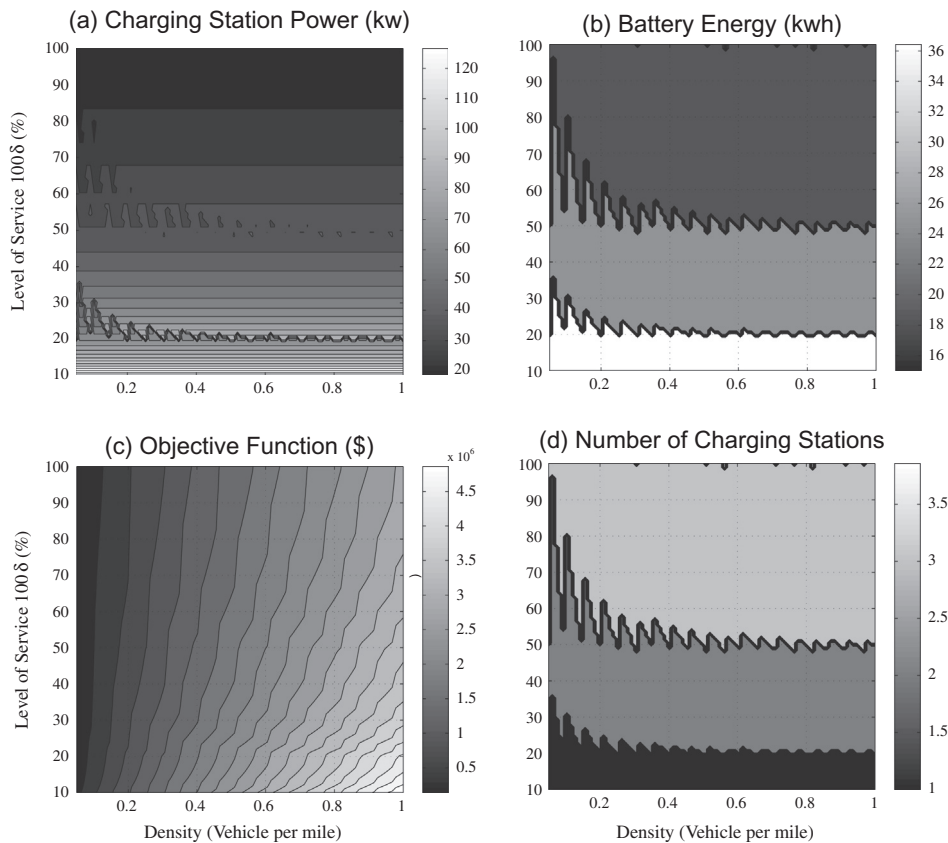
Similarly, Fig. 6a–d demonstrates the sensitivity analysis results with respect to C_e (unit battery cost) and C_s (unit charging power cost). From Fig. 6c, one can easily see how the total cost decreases as C_e and C_s are reduced from their current

⁵ LoopNet Data, Gas stations and convenience stores for sale, <http://www.loopnet.com/Gas-Stations-For-Sale/>, last visited: 07.03.12.

Table 3

Optimum battery energy and charging station power in the base scenario.

Level of service 100δ (%)	Total travel time (h)	Energy E (kW h)	Battery range (mile)	Charging Power P (kW)	Number of charging stations m
0	2.7	75.0	187.50	0	0
5	2.9	37.5	93.75	286.0	1
15	3.1	37.5	93.75	95.3	1
25	3.4	25.0	62.50	76.3	2
50	4.1	25.0	62.50	38.1	2
85	5.0	18.7	46.87	25.2	3
100	5.5	18.7	46.87	21.4	3

**Fig. 5.** Sensitivity of demand variations in the base model.

values. A less obvious result is that the cost is as twice sensitive to C_e as to C_s . To see this, note that the cost is about same for a 400 ($\frac{\$}{\text{kWh}}$) reduction in C_s (with C_e remaining at 600 ($\frac{\$}{\text{kWh}}$)) as for 200 ($\frac{\$}{\text{kWh}}$) reduction in C_e (with C_s remaining at 500 ($\frac{\$}{\text{kWh}}$)). It seems from this example that the impacts from advancing the battery technology seems higher for the same. From Fig. 6d, one can also see that no charging station is needed when the battery cost C_e is very low ($\ll 100$ ($\frac{\$}{\text{kWh}}$)): note that the total cost is no longer sensitive to C_s when C_e falls into that range.

Fig. 6a, b and d shows that a lower battery cost leads to a larger battery size and a smaller number of charging stations. These plots also suggest that the optimal charging solution and battery size only depend on the ratio between C_e and C_s , not their absolute values. Roughly speaking, when $C_e/C_s < 0.2$, no charging station is needed, i.e. $m^* = 0$; when $0.2 < C_e/C_s < 1$, $m^* = 1$; when $1 < C_e/C_s < 2$, $m^* = 2$; and for even larger ratio, $m^* = 3$. This is expected given C_e and C_s are the two primary coefficients in a linear objective function.

6.3. Discrete charging power

This experiment is concerned with the case where the charging power P is a discrete variable that depends on the charging method (level 1–level 3). It is designed to examine the most appropriate charging method under different scenarios. As per discussions in Section 2, the average power for level 1, 2 and 3 charging is set as 1.5 kW, 10 kW and 90 kW, respectively. All other parameters take the default in Table 2. Table 4 reports the optimal battery range and average user cost for the three

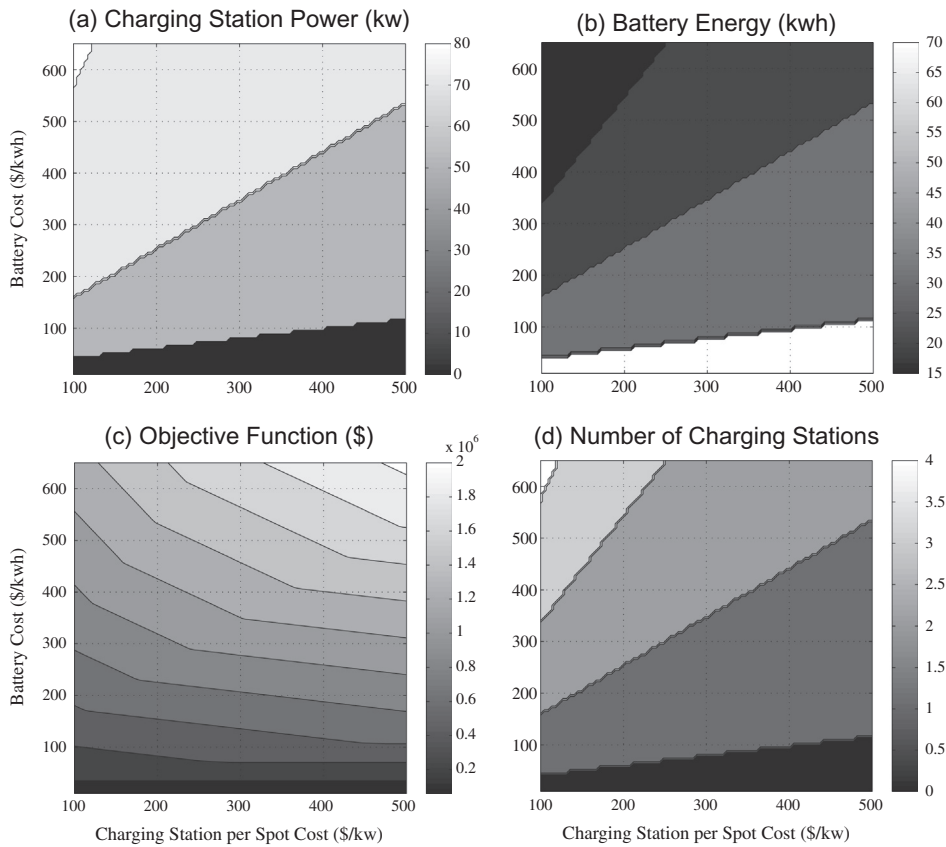


Fig. 6. Sensitivity of technology variations in the base model.

Table 4

Optimal battery range and per driver cost for three levels of discrete charging power.

Charging station level	Charging station power (kW)	Level of service 100δ							
		1	5	15	25	50	85	100	300
		Battery range (mile) (Cost per driver (k\$))							
1	1.5	187.50	187.50	187.50	187.50	187.50	187.50	187.50	187.50
		(48.75)	(48.75)	(48.75)	(48.75)	(48.75)	(48.75)	(48.75)	(48.75)
2	10	187.50	187.50	187.50	187.50	187.50	187.50	187.50	31.25
		(48.75)	(48.75)	(48.75)	(48.75)	(48.75)	(48.75)	(48.75)	(15.45)
3	90	187.50	187.50	187.50	62.50	62.50	62.50	62.50	62.50
		(48.75)	(48.75)	(48.75)	(29.50)	(29.50)	(29.50)	(29.50)	(29.50)

levels of discrete power. First of all, the battery range (hence the user cost) decreases as δ rises. For level one, this trend is not revealed from the table because the battery range decreases only after δ becomes as large as 10. Second, a level 3 charging station would help reduce social cost when the delay tolerance is greater or equal 20%. In comparison, a level 2 station would make sense only if δ is as high as 3. Third, for most values of δ , the lowest user cost is achieved by level 3 charging. Only when δ reaches 3 does the level 2 charging become a more competitive option, evidently because of the lower construction cost. Thus, we can safely conclude that level 3 charging is the most attractive option for the parameters adopted in this study.

Moreover, Fig. 7 examines how the average user cost changes with density levels for the three levels of charging power. Fig. 7a suggests that for very low density ($\lambda = 0.01$), level 2 charging could be a viable option, provided the users are willing to accept extremely low level of service ($\delta \sim 1.5$). Interestingly, this case seems to explain why level 2 charging currently dominates in the US. It does seem to make sense to start with level 2 when the market penetration rate of EVs is very low and the enthusiastic initial adopters do not care much about the level of service. Yet, for larger densities and reasonable δ (cf. Fig. 7b and c), level 3 charging always emerges as a better choice in our model. Hence, a transition to level 3 seems necessary to foster the growth of the EV market and to achieve an optimal resource allocation in the long run.

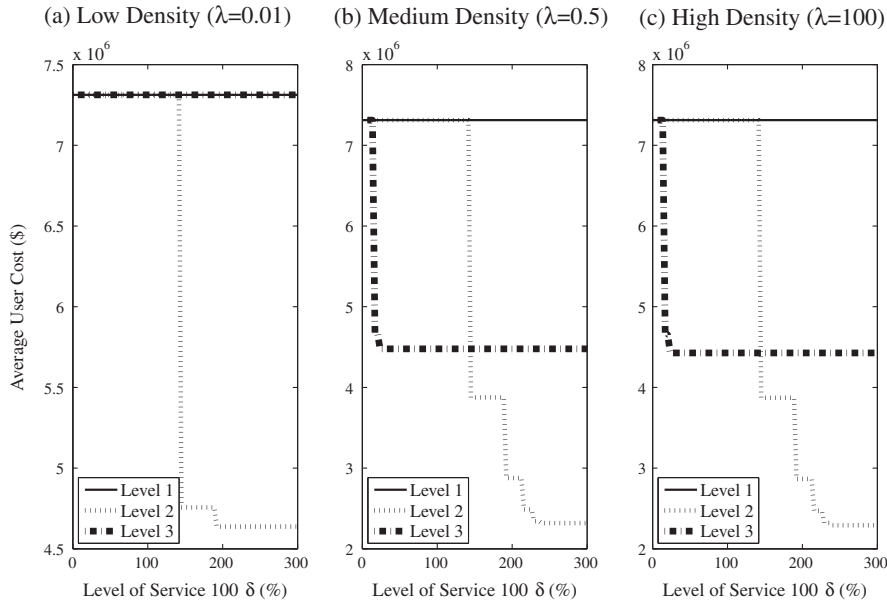


Fig. 7. Objective function value vs. delay tolerance for three discrete levels of charging power at various demand densities.

6.4. Battery swapping problem

We assume that new construction is necessary to offer battery swapping service, and that the construction area $A_0 = 2000$ sqf. We suspect that providing a state-of-the-art swapping service such as portrayed by Better Place Inc. (www.betterplace.com) would cost much more than what is needed to build a charging station (estimated at 104 per sqf in this study, and is the same as the cost for building a gas station). Since the data is not available, we assume the same unit construction cost 104 per sqf applies, which likely underestimates the capital cost of the battery swapping option. It is further assumed (1) that each battery swapping requires $t_e = 5$ min⁶ (Cocchi and Rizkalla, 2012), and (2) that r is equal to 0.5, which means the number of level 3 chargers needed equals 50% of the number of batteries stored by the stations. A quick calculation shows that the optimal battery size for the swap problem (based on default values reported in Table 2) is 27.5 (kW h), corresponding to a range of 69 mile. Note that to achieve the same level of service ($\delta = 0.15\%$) with charging services, the optimal battery size has to be as high as 37.5 (kW h), a more than 35% increase. Thus, smaller batteries seem better for battery swapping than for charging.

Does battery swapping necessarily help reduce the total system cost? To answer this question, we compare the total costs obtained from the base charging model and the battery swapping model for different combinations of λ , δ and r , as shown in Fig. 8. First, it is easy to observe from the plots that a very low density would deprive the scale of economy and hence neither swapping nor charging would be beneficial from the society point of view. Hence, in those cases, the costs of the two options are the same. Second, the plots suggest that swapping is generally more competitive than charging for high density and high level of service (i.e. large λ and small δ). For larger values of r (i.e. $r \geq 0.3$ in this case), which implies that the swapping stations need to build more level 3 chargers for each battery they keep, charging seems always better for modest level of service ($\delta > 0.25$, roughly). Also, for the same r , the larger δ is, the more significant the advantage of the charging option becomes. Third, for small r ($r = 0.1, 0.3$), the charging option always becomes less competitive as the density increases; for larger r ($r = 0.5, 0.7$), the charging option first becomes more competitive as the density increases, then its advantages decline when λ exceeds certain threshold (roughly at $\lambda = 1.0$).

6.5. Impacts of station construction costs

So far it is assumed that the system can make use of existing gasoline stations or other structures to host battery charging or swapping operations. This section examines how the construction cost might affect the results of analysis. Interestingly, even with the additional construction cost, the optimal solution remains the same for $\delta = 0.15$ in the baseline model. Unlike in the baseline model, however, the number of charging station does not increase when the level of service requirement is relaxed. Even for $\delta = 1$, the optimal battery range is still 93.75 miles and the number of charging station is still 1. In comparison, the battery range would have been reduced to 46.87 miles with three charging stations, when the construction cost is ignored (see Table 3). Certainly, the required power at charging stations still decreases as δ increases.

⁶ www.betterplace.com, last visited 04.04.13.

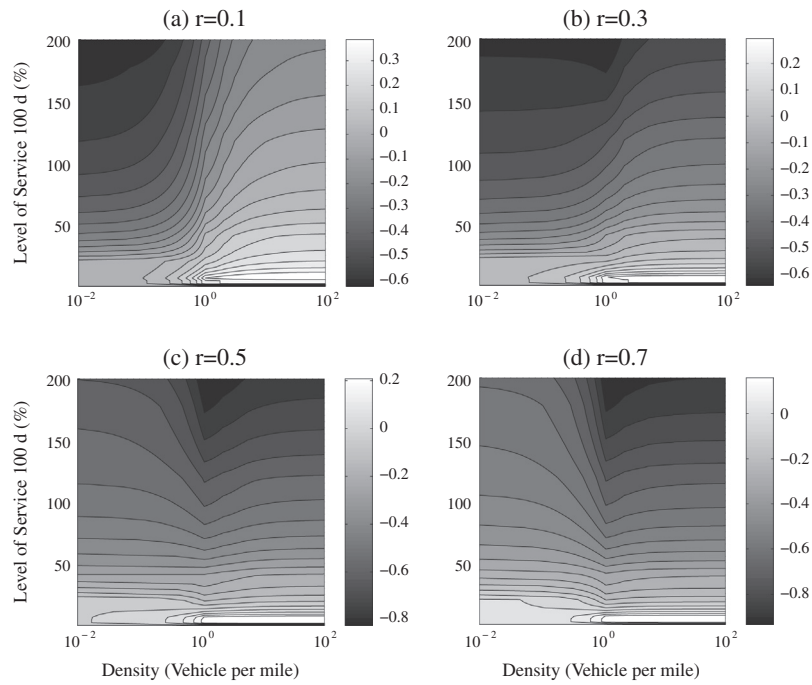


Fig. 8. Relative performance of the base and battery swapping model for different values of λ and δ (Let z_r and z_e denote the optimal system cost for the charging and swapping options, respectively. Then the color in the contour plots represents $(z_r - z_e)/z_r$).

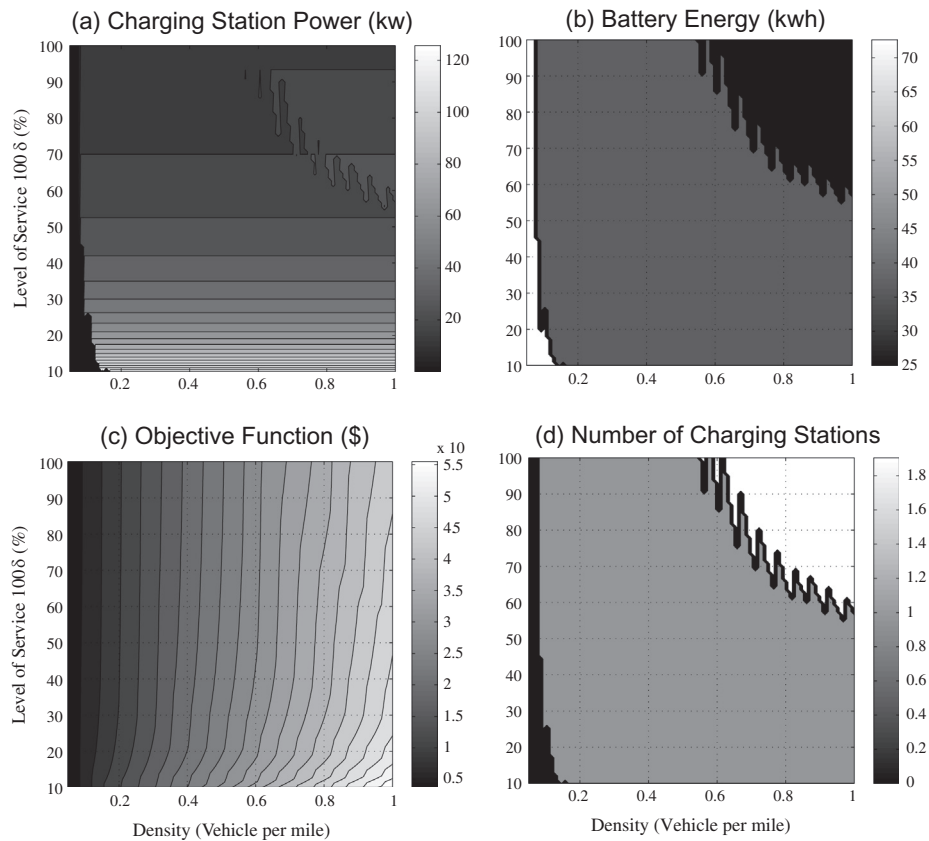


Fig. 9. Sensitivity of demand variations in the base model (station construction cost included).

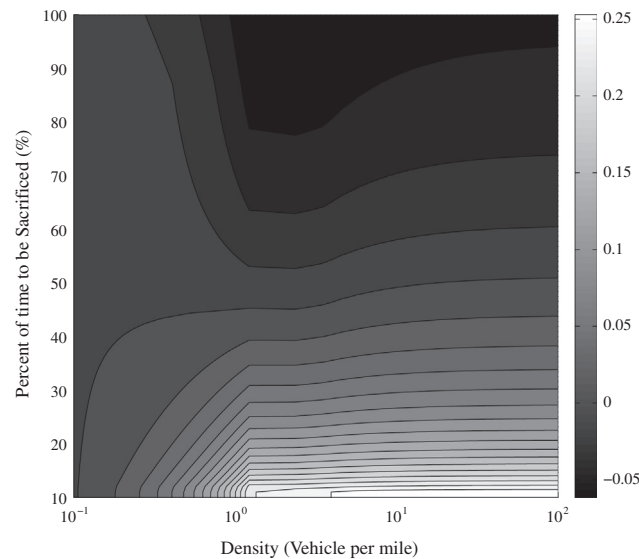


Fig. 10. Relative performance of the base and battery swapping model for different values of λ and δ (station construction cost included, $r = 0.5$).

From previous analysis, we know that a charging station would only be built if $c_2 c_3 > C_p$, which actually provides a lower bound for the battery cost C_e . Without the construction cost, this lower bound is about $10 \left(\frac{\$}{\text{kWh}} \right)$. That is, if the battery cost is lower than 10 (while all other parameters remain the same), there is no need to build charging station. When the construction cost is included, this lower bound becomes 200 for the baseline model.

Fig. 9 performs the same demand sensitivity analysis as in Fig. 5, but includes the station construction cost. It is easy to see by comparing Figs. 9 and 5 that the optimal number of charging stations decreases significantly when the construction cost is included. Another difference revealed in Fig. 9 is that the optimal solution depends much more on demand density than the level of service. Specifically, for very low density, no charging station is needed at optimum; and for medium density (roughly between 0.2 and 0.6), one station is optimal regardless of the level of service. Intuitively, when the construction cost is included, it is much more difficult to justify building new stations. It would only make sense when the capital investment on the infrastructure can be shared by a sufficiently large number of EV drivers such that the average investment is lower than the cost of building larger batteries.

Finally, we examine the relative performance of charging and swapping options with the station construction cost included in Fig. 10. In this case, the charging option is better only for high density and low level of service, i.e. δ close to 1. Another observation is that the relative performance is almost independent of demand density λ after λ exceeds certain threshold. Obviously, the reason why the charging option becomes much less competitive is that the capital investment is much higher in the case of new construction. Compared to a swapping station, a new charging station would require $84 \times 300 = \$25,200$ more per charger. Such a cost is much higher than the cost of a typical EV battery today. Therefore, if new construction is needed anyway, the society seems better off by building the swapping instead of charging stations. We caution that, however, this conclusion is based on an assumption that likely underestimates the construction cost of the swapping option.

7. Conclusions

The current generation of EV batteries remain to be expensive and limited by range. The low density of fast charging facilities severely restricts the use of EVs for medium and long distance travel. Together, these two hurdles prevent the rapid growth of electric vehicles in the US, despite substantial federal investment in research and development, as well as policy supports. To gain insights regarding how these hurdles may be overcome, a conceptual optimization model was built and analyzed in this paper to explore travel by EVs along a long corridor. The objective of the model was to select the battery size and charging capacity to meet a given level of service in such a way that the total social cost is minimized. The main findings from our analysis are summarized below.

- The current paradigm of charging facility development that focuses on level 2 charging does not well serve the traditional long-distance trips made using automobiles. Interestingly, our model suggests that level 2 charging is indeed socially optimal for very low EV market penetrate rates, which closely resembles the present reality. However, if the level of service is kept at such a low level, the growth of EVs will likely continue its sluggish pace.

- To achieve a reasonable level of service, the level 3 charging method is needed to minimize the social cost. This finding justifies the investment on level 3 charging, which will not only help the EV adoption, but also reduce social cost because of the savings on batteries.
- Our baseline model shows that reducing the unit battery manufacturing cost offers larger benefits than reducing the unit charging power installation cost. Therefore, advancing battery technology seems to promise larger impacts than the charging technology. We caution that other parameters might affect this relative sensitivity, which are not fully explored in this study.
- Battery swapping enables the use of smaller batteries and to achieve higher level of service. Yet, when the level of service requirement is modest, battery swapping can be more costly than charging, especially when the cost of constructing swapping and charging stations (excluding the installation cost of chargers) is close. If existing infrastructure can be remodeled to support battery swapping and charging operations, charging could be a socially optimal solution for modest levels of service.

For future research, adding a few important features that are currently missing could make the model more useful for policy analysis.

- First, our model assumes that the total battery and charging costs are linear functions of the battery capacity and charging power, respectively. The linear assumption evidently does not hold for the charging cost, as shown in Fig. 2. On the other hand, as the battery size increases, the marginal production cost will likely rise due to the law of diminishing returns. Thus, the average unit production cost should be an increasing function of E instead of a constant. Moreover, unless a technology breakthrough takes place, increasing the battery capacity also means adding extra weight to the vehicle, which in turn affects the range. Therefore, the parameter β should depend on the battery energy capacity E , which introduces still more nonlinearity into the model.
- Second, the current analysis ignores the impacts of the arriving pattern of EVs at charging and/or swapping stations. Instead, the most conservative scenario is used to guide the design, which assumes that the stations have to accommodate all EV trips simply based on the EV ownership and trip frequency, regardless of the temporal and spatial distributions of these trips. While relaxing this assumption is likely to change the relative performance of charging and swapping options, it is not easy to predict which option will be better off.
- Last but not least, a more realistic mode should take the network effects into account. Specifically, when multiple corridors are planned between different origin destination pairs, the estimated demand pattern and even route choices could affect the infrastructure decisions considered in this paper.

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