Proceedings of the 34th Chinese Control Conference
July 28-30, 2015, Hangzhou, China

Charging Station Selection and Charging Price Decision for PEV: A Two Level Game Approach

Jie Chen, Bo Yang, Hongbin Zhou, Cailian Chen and Xinping Guan
Department of Automation, Shanghai Jiao Tong University
Key Laboratory of System Control and Information Processing Ministry of Education of China
800 Dongchuan Road, Shanghai, China, 200240
Email: {euthanasia, bo.yang, vincent_sjtu, cailianchen, xpguan}@sjtu.edu.cn

***Abstract*—In this paper, we consider the problem that a group of plug-in electric vehicles (PEVs) seek to maximize their payoff by selecting a proper charging station and the charging stations try to maximize their revenue by adjusting the electricity price according to the total demands they received. Since both the decisions of the charging stations and the PEVs should be made dynamically under competition environments, we propose a two-level game framework to model this issue. At the lower level, the PEVs need to adapt their strategies, i.e. the charging station selection, according to the service they can get from each charging station, and we formulate it as an evolutionary game and use replicator dynamics to model the charging station selection process. Also, we propose an algorithm to arrive at the evolutionary equilibrium. At the upper level, each charging station needs to set the electricity price appropriately to maximize its revenue according to the received total amount of electricity requirement from PEVs. We formulate a differential game to describe the competition among the charging stations and analyze the Nash equilibrium for both open-loop strategies and closed-loop strategies. Simulation results demonstrate that the proposed mechanism can gain a higher profit and faster convergence speed than static control.**

I. Introduction

Due to the growing concerns for energy conservation and the environment, it is expected that PEVs will play a significant role in future smart grid technology. As PEVs are being widely deployed, they may also lead to serious impacts on the grid. First, due to the considerable amount of their energy consumption, PEVs are excepted to generate a huge amount of new load on the grid, which will result in new load peaks and electricity price adjustment. Also it is noted in [1] that a complete conversion to electric vehicles would require 640 new large (1000MW) power plants. Besides the additional demand for electricity, PEVs can violate line capacities if charging demands are not properly coordinated [2].

A lot of works have been done on how to coordinate the charging demands between the PEVs and the charging stations, among which game theory and auction method are widely used. A noncooperative game model for pricing and frequency regulation in smart grids with electric vehicles is studied in [3]. An optimization framework is proposed in [4] for enabling the smart grid to determine the time and duration of the PEV charging strategies. In [5], a control algorithm is developed based on queuing theory to control the charging of PEVs. In [6] and [7], a mean field game is proposed to investigate the competitive interaction between electric vehicles in a Cournot market consisting of the electricity transactions to/from an electricity distribution network. Although some of these work ever thought about modeling the competition nature in smart grid as a multi-stage dynamic game structure, they mainly focus on the final results, i.e., the stage game in these works is static. Their dynamic is reflected by the play order of the game and they ignore the time dependency.

In this paper, we consider a situation that the technology of PEV is mature and it is used in taxis, which means that the PEVs usually have the same engine and battery. In [8], the authors analyzed the properties of PEVs and pointed out when to use fuel and when to use electricity to maximize each PEV's profit. They also got the conclusion that each PEV can be represented by its unique type, and this type value is related to PEV battery coefficients, engine and electricity motor coefficients and fuel price. They have proved that as long as the type value is positive, PEV can gain economy from buying electricity compared with buying fuel. What we consider here is that the taxis have the same type with positive value and their charging request is regular, i.e., when their battery level is under a predefined level set by the taxi company, they will choose to submit a request of a_r amount electricity to a certain charging station.

We propose a two-level game framework considering time dependency to describe the dynamic relationship between the PEVs and the charging stations. At the lower level of the game framework, each PEV selects charging station competitively based on the charging price and the queueing delay it incurs at that station and the queueing delay is affected by PEV's strategy. We use an evolutionary game to represent the dynamic competition among the PEVs. The reason for applying evolutionary game is that an evolutionary game can explicitly capture the dynamics of interaction among the players. The strategies of the PEVs are subject to the control of the charging stations in terms of electricity price and we adopt the replicator dynamics to analyze the strategy adaption process and an evolutionary stable strategy (ESS) is considered to be the

[0]This work was supported in part by the National Nature Science Foundation of China under Grant 61174127, 61221003, 61290322, 61273181, 61374107, and U1405251; by the Research Fund for the Doctoral Program of Higher Education under Grant 20110073120025 and 20110073130005; by the Shanghai Municipal Science and Technology Commission, China, under Grant 15QA1402300, 14511107903 and 13QA1401900.

solution of this game. Then the charging stations at the upper level will decide their optimal prices according to the received electricity requests from the PEVs. We model the upper level game as a differential game, where the state of the differential game is the system state, i.e., the population share of the evolutionary game. In this two level game framework, the players of two levels (i.e., the PEVs and the charging stations) are connected through the differential equation. This is how the transient states of the lower level impact the upper level.

The rest of this paper is organized as follows: Section II describes the system model. In Section III, we formulate the lower level evolutionary game for strategy adaption of selecting the charging stations and we discuss the relationship between the evolutionary equilibrium and evolutionary stable strategy. In Section IV, we formulate the upper level differential game and discuss its Nash equilibrium. Numerical results are analyzed in Section V and conclusions are drawn in Section VI.

II. System Model

We consider an area where there are N active PEVs and M charging stations located at particular geography positions. Each of the N PEVs will choose to charge at a certain charging station among the M charging stations when the residual electricity in the battery is under a predefined level. We suppose that in the near future, PEV will be quite popular and can be used in various ways and an important application is be used as taxi. These taxis have the same engine and battery or in other words, they have the same type. Since these taxis have strong consistency, we assume that once their battery level is under a predefined level, they will submit a charging request of a_r amount of electricity.

According to the number of PEVs that submit charging request, charging station i will broadcast the average delay time $\tau_i(t)$ and charging price $p_i(t)$ to the vehicles that choose to charge at charging station i. After receiving the broadcasted information, PEV j will calculate its corresponding payoff by selecting charging station i and we denote it as u_{ji}. We model the charging stations as a multi-queue system, where each station corresponds to a single queue. The demand backlog dynamics at charging station i is stored in queue i and reflects the basic supply and demand relationship. We denote w_i the station service amount and a_i represents the total demand amount of charging station i and n_i represents the total number of PEV that submit request to charging station i and it satisfies $\sum_{i=1}^{M} n_i = N$. Notice that a_i is proportional to n_i as these taxis require the same charging rate a_0, i.e. $a_i = n_i a_0$.

In smart grid, the supply and demand control are usually used to guarantee system reliability. In this paper, we suppose w_i is fixed and this is usually fulfilled by the station through a day-ahead purchasing according to history statistical data. We suppose each queue in the system is an M/M/1 queue. From the stationary distribution of M/M/1 queue, we know that the average waiting time is given by $\frac{a_i/w_i}{w_i-a_i}$ and notice that in order to maintain the stability of the system the charging station will make sure that the total demand amount should be smaller than the service amount (i.e., $w_i > a_i$). Then the average delay time is constituted by the average waiting time plus service time, so we have $\tau_i = \frac{a_i/w_i}{w_i-a_i} + \frac{1}{w_i} = \frac{1}{w_i-a_i}$.

We assume that each PEV can select the charging stations dynamically and independently, furthermore it is reasonable to suppose that each PEV can submit charging request to only one charging station at a time. The payoff u_{ji} for different j are not always the same because the distance to station i are not the same, yet in this paper we treat all the u_{ji} the same as u_i and the reason is that initially the taxis do not consider the distance problem and its payoff just take price and delay time into consideration and then when the evolution comes to an equilibrium, n_i of the taxis will choose to charge at station i.

III. Modeling the Evolution of The Charging Strategy

What we consider here is a homogeneous scene, which means any PEV goes to charging station i can get the same payoff u_i as we discussed above.

A. Evolutionary Game of The Lower Level

In our proposed two-level framework, the lower level game is the dynamic charging station selection game among all the N PEVs and is formulated as an evolutionary game as mentioned before. Notice that the population of the evolutionary game is also the players, i.e. the N PEVs. The strategy of each player is the particular selection of a charging station and the payoff quantifies the players' performance satisfaction level. In this paper, we denote the payoff function as follows:

$$u_i(t) = u_0 - \alpha\tau_i(t) - \beta p_i^2(t) \tag{1}$$

where u_0 is the utility that each PEV can get after its demand a_o is satisfied. An intuitive way to measure this utility is to calculate the distance this a_o amount of electricity can drive and calculate the saved money when driving the same distance with electricity in stead of fuel. Thus in this paper, we assume the fuel is more expensive than electricity. And in (1) α is a parameter that converts the delay time into disutility and a bigger α means the PEVs are more time-sensitive and $\tau_i(t) = \frac{1}{w_i(t)-a_i(t)}$. β is the parameter that describes the price sensitivity and the last term in Eq. (1) is set to be quadratic, which is the economic cost when the PEV charges at station i. The reason that we select the quadratic cost is that when the price is low the cost is almost linear to the price and when the price becomes larger the cost increase drastically and thus the charging station will not set its charging price too high to avoid loosing customers. Besides we give the feasible price set P_i as:

$$P_i = \{p_i > 0 \mid u_0 - \alpha\tau_i(t) - \beta p_i^2(t) > 0\}$$

Denote by $n_i(t)$ the number of PEVs choosing station i at time t, then $x_i(t) = n_i(t)/N$ is the population share. The population state is constituted by all the population shares and can be denoted by $\mathbf{x}(t) = [x_1(t) \cdots x_i(t) \cdots x_M(t)]^T$, where

$x_i(t) \in [0,1]$ and satisfy $\sum_{i=1}^{M} x_i(t) = 1$. With the definition of population state, we can calculate the average payoff of the population as

$$\bar{u}(t) = \frac{1}{N}\sum_{i=1}^{M}(n_i(t)u_i(t)) \tag{2}$$

Substituting $u_i(t)$ into the right hand side of (2) and noticing that $n_i(t)/N = x_i(t)$, we have:

$$\bar{u}(t) = \sum_{i=1}^{M} x_i(t)(u_0 - \alpha\frac{1}{w_i(t) - Nx_i(t)a_0} - \beta p_i^2(t)) \tag{3}$$

From the perspective of evolution, only the stronger species will survive. In our case, the strategy with higher payoff than the average payoff will be learned and then replicated by other PEV owners. This is referred to as the evolution of the charging station selection game during which the strategy adaption will change the population share and then the population state evolves over time. In a very short time, the change rate of selection of the charging station can be modeled by replicated dynamics [9][10][11], and we define it as a differential equation as follows:

$$\frac{\partial x_i(t)}{\partial t} = \delta x_i(t)(u_i(t) - \bar{u}(t)) \tag{4}$$

where δ is the learning rate of the population and can control the change speed of the choice of the charging station. The initial condition for Eq. (4) is $\mathbf{x}(0) = \mathbf{x}_0 \in \mathbf{X}$, for all $i \in \{1, 2, \cdots, M\}$, and $\mathbf{X} \in R^M$ represents all the possible population state. Notice that $u_i(t)$ is in fact a function of $p_i(t)$ and $x_i(t)$, i.e., $u_i(t) = u_i(p_i(t), x_i(t), i)$ and $\bar{u}(t) = \bar{u}(\mathbf{p}(t), \mathbf{x}(t), \mathbf{x}(t))$.

According to the evolution theory and replicator dynamics, if the corresponding payoff of choosing charging station i is higher than the average payoff, the strategy of choosing this charging station is the so-called stronger specie and will survive finally. The population will learn from each other and result in the number of PEVs choosing to go to charging station i increases. The increasing rate is proportional to the payoff difference between the selected station and the population's average and it is also proportional to the size of the population share.

B. Evolutionary Stable Strategy and Evolutionary Equilibrium

At the lower level evolutionary game, the replicator dynamics for station selection describes the variation of all population shares and the evolution of a population state described by the replicated dynamics is affected by the price of the charging stations.

We first give the definition of ESS as follows:

Definition 1: A strategy $\mathbf{x}$ is an ESS, if for any $\mathbf{y} \neq \mathbf{x}$, there exists $\varepsilon_y \in (0,1)$ such that for all $\varepsilon \in (0, \varepsilon_y)$, the following inequality holds:

$$\bar{u}(\mathbf{p}, (1-\varepsilon)\mathbf{x} + \varepsilon\mathbf{y}, \mathbf{x}) > \bar{u}(\mathbf{p}, (1-\varepsilon)\mathbf{x} + \varepsilon\mathbf{y}, \mathbf{y})$$

This means that if a small amount of a new state $\mathbf{y}$ invades to form a new state $(1-\varepsilon)\mathbf{x} + \varepsilon\mathbf{y}$ then the intuder $\mathbf{y}$ has a lower payoff than the original state. We consider the ESS as the solution of this evolutionary game and consider the evolutionary equilibrium (EE) as the solution of the strategy adaption process at which none of the players has an incentive to change its strategy. The EE is defined as the fixed point where all population shares do not change and is usually obtained by solving the differential equation $\dot{x}_i(t) = 0$. In [12], Friedman points out that an EE must be a Nash equilibrium. In general, an EE obtained from the replicator dynamics may not be an ESS [13][14]. In our case, we will show the relationship between the EE and ESS later. As Eq. (4) shows, at the equilibrium, all the PEVs will get the same payoff (i.e., the average population payoff). Since we consider the PEVs have strong consistency, the fairness among the PEVs is of vital importance, and in this way the fairness is achieved. Now we come to solve $\dot{x}_i(t) = 0$. First we substitute $u_i(t) = u_0 - \alpha\tau_i(t) - \beta p_i^2(t)$ and $\bar{u}(t) = \sum_{i=1}^{M} x_i(t)(u_0 - \alpha\tau_i(t) - \beta p_i^2(t))$ into the right hand side of equation (4) and set it to zero. Remember that $x_i(t) \in [0,1]$ then we get the solutions as :

$x_i^1 = 1$, $x_i^2 = x_i^*$,

where x_i^1 is the boundary EE and x_i^2 is the interior EE and x_i^* is a very complex expression. In order to show how to get the value of x_i^*, we consider a simpler situation where $M = 2$ and point out that the similar method can be adopted for $M > 2$ in our technical report [15] and give an algorithm to arrive at the interior EE.

The algorithm to arrive at the EE is given as follows:

1) : All the PEVs choose a charging station randomly;

2) : The taxi company collects the state (i.e., p_i and τ_i) of all the charging stations and calculates each payoff u_i according to Eq. (1) and then the average payoff $\bar{u}$;

3) : The taxi company broadcasts the payoff information to all the taxis;

4) : **If** $u_i < \bar{u}$ and $(\bar{u} - u_i)/\bar{u} > rand()$, then PEVs chosing station i turn to submit charging requirements to charging station j, where $u_j > u_i$ and $j \neq i$. Return to step (2)

else return $\mathbf{x}^*$

In the simulation part, we will show this algorithm can find the EE efficiently.

The boundary equilibria are not stable because any small perturbation will make the system deviate from the equilibrium state and now we show the relationship between interior EE and ESS of our considered situation in the following theorem:

Theorem 1: The interior EE $\mathbf{x}^*$ of the replicator dynamics for charging station selection is also the ESS of the game.

Proof: See technical report in [15]. ■

IV. Dynamic Electricity Price Adjustment

At the upper level of our proposed two-level game framework, the charging stations need to optimally decide the electricity price considering the dynamic actions of the PEV owners. Intuitively, the lower the electricity price is, the more attractive this charging station will be to the PEVs, yet at the same time the revenue the station can get from one individual PEV will decrease. So the charging station need to decide the optimal price to gain the maximum revenue. In this paper, we formulate this competition of setting the electricity price among charging stations as a differential game. We just consider the simultaneous play model in this paper.

A. Noncooperative Dynamic Electricity Price Adjustment As a Differential Game

We modeled the upper level as a simultaneous play model and in the noncooperative differential game, every player (i.e., the charging station) tries to maximize its objective function over some certain time interval. This interval can be represented by either bounded interval $[0, T]$ or unbounded interval $[0, \infty)$, where a bounded time interval means the player wants to maximize a short-term revenue while an unbounded time interval indicates that the player tries to gain a maximum long-term revenue. For simplicity of notation, we use $[0, T\rangle$ to represent both the finite and infinite time interval.

In this game, the players of the upper-level are composed of the M charging stations, and their strategies are the electricity price $p_i(t)$ at time t. As is commonly used in game theory, we use $\Phi = \{p_i(t), \mathbf{p}_{-i}(t)\}$ to represent the strategy set of the dynamic differential game, where $\mathbf{p}_{-i}(t)$ is the strategy of the rest players except player i. According to the existing information technology and devices, we can divide the dynamic control strategy into two different types. The first one is called an open-loop strategy, which is suitable for situations where the state measurement is costly. In the open-loop strategy, the charging stations determines their strategies in advance according to the statistical data and do not need to measure the system state. The other one is called a closed-loop strategy, where there is an underlying communication infrastructure that collects the system state and broadcasts the state information to every charging station. Both the charging stations and the PEV owners will adjust their strategy timely according to the received state information. In this paper, we consider the open-loop strategy in more details and the closed loop strategy will be introduced briefly due to space limitation.

Notice that in the upper level differential game, the state is actually the population share $\mathbf{x}(t)$ of the lower level evolutionary game. The differential Eq. (4) the lower level can also show how the system state changes. The upper level players' strategies can influence not only the system state (i.e., the evolution of the lower level game) but also other charging stations' revenue. Now the problem for each station under dynamic competition becomes an revenue maximum problem subject to differential Eq. (4). We can formulate the revenue function of player i with price $p_i(t)$ at time t as $J_i(p_i(t), \mathbf{p}_{-i}(t)) = p_i(t)a_i(t) = p_i(t)Nx_i(t)a_0$.

Based on what is discussed above, we formulate the optimal control of each charging station as follows:

$$\max_{p_i} J_i = \int_0^T e^{-\rho t} J_i(p_i, \mathbf{p}_{-i})dt \tag{5}$$
$$s.t. \quad \dot{x}_i = \delta x_i(u_i - \bar{u}), \; \mathbf{x}(0) = \mathbf{x}_0, \; and \; p_i \in P_i$$

where ρ is the discount rate influencing the discounted present value of future profit and P_i is the feasible price set.

B. Nash Equilibrium Analysis

We point out that in practice, the charging stations cannot make agreements with each other privately, otherwise they may be punished seriously by their supervisor. Under this condition, the Nash equilibrium is considered to be an reasonable optimal solution for the electricity price adjustment differential game. At the equilibrium, each charging station can get its maximum revenue given other competitors' strategies. We first give the definition of best response and Nash equilibrium as follows:

Definition 2: An electricity price adjustment strategy p_i^* is called the best response of charging station i if the inequality $J_i(p_i^*, \mathbf{p}_{-i}) \geq J_i(p_i, \mathbf{p}_{-i})$ holds for all feasible strategies $p_i \in P_i$ in the electricity price adjustment differential game.

After giving the definition of best response, we then can study the Nash equilibrium. At the Nash equilibrium, no one can gain a higher revenue by changing his own strategy unilaterally and so we give the definition of Nash equilibrium of the differential game as follows:

Definition 3: Denoting $p_i \in P_i$ as the strategy of charging station i, the strategy set $\{p_i^*, \mathbf{p}_{-i}^*\}$ is an Nash equilibrium if to every charging station i, p_i^* is the best response given other charging stations' optimal strategies $\mathbf{p}_{-i}^*$.

C. Solution to the Differential Game

In this section,we will show how to find the solution of the electricity price adjustment differential game. In this case, each player needs to solve an optimal control problem when the other competitors' strategies are given. As is commonly used, Pontryagin's maximum principle can be adopted as a necessary condition to find the candidate optimal strategies. Later we will give the sufficient conditions for proving the optimality.

According to the Pontryagin's principle [16], we first give the Hamilton function of player i as:

$$H_i(\mathbf{x}, p_i, \mathbf{p}_{-i}, \boldsymbol{\lambda}_i, t) = J_i(p_i, \mathbf{p}_{-i}) + \sum_{j=1}^{M} \lambda_{ij}\dot{x}_j$$

where $\boldsymbol{\lambda}_i = [\lambda_{i1} \; \cdots \; \lambda_{ij} \; \cdots \lambda_{iM}]^T$and λ_{ij} represents the costate variable of charging station i associated with state j. Now we can define the maximum Hamilton function H^* as

$$H_i^*(\mathbf{x}, \boldsymbol{\lambda}_i, t) = \max_{p_i}\{H_i(\mathbf{x}, p_i, \mathbf{p}_{-i}, \boldsymbol{\lambda}_i, t) \mid p_i \in P_i\}$$

The optimal control theory tells us that the optimal control strategy for problem (5) should also optimize its corresponding Hamilton function given any triple $(\mathbf{x}, \boldsymbol{\lambda}_i, t)$. Also, any candidate optimal strategy should satisfy the following necessary optimality conditions [18]:

$$p_i^* = arg \max_{p_i} H_i(\mathbf{x}, p_i, \mathbf{p}_{-i}, \boldsymbol{\lambda}_i, t) \tag{6}$$

$$\dot{\lambda}_{ij} = \rho\lambda_{ij} - \frac{\partial H_i^*(\mathbf{x}, \boldsymbol{\lambda}_i, t)}{\partial x_j} \tag{7}$$

$$\dot{x}_i = \delta x_i(u_i - \bar{u}) \tag{8}$$

for all $i, j \in \{1, 2, \ldots, M\}$, where Eq. (7) is the adjoint equation to describe the dynamics of a costate variable. The transversality condition is $\lambda_{ij}(T) = 0$ for finite time interval and $\lim_{T\to\infty} \lambda_{ij}(T) = 0$ for infinite time interval. According to Eq. (6), we can get the candidate optimal strategy for charging station i by taking the first-order derivative of $H_i(\mathbf{x}, p_i, \mathbf{p}_{-i}, \boldsymbol{\lambda}_i, t)$ on p_i and setting it to 0, and then we get

$$p_i^* = \frac{Na_0}{2\delta\beta} \frac{1}{(\lambda_{ii} - \sum_{j=1}^{M} \lambda_{ij} x_j)} \tag{9}$$

Notice that if the calculated p_i^* given by Eq. (9) does not lie in the feasible price set P_i, then charging station i will choose to set a high price to declare that he quits the game. Substituting p_i^* into the Hamilton function, we can get H_i^*, and substitute H_i^* into Eq. (7), we then have the following equations:

$$\dot{\lambda}_{ii} = \rho\lambda_{ii} - p_i N a_0 - \delta\lambda_{ii} f(x_i, p_i) + \delta g(x_i, p_i) \sum_{j=1, j\neq i}^{M} \lambda_{ij} x_j \tag{10}$$

$$\dot{\lambda}_{ij} = \rho\lambda_{ij} - \lambda_{ij}\delta[f(x_j, p_j) - \sum_{k=1, k\neq j}^{M} x_k u_k] \tag{11}$$

where $f(x_i, p_i) = (1 - 2x_i)(u_0 - \beta p_i^2 - \frac{\alpha}{w_i - N a_0 x_i}) - \frac{\alpha N x_i (1 - x_i) a_0}{(w_i - N x_i a_0)^2}$ and $g(x_i, p_i) = u_o - \beta p_i^2 - \frac{\alpha}{w_i - N a_0 x_i} - \frac{\alpha N x_i a_0}{(w_j - N a_0 x_i)^2}$. We can observe that the differential equation (10) is complex and not so easy to solve since λ_{ii} is coupled with λ_{ij} while Eq. (11) is a quite conventional differential equation and is not difficult to solve. We denote the solution as $\lambda_{ij}^*, i \neq j$. Then substitute λ_{ij}^* into Eq. (10) and we get the solution of Eq. (10) as λ_{ii}^*.

The initial problem subject to differential equation become a two point boundary value problem and the variable turns to be the system state $\mathbf{x}$ and the costate $\boldsymbol{\lambda}$. Denote $\mathbf{x}^*$ and $\boldsymbol{\lambda}^*$ as the optimal solution to the above system, where $\mathbf{x}^* = [x_1^*, \cdots, x_i^*, \cdots, x_M^*]$ and $\boldsymbol{\lambda}^* = [\lambda_{ij}^* \mid \forall i, j \in \{1, 2, \cdots, M\}]$. As calculated in Eq. (9), we now can give the candidate optimal strategy as

$$p_i^* = \frac{Na_0}{2\delta\beta} \frac{1}{(\lambda_{ii}^* - \sum_{j=1}^{M} \lambda_{ij}^* x_j^*)}$$

We argue that the Pontryagin's maximum principle can result in the candidate optimal solution before and next we show the candidate solution is indeed the Nash equilibrium in the following theorem.

Theorem 2: For the electricity price adjustment differential game, the strategy profile $\Phi^* = \{p_i^*, \mathbf{p}_{-i}^*\}$ is an open-loop Nash equilibrium.

Proof: See technical report in [15]. ■

Now we come to the discussion of the closed-loop Nash equilibrium. Similarly with how we get the open-loop optimal strategy, we can just use the measured current system state $\mathbf{x}$ to replace the computed optimal state path $\mathbf{x}^*$, so a closed-loop strategy can be given by $p_{i,close}^* = \frac{Na_0}{2\delta\beta} \frac{1}{(\lambda_{ii}^* - \sum_{j=1}^{M} \lambda_{ij}^* x_j)}$. Then the closed-loop Nash equilibrium strategy profile is $\phi = \{p_i(\mathbf{x}, t), \mathbf{p}_{-i}(\mathbf{x}, t)\}$. However, the Nash equilibrium obtained in this way is not Markov perfect which means that if the states are not in the optimal path then the optimality of ϕ cannot be guaranteed. In order to get the Markov perfect Nash equilibrium [19], we can use the Hamilton Jacobi Bellman equation and interested readers can find more details related with the closed-loop Nash equilibrium in [20].

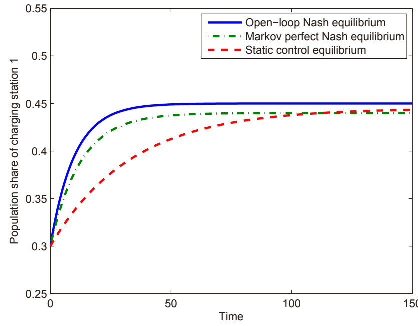

Figure 1. Population state evolution under different strategies

V. Simulation Results

A. Parameter Setting

We consider an area with $M = 2$ charging stations and $N = 50$ PEVs. We assume the PEVs will submit charging requests when the state of charging (SOC) is under 0.3 and they will require the charging station to charge their battery to $SOC = 0.9$. The charging rate of each PEV is set to be $a_0 = 1.6$ and the service amount is supposed to be $w_1 = 40$ and $w_2 = 60$. The learning rate of the replicator dynamics is set to be $\delta = 1$ and the discount rate is set to be $\rho = 0.01$. The initial population shares of the charging stations are supposed to be $x_1(0) = 0.3$ and $x_2(0) = 0.7$.

B. Numerical Results

1) Population state evolution : In this paper we formulate and analyze the open-loop Nash equilibrium (OLNE) strategy and the Markov perfect Nash equilibrium (MPNE) strategy. For comparison purpose, we also consider another strategy called static equilibrium (SE) strategy where the lower level is still the evolutionary game and similar to [21] the price of charging station i is given as $p_i = a(\frac{N x_i a_0}{w_i})^\theta$. The evolution trajectory from initial state under different control strategies is given in Fig. 1. The curves describes how the proportion of PEVS choosing charging station 1 increases from the initial state $x_1(0) = 0.3$ to the equilibrium state and $x_2 = 1 - x_1$.

2) Price adjustment: The optimal charging price adjustment is shown in Fig. 2. As the results show, the price increases with the population share increases and until the evolution comes to an equilibrium. At the beginning, the

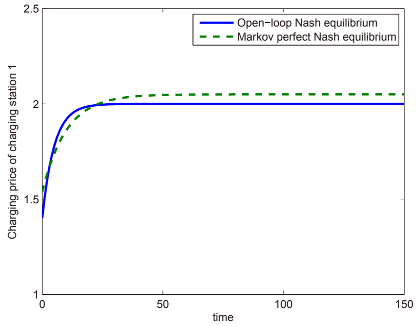

Figure 2. Optimal price adjustment of charging station 1

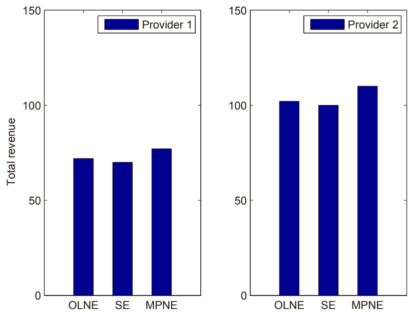

Figure 3. Comparison of total revenue

population share of charging station 1 is lower which means the delay time at charging station 1 is shorter and this will attract more PEVs. As a result, the charging station will increase its charging price to gain a higher revenue. And the increased price will cause a dissatisfaction level to the PEVs, which slows down the increasing rate of the population share until the price adjustment process reaches the equilibrium.

3) Revenue comparison: As can be seen in Fig. 3, the MPNE strategy yields the highest revenue and the SE yields the least revenue. Besides, charging station 2 can gain a higher revenue than station 1 because its maximum service amount is bigger.

VI. Conclusion

We have proposed a two-level game framework to formulate the hierarchical problem where both the charging stations and a group of PEVs want to maximize their profit. As for the PEVs, they will choose one of the charging stations according to the charging price and the delay time that they incurs at each charging station. We use an evolutionary game to model the selection behavior and the selection adaption process has been analyzed using the replicator dynamics. We also analyze the evolutionary equilibrium of this game and propose an algorithm to find the equilibrium. At the upper level, the optimal charging price is obtained based on solving the differential game in order to maximize each charging stations' revenue. The simulation results show that the proposed two-level game framework yields the improved performance gains compared to static control.

References

[1] Decker, "Who killed the electric grid? fast-charging electric cars." [Online]. Available: http://www.lowtechmagazine.com/2009/03/fast-charging-electric-cars-off-peak-grid.html

[2] S. Shao, M. Pipattanasomporn, and S. Rahman, "Demand response as a load shaping tool in an intelligent grid with electric vehicles," *Smart Grid, IEEE Transactions on*, vol. 2, no. 4, pp. 624–631, 2011.

[3] C. Wu, H. Mohsenian-Rad, and J. Huang, "Vehicle-to-aggregator interaction game," *Smart Grid, IEEE Transactions on*, vol. 3, no. 1, pp. 434–442, 2012.

[4] S. Sojoudi and S. H. Low, "Optimal charging of plug-in hybrid electric vehicles in smart grids," in *Power and Energy Society General Meeting, 2011 IEEE*. IEEE, 2011, pp. 1–6.

[5] K. Turitsyn, N. Sinitsyn, S. Backhaus, and M. Chertkov, "Robust broadcast-communication control of electric vehicle charging," in *Smart Grid Communications (SmartGridComm), 2010 First IEEE International Conference on*. IEEE, 2010, pp. 203–207.

[6] R. Couillet, S. M. Perlaza, H. Tembine, and M. Debbah, "Electrical vehicles in the smart grid: A mean field game analysis," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 6, pp. 1086–1096, 2012.

[7] Z. Yang, L. Sun, J. Chen, Q. Yang, X. Chen, K. Xing, "Profit Maximization for Plug-In Electric Taxi With Uncertain Future Electricity Prices," *Power System,IEEE Transactions on*, vol. 29,no. 6, pp.3058 - 3068,2014.

[8] W. Cao, B. Yang, C. Chen, and X. Guan, "Phev charging strategy with asymmetric information based on contract design," *Proc. the 13th IFAC Symposium on Large Scale Complex Systems: Theory and Applications*, 2013.

[9] J. Hofbauer and K. Sigmund, "Evolutionary game dynamics," *Bulletin of the American Mathematical Society*, vol. 40, no. 4, pp. 479–519, 2003.

[10] P. D. Taylor and L. B. Jonker, "Evolutionary stable strategies and game dynamics," *Mathematical biosciences*, vol. 40, no. 1, pp. 145–156, 1978.

[11] Z. Yang, L. Sun, M. Ke,Z. Shi,J. Chen, " Optimal Charging Strategy for Plug-In Electric Taxi With Time-Varying Profits, " *Smart Grid, IEEE Transactions on*, vol. 5,no. 6, pp.2787 - 2797,2014.

[12] D. Friedman, "On economic applications of evolutionary game theory," *Journal of Evolutionary Economics*, vol. 8, no. 1, pp. 15–43, 1998.

[13] J. M. Smith, *Evolution and the Theory of Games*. Springer, 1993.

[14] R. Deng, Z. Yang, J. Chen, and M. Chow, "Load scheduling with price uncertainty and temporally-coupled constraints in smart grids," *Power Syst., IEEE Transactions on*, no. 99, Mar. 2014.

[15] J. Chen, B. Yang, C. Chen, and X. Guan, "Charging station selection and charging price decision for phev: A two level game approach." [Online]. Available: Tech. Rep. http://pan.baidu.com/s/1bn1wvSF

[16] L. Rozonoer, "Ls pontryagin maximum principle in the theory of optimum systems. i, ii, iii," *Automat. Remote Control*, vol. 20, pp. 1288–1302, 1959.

[17] J. Yue,B. Yang,C. Chen, and X. Guan, "Chasing the Most Popular Video: An Evolutionary Video Clip Selection, " *Communications Letters, IEEE*, vol. 18,no. 5,pp.781-784,Mar. 2014.

[18] G. M. Torres and L. G. Esparza, "A brief introduction to differential games," *International Journal of Physical and Mathematical Sciences*, vol. 4, no. 1, 2013.

[19] E. Maskin and J. Tirole, "Markov perfect equilibrium: I. observable actions," *Journal of Economic Theory*, vol. 100, no. 2, pp. 191–219, 2001.

[20] K. Zhu, D. Niyato, P. Wang, and Z. Han, "Dynamic spectrum leasing and service selection in spectrum secondary market of cognitive radio networks," *Wireless Communications, IEEE Transactions on*, vol. 11, no. 3, pp. 1136–1145, 2012.

[21] Z. Fan, "A distributed demand response algorithm and its application to phev charging in smart grids," *Smart Grid, IEEE Transactions on*, vol. 3, no. 3, pp. 1280–1290, 2012.