Robust Optimal Sizing & Energy Management for Solar + Storage + EV Chargers

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In this note, we formulate a mixed integer second-order cone program (MISOCP) for optimal sizing and energy management of a facility with rooftop solar PV, stationary energy storage, and EV charging poles. The objective involves two components. First, optimally size solar panels, energy storage, and EV chargers. This reflects *capital costs*. Second, optimally manage power flow through the facility's electrical network. This reflects *operating costs*. The unique and important features of this formulation include:

1. explicitly accounts for uncertainty in solar production, local electricity consumption, and EV charging demand based on statistical data, thus ensuring robust solutions.

2. incorporates electric vehicle supply equipment (EVSEs) into the formulation, including the size (L2 & L2) and quantity of charging piles.

3. allows for multiple electricity tariffs, e.g. time-of-use (TOU) rates, tariffs with demand charges, or flat rates.

4. allows for multiple revenue generating export arrangements, e.g. net energy metering (NEM) or community choice aggregation (CCA).

5. allows for revenue generated from EV charging.

6. incorporates grid import/export constraints due to grid-tied transformer capacity.

7. provides a mathematically elegant yet comprehensive optimization formulation.

1 Problem Statement

We consider a facility with rooftop solar PV, stationary energy storage, EV charging poles, and a grid connection supported by a transformer. The combined optimal sizing and energy management problem can be formulated as:

$$
\begin{array}{lll}
\text{minimize} & [c_b \cdot b + c_s \cdot s + c_2 \cdot n_2 + c_3 \cdot n_3] \cdots & \text{CAPEX} \\
& + \sum_{k=0}^{N} [c_I(k) G_I(k)] \,\Delta t + c_D G_D & \text{OPEX-costs} \\
& - \sum_{k=0}^{N} [r_E(k) G_E(k) + r_2 C_2(k) + r_3 C_3(k)] \,\Delta t & \text{OPEX-revenues} \quad (1) \\
\text{subject to:} & s \cdot S(k) - S_c(k) + B_d(k) + G_I(k) \cdots & \text{Supply} \\
& = L(k) + B_c(k) + C_2(k) + C_3(k) + G_E(k) & \text{Consumption} \quad (2) \\
\hline
& 0 \leq S_c(k) & \text{Curtail PV} \quad (3) \\
\hline
& E(k+1) = E(k) + \left[\eta_c B_c(k) - \frac{1}{\eta_d} B_d(k) \right] \Delta t & \text{ESS dynamics} \quad (4) \\
& E(0) = E_0 & \text{ESS energy IC} \quad (5) \\
& 0 \leq E(k) \leq b \cdot E_{\max} & \text{ESS energy limits} \quad (6) \\
& 0 \leq B_c(k) \leq b \cdot B_{\max}, \quad 0 \leq B_d(k) \leq b \cdot B_{\max} & \text{ESS power limits} \quad (7) \\
\hline
& Z(k+1) = Z(k) + [\eta_2 C_2(k) + \eta_3 C_3(k)] \,\Delta t & \text{Delivered EV energy dyn} \quad (8) \\
& Z(0) = 0, & \text{EV energy IC} \quad (9) \\
& Z_{\min}(k) \leq Z(k), \quad \forall\, k & \text{EV energy lower bound} \quad (10) \\
& 0 \leq C_2(k) \leq n_2 C_{2,\max}, \ 0 \leq C_3(k) \leq n_3 C_{3,\max} & \text{EVSE power limits} \quad (11) \\
\hline
& 0 \leq G_I(k) \leq G_{I,\max}, \quad 0 \leq G_E(k) \leq G_{E,\max} & \text{import, export limit} \quad (12) \\
& G_I(k) \leq G_D & \text{demand charge} \quad (13) \\
\hline
& s_{\min} \leq s \leq s_{\max}, \quad b_{\min} \leq b \leq b_{\max} & \text{solar, ESS scale limits} \quad (14) \\
& n_2, n_3 \in \mathbb{Z}, \ 0 \leq n_2 \leq n_{2,\max}, \ 0 \leq n_3 \leq n_{3,\max} & \text{no. of L2, L3 EVSEs} \quad (15)
\end{array}
$$

The notation is defined in Table 1. Optimization variables are color-coded in blue. Notice that (1)-(15) is a mixed integer linear program (MILP) with respect to the optimization variables. The integer variables are n_2, n_3. Optionally, variables $b, s, C_2(k), C_3(k)$ can be integer as well, depending on practical considerations. For clarity, we highlight that $L(k), S(k), Z_{\min}(k)$ are NOT optimization variables, as the environment imposes these inputs on our system. That is, electricity consumption $L(k)$ and solar generation $S(k)$ cannot be predicted perfectly nor controlled. They are uncertain. Symbol $Z_{\min}(k)$ represents the absolute minimum energy that must be delivered to the EVs. This is uncertain as well, since it depends on vehicle mobility patterns.

Table 1: Notation

Variable	**Units**	**Description**
s, b	[-]	Scale factors for solar and ESS sizes
$G_I(k), G_E(k)$	[kW]	Power imported and exported from/to grid
G_D	[kW]	Maximum power for demand charge
$S(k)$	[kW]	(r.v.) Power generated from solar
$S_c(k)$	[kW]	Curtailed solar power
$B_d(k), B_c(k)$	[kW]	Power discharged from / charged into battery
$C_2(k), C_3(k)$	[kW]	Total charging load of L2, L3 EVSEs
$L(k)$	[kW]	(r.v.) Power load of facility
$E(k)$	[kWh]	Energy level of ESS
$Z(k)$	[kWh]	Cumulative energy delivered to EVs
$Z_{\min}(k)$	[kWh]	(r.v.) Minimum delivered energy to EVs
c_b, c_s	[USD]	Marginal cost of scaling battery, solar size
$c_I(k)$	[USD/kW]	Time-of-use price of grid-imported power
$r_E(k)$	[USD/kW]	Time-of-use revenue of exported power to grid
r_2, r_3	[USD/kW]	Revenue for L2, L3 EV charging (can be TOU)
c_D	[USD/kW]	Demand charge
Δt	[hr]	Time step
η_c, η_d	[-]	ESS charge, discharge efficiency $\in [0, 1]$
E_0	[kWh]	Initial ESS energy level
$E_{\max}$	[kWh]	Nominal ESS energy capacity
$B_{\max}$	[kW]	Nominal ESS power capacity
η_2, η_3	[-]	EVSE charge efficiency $\in [0, 1]$
$C_{2,\max}, C_{3,\max}$	[kW]	L2, L3 EVSE power capacity
$G_{I,\max}, G_{E,\max}$	[kW]	Maximum grid power (import & export)
$s_{\min}, s_{\max}$	[-]	Solar scale limits
$b_{\min}, b_{\max}$	[-]	Battery scale limits
$n_{2,\max}, n_{3,\max}$	[-]	Max number of L2, L3 charging poles
α_Z	[-]	Prob of satisfying EV energy limits
α_{G1}, α_{G2}	[-]	Prob of satisfying grid import limits

2 Robust Program Reformulation

2.1 Statistics on Random Variables

The red variables denote *random variables*: solar generation $S(k)$, load $L(k)$, and the EV energy lower bound $\underline{E}(k)$. We assume that $S(k)$ and $L(k)$ have independent Gaussian distributions:

$$S(k) \sim \mathcal{N}\left(\overline{S}(k), \sigma_S^2(k)\right), \quad \forall\ k \tag{16}$$

$$L(k) \sim \mathcal{N}\left(\overline{L}(k), \sigma_L^2(k)\right), \quad \forall\ k \tag{17}$$

Additionally, we consider that random variables $Z_{\min}(k)$ have known cumulative distribution functions (which need not be normally distributed):

$$Z_{\min}(k) \sim \text{cdf: } F_{Z_{\min}(k)}(z) = \Pr\left(Z_{\min}(k) \leq z\right), \quad \forall\ k \tag{18}$$

2.2 Chance Constraints

2.2.1 EV energy lower bound

Next, we focus on (10) which includes random variables $Z_{\min}(k)$. Instead of requiring that (10) hold with probability one (i.e., for all possible realizations of $Z_{\min}(k)$), we require that each constraint holds with a probability (i.e. confidence or reliability) α_Z. For example,

$$\Pr\left(Z_{\min}(k) \leq Z(k)\right) \geq \alpha_Z \tag{19}$$

We now re-express this constraint using the cdf:

$$F_{Z_{\min}(k)}(Z(k)) \geq \alpha_Z \tag{20}$$

Now, assume the cdf is invertible over its domain. Then we have

$$F^{-1}_{Z_{\min}(k)}(\alpha_Z) \geq Z(k) \tag{21}$$

Observe that the red random variables $Z_{\min}(k)$ have been re-formulated out of the EV energy lower limit constraint. All that remains are their cdfs $F_{Z_{\min}(k)}$ and reliability parameter α_Z. Next we pursue a similar process for grid power limits.

2.2.2 Grid Power Limits

Next, we focus on (12) which models the grid import and export limits. As a preliminary step, let us solve the power balance equality constraint (2) for grid import power

$$G_I(k) = L(k) + B_c(k) + C_2(k) + C_3(k) + G_E(k) - s \cdot S(k) + S_c(k) - B_d(k) \tag{22}$$

Substituting this expression into the lower and upper bounds on $G_I(k)$ in (12) yields:

$$0 \leq L(k) + B_c(k) + C_2(k) + C_3(k) + G_E(k) - s \cdot S(k) + S_c(k) - B_d(k) \leq G_{I,\max} \tag{23}$$

Instead of requiring that (23) hold with probability one (i.e., for all possible realizations of $L(k), S(k)$), we require that each constraint holds with a probability (i.e. confidence or reliability) α. For example, the upper bound becomes:

$$\Pr\left(L(k) + B_c(k) + C_2(k) + C_3(k) + G_E(k) - s \cdot S(k) + S_c(k) - B_d(k) \leq G_{I,\max}\right) \geq \alpha_{G2} \tag{24}$$

We now re-express this constraint as a second order cone constraint. Let $u = L(k) - s \cdot S(k)$ with mean $\overline{u} = \overline{L}(k) - s \cdot \overline{S}(k)$ and variance $\sigma_u^2 = \sigma_L^2(k) + s^2\sigma_S^2(k)$ (note, we use properties of adding, subtracting, and scaling Gaussian random variables [1]). This constraint can be re-written as

$$\Pr\left(\frac{u - \overline{u}}{\sigma_u} \leq \frac{G_{I,\max} - B_c(k) - C_2(k) - C_3(k) - G_E(k) - S_c(k) + B_d(k) - \overline{u}}{\sigma_u}\right) \geq \alpha_{G2} \tag{25}$$

Since $(u - \overline{u})/\sigma_u$ is a zero mean, unit variance Gaussian random variable, the probability in (25) is given by the normal CDF function

$$\Phi\left(\frac{G_{I,\max} - B_c(k) - C_2(k) - C_3(k) - G_E(k) - S_c(k) + B_d(k) - \overline{u}}{\sigma_u}\right) \geq \alpha_{G2} \tag{26}$$

where $\Phi(z) = \frac{-1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2} dt$. Consequently, (25) can be expressed as

$$\frac{G_{I,\max} - B_c(k) - C_2(k) - C_3(k) - G_E(k) - S_c(k) + B_d(k) - \overline{u}}{\sigma_u} \geq \Phi^{-1}(\alpha_{G2}) \tag{27}$$

or equivalently,

$$\overline{u} + \Phi^{-1}(\alpha_{G2}) \cdot \sigma_u \leq G_{I,\max} - B_c(k) - C_2(k) - C_3(k) - G_E(k) - S_c(k) + B_d(k) \tag{28}$$

Replacing mean $\overline{u} = \overline{L}(k) - s \cdot \overline{S}(k)$ and variance $\sigma_u^2 = \sigma_L^2(k) + s^2\sigma_S^2(k)$ we get

$$\begin{aligned} & \overline{L}(k) - s \cdot \overline{S}(k) + \Phi^{-1}(\alpha_1) \cdot \sqrt{\sigma_L^2(k) + s^2\sigma_S^2(k)} \\ \leq \quad & G_{I,\max} - B_c(k) - C_2(k) - C_3(k) - G_E(k) - S_c(k) + B_d(k) \end{aligned} \tag{29}$$

Rearranging this expression and assuming $\Phi^{-1}(\alpha_{G2}) > 0$ (i.e. $\alpha_{G2} > 0.5$), we get a second order cone constraint with respect to optimization variable s:

$$\begin{aligned} & \sqrt{\sigma_S^2(k) \cdot s^2 + \sigma_L^2(k)} \leq \\ & \frac{1}{\Phi^{-1}(\alpha_{G2})}\left[s \cdot \overline{S}(k) - \overline{L}(k) + G_{I,\max} - B_c(k) - C_2(k) - C_3(k) - G_E(k) - S_c(k) + B_d(k)\right] \end{aligned} \tag{30}$$

An identical process applied to lower bound

$$\Pr\left(0 \leq L(k) + B_c(k) + C_2(k) + C_3(k) + G_E(k) - s \cdot S(k) + S_c(k) - B_d(k)\right) \geq \alpha_{G1} \tag{31}$$

yields second order cone constraint

$$\sqrt{\sigma_S^2(k) \cdot s^2 + \sigma_L^2(k)} \leq \frac{1}{\Phi^{-1}(\alpha_{G1})} \left[\overline{L}(k) - s \cdot \overline{S}(k) + B_c(k) + C_2(k) + C_3(k) + G_E(k) + S_c(k) - B_d(k) \right] \quad (32)$$

Observe that all the red random variables have been re-formulated out of the imported grid power limit constraints. All that remains are their statistical parameters $\bar{S}(k), \bar{L}(k)$ and $\sigma_S^2(k), \sigma_L^2(k)$ and reliability parameters α_{G1}, α_{G2}.

2.3 Expected Cost

We now turn our attention to random variables that will appear in the objective function. As before, consider solving the power balance equality constraint (2) for grid import power $G_I(k)$. Substituting this expression into the imported energy cost term in the OPEX portion of the objective function (1) yields:

$$c_I(k) G_I(k) = c_I(k) \left[L(k) + B_c(k) + C_2(k) + C_3(k) + G_E(k) - s \cdot S(k) + S_c(k) - B_d(k) \right] \quad (33)$$

Since $L(k), S(k)$ are random, our imported energy cost will also be random and the objective function is therefore random. Minimizing a random number does not make mathematical sense. Consequently, we consider minimizing the *expected* imported energy costs:

$$\mathbb{E} \left[c_I(k) G_I(k) \right] = c_I(k) \left[\overline{L}(k) + B_c(k) + C_2(k) + C_3(k) + G_E(k) - s \cdot \overline{S}(k) + S_c(k) - B_d(k) \right] \quad (34)$$

2.4 Mixed Integer Second Order Cone Program

This arrives at our final MISOCP, which is now ready for implementation into `CPLEX`:

$$
\begin{aligned}
\text{minimize} \quad & [c_b \cdot b + c_s \cdot s + c_2 \cdot n_2 + c_3 \cdot n_3] \cdots && \text{CAPEX} \\
& + \sum_{k=0}^{N} c_I(k) \left[\overline{L}(k) + B_c(k) + C_2(k) + C_3(k) \right. \\
& \qquad \left. + G_E(k) - s \cdot \overline{S}(k) + S_c(k) - B_d(k) \right] \Delta t && \text{Energy Costs} \\
& + c_D G_D && \text{Demand charges} \\
& - \sum_{k=0}^{N} \left[c_E(k) G_E(k) + r_2 C_2(k) + r_3 C_3(k) \right] \Delta t && \text{Export Revenue} \quad (35) \\
\text{subject to:} \quad & 0 \leq S_c(k) && \text{Curtail PV} \quad (36) \\
& E(k+1) = E(k) + \left[\eta_c B_c(k) - \frac{1}{\eta_d} B_d(k) \right] \Delta t && \text{ESS dynamics} \quad (37) \\
& E(0) = E_0 && \text{ESS energy IC} \quad (38) \\
& 0 \leq E(k) \leq b \cdot E_{\max} && \text{ESS energy limits} \quad (39) \\
& 0 \leq B_c(k) \leq b \cdot B_{\max}, \quad 0 \leq B_d(k) \leq b \cdot B_{\max} && \text{ESS power limits} \quad (40) \\
& Z(k+1) = Z(k) + \left[\eta_2 C_2(k) + \eta_3 C_3(k) \right] \Delta t && \text{Delivered EV energy dyn} \quad (41) \\
& Z(0) = 0, && \text{EV energy IC} \quad (42) \\
& F^{-1}_{Z_{\min}(k)}(\alpha_Z) \geq Z(k), \forall\, k && \text{EV energy lower bound} \quad (43) \\
& 0 \leq C_2(k) \leq n_2 C_{2,\max}, \quad 0 \leq C_3(k) \leq n_3 C_{3,\max} && \text{EVSE power limits} \quad (44) \\
& \sqrt{\sigma_S^2(k) \cdot s^2 + \sigma_L^2(k)} \leq \\
& \frac{1}{\Phi^{-1}(\alpha_{G1})} \left[\overline{L}(k) - s \cdot \overline{S}(k) + B_c(k) + C_2(k) \right. \\
& \left. + C_3(k) + G_E(k) + S_c(k) - B_d(k) \right] && \text{min imported power} \quad (45) \\
& \sqrt{\sigma_S^2(k) \cdot s^2 + \sigma_L^2(k)} \leq \\
& \frac{1}{\Phi^{-1}(\alpha_{G2})} \left[s \cdot \overline{S}(k) - \overline{L}(k) + G_{I,\max} - B_c(k) \right. \\
& \left. - C_2(k) - C_3(k) - G_E(k) - S_c(k) + B_d(k) \right] && \text{max imported power} \quad (46) \\
& 0 \leq G_E(k) \leq G_{E,\max} && \text{grid export power limit} \quad (47) \\
& G_I(k) \leq G_D && \text{demand charge} \quad (48) \\
& s_{\min} \leq s \leq s_{\max}, \quad b_{\min} \leq b \leq b_{\max} && \text{solar, ESS scale limits} \quad (49) \\
& n_2, n_3 \in \mathbb{Z}, \ 0 \leq n_2 \leq n_{2,\max}, \ 0 \leq n_3 \leq n_{3,\max} && \text{no. of L2, L3 EVSEs} \quad (50)
\end{aligned}
$$

Remark 1 Intuitively, one seeks to avoid $B_c(k), B_d(k)$ simultaneously being non-zero. In words, this prevents charging and discharging in the same time period. Although not shown here, one can prove the optimal solution to (33)-(39) satisfies this property, without enforcing it explicitly. To prove this, one can use monotonicity analysis [2] and exploit the fact that (33) and (34) are linear with respect to $B_c(k), B_d(k)$. In fact, this makes intuitive sense because $B_c(k) > 0, B_d(k) > 0$ would unnecessarily waste energy due to round-trip power conversion losses in (34), thereby inflating the operating cost.

3 Applications

This optimization model can be used for a variety of purposes, including

- Determining the optimal PV size, ESS size, and number of charging poles.
- Ensuring infrastructure planning robustness due to environmental uncertainty.
- Quantifying the cost, energy, and emissions savings relative to a baseline.
- Selecting the best tariff structure amongst a menu of options
- Selecting the best electricity export structure amongst a menu of options
- Selecting EV charging rates

4 Extensions

Several extensions are possible to integrate more practical features. They include:

- Vehicle-to-grid (V2G), i.e. considering bi-directional EVSEs
- Other local generators, e.g. fuel cells, wind turbines, and engines.
- Smart electric water heaters, smart thermostats, and/or other smart appliances
- Risk-based optimization, i.e. minimizing a weighted sum of expected operating costs and variance of operating costs

This technical note describes a *planning tool.* In other words, its purpose is for planning infrastructure and a separate tool is required for real-time controls.

References

[1] J. A. Gubner, *Probability and random processes for electrical and computer engineers.* Cambridge University Press, 2006.

[2] P. Y. Papalambros and D. J. Wilde, *Principles of Optimal Design: Modeling and Computation.* Cambridge University Press, 2000.