Rough ideas for talk

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Overarching idea: give an example with $K = \mathbb{Q}(i)$, sketching the key ideas behind the proof. Run through an example of computing $\lim_{s\to 1^+}(s-1)\zeta_{\mathbb{Q}(i)}(s)$. Todo:

1. Introduce the $\zeta_{\mathbb{Q}(i)}(s) = \sum_{0\neq I\subseteq\mathbb{Z}[i]} \frac{1}{[\mathbb{Z}[i]:I]^s} = \sum_{a\geq 0, b>0} \frac{1}{(a^2+b^2)^{s/2}}$.
2. Rewrite $\zeta_{\mathbb{Q}(i)}$ in terms of point counting over a cone in $\mathbb{C}$.
3. Bring in theorem 3 (in weak generality) and sketch proof.
4. Compute the volumes v and Δ (quite straightforward with $\mathbb{Z}[i] \hookrightarrow \mathbb{Q}(i) \hookrightarrow \mathbb{C}$) to show
$$\lim_{s\to 1^+}(s-1)\zeta_{\mathbb{Q}(i)}(s) = \frac{\pi}{2}$$
5. Talk a little bit about the generalisation