Rough ideas for talk

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May 13, 2023

Overarching idea: give an example with $K = \mathbb{Q}(i)$, sketching the key ideas behind the proof. Run through an example of computing $\lim_{s\to 1^+}(s-1)\zeta_{\mathbb{Q}(i)}(s)$. Todo:

1. Introduce the $\zeta_{\mathbb{Q}(i)}(s) = \sum_{0\neq I\subseteq \mathbb{Z}[i]} \frac{1}{[\mathbb{Z}[i]:I]^s} = \sum_{a\geq 0, b>0} \frac{1}{(a^2+b^2)^{s/2}}$.
2. Rewrite $\zeta_{\mathbb{Q}(i)}$ in terms of point counting over a cone in $\mathbb{C}$.
3. Bring in theorem 3 (in weak generality) and sketch proof.
4. Compute the volumes v and Δ (quite straightforward with $\mathbb{Z}[i] \hookrightarrow \mathbb{Q}(i) \hookrightarrow \mathbb{C}$) to show
$$\lim_{s\to 1^+}(s-1)\zeta_{\mathbb{Q}(i)}(s) = \frac{\pi}{4}$$
5. Talk a little bit about the generalisation – residues of arbitrary Dedekind zeta functions.

1 Introduction

The key idea: introduce $\zeta_{\mathbb{Q}(i)}(s)$, reason about convergence for $s > 1$ (hence $\mathrm{Re}(s) > 1$), compute $\mathrm{Res}_{s=1}\,\zeta_{\mathbb{Q}(i)}$ [and outline procedure that generalises with some slightly more annoying parts]

2 Point-counting

Rewrite $\zeta_{\mathbb{Q}(i)}$ to rely on point-counting over some cone; introduce this cone informally

3 Theorem 3

Statement:

4 Volume computation

5 Generalisation