Rough ideas for talk

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Most likely – introduce the background, give intuition for the proof (and sketch some of the other structure results in ANT?). [[Sketch overarching ideas?]]

Overview: Let K be a fixed number field [finite extension of $\mathbb{Q}$] of degree $[K : \mathbb{Q}] = n = r + 2s$ (r the number of embeddings $K \hookrightarrow \mathbb{R}$, s the number of conjugate pairs of embeddings $K \hookrightarrow \mathbb{C}$) – Talk a bit about how these embedding numbers are well defined (separability implies $K = \mathbb{Q}(\alpha)$, and look at $f^{\alpha}_{\mathbb{Q}} \in \mathbb{Q}[x]$)

1. *Background:*
   (a) Number field objects and constants
      i. Ring of integers [[Integral closure?]]; properties (unique prime ideal factorisation)
      ii. Ideal group (fractional ideals; every ideal is "invertible"); the ideal norm (reasonable notion of "size"); ideal class group, the class number (finiteness – "every ideal class contains an integral ideal of bounded norm")
      iii. Unit group (structure theorem and regulator)
      iv. Discriminant (description with factorisation – getting less primes than expected)
   (b) Dedekind zeta function – Motivate as a generalisation of Riemann zeta.