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Scientific Computing 372
Exercises 5: Numerical differentiation and integration

1. (a) Given the data,

x	1.5	1.9	2.1	2.4	2.6	3.1
$f(x)$	1.0628	1.3961	1.5432	1.7349	1.8423	2.0397

compute $f'(2)$ and $f''(2)$ using

i. polynomial interpolation over three nearest-neighbour points, and
ii. the natural cubic spline interpolant spanning all these data points.

(b) Determine $f'(0)$ and $f'(1)$ from the following noisy data:

x	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$f(x)$	1.9934	2.1465	2.2129	2.1790	2.0683	1.98448	1.7655	1.5891

i. First, determine the best polynomial fit in the least-squares sense; then write down an equation for the derivative, and substitute for the required x.
ii. Plot the interpolating function together with the set of original data points.

(c) Use polynomial interpolation to compute f' and f'' at $x = 0$, using the following data:

x	−2.2	−0.3	0.8	1.9
$f(x)$	15.180	10.962	1.920	−2.040

Given that $f(x) = x^3 - 0.3x^2 - 8.56x + 8.448$, gauge the accuracy of the result.

2. For parts (a) to (c), also solve the integrals by using Scientific Python. In particular, consider how to to use `scipy.integrate.quad`.

(a) Determine

$$\int_1^\infty \frac{dx}{1+x^4},$$

with the trapezoidal rule using five panels and compare the result with the “exact” integral 0.24375. *Hint:* Use the transformation $x = 1/t$.

(b) Evaluate the integral

$$\int_0^\infty \frac{x\,dx}{e^x+1}$$

by Gauss–Legendre quadrature to six decimal places. *Hint:* Substitute $e^x = \ln(1/t)$.

(c) Evaluate

$$\int_0^{\pi/4} \frac{dx}{\sqrt{\sin x}}$$

with Romberg integration. *Hint:* Use the transformation $\sin x = t^2$.

(d) A power spike in an electric circuit results in the current

$$i(t) = i_0 e^{-t/t_0} \sin\frac{2t}{t_0}$$

across a resistor. The energy E dissipated by the resistor is

$$E = \int_0^\infty R[i(t)]^2\,dt.$$

Find E using the data $i_0 = 100$ A, $R = 0.5\,\Omega$, and $t_0 = 0.01$ s.

(e) Determine how many nodes are required to evaluate

$$\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$$

with Gauss–Legendre quadrature to six decimal places. The exact integral, round to six decimal places, is 1.41815.