HPCA 2025 Tutorial

Topic 4. QuFEM: Fast and Accurate Quantum Readout Calibration Using the Finite Element Method

JanusQ Cloud

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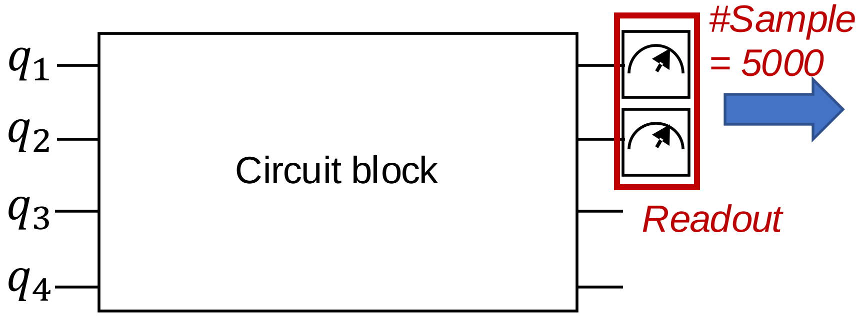
https://janusq.github.io/HPCA_2025_Tutorial/

Outline of Presentation

- **Background and challenges**
- Overview of QuFEM
- QuFEM characterization and calibration
- Experiment
- API of QuFEM

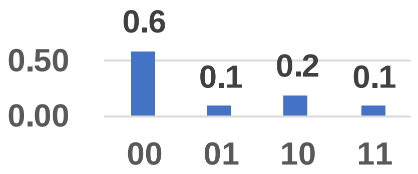
Background

Quantum readout is an operation to **read the information from quantum bits to classical bits**.



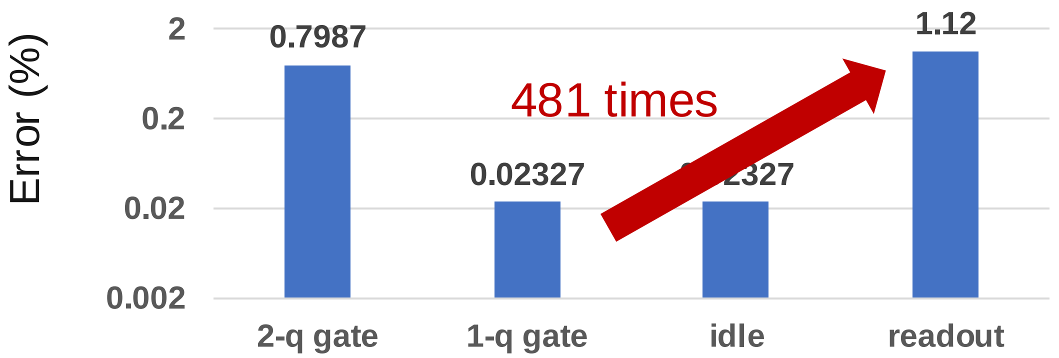
sample	1	2	3	4	4	…	4999	5000
q_1	0	1	1	1	0	…	1	0
q_2	1	0	1	1	1		1	1

Measured bitstrings in different samples

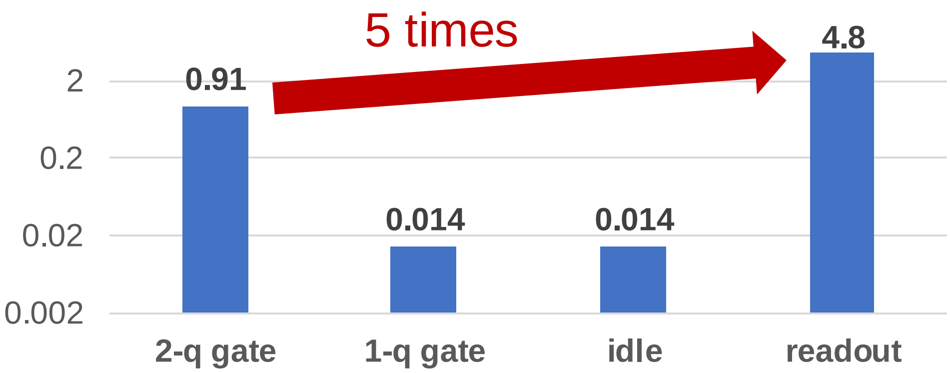


Probability distribution

Readout error is significant on current quantum hardware.



Noise on 127-qubit IBM Sherbrooke quantum device

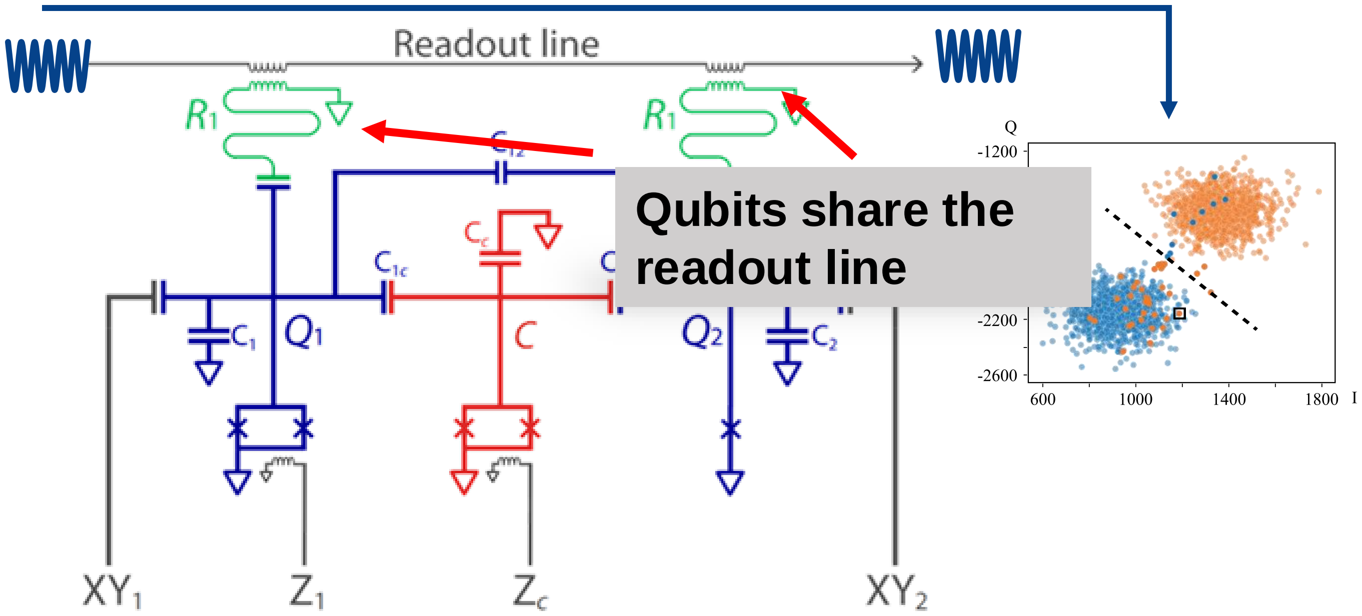


Noise on 10-qubit Tianmu quantum device

Background

Implementation of readout on superconducting qubits

(1) Pulse in *(2) Pulse takes the information out.* *FFT*



e.g. Das, et al. JigSaw: Boosting Fidelity of NISQ Programs via Measurement Subsetting. MICRO 2021

Source of readout error

from 1 to 0

Relaxation error

from 0 to 1

Excitation error

from 1/0 to 0/1

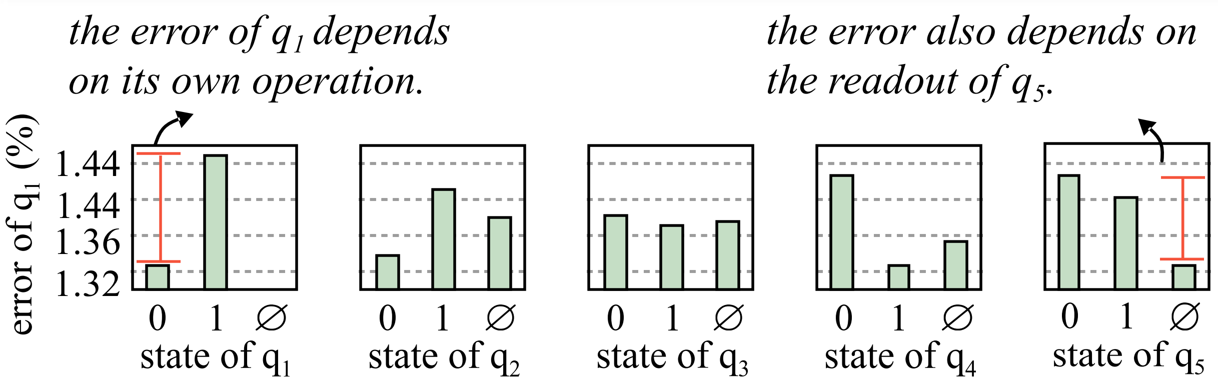
Crosstalk

State-dependent Readout Error

Readout errors vary in different combinations of measured qubits due to crosstalk.

Crosstalk has different frequencies when Q2 is measured 0, 1 or not measured

Example of state-dependent and readoutdependent noises on the IBMQ Perth quantum device.

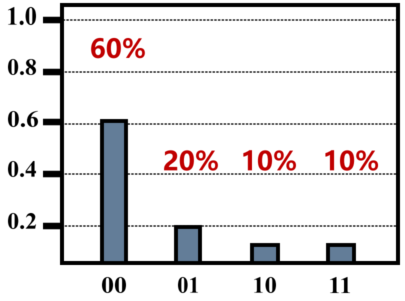


Similar to entanglement

The readout output of qubits has correlations similar to the entanglement, making the calibration difficult.

Basic Matrix-based readout calibration

Ideal readout

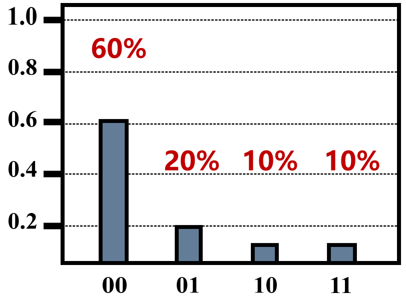


State vector

$$\begin{bmatrix} 0.6 \\ 0.2 \\ 0.1 \\ 0.1 \end{bmatrix}$$

Ideal distribution
(ideal program output)

Readout with noise

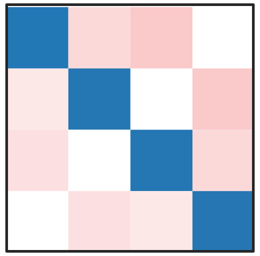


State vector

$$\begin{bmatrix} 0.5 \\ 0.1 \\ 0.09 \\ 0.31 \end{bmatrix}$$

Noisy distribution
(noisy program output)

Matrix-based readout error calibration

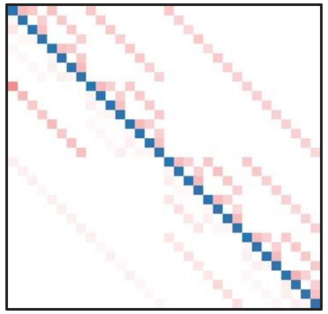


$$\left[\text{Noise matrix}\right]^{-1} \cdot \begin{bmatrix} 0.5 \\ 0.1 \\ 0.09 \\ 0.31 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.2 \\ 0.1 \\ 0.1 \end{bmatrix}$$

Noise matrix

Noisy distribution

Calibrated distribution



Calibration matrix of a 5-qubit readout

The size exponentially increases!

$2^5 \times 2^5$

Basic Matrix-based readout calibration

Step 1. Matrix characterization

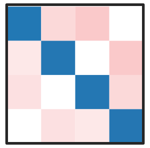
Prepares qubits to different basis states and apply measurement.

Fill in a noise matrix.

$$M = \begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0 & \boxed{0.7} & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}$$

Inverse the noise matrix

$$M^{-1} =$$

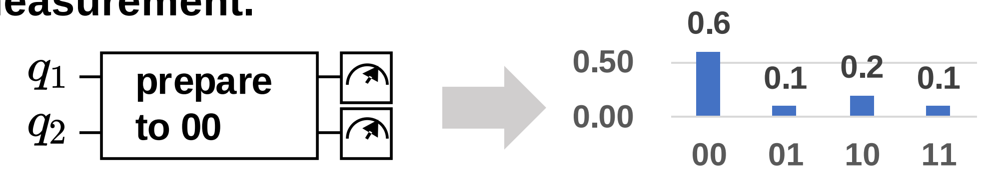


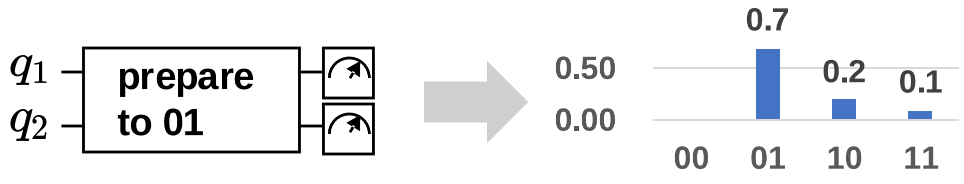
Calibration matrix

Basic Matrix-based Readout Calibration

Step 1. Matrix characterization

Prepares qubits to different basis states and apply measurement.



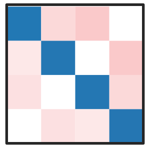


Fill in a noise matrix.

$$\mathrm{M} = \begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0 & 0.7 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}$$

Inverse the noise matrix

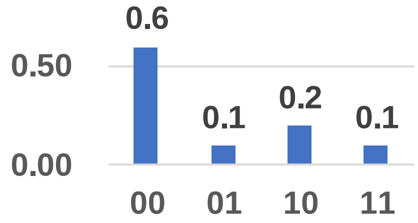
$M^{-1} =$



Calibration matrix

Step 2. Calibration for any input

Represent the measured distribution as a vector.

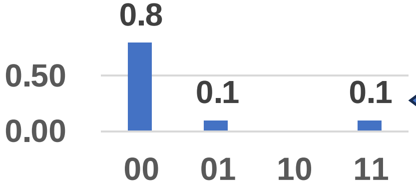


$$\begin{bmatrix} 0.6 \\ 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}$$

Apply matrix-vector multiplication.

$$\begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0 & 0.7 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0.6 \\ 0.1 \\ 0.2 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

$$M^{-1} \cdot P_{\text{read}} = P_{\text{calibrated}}$$



Complexity Analysis

Step 1. Matrix characterization

Prepares qubits to different basis states and apply measurement.

2^N circuits are executed to measure qubits on all basis states.

Fill in a noise matrix.

The size of the noise matrix is $2^N \times 2^N$.

Inverse the noise matrix

Calcauting the inverse has $\mathcal{O}(4^N)$ complexity.

Step 2. Calibration for any input

Represent the measured distribution as a vector.

The transformation has linear complexity.

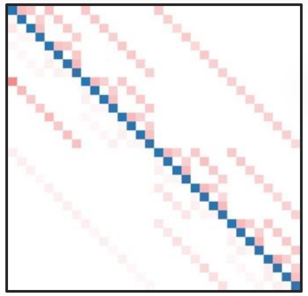
Apply matrix-vector multiplication.

The multiplication has $\mathcal{O}(4^N)$ complexity.

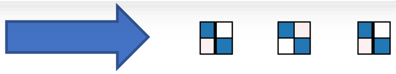
8.8 TB and 10 hours for a 32-qubit calibration on a server with AMD EPYC 2.25GHz 64-core CPUs

Limitations of Current Methods

IBU (Google Science 2021) Realizing topologically ordered states on a quantum processor.

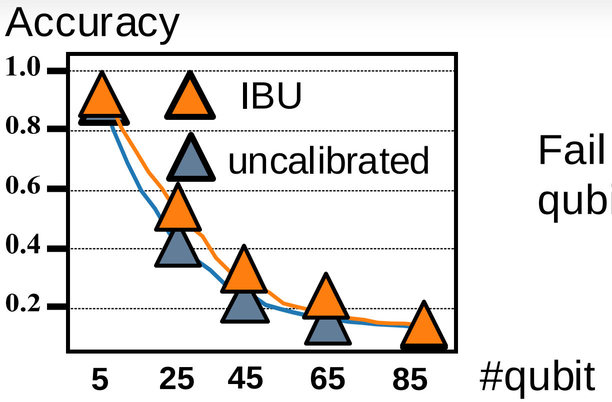


Crosstalk makes the matrix not simple tensor-product result.



Real calibration matrix   Single-qubit matrix

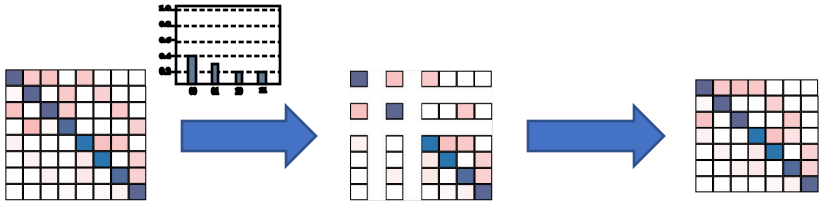
Use tensor product of a series of single-qubit meta-matrices



Fail to calibrate on 80-qubit readout output

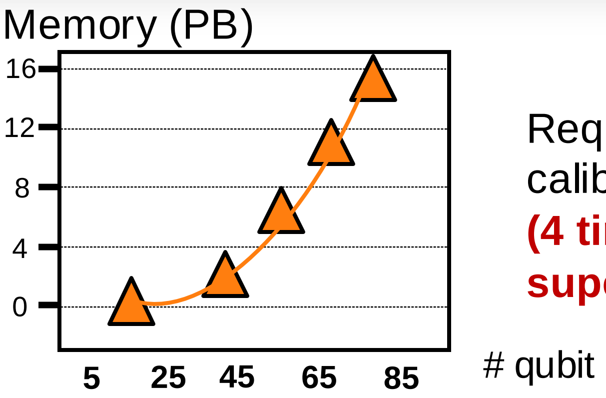
Fast but not accurate: ignore the qubit interactions.

M3 (IBM PRA 2021): Scalable mitigation of measurement errors on quantum computers



Before pruning   Pruning based on program output   After pruning

Use a sparsity-aware method to prune on the matrix under a threshold of Hamming distance



Require 16PB to calibrate a 85-qubit result. **(4 times the Fugaku supercomputer)**

Accurate but not fast: many matrix elements cannot be ignored

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- **Overview of QuFEM**
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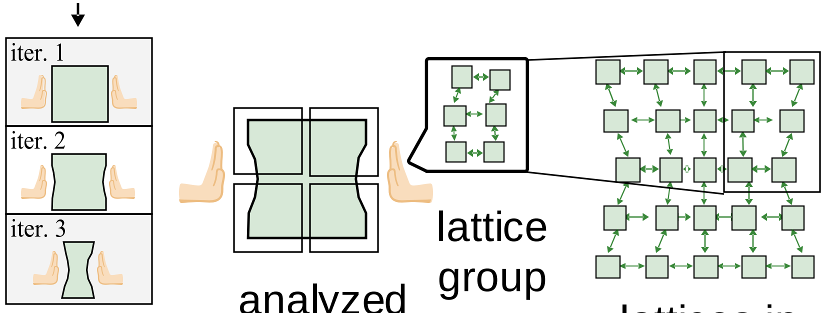
- Experiment
- API of QuFEM

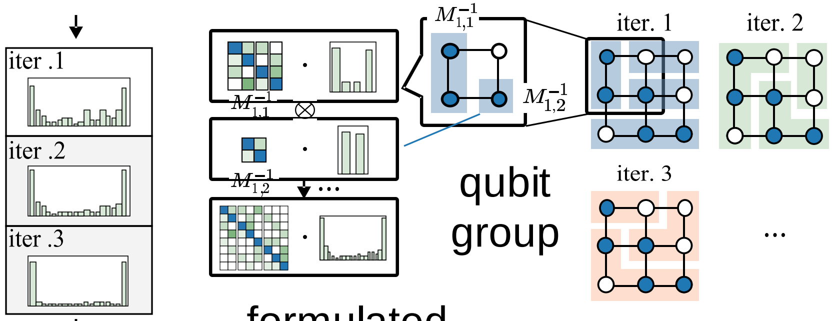
Calibration based on Finite Element method (FEM)

ZHEJIANG UNIVERSITY

Classical Finite Element Method



Quantum Finite Element Method



① partitions the sponge into lattices → ① partitions qubits into groups

② analyzes the state of each lattice independently → ② analyze the noise in each group independently

③ simulate the interaction → ③ simulate the interaction

④ update the state of sponge → ④ update the calibration result of qubits

A **divide-and-conquer strategy** to calibrate measured distribution

Calibration formulation

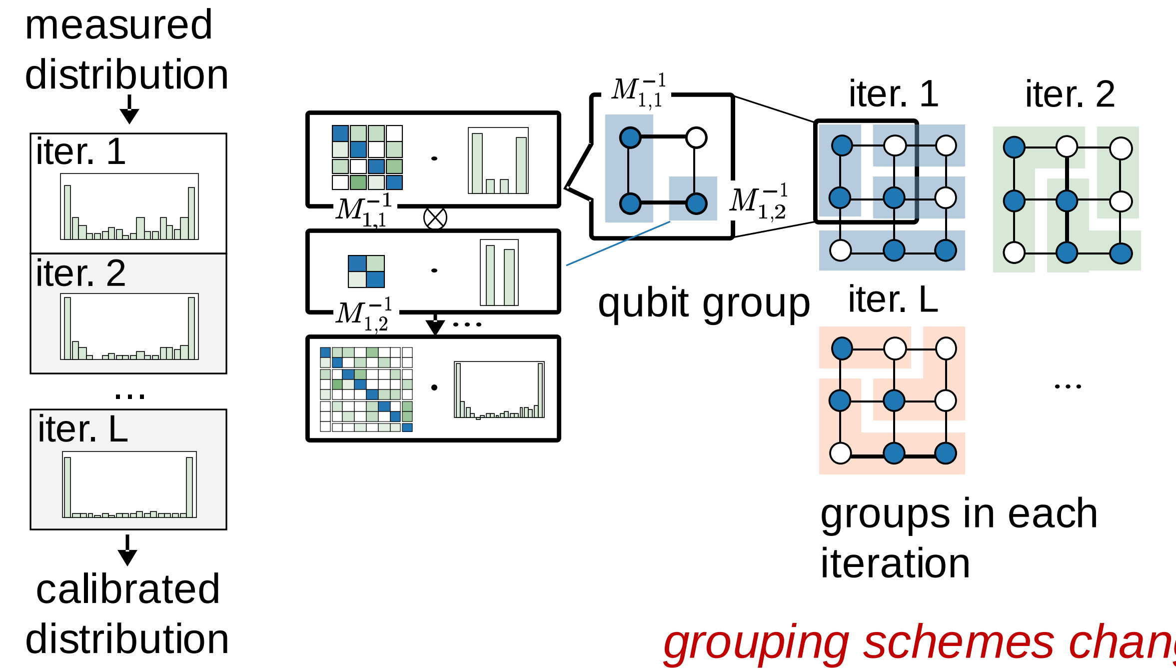
QuFEM reformulates the calibration as an iterative process with a series of sub-noise matrices.

$$\text{Iter. 1: } P_2 = (\overbrace{M_{1,1} \otimes M_{1,2} \otimes \cdots M_{1,k}}^{\text{sub-noise matrices}})^{-1} \overbrace{P_1}^{\text{measured distribution}}$$

$$\text{Iter. 2: } P_3 = (M_{2,1} \otimes M_{2,2} \otimes \cdots M_{2,k})^{-1} P_2$$

$$\cdots$$

$$\text{Iter. L: } \underbrace{P_{L+1}}_{\text{calibrated distribution}} = (M_{L,1} \otimes M_{L,2} \otimes \cdots M_{L,k})^{-1} P_L$$



readout error is iteratively mitigated

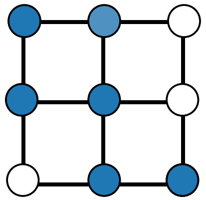
grouping schemes change in different iterations

- Reason for fast: adopt finite element method
- Reason for accurate: dynamically generate noise matrices for different measured qubits

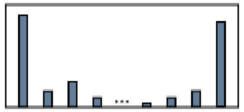
An example

Input :

measured qubits

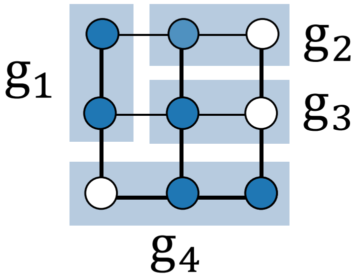


measured distribution

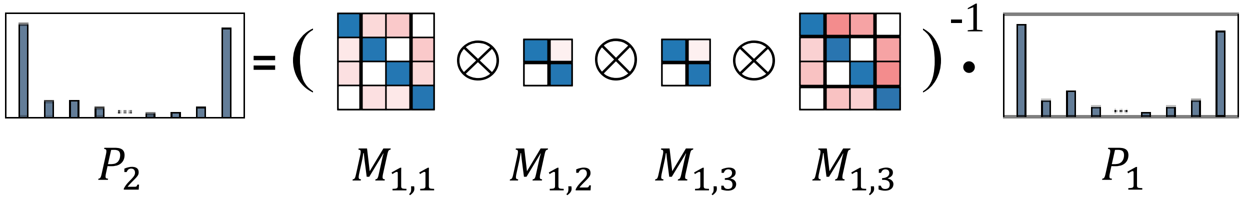


Iteration 1 :

grouping scheme

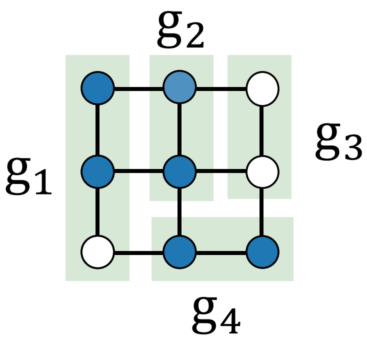


formulation

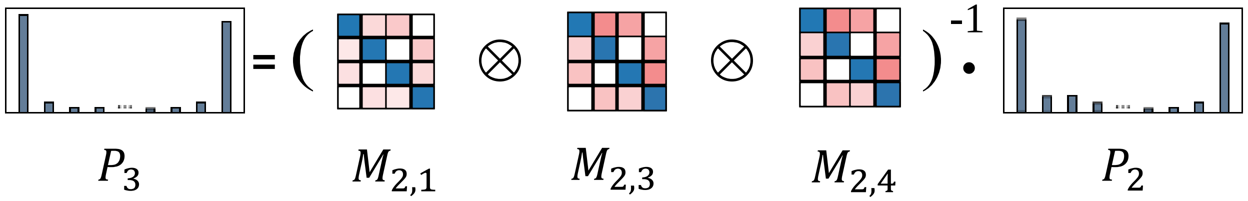


Iteration 2 :

grouping scheme



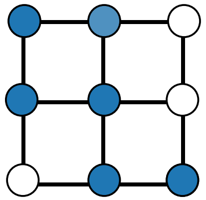
formulation



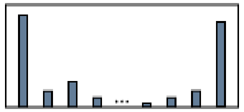
An example

Input :

measured qubits



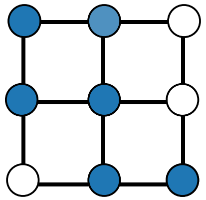
measured distribution



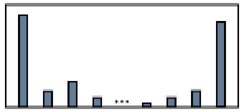
An example

Input :

measured qubits

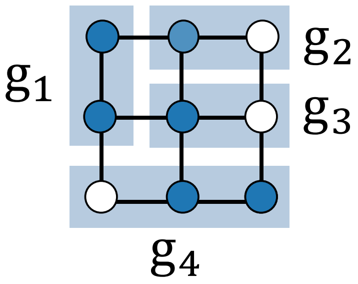


measured distribution

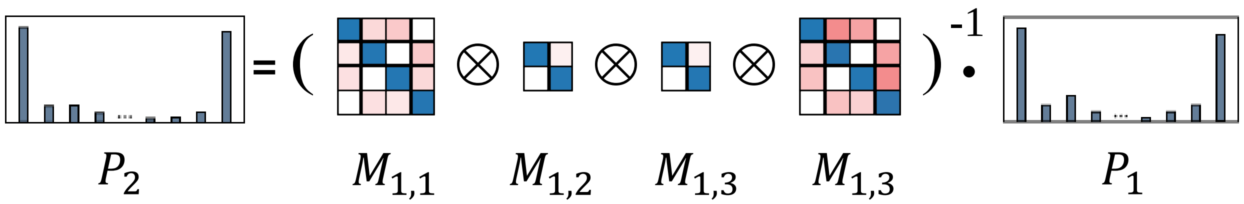


Iteration 1 :

grouping scheme



formulation



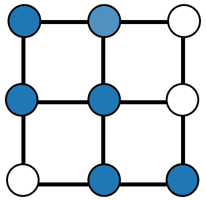
Matrices are generated according to the measured qubits.

Since crosstalk varies in different combinations of measured qubits

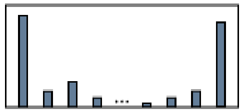
An example

Input :

measured qubits

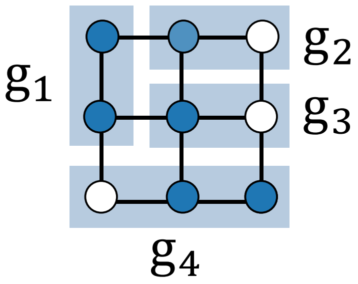


measured distribution

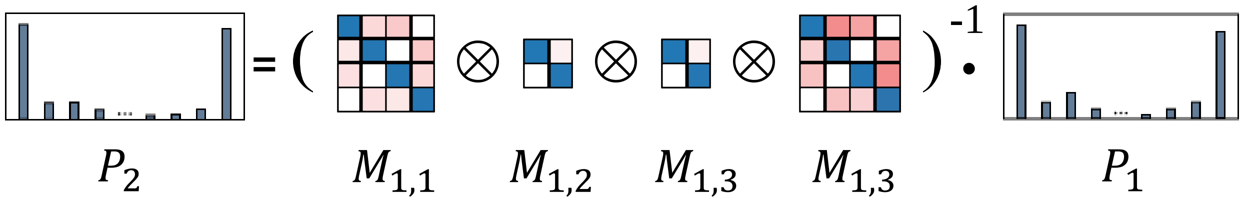


Iteration 1 :

grouping scheme

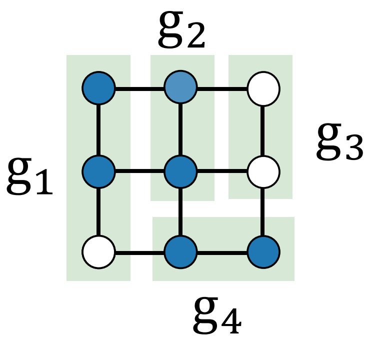


formulation

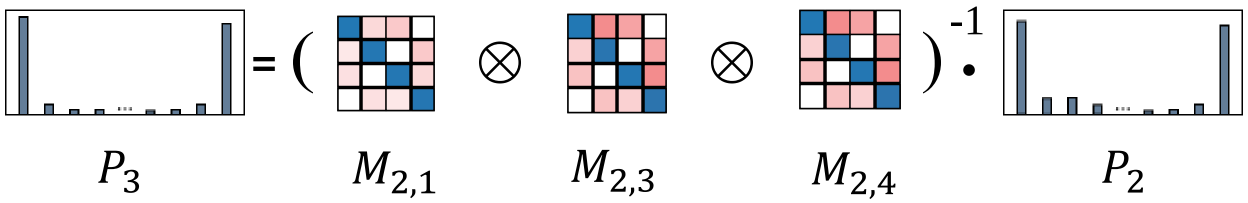


Iteration 2 :

grouping scheme



formulation



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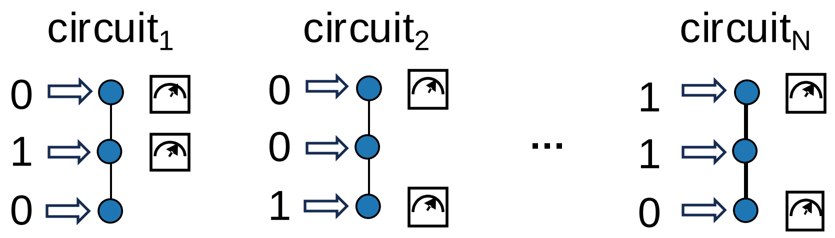
Deployment on Specific Quantum Device

Technique 1: determine the grouping scheme

Data collection

Run benchmarking circuits.



Possible states of a qubit in a benchmarking circuit:

- 0: qubit is set 0 and measured
- 1: qubit is set 1 and measured
- 2: qubit is set 0 or 1 and not measured

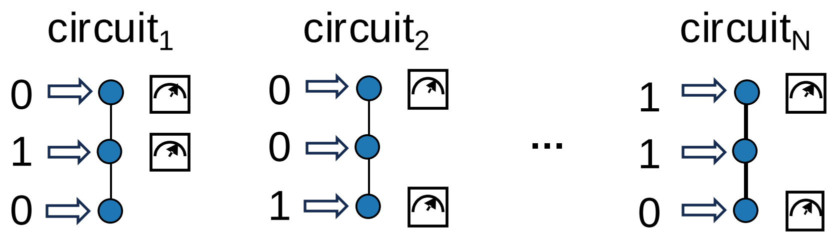
Not all qubits are measured to maximize the variety.

Deployment on Specific Quantum Device

Technique 1: determine the grouping scheme

Data collection

Run benchmarking circuits.



Possible states of a qubit in a benchmarking circuit:

- 1: qubit is set 0 and measured
- 2: qubit is set 1 and measured
- 3: qubit is set 0 or 1 and not measured

Not all qubits are measured to maximize the variety.

Qubit partition

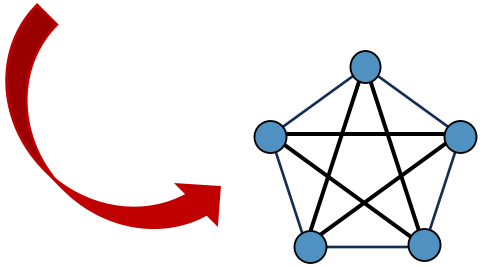
Characterize the **interaction** from one qubit to another qubit under different states:

$$\mathrm{interact}(q_i.\mathrm{state} = x \rightarrow q_j.\mathrm{state} = x)$$

$$= \left|\underbrace{P(q_j.\mathrm{error} = 1 \mid C1, C2)}_{\text{error rate of } q_j \text{ under } C1, C2} - \underbrace{P(q_j.\mathrm{error} = 1 \mid C2)}_{\text{average error rate of } qz_j}\right|$$

$$C1:\ q_i.\mathrm{state} = x, \qquad C2:\ q_j.\mathrm{state} = y$$

Construct weighted graph

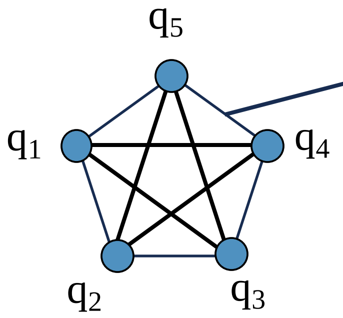


Deployment on Specific Quantum Device

Technique 1: determine the grouping scheme

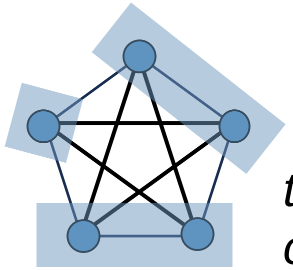
Qubit partition

Construct a **weighted qubit graph**:



labeled by the sum of interactions under all operations

Partitions with a **MAX-CUT solver**:



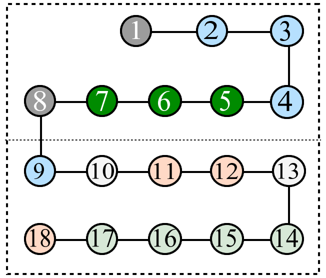
try to comprehensively capture the interactions between qubits

An Example

Prior knowledge of hardware helps grouping

Readout resonator 1

Readout resonator 2



18-qubit topology

group 1: 14 15 16 17

same readout resonator

group 2: 2 3 4 9

similar readout frequency

group 3: 1 8

overlapping frequency shift region

- **demonstrated in the results from other quantum devices**
- **can be used as prior knowledge to facilitate the partition.**

Deployment on Specific Quantum Device

Technique 2: sub-noise matrix generation

Perform matrix-vector multiplication

$$\text{Iter. i: } P_{i+1} = (M_{i,1} \otimes M_{i,2} \otimes \cdots M_{i,k})^{-1} P_i$$

Matrix generation

Noise matrix formulates the transformation probability from the ideal state to measured state.

$$\begin{bmatrix} 0.6 & 0 & 0.1 & 0 \\ 0.1 & 0.7 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.8 \end{bmatrix} =$$

set state

read state

	00	01	10	11
00	0.6	0	0.1	0
01	0.1	0.7	0.2	0.1
01	0.2	0.2	0.6	0.1
11	0.1	0.1	0.1	0.8

Deployment on Specific Quantum Device

Technique 2: sub-noise matrix generation

Perform matrix-vector multiplication

$$\text{Iter. i: } P_{i+1} = (M_{i,1} \otimes M_{i,2} \otimes \cdots M_{i,k})^{-1} P_i$$

Matrix generation

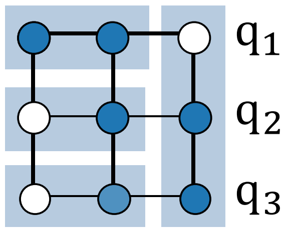
Noise matrix formulates the transformation probability from the ideal state to measured state.

$$\begin{bmatrix} 0.6 & 0 & 0.1 & 0 \\ 0.1 & 0.7 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.8 \end{bmatrix} = \text{read state}$$

set state

	00	01	10	11
00	0.6	0	0.1	0
01	0.1	0.7	0.2	0.1
01	0.2	0.2	0.6	0.1
11	0.1	0.1	0.1	0.8

Sub-noise matrices of QuFEM formulates the transformation probability of states inside the qubit groups.



set state of q_2, q_3

read state of q_2, q_3

	00	01	10	11
00	…	…		
01	…	…		
01	…	…		
11	…	…		

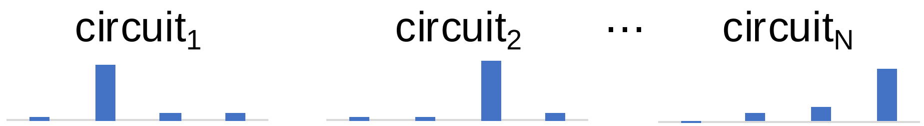
$M[x][y] = P(\{q_2, q_3\}.\text{read} = x \mid \{q_2, q_3\}.\text{set} = y, \boxed{q_1 = 2})$

Transformation probability when q_1 is not measured

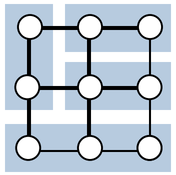
Put all together

Characterization

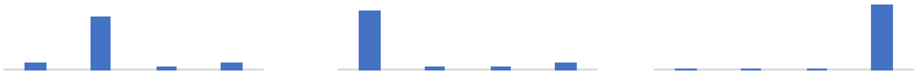
Iteration 1. *Run benchmarking circuits.*



Iteration 1. *Partition qubits.*

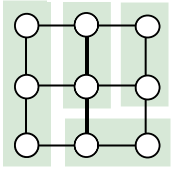


Iteration 1. *Calibrate.*



distributions, grouping scheme of iteration 1.

Iteration 2. *Partition qubits.*



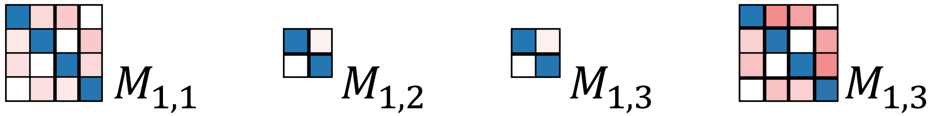
Iteration 2. *Calibrate.*

distributions, grouping scheme of iteration 2.

Calibration

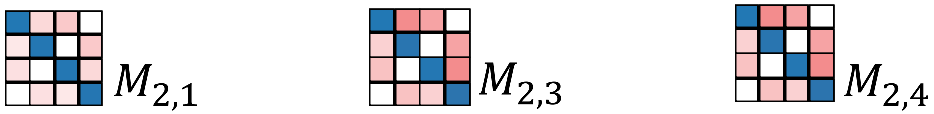
Input. *measured qubits* *measured distribution*

Iteration 1. *Generate sub-noise matrices.*

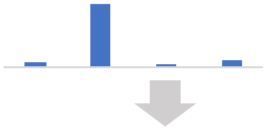


Iteration 1. *Calibrate.*

Iteration 2. *Generate sub-noise matrices.*



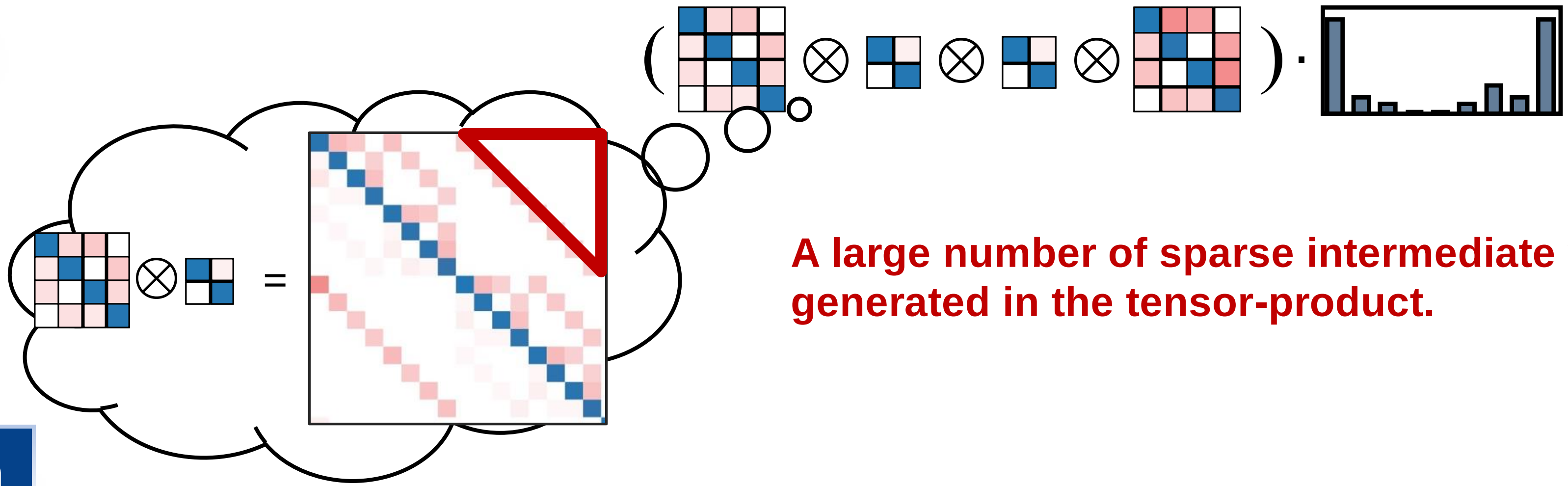
Iteration 2. *Calibrate.*



Output

Sparse Tensor-Product Engine

Observation



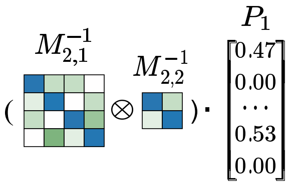
A large number of sparse intermediate vectors is generated in the tensor-product.

Implementation

Use a key-value table to store sparse vector

Calculate the tensor product

Aggregate the tensor-product result



target equation

x	$prob.$
$P_1(000)$	0.47
$P_1(011)$	0.53

① →

	$value$
00	0.50
01	-0.02
10	0.01
11	0.01

② ⊗

	$value$
0	0.99
1	0.01

$\cdot\ 0.47 =$

	$value$
000	0.49
001	0.01
010	-0.01
100	0.01
~~101~~	~~10^{-4}~~
~~110~~	~~10^{-4}~~

…

③ $|value| < \beta$

→ ④ $\sum$ →

x	$prob.$
$P_2(000)$	0.48
$P_2(001)$	6×10^{-3}
$P_2(010)$	6×10^{-3}
$P_2(011)$	0.50
$P_2(111)$	6×10^{-3}

Prune values < threshold (e.g.， 10^{-5})

For each basis states
① calculate the matrix-vector multiplication
② calculate the tensor-product
③ prune intermediate values
④ sum intermediate values to obtain output.

Compute the tensor-product of other basis states

Outline of Presentation

- Background and challenges
- Overview of QuFEM
- QuFEM characterization and calibration
- **Experiment**
- API of QuFEM

Experiment

ZHEJIANG UNIVERSITY

Setup

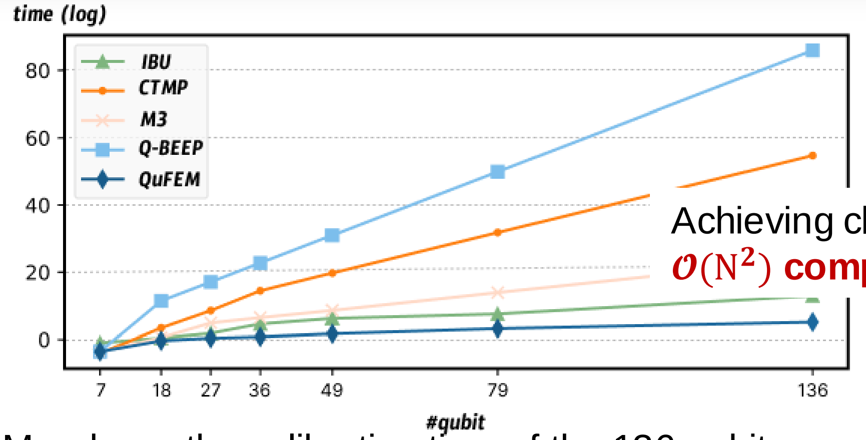
Platform	#Qubits	1-q fidelity	2-q fidelity	Instructions
Quafu	136	94.6±3.1%	94.6±3.0%	ID,RX,RY,RZ,H,CX
	18	95.9±1.3%	95.9±1.3%	ID,RX,RY,RZ,H,CX
Rigetti	79	99.5±1.1%	90.0±6.4%	CPHASE,XY
Self-developed	36	99.9±0.1%	98.7±0.8%	U3,CZ
IBMQ	7	99.9±0.1%	99.2±0.1%	CX,ID,RZ,SX,X

Evaluated hardware

IBU: KJ Satzinger, et al. Realizing topologically ordered states on a quantum processor. Science 2021
CTMP: Sergey, et al. Mitigating measurement errors in multiqubit experiments. PRA 2021.
M3: Paul D Nation , et al. Scalable mitigation of measurement errors on quantum computers. PRX Quantum 2021.
Q-BEEP: Nathan Wiebe, et al. QBEEP: Quantum Bayesian error mitigation employing Poisson modeling over the hamming spectrum. ISCA 2023.

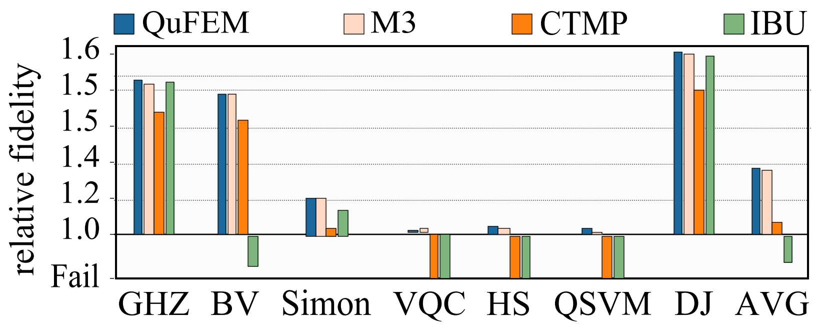
Baselines

Result



Achieving closed to $\mathcal{O}(\mathrm{N}^2)$ **complexity**

QuFEM reduces the calibration time of the 136-qubit program output from **119.44 hours** (IBU) to **169.65 seconds** (**119.44 × reduction**).

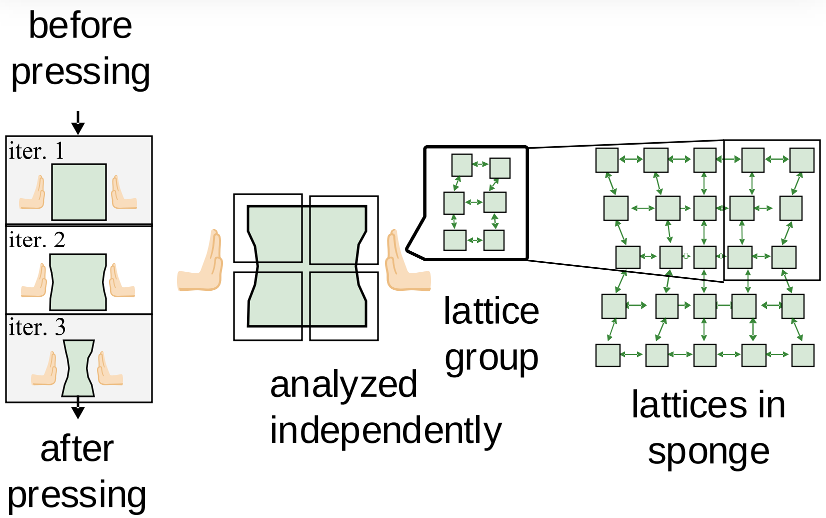


QuFEM shows an average improvement in relative fidelity of **1.003×, 1.2×,** and **1.4×** compared to M3, CTMP, and IBU, respectively.

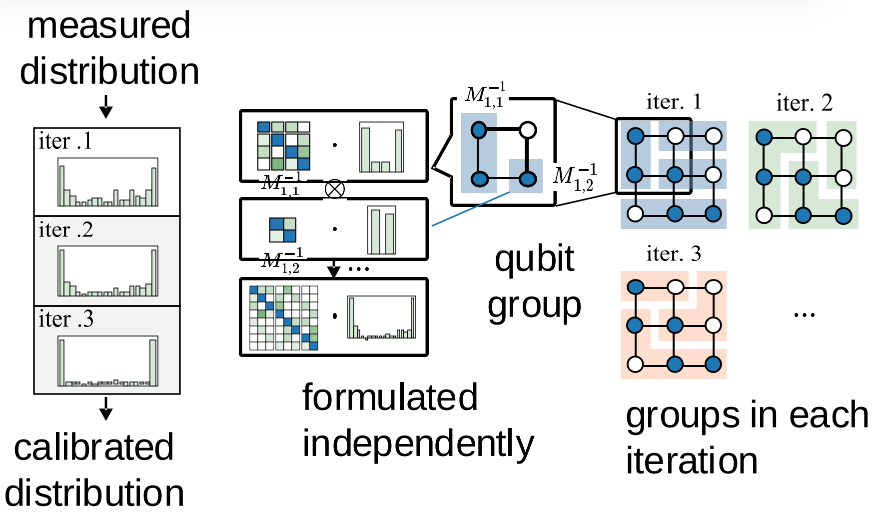
Conclusion

1. Limitations of prior matrix-based calibration methods: slow and inaccurate
2. Finite element method: a divide and conquer strategy
3. Detailed techniques to partition qubits and generate noise matrix
4. Sparse tensor product engine to speed up the computation

Classical Finite Element Method



Quantum Finite Element Method



Outline of Presentation

- Background and challenges
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- **API of QuFEM**

API of QuFEM

File:
- JanusQ/examples/ipynb/4_1_readout_calibration_simulator.ipynb
- JanusQ/examples/ipynb/4_2_readout_calibration_realqc.ipynb
- https://janusq.github.io/tutorials/demo/4_1_readout_calibration_simulator
- https://janusq.github.io/tutorials/demo/4_2_readout_calibration_realqc

construct mitigator

calibrate output of GHZ circuit

```
from janusq.dataset import ghz_error, benchmark_circuits_and_results

from janusq.calibration.readout_mitigation.qufem import Mitigator
from janusq.calibration.readout_mitigation.qufem.tools import npformat_to_statuscnt

mitigator = Mitigator(n_qubits=8, n_iters=2)
scores = mitigator.init(benchmark_circuits_and_results, group_size=2,
multi_process=False, draw_grouping=True)

n_qubits = 4
outout_ideal = { '1'*n_qubits: 0.5, '0'*n_qubits: 0.5 }
output_fem = mitigator.mitigate(ghz_error[0], [ _ for _ in range(n_qubits) ], cho = 1)
output_fem = npformat_to_statuscnt(output_fem)
```

Thanks for listening

QuFEM: Fast and Accurate Quantum Readout Calibration Using the Finite Element Method

Siwei Tan, Liqiang Lu*, Hanyu Zhang, Jia Yu, Congliang Lang, Yongheng Shang, Xinkui Zhao, Mingshuai Chen, Yun Liang, and Jianwei Yin*