WELCOME TO TUTORIAL

Session 4.1 Janus-FEM: Fast and Accurate Quantum Readout Calibration Using the Finite Element Method

ACES Lab

JanusQ Cloud

https://janusq.github.io/tutorials/

College of Computer Science and Technology,
Zhejiang University

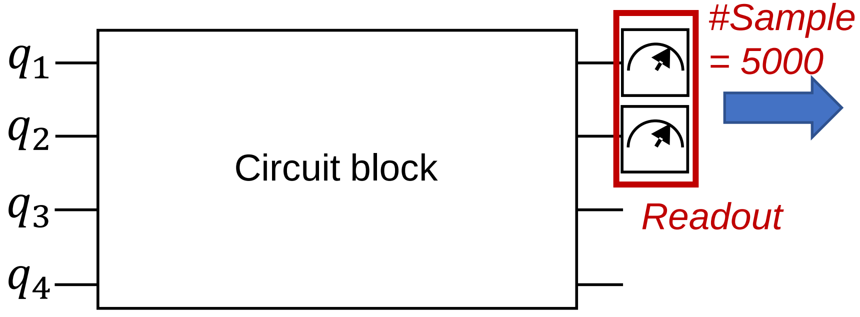
ASPLOS 2024

Outline of Presentation

- **Background and challenges**
- Overview of Janus-FEM
- Janus-FEM Characterization And Calibration
- Experiment
- API Of Janus-FEM

Background

Quantum readout is an operation to **read the information from quantum bits to classical bits.**

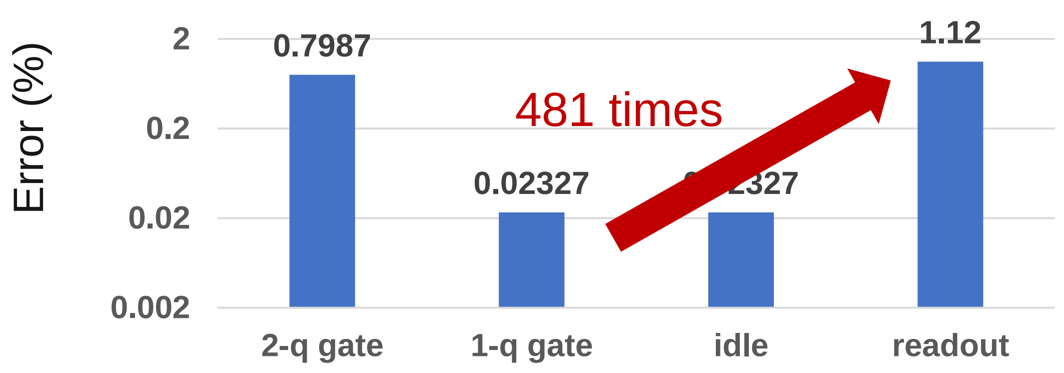


sample	1	2	3	4	4	…	4999	5000
q_1	0	1	1	1	0	…	1	0
q_2	1	0	1	1	1		1	1

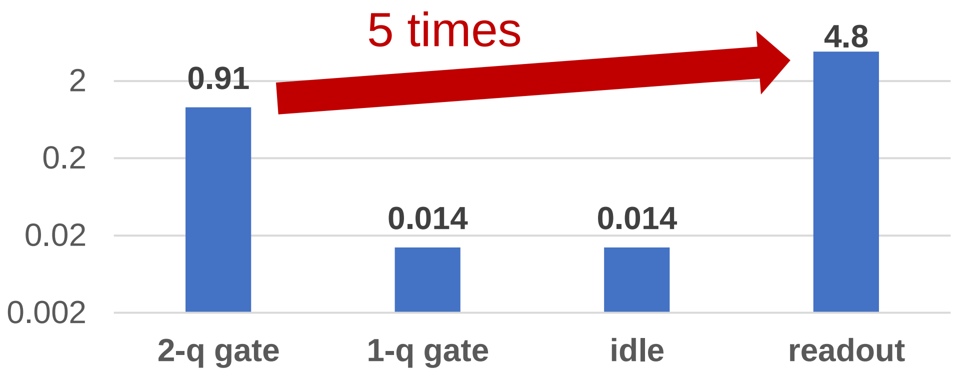
Measured bitstrings in different samples

Probability distribution

Readout error is significant on current quantum hardware.



Noise on 127-qubit IBM Sherbrooke quantum device

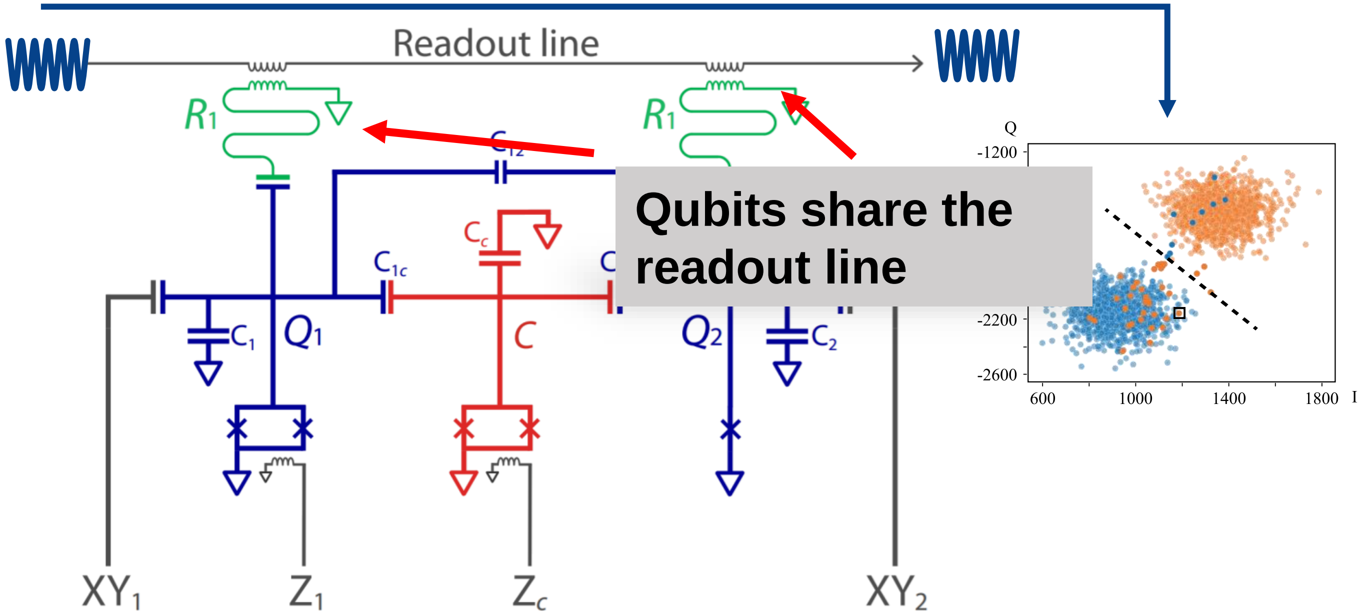


Noise on 10-qubit Tianmu quantum device

Background

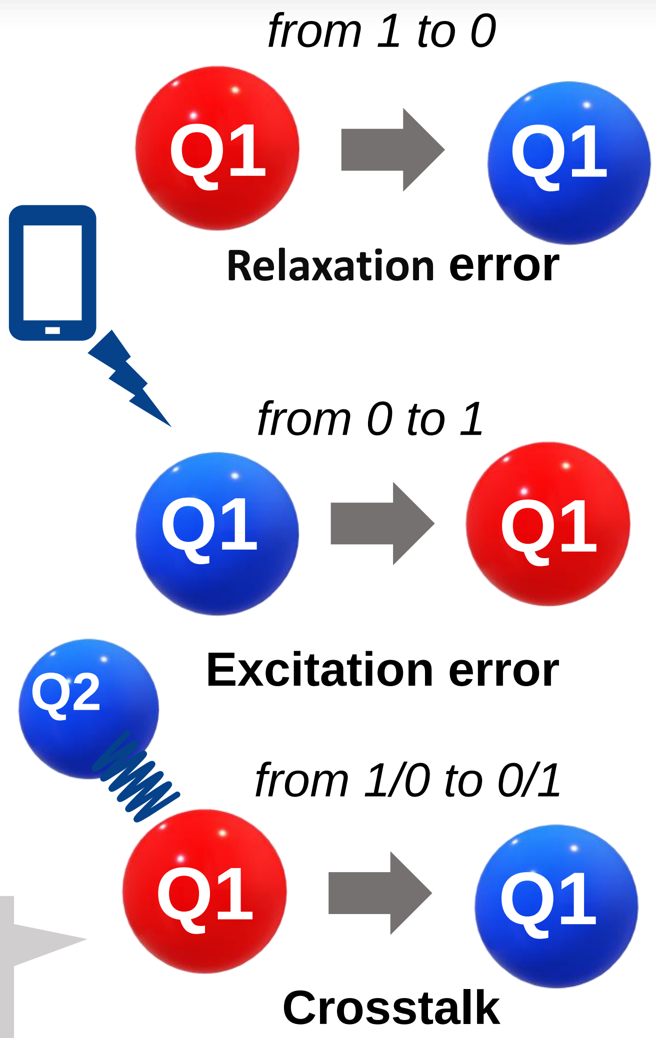
Implementation of readout on superconducting qubits

(1) Pulse in *(2) Pulse takes the information out.* *FFT*



e.g. Das, et al. JigSaw: Boosting Fidelity of NISQ Programs via Measurement Subsetting. MICRO 2021

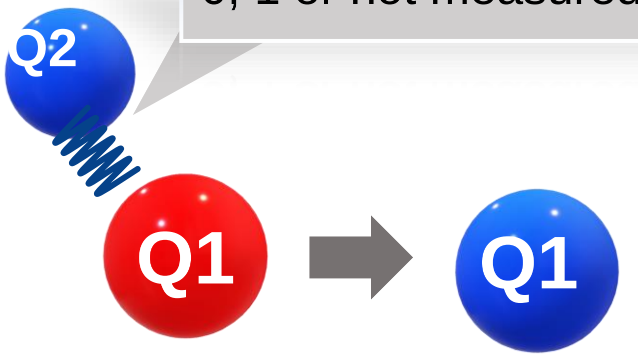
Source of readout error



State-dependent Readout Error

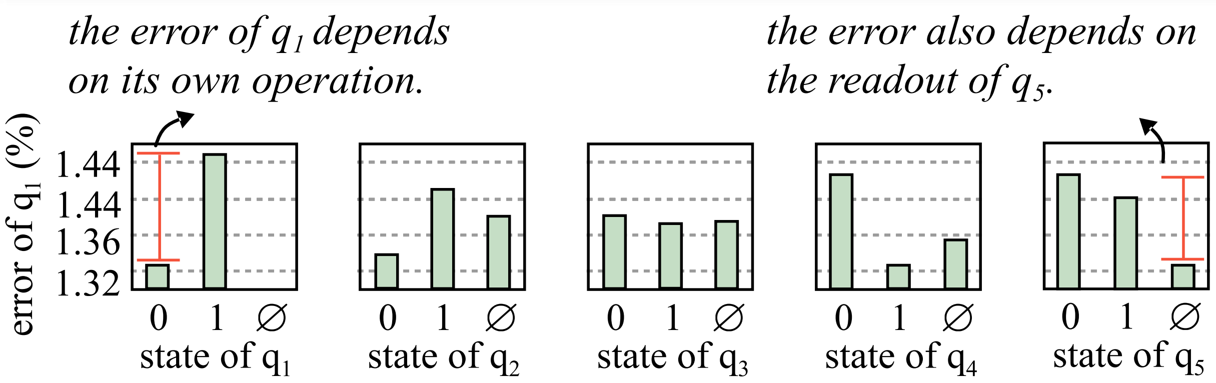
Readout errors vary in different combinations of measured qubits due to crosstalk.

Crosstalk has different frequencies when Q2 is measured 0, 1 or not measured



Similar to entanglement

Example of state-dependent and readoutdependent noises on the IBMQ Perth quantum device.



The readout output of qubits has correlations similar to the entanglement, making the calibration difficult.

Basic Matrix-based readout calibration

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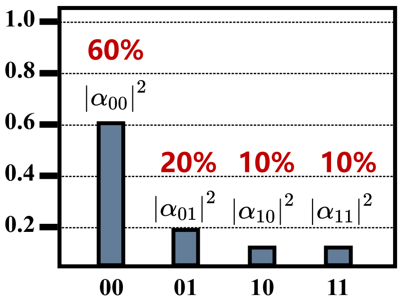
Ideal readout

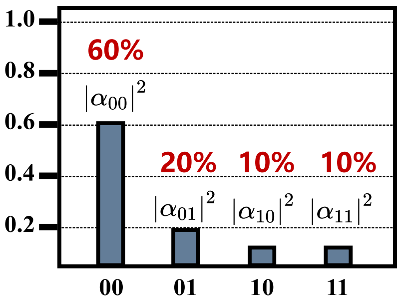


State vector

$$\begin{bmatrix} |\alpha_{00}|^2 \\ |\alpha_{01}|^2 \\ |\alpha_{10}|^2 \\ |\alpha_{11}|^2 \end{bmatrix} \quad \begin{bmatrix} 0.6 \\ 0.2 \\ 0.1 \\ 0.1 \end{bmatrix}$$

Ideal distribution
(ideal program output)

Readout with noise



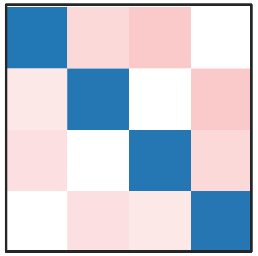
State vector

☹

$$\begin{bmatrix} 0.5 \\ 0.1 \\ 0.09 \\ 0.31 \end{bmatrix}$$

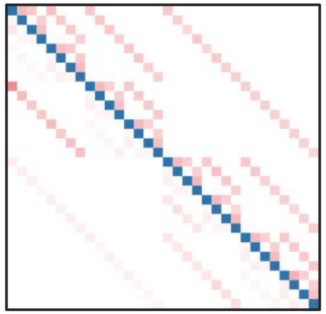
Noisy distribution
(noisy program output)

Matrix-based readout error calibration



$$(\cdot)^{-1} \cdot \begin{bmatrix} 0.5 \\ 0.1 \\ 0.09 \\ 0.31 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.2 \\ 0.1 \\ 0.1 \end{bmatrix}$$

Noise matrix | **Noisy distribution** | **Calibrated distribution**



Calibration matrix of a 5-qubit readout

The size exponentially increases!

$2^5 \times 2^5$

Basic Matrix-based Readout Calibration

Step 1. Matrix characterization

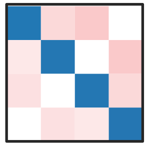
Prepares qubits to different basis states and apply measurement.

Fill in a noise matrix.

$$\mathrm{M} = \begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0 & \boxed{0.7} & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}$$

Inverse the noise matrix

$M^{-1} =$



Calibration matrix

Step 2. Calibration for any input

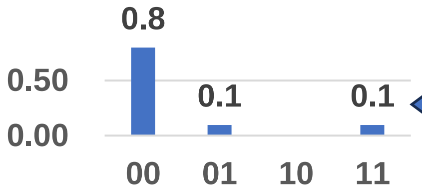
Represent the measured distribution as a vector.

$$\begin{bmatrix} 0.6 \\ 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}$$

Apply matrix-vector multiplication.

$$\begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0 & 0.7 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0.6 \\ 0.1 \\ 0.2 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

$$M^{-1} \cdot P_{\text{read}} = P_{\text{calibrated}}$$



Complexity Analysis

Step 1. Matrix characterization

Prepares qubits to different basis states and apply measurement.

2^N circuits are executed to measure qubits on all basis states.

Fill in a noise matrix.

The size of the noise matrix is $2^N \times 2^N$.

Inverse the noise matrix

Calcauting the inverse has $\mathcal{O}(4^N)$ complexity.

Step 2. Calibration for any input

Represent the measured distribution as a vector.

The transformation has linear complexity.

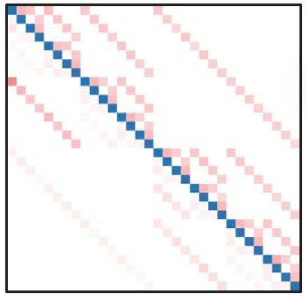
Apply matrix-vector multiplication.

The multiplication has $\mathcal{O}(4^N)$ complexity.

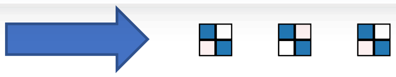
8.8 TB and 10 hours for a 32-qubit calibration on a server with AMD EPYC 2.25GHz 64-core CPUs

Limitations of Current Methods

IBU (Google Science 2021) Realizing topologically ordered states on a quantum processor.

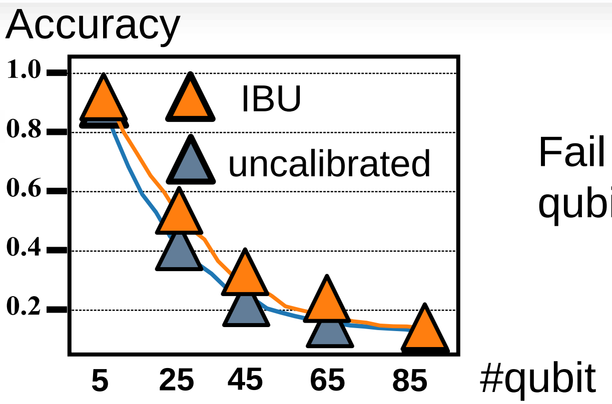


Crosstalk makes the matrix not simple tensor-product result.



Real calibration matrix — Single-qubit matrix

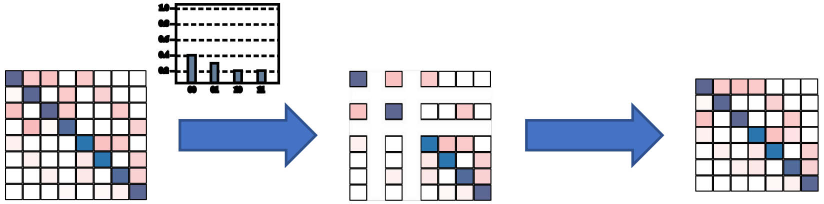
Use tensor product of a series of single-qubit meta-matrices



Fail to calibrate on 80-qubit readout output

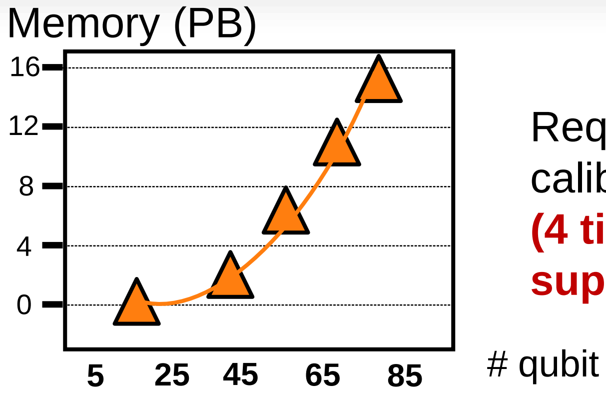
Fast but not accurate: ignore the qubit interactions.

M3 (IBM PRA 2021): Scalable mitigation of measurement errors on quantum computers



Before pruning — Pruning based on program output — After pruning

Use a sparsity-aware method to prune on the matrix under a threshold of Hamming distance



Require 16PB to calibrate a 85-qubit result. **(4 times the Fugaku supercomputer)**

Accurate but not fast: many matrix elements cannot be ignored

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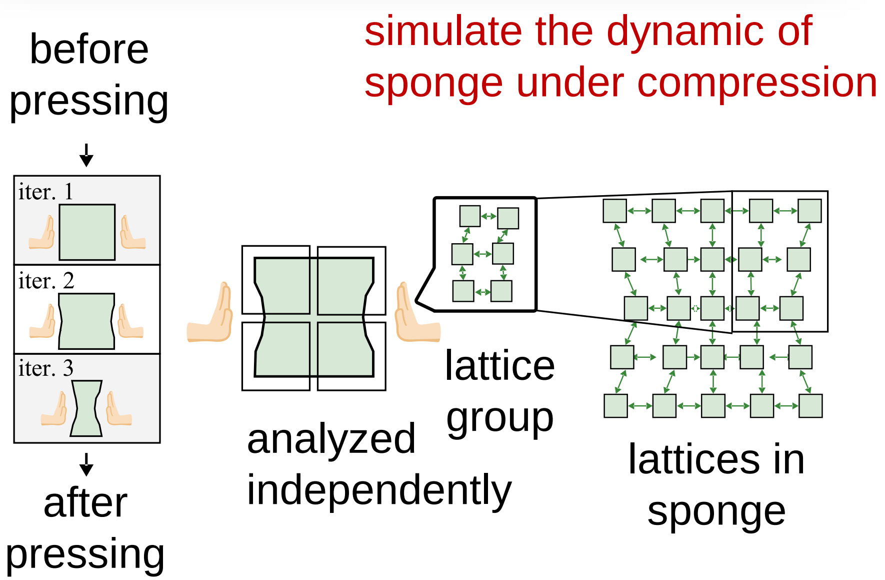
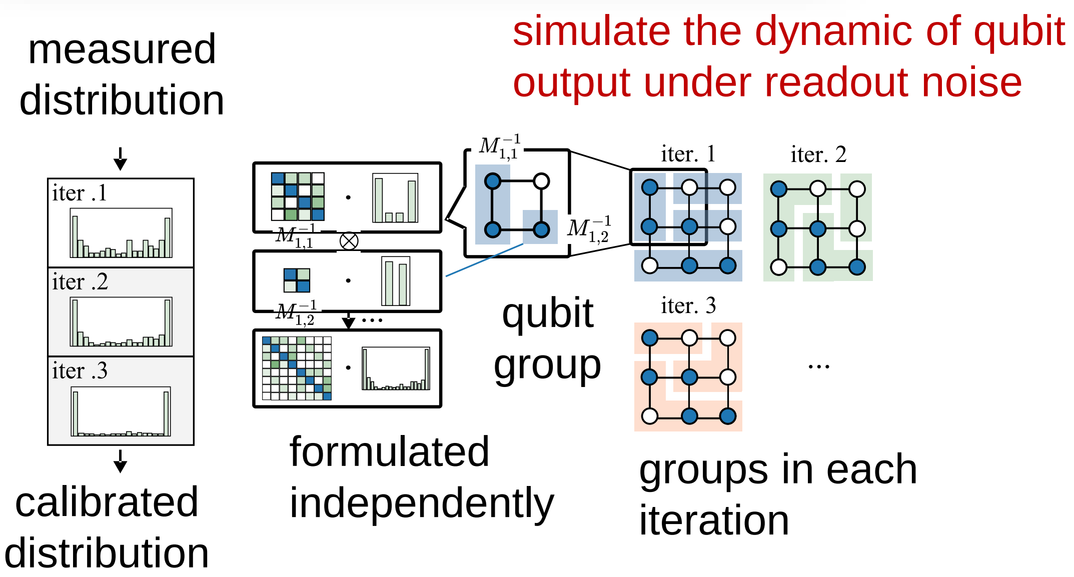
Calibration based on Finite Element method (FEM)

Classical Finite Element Method



Quantum Finite Element Method



① partitions the sponge into lattices
② analyzes the state of each lattice independently
③ simulate the interaction
④ update the state of sponge

① partitions qubits into groups
② analyze the noise in each group independently
③ simulate the interaction
④ update the calibration result of qubits

A **divide-and-conquer strategy** to calibrate measured distribution

Calibration formulation

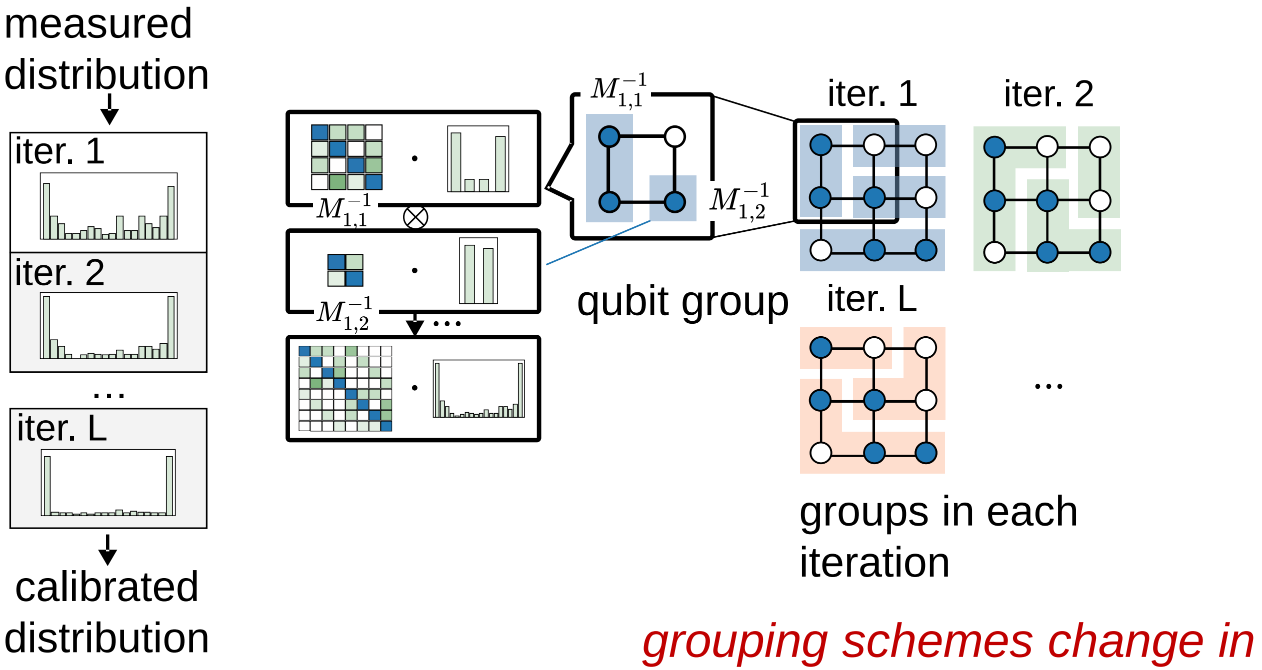
Janus-FEM reformulates the calibration as an iterative process with a series of sub-noise matrices.

$$\text{Iter. 1: } P_2 = \overbrace{(M_{1,1} \otimes M_{1,2} \otimes \cdots M_{1,k})^{-1}}^{\text{sub-noise matrices}} \overbrace{P_1}^{\text{measured distribution}}$$

$$\text{Iter. 2: } P_3 = (M_{2,1} \otimes M_{2,2} \otimes \cdots M_{2,k})^{-1} P_2$$

$$\cdots$$

$$\text{Iter. L: } \underbrace{P_{L+1}}_{\text{calibrated distribution}} = (M_{L,1} \otimes M_{L,2} \otimes \cdots M_{L,k})^{-1} P_L$$



readout error is iteratively mitigated

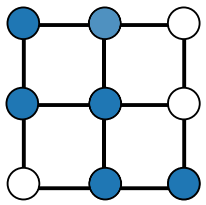
grouping schemes change in different iterations

- Reason for fast: adopt finite element method
- Reason for accurate: dynamically generate noise matrices for different measured qubits

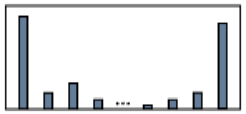
An example

Input :

measured qubits

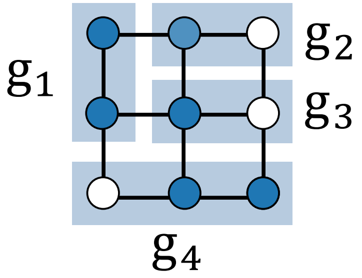


measured distribution

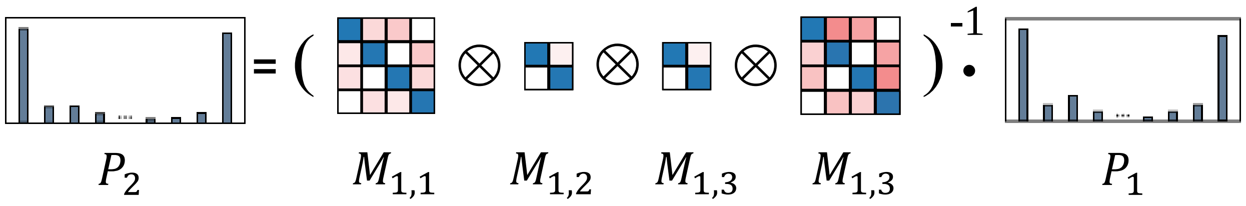


Iteration 1 :

grouping scheme

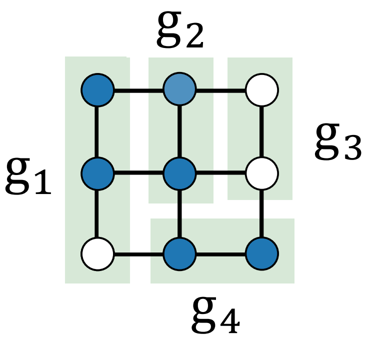


formulation

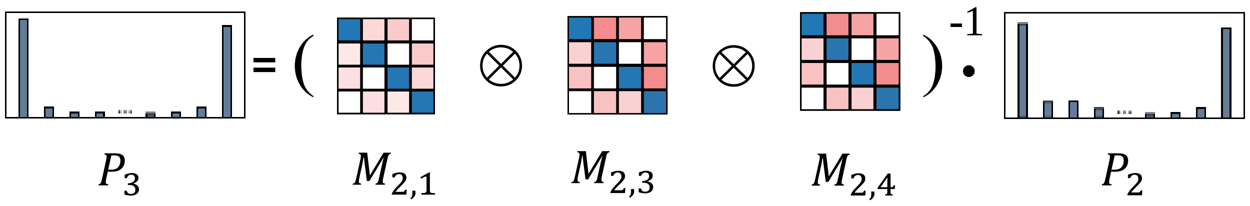


Iteration 2 :

grouping scheme



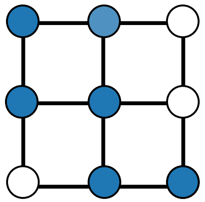
formulation



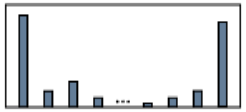
An example

Input :

measured qubits

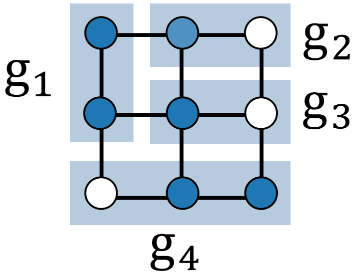


measured distribution

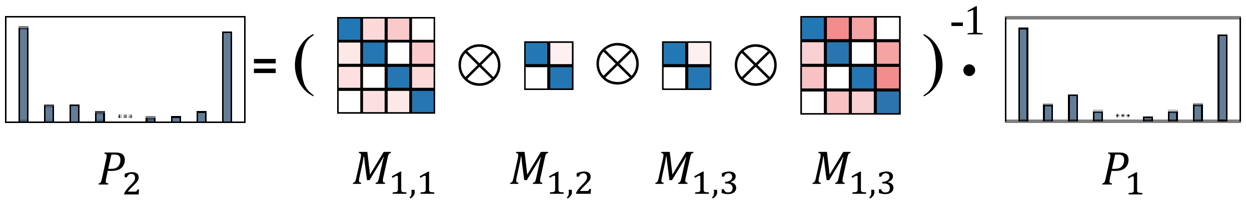


Iteration 1 :

grouping scheme

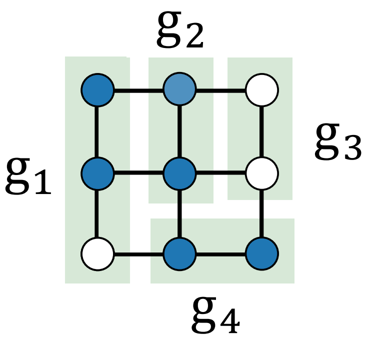


formulation

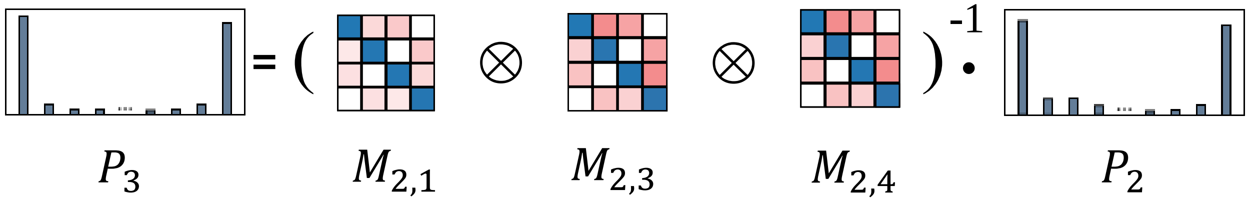


Iteration 2 :

grouping scheme



formulation



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- **Janus-FEM Characterization And Calibration**
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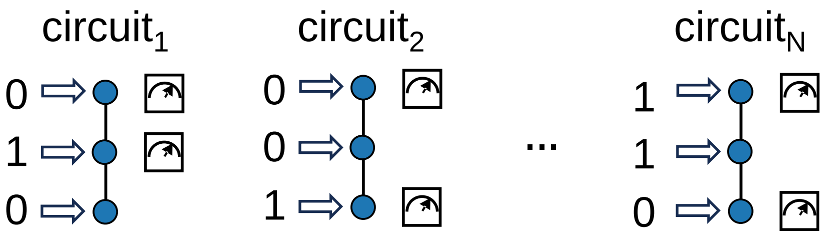
- API Of Janus-FEM

Deployment on Specific Quantum Device

Technique 1: determine the grouping scheme

Data collection

Run benchmarking circuits.



Possible states of a qubit in a benchmarking circuit:

- 1: qubit is set 0 and measured
- 2: qubit is set 1 and measured
- 3: qubit is set 0 or 1 and not measured

Not all qubits are measured to maximize the variety.

Qubit partition

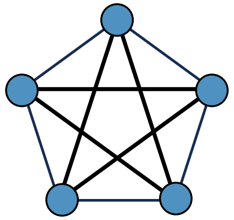
Characterize the **interaction** from one qubit to another qubit under different states:

$$\mathrm{interact}(\mathrm{q_i.state} = x \rightarrow \mathrm{q_j.state} = x)$$

$$= \left| \underbrace{\mathrm{P}\left(\mathrm{q_j.error} = 1 \,\middle|\, C1, C2\right)}_{\text{error rate of } \mathrm{q_j} \text{ under } C1, C2} - \underbrace{\mathrm{P}\left(\mathrm{q}_j\mathrm{.error} = 1 \,\middle|\, \mathrm{C2}\right)}_{\text{average error rate of } \mathrm{qz_j}} \right|$$

$$\mathrm{C1:\ q_i.state = x}, \qquad \mathrm{C2:\ q_j.state = y}$$

Construct weighted graph

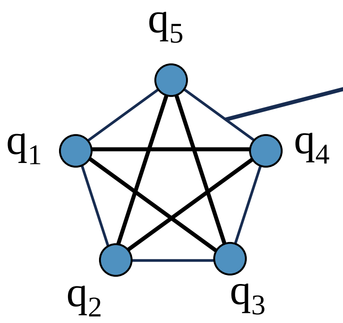


Deployment on Specific Quantum Device

Technique 1: determine the grouping scheme

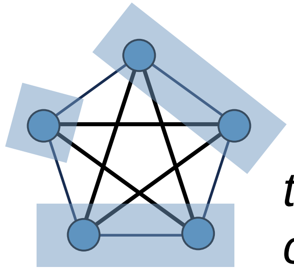
Qubit partition

Construct a **weighted qubit graph**:



labeled by the sum of interactions under all operations

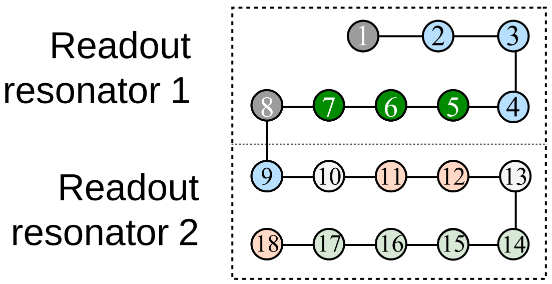
Partitions with a **MAX-CUT solver**:



try to comprehensively capture the interactions between qubits

An Example

Prior knowledge of hardware helps grouping



18-qubit topology

group 1: 14 15 16 17

same readout resonator

group 2: 2 3 4 9

similar readout frequency

group 3: 1 8

overlapping frequency shift region

- **demonstrated in the results from other quantum devices**
- **can be used as prior knowledge to facilitate the partition.**

Deployment on Specific Quantum Device

Technique 2: sub-noise matrix generation

Perform matrix-vector multiplication

$$\text{Iter. i: } P_{i+1} = (M_{i,1} \otimes M_{i,2} \otimes \cdots M_{i,k})^{-1} \; P_i$$

Matrix generation

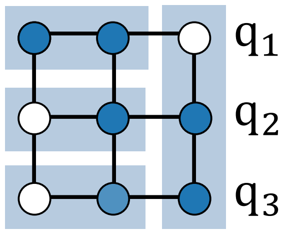
Noise matrix formulates the transformation probability from the ideal state to measured state.

$$\begin{bmatrix} 0.6 & 0 & 0.1 & 0 \\ 0.1 & 0.7 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.8 \end{bmatrix} =$$

set state

read state	00	01	10	11
00	0.6	0	0.1	0
01	0.1	0.7	0.2	0.1
01	0.2	0.2	0.6	0.1
11	0.1	0.1	0.1	0.8

Sub-noise matrices of Janus-FEM formulates the transformation probability of states inside the qubit groups.



set state of q_2, q_3

read state of q_2, q_3	00	01	10	11
00	…	…		
01	…	…		
01	…	…		
11	…	…		

$$M[x][y] = P(\{q_2, q_3\}.\text{read} = x \mid \{q_2, q_3\}.\text{set} = y, \boxed{q_1 = 2)}$$

Transformation probability when q_1 is not measured

Put all together

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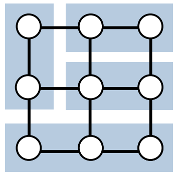
Characterization

Iteration 1. *Run benchmarking circuits.*

circuit_1 circuit_2 … circuit_N

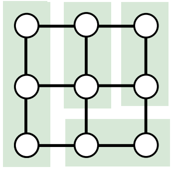
Iteration 1. *Partition qubits.*



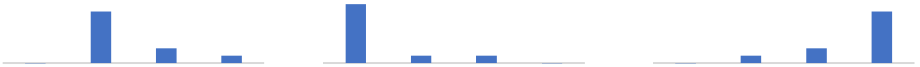
Iteration 1. *Calibrate.*

distributions, grouping scheme of iteration 1.

Iteration 2. *Partition qubits.*



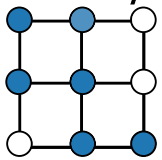
Iteration 2. *Calibrate.*



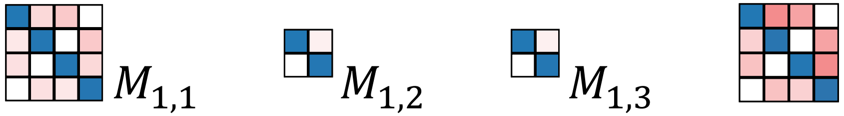
distributions, grouping scheme of iteration 2.

Calibration

Input. *measured qubits* *measured distribution*



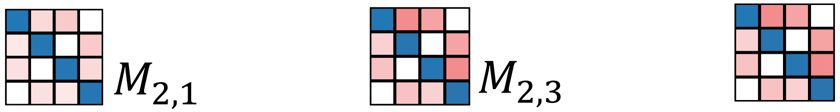
Iteration 1. *Generate sub-noise matrices.*



$M_{1,1}$ $M_{1,2}$ $M_{1,3}$ $M_{1,3}$

Iteration 1. *Calibrate.*

Iteration 2. *Generate sub-noise matrices.*



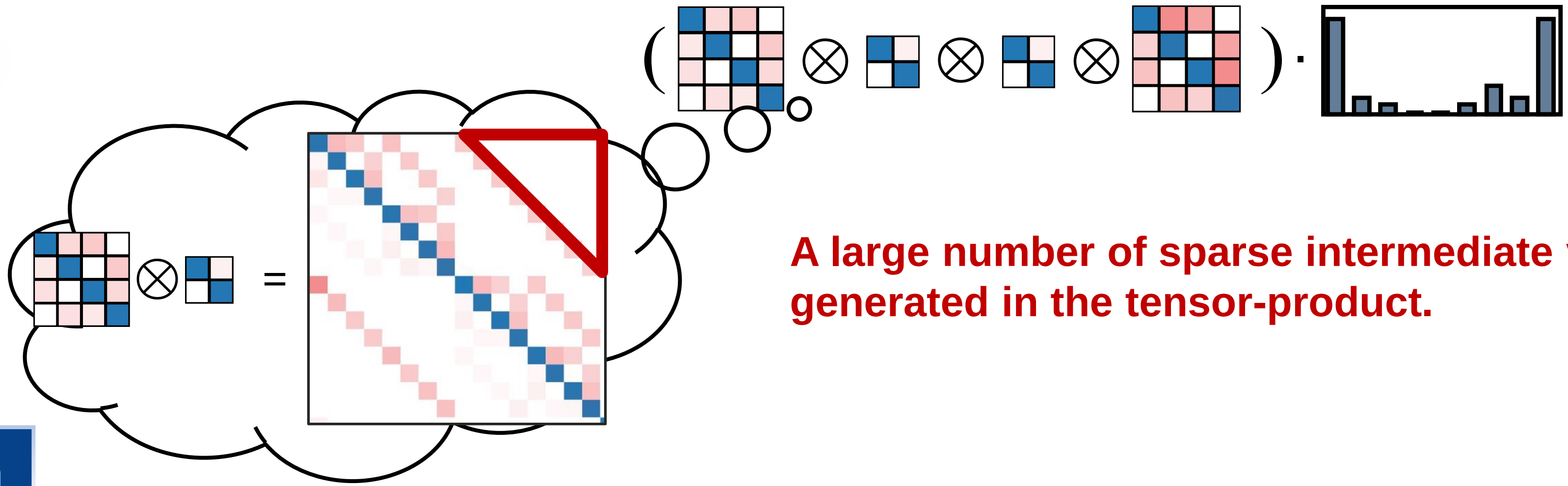
$M_{2,1}$ $M_{2,3}$ $M_{2,4}$

Iteration 2. *Calibrate.*

Output

Sparse Tensor-Product Engine

Observation



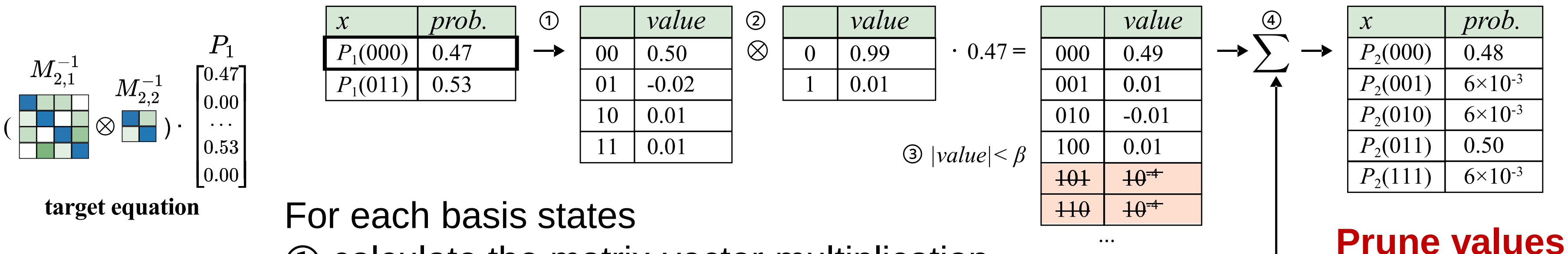
A large number of sparse intermediate vectors is generated in the tensor-product.

Implementation

Use a key-value table to store sparse vector

Calculate the tensor product

Aggregate the tensor-product result



For each basis states
① calculate the matrix-vector multiplication
② calculate the tensor-product
③ prune intermediate values
④ sum intermediate values to obtain output.

Prune values < threshold (e.g., 10^{-5})

Compute the tensor-product of other basis states

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Experiment

Setup

Platform	#Qubits	1-q fidelity	2-q fidelity	Instructions
Quafu	136	94.6±3.1%	94.6±3.0%	ID,RX,RY,RZ,H,CX
	18	95.9±1.3%	95.9±1.3%	ID,RX,RY,RZ,H,CX
Rigetti	79	99.5±1.1%	90.0±6.4%	CPHASE,XY
Self-developed	36	99.9±0.1%	98.7±0.8%	U3,CZ
IBMQ	7	99.9±0.1%	99.2±0.1%	CX,ID,RZ,SX,X

Evaluated hardware

IBU: KJ Satzinger, et al. Realizing topologically ordered states on a quantum processor. Science 2021

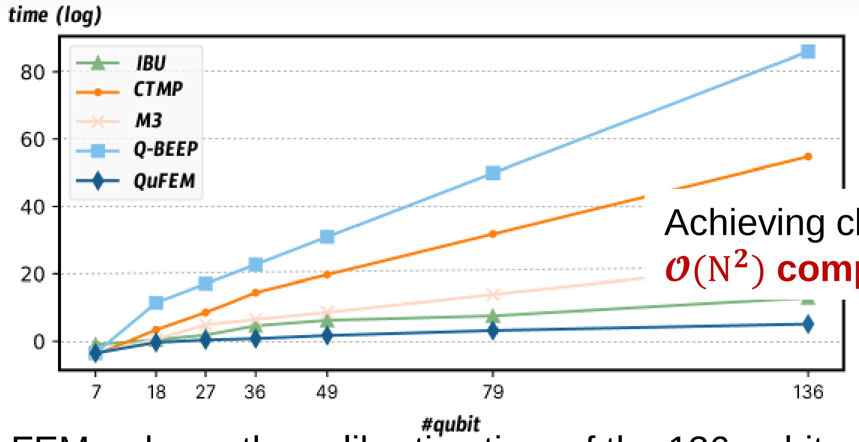
CTMP: Sergey, et al. Mitigating measurement errors in multiqubit experiments. PRA 2021.

M3: Paul D Nation , et al. Scalable mitigation of measurement errors on quantum computers. PRX Quantum 2021.

Q-BEEP: Nathan Wiebe, et al. QBEEP: Quantum Bayesian error mitigation employing Poisson modeling over the hamming spectrum. ISCA 2023.

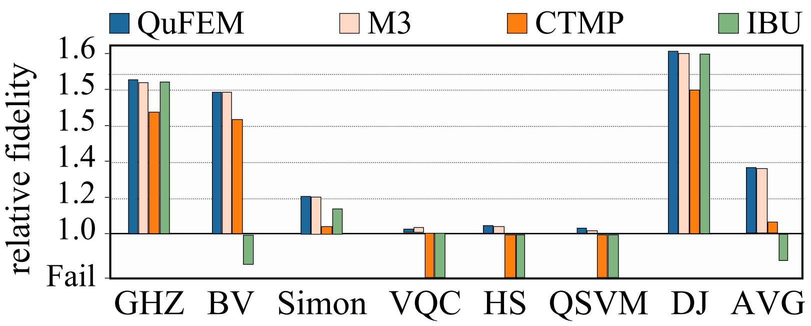
Baselines

Result



Achieving closed to $\mathcal{O}(\mathrm{N}^2)$ **complexity**

Janus-FEM reduces the calibration time of the 136-qubit program output from **119.44 hours** (IBU) to **169.65 seconds** (**119.44 × reduction**).

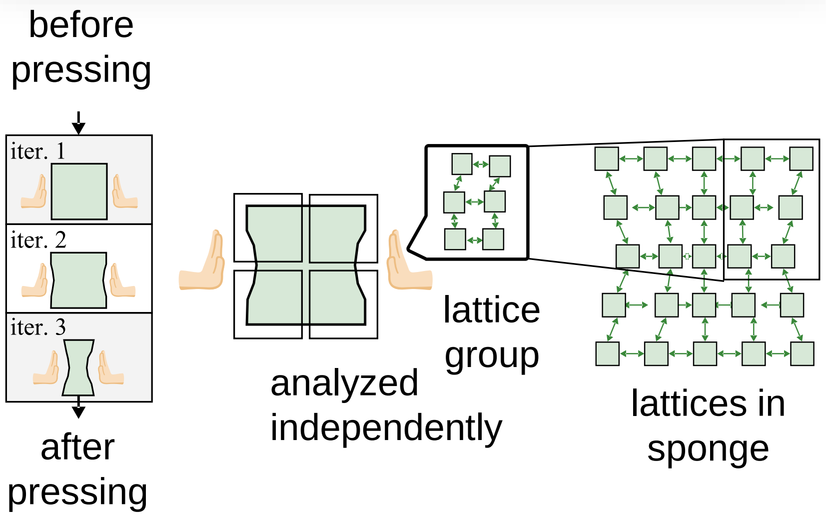


Janus-FEM shows an average improvement in relative fidelity of **1.003×, 1.2×,** and **1.4×** compared to M3, CTMP, and IBU, respectively.

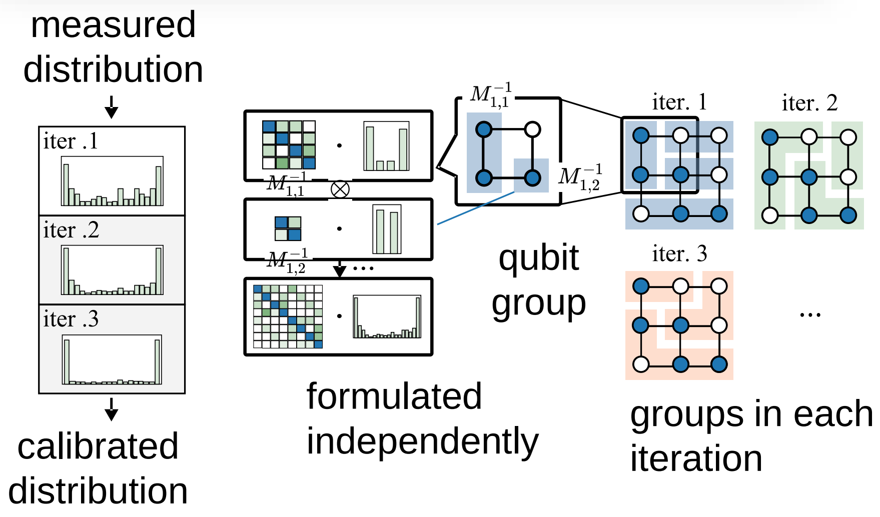
Conclusion

1. Limitations of prior matrix-based calibration methods: slow and inaccurate
2. Finite element method: a divide and conquer strategy
3. Detailed techniques to partition qubits and generate noise matrix
4. Sparse tensor product engine to speed up the computation

Classical Finite Element Method



Quantum Finite Element Method



Outline of Presentation

- Background and challenges
- Overview of Janus-FEM
  - Characterization
  - Calibration
- Experiment
- **API Of Janus-FEM**

API Of Janus-FEM

File:
- examples/4-2.readout_calibration_realqc.ipynb
- https://janusq.github.io/tutorials/Demonstrations/4-2.readout_calibration_realqc

Import package and data

```
from janusq.optimizations.readout_mitigation.fem import Mitigator
from qiskit.quantum_info.analysis import hellinger_fidelity
from janusq.optimizations.readout_mitigation.fem.tools import npformat_to_statuscnt
from janusq.dataset import protocol_8 as benchmark_circuits_and_results, ghz_8qubit as ghz_output

qubits = 8
```

Construct mitigator

```
mitigator = Mitigator(qubits, n_iters = 2)
scores = mitigator.init(benchmark_circuits_and_results, group_size = 2,multi_process=False, draw_grouping = True)
```

Calibrate GHz circuit output

```
n_qubits = 4
outout_ideal = {'1'*n_qubits:0.5,'0'*n_qubits:0.5}
output_fem = mitigator.mitigate(ghz_output[0],[i for i in range(n_qubits)], cho = 1 )
output_fem = npformat_to_statuscnt(output_fem)
```

Compute the fidelity

```
print("Janus-FEM fidelity: ",hellinger_fidelity(outout_ideal,output_fem))
```

Thanks for listening

Janus-FEM: Fast and Accurate Quantum Readout Calibration Using the Finite Element Method

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