WELCOME TO TUTORIAL

Session 4.2 Janus-TC:
Simulate time crystal on Janus quantum platform

ACES Lab
ADVANCED COMPUTING AND EMERGING SERVICE LAB

JanusQ Cloud

浙江大學 计算机科学与技术学院
COLLEGE OF COMPUTER SCIENCE AND TECHNOLOGY
ZHEJIANG UNIVERSITY

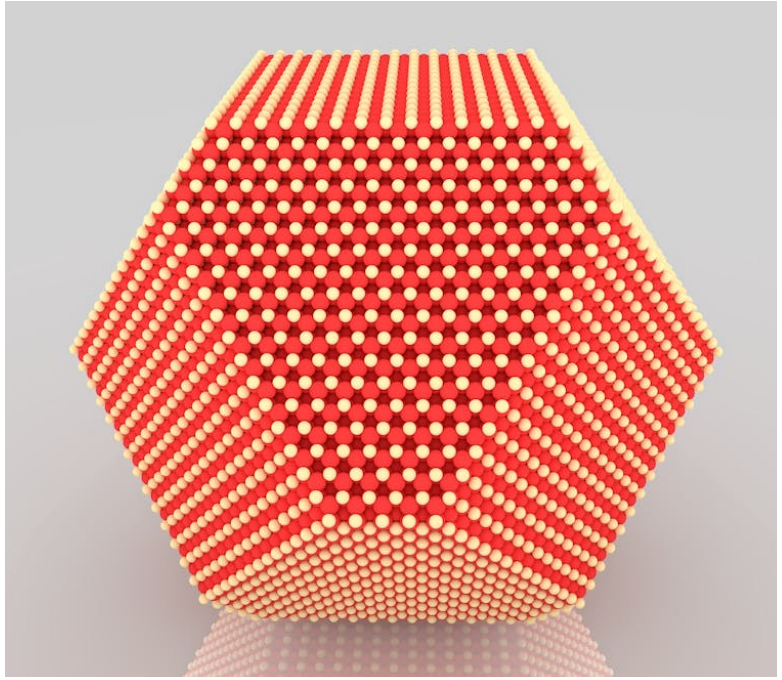
https://janusq.github.io/tutorials/

College of Computer Science and Technology,
Zhejiang University

ASPLOS 2024

What is time crystal?

Common crystal



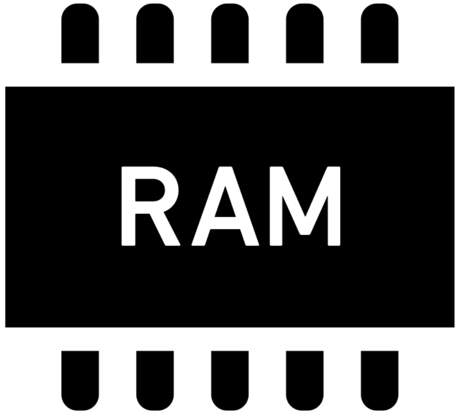
- Particles arrange periodically in space.

Time crystal

- Particles arrange periodically in **time** and space.

Application

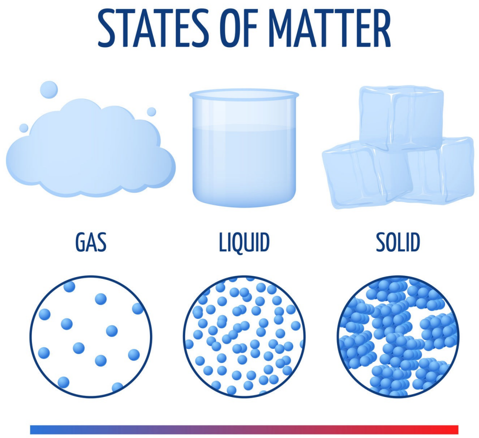
Quantum Memory



- Resilient to heat entropy
- Repeat themselves in time

Time Crystal

Different phases of matter



Time crystal: A new phase

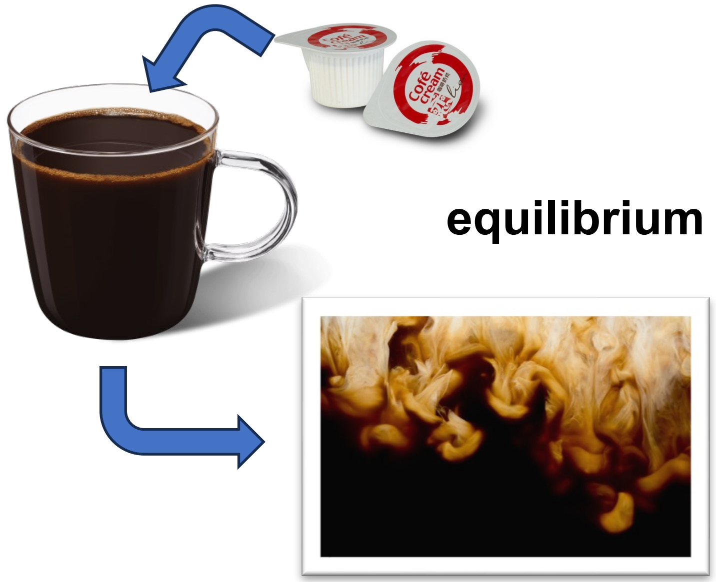
Time crystals are described as: **spatio-temporally ordered phases of matter** (arxiv: 1910.10745)

→ **out-of-equilibrium** phase

× Lose energy

× Requires supply of energy

Intuitive Example



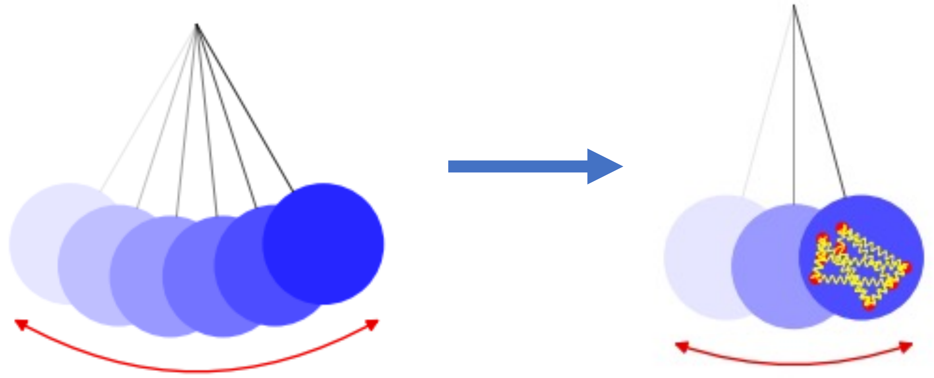
equilibrium

coffee and cream will never mix together

entropy remains stationary

Time Crystal

Second law example:



× Vanished with time evolution (thermalized)

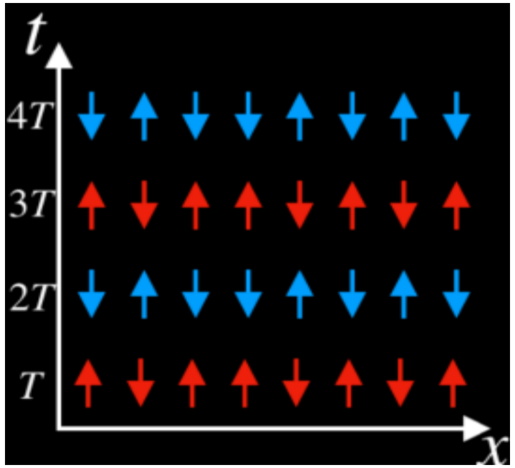
Almost violate!

Laws of thermodynamics

- **The 1st law:** energy is not created or destroyed
- **The 2nd law:** things left to themselves can only become more disordered over time.

Example of time crystal

Driven Ising chain: glassy in the horizontal (space) direction, and period-doubled in the vertical (time) direction.



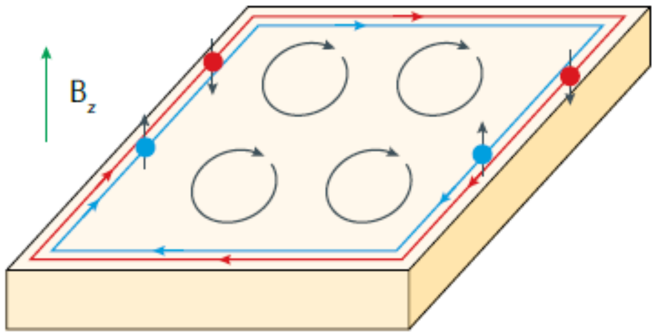
'loophole'

maintain their current level of disorderedness forever

Topological Time Crystal

Example of topological phase

Quantum Hall phase



Topological invariant

- Stable on the boundary
- Can have different state on the inner and boundary

FSPT phases -- Topological Time Crystal

Floquet symmetry-protected topological

- Non-equilibrium topological phase
- Enabled by time-periodic driving

Evolution and Hamiltonian

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Time-periodic Hamiltonian

$$H(t) = \begin{cases} H_1, & for\ 0 \le t < T_1 \\ H_2, & for\ T_1 \le t < T \end{cases}$$

$$H_1 \equiv \left(\frac{\pi}{2}\right) \sum_k \sigma_k^x$$

$$H_2 \equiv -\sum_k J_k\, \sigma_{k-1}^z \sigma_k^x \sigma_{k+1}^z$$

$$J_k \in [0, 2]$$

Evolution and Hamiltonian

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Introduce a periodic driving

- Sum of one-body operators on different sites

$0 \le t < T_1$

Evolution and Hamiltonian

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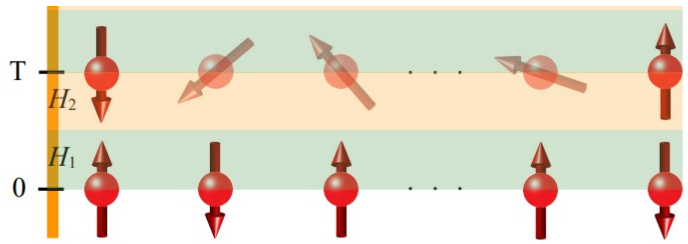
Construct a physical state

- Interactions around neighboring sites
- subtle many-body properties



$T_1 \le t < T$

$0 \le t < T_1$

Evolution and Hamiltonian

Time-periodic Hamiltonian

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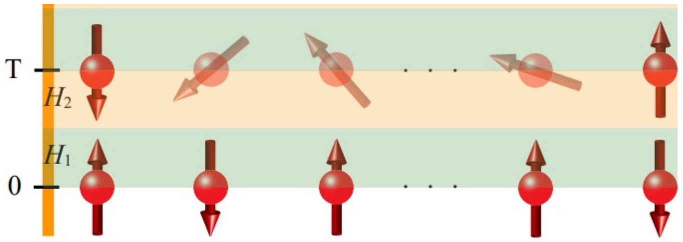

The eigenstates of H_2 :

$$\{|A_k\rangle = |\uparrow \cdots \uparrow\rangle\},$$
$$\{|B_k\rangle = |\downarrow \cdots \downarrow\rangle\},$$
$$\{|C_k\rangle = |\uparrow \cdots \downarrow\rangle\},$$
$$\{|D_k\rangle = |\downarrow \cdots \uparrow\rangle\}.$$

Construct a physical state

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Evolution and Hamiltonian

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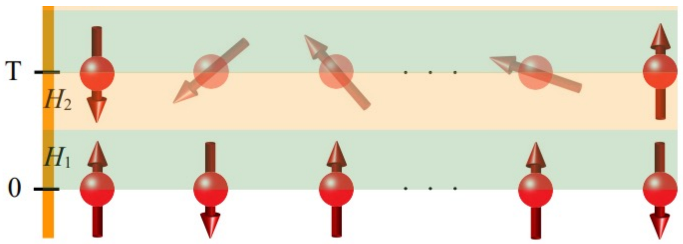

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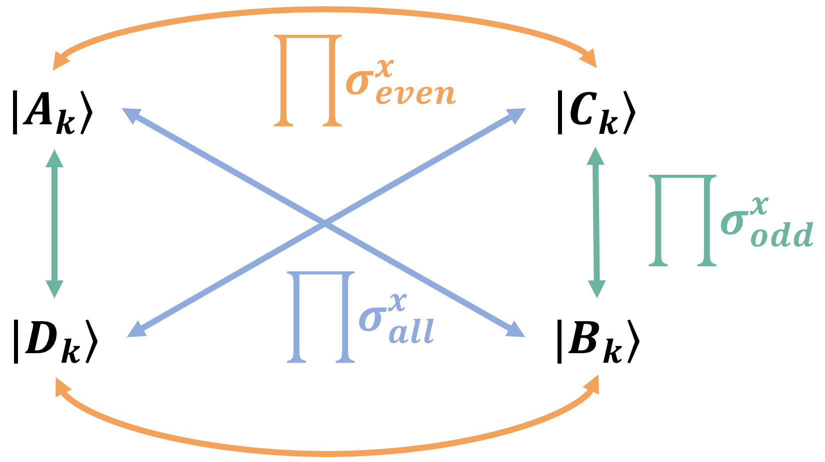

H_2 has $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry:

$$\left[H_2, \prod \sigma_{even}^x\right] = 0,$$
$$\left[H_2, \prod \sigma_{odd}^x\right] = 0.$$
$$\prod \sigma_{even}^x \equiv \prod_k \sigma_{2k}^x ,$$
$$\prod \sigma_{odd}^x \equiv \prod_k \sigma_{2k+1}^x .$$

Evolution and Hamiltonian

Time-periodic Hamiltonian

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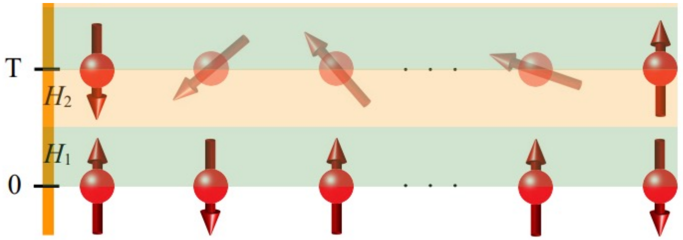

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The eigen-energies:

$$E_{A_k} = E_{B_k} = E_{C_k} = E_{D_k}$$

Floquet operator $U_F = e^{-iH_2}\, e^{-iH_1}$:

$$U_F|A_k\rangle = e^{-iE_{B_k}}|B_k\rangle,$$
$$U_F|B_k\rangle = e^{-iE_{A_k}}|A_k\rangle,$$
$$U_F|C_k\rangle = e^{-iE_{D_k}}|D_k\rangle,$$
$$U_F|D_k\rangle = e^{-iE_{C_k}}|C_k\rangle.$$

Floquet eigenstates:

$$|A_k\rangle \pm |D_k\rangle,$$
$$|B_k\rangle \pm |C_k\rangle.$$

Floquet eigen-energies:

$$\left(E_{A_k} + E_{B_k}\right)/2 = \left(E_{C_k} + E_{D_k}\right)/2,$$
$$\left(E_{A_k} + E_{B_k}\right)/2 + \pi = \left(E_{C_k} + E_{D_k}\right)/2 + \pi.$$

Cat-like linear combinations of topological eigenstates

Long-range correlations between the boundaries

Circuit to Simulate Topological Time Crystal

Circuit to Simulate Topological Time Crystal

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Mathematical formulation

$$H(t) = \begin{cases} H_1, & for\ 0 \le t < T_1 \\ H_2, & for\ T_1 \le t < T \end{cases}$$

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$$H_2 \equiv -\sum_k J_k \sigma_{k-1}^z \sigma_k^x \sigma_{k+1}^z$$

$$J_k \in [0, 2]$$

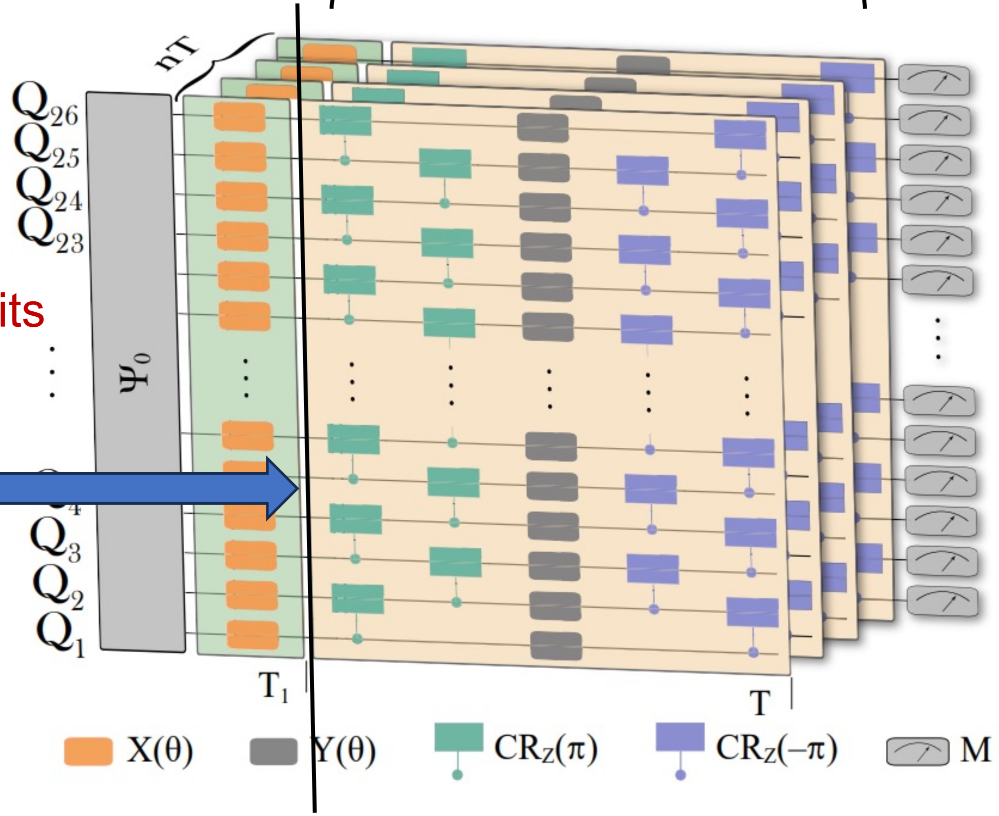
- variational quantum circuits
- Iterative optimization

$$U_2(t) = e^{-iH_2}$$

20 random disorder instances $\{J_k\}$

Circuit Implementation

H_1 t H_2

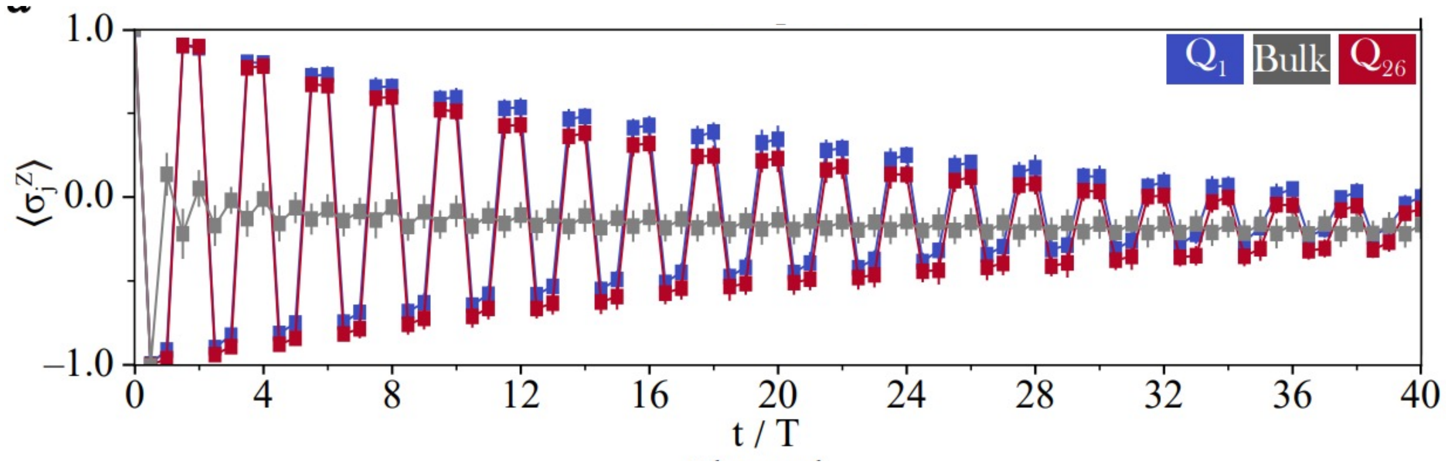


Circuit to Simulate Topological Time Crystal

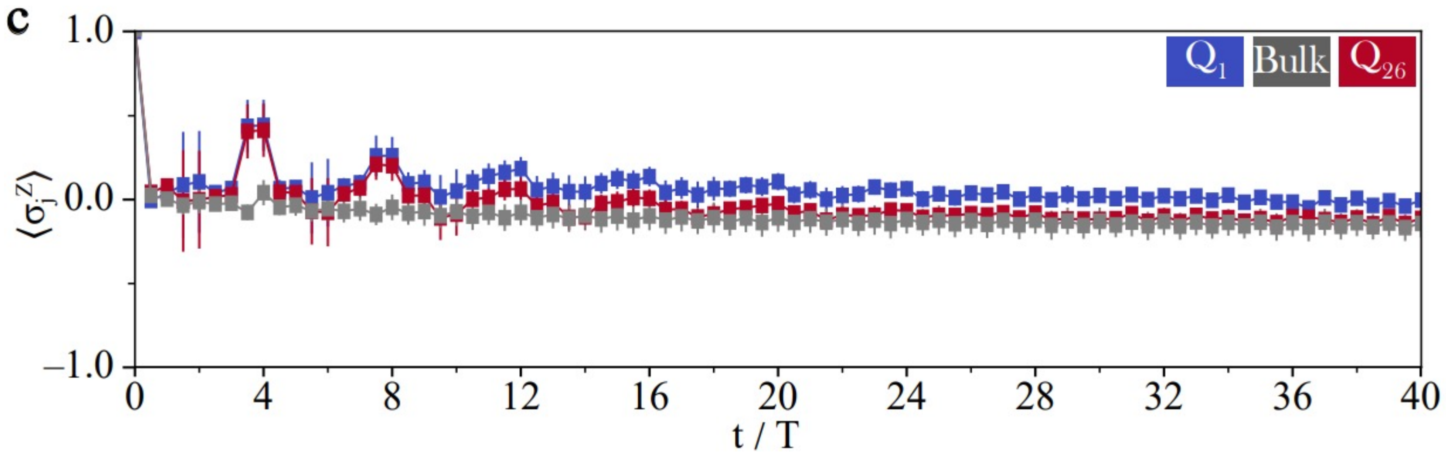
Jupyter notebook: 4-2

Result Interpretation

FSPT phase



thermal phase



$\langle \sigma^Z \rangle$: the expectation of σ^Z, equals to $P_0 - P_1$

- Edge qubits oscillate periodically
- Outcome is independent of the initial state

- The results of all qubits are the same
- No apparent pattern for all qubits

Thanks for listening

Digital quantum simulation of
Floquet symmetry-protected topological phases

Xu Zhang, Wenjie Jiang, Jinfeng Deng, Ke Wang, Jiachen Chen, Pengfei Zhang, Wenhui Ren, Hang Dong, Shibo Xu, Yu Gao, Feitong Jin, Xuhao Zhu, Qiujiang Guo, Hekang Li, Chao Song, Alexey V. Gorshkov, Thomas Iadecola, Fangli Liu, Zhe-Xuan Gong, Zhen Wang*, Dong-Ling Deng* & H. Wang