



WELCOME TO TUTORIAL

Session 4.2 Janus-TC: Simulate time crystal on Janus quantum platform



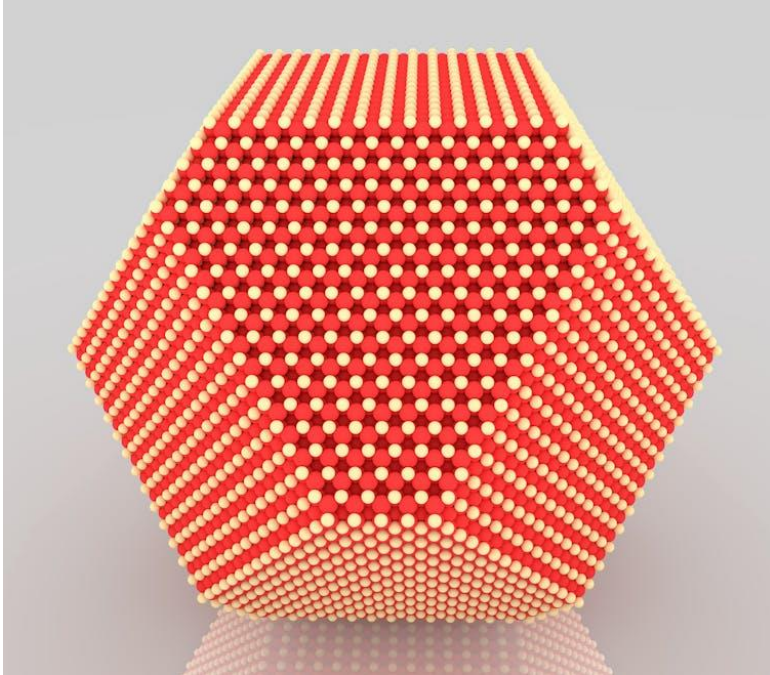
<https://janusq.github.io/tutorials/>

College of Computer Science and Technology,
Zhejiang University

What is time crystal?



Common crystal



- Particles arrange periodically in space.

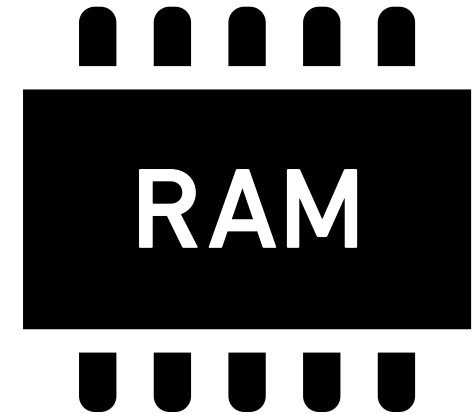
Time crystal



- Particles arrange periodically in **time** and space.

Application

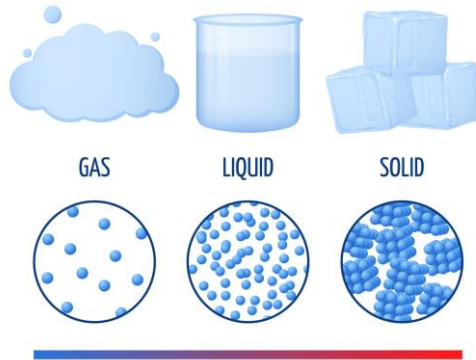
Quantum Memory



- Resilient to heat entropy
- Repeat themselves in time

Different phases of matter

STATES OF MATTER



Time crystal: A new phase

Time crystals are described as: **spatio-temporally ordered phases of matter**
(arxiv: 1910.10745)

out-of-equilibrium phase

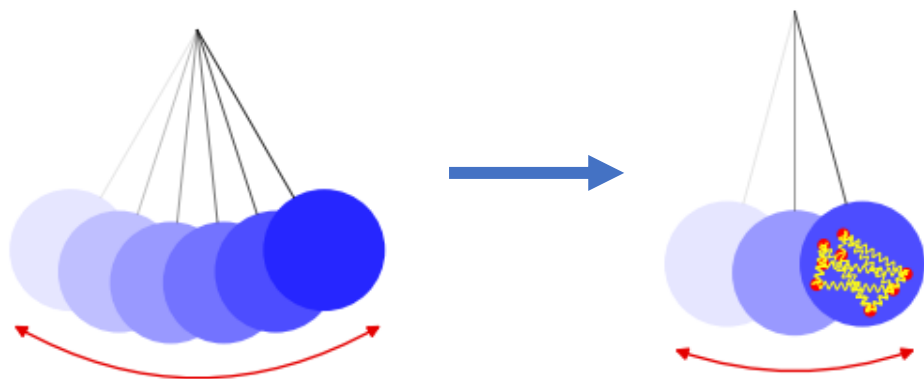
- × Lose energy
- × Requires supply of energy

Intuitive Example



coffee and cream
will never mix together
**entropy remains
stationary**

Second law example:



× Vanished with time evolution (thermalized)

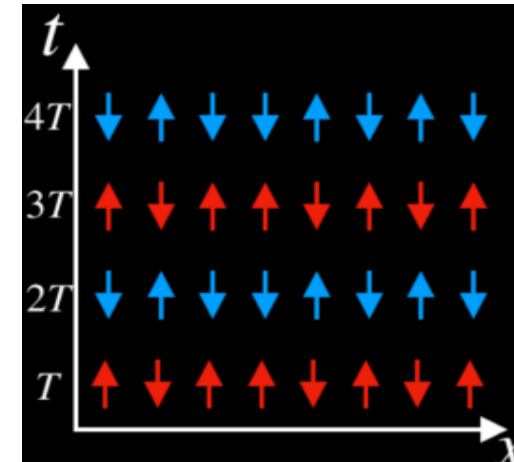
Almost violate!

Laws of thermodynamics

- **The 1st law:** energy is not created or destroyed
- **The 2nd law:** things left to themselves can only become more disordered over time.

Example of time crystal

Driven Ising chain:
glassy in the horizontal (space) direction, and period-doubled in the vertical (time) direction.

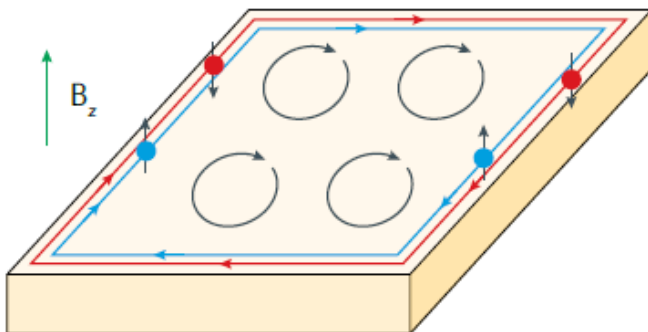


‘loophole’

maintain their current level of disorderedness forever

Example of topological phase

Quantum Hall phase



Topological invariant

- Stable on the boundary
- Can have different state on the inner and boundary

FSPT phases -- Topological Time Crystal

Floquet symmetry-protected topological

- Non-equilibrium topological phase
- Enabled by time-periodic driving

Time-periodic Hamiltonian

$$H(t) = \begin{cases} H_1, & \text{for } 0 \leq t < T_1 \\ H_2, & \text{for } T_1 \leq t < T \end{cases}$$

$$H_1 \equiv \left(\frac{\pi}{2}\right) \sum_k \sigma_k^x$$

$$H_2 \equiv - \sum_k J_k \sigma_{k-1}^z \sigma_k^x \sigma_{k+1}^z$$

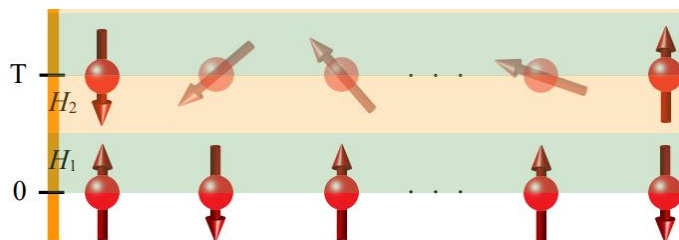
$J_k \in [0, 2]$

The eigenstates of H_2 :

$$\begin{aligned} \{|A_k\rangle &= |\uparrow \cdots \uparrow\rangle\}, \\ \{|B_k\rangle &= |\downarrow \cdots \downarrow\rangle\}, \\ \{|C_k\rangle &= |\uparrow \cdots \downarrow\rangle\}, \\ \{|D_k\rangle &= |\downarrow \cdots \uparrow\rangle\}. \end{aligned}$$

Construct a physical state

- Interactions around neighboring sites
- subtle many-body properties



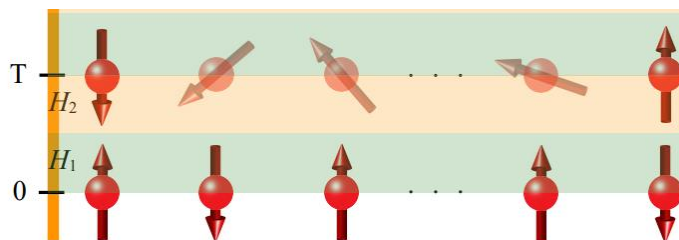
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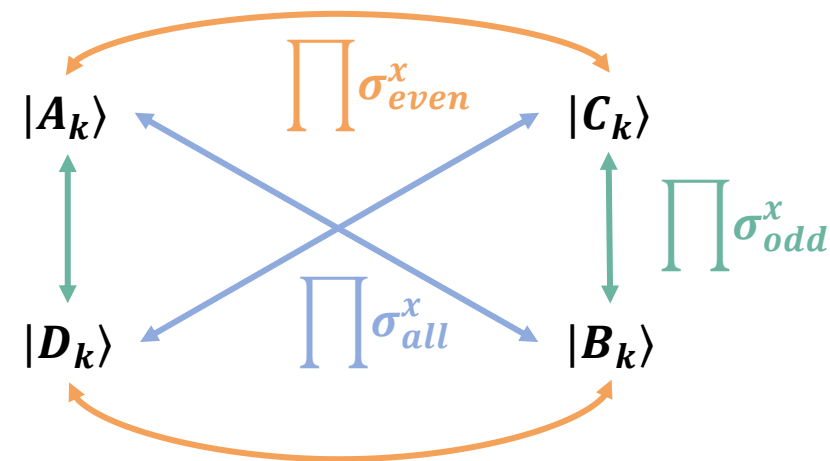
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H_2 has $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry:

$$\begin{aligned} \left[H_2, \prod \sigma_{even}^x \right] &= 0, \\ \left[H_2, \prod \sigma_{odd}^x \right] &= 0. \\ \prod \sigma_{even}^x &\equiv \prod_k \sigma_{2k}^x, \\ \prod \sigma_{odd}^x &\equiv \prod_k \sigma_{2k+1}^x. \end{aligned}$$

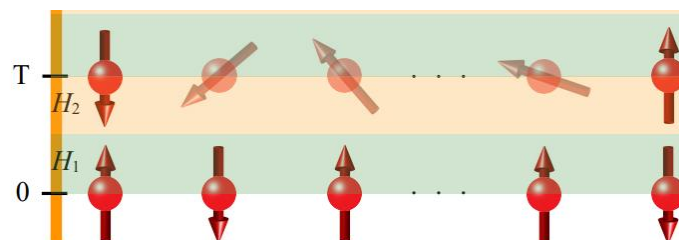
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The eigen-energies:

$$E_{A_k} = E_{B_k} = E_{C_k} = E_{D_k}$$

Floquet eigen-energies:

$$\begin{aligned} (E_{A_k} + E_{B_k})/2 &= (E_{C_k} + E_{D_k})/2, \\ (E_{A_k} + E_{B_k})/2 + \pi &= (E_{C_k} + E_{D_k})/2 + \pi. \end{aligned}$$

Floquet operator $U_F = e^{-iH_2} e^{-iH_1}$:

$$\begin{aligned} U_F |A_k\rangle &= e^{-iE_{B_k}} |B_k\rangle, \\ U_F |B_k\rangle &= e^{-iE_{A_k}} |A_k\rangle, \\ U_F |C_k\rangle &= e^{-iE_{D_k}} |D_k\rangle, \\ U_F |D_k\rangle &= e^{-iE_{C_k}} |C_k\rangle. \end{aligned}$$

Floquet eigenstates:

$$\begin{aligned} |A_k\rangle \pm |D_k\rangle, \\ |B_k\rangle \pm |C_k\rangle. \end{aligned}$$

Cat-like linear combinations of topological eigenstates

Long-range correlations between the boundaries

Circuit to Simulate Topological Time Crystal



Mathematical formulation

$$H(t) = \begin{cases} H_1, & \text{for } 0 \leq t < T_1 \\ H_2, & \text{for } T_1 \leq t < T \end{cases}$$

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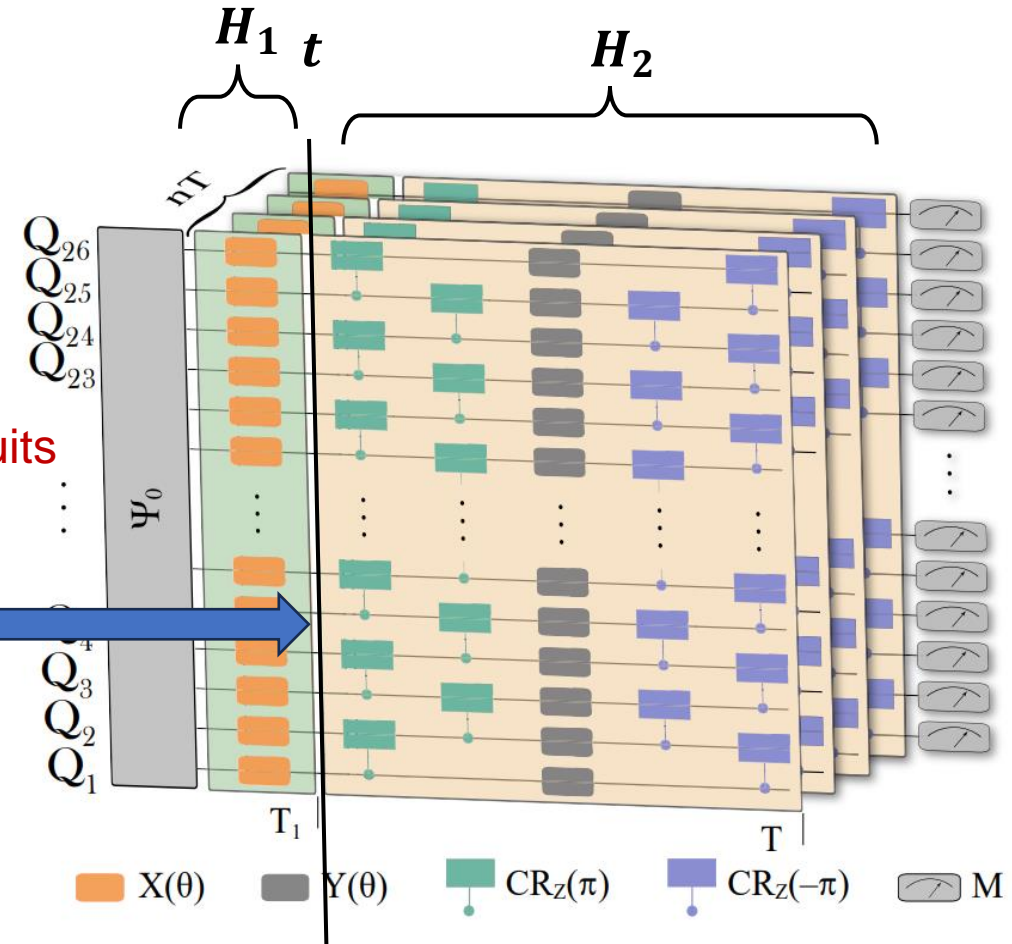
$$J_k \in [0, 2]$$

- variational quantum circuits
- Iterative optimization

$$U_2(t) = e^{-iH_2}$$

20 random disorder instances $\{J_k\}$

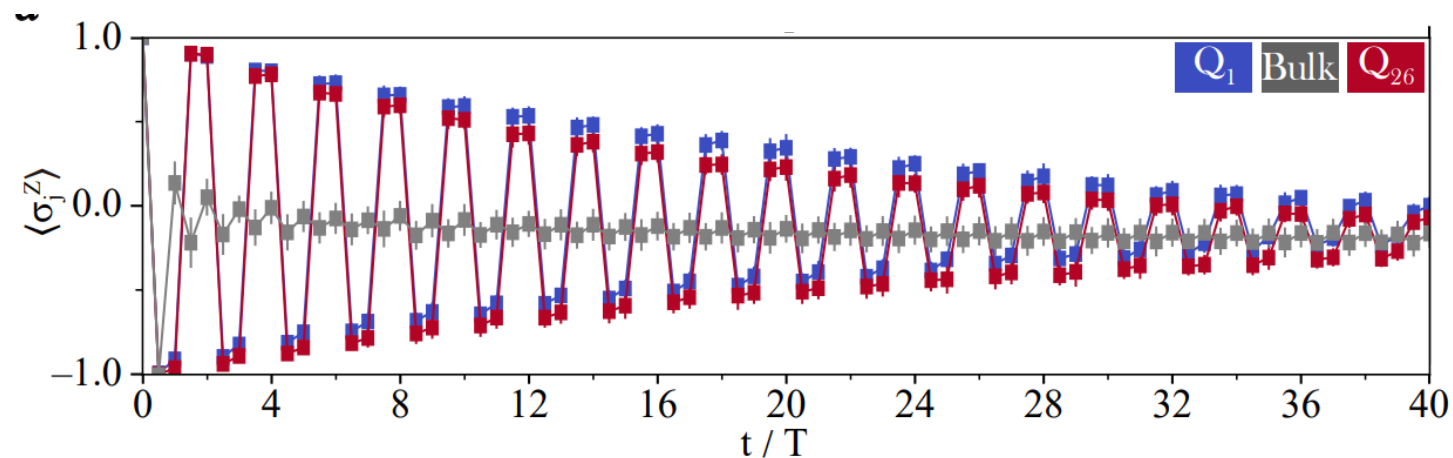
Circuit Implementation





Jupyter notebook: 5-2

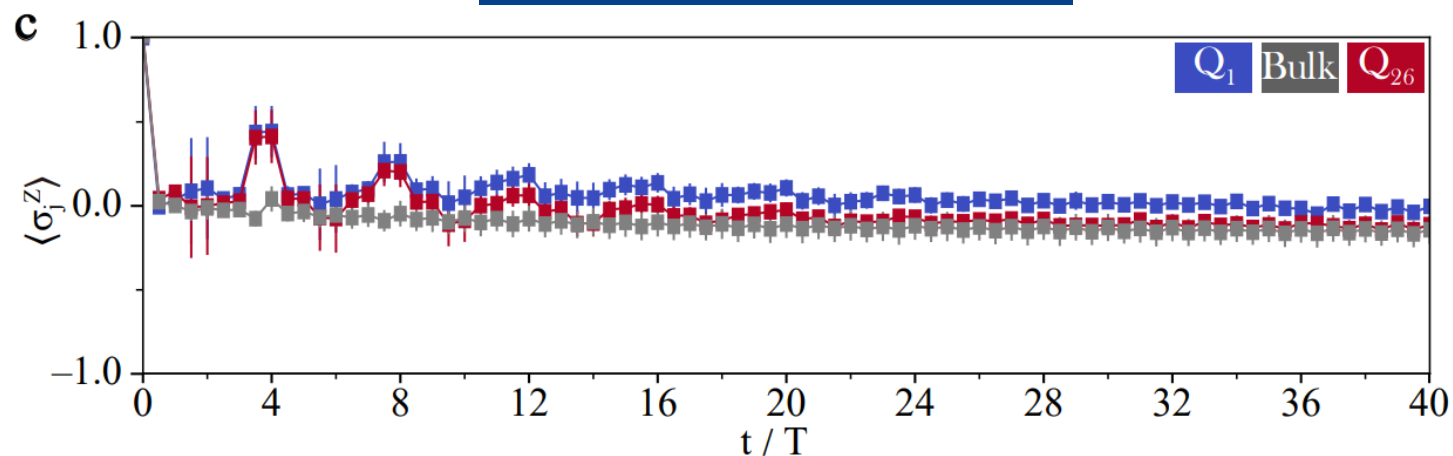
FSPT phase



$\langle \sigma^Z \rangle$: the expectation of σ^Z , equals to $P_0 - P_1$

- Edge qubits oscillate periodically
- Outcome is independent of the initial state

thermal phase



- The results of all qubits are the same
- No apparent pattern for all qubits



Thanks for listening

Digital quantum simulation of Floquet symmetry-protected topological phases

Xu Zhang, Wenjie Jiang, Jinfeng Deng, Ke Wang, Jiachen Chen, Pengfei Zhang,
Wenhui Ren, Hang Dong, Shibo Xu, Yu Gao, Feitong Jin, Xuhao Zhu, Qiujiang Guo,
Hekang Li, Chao Song, Alexey V. Gorshkov, Thomas Iadecola, Fangli Liu,
Zhe-Xuan Gong, Zhen Wang*, Dong-Ling Deng* & H. Wang