Assignment 3

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1 Question 1

1.1 a

$$\begin{aligned} Y_t &= \theta X_{t-1} + Z_t + W_t \\ &= \theta^2 X_{t-2} + Z_t + \theta^1 Z_{t-1} + W_t \\ &= \theta^3 X_{t-3} + Z_t + \theta^1 Z_{t-1} + \theta^2 Z_{t-2} + W_t \\ &= \theta^t X_0 + \sum_{i=0}^{t-1} \theta^i Z_{t-i} + W_t \end{aligned}$$

$$\begin{aligned} E[Y_t] &= E[\theta^t X_0 + \sum_{i=0}^{t-1} \theta^i Z_{t-i} + W_t] \\ &= E[\theta^t X_0] + E[\sum_{i=0}^{t-1} \theta^i Z_{t-i}] + E[W_t] \\ &= 0 + 0 + 0 = 0 \end{aligned}$$

$$\gamma_y(t+h,t) = E[(\theta^t X_0 + \sum_{i=0}^{t-1} \theta^i Z_{t-i} + W_t - 0) * (\theta^{t+h} X_0 + \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} + W_{t+h} - 0)]$$

$$\begin{aligned}
&= E[(\theta^{2t+h} X_0^2 + \theta^t X_0 * \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} + \theta^{t+h} X_0 * \sum_{i=0}^{t-1} \theta^i Z_{t-i} + \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} * \sum_{i=0}^{t-1} \theta^i Z_{t-i} \\
&\qquad + \theta^t X_0 W_{t+h} + \theta^{t+h} X_0 W_t + \sum_{i=0}^{t-1} \theta^i Z_{t-i} * W_{t+h} + \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} * W_t] \\
&= E[\sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} * \sum_{j=0}^{t-1} \theta^i Z_{t-j}] \\
&= \sum_{i=h}^{t-1} \theta^i \theta^{i-h} \sigma_z^2 \\
&= \theta^h \sigma_z^2 \sum_{i=0}^{t-1} \theta^{2i} \sigma_z^2 \\
&= \theta^h \sigma_z^2 / (1-\theta^2)
\end{aligned}$$

So its stationary.

1.2 b

1-correlated if $\gamma_y(0) \neq 0$ **and** $\gamma_y(h) = 0$ for $|h| > 1$

$$\begin{aligned}
\gamma_u &= Cov(Y_{t+h} - \theta Y_{t-1+h}, Y_t - \theta Y_{t-1}) && (1) \\
&= E((Y_{t+h} - \theta Y_{t-1+h} - 0)(Y_t - \theta Y_{t-1} - 0)) && (2) \\
&= E(Y_{t+h} Y_t + Y_{t+h} \theta Y_{t-1} - \theta Y_{t-1+h} Y_t + Y_{t-1+h} \theta Y_{t-1}) && (3)
\end{aligned}$$

1.3 c

2 Question 2

2.1 a

2.2 b

2.3 c

3 Question 3

3.1 a

3.2 b

3.3 c