Assignment 3

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1 Question 1

1.1 a

$$Y_t = \theta X_{t-1} + Z_t + W_t$$
$$= \theta^2 X_{t-2} + Z_t + \theta^1 Z_{t-1} + W_t$$
$$= \theta^3 X_{t-3} + Z_t + \theta^1 Z_{t-1} + \theta^2 Z_{t-2} + W_t$$
$$= \theta^t X_0 + \sum_{i=0}^{t-1} \theta^i Z_{t-i} + W_t$$

$$E[Y_t] = E[\theta^t X_0 + \sum_{i=0}^{t-1} \theta^i Z_{t-i} + W_t]$$
$$= E[\theta^t X_0] + E[\sum_{i=0}^{t-1} \theta^i Z_{t-i}] + E[W_t]$$
$$= 0 + 0 + 0 = 0$$

$$\gamma_y(t+h,t) = E[(\theta^t X_0 + \sum_{i=0}^{t-1} \theta^i Z_{t-i} + W_t - 0) * (\theta^{t+h} X_0 + \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} + W_{t+h} - 0)]$$

$$= E[(\theta^{2t+h} X_0^2 + \theta^t X_0 * \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} + \theta^{t+h} X_0 * \sum_{i=0}^{t-1} \theta^i Z_{t-i} + \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} * \sum_{i=0}^{t-1} \theta^i Z_{t-i}$$
$$+ \theta^t X_0 * W_t + h + \theta^{t+h} W_t + \sum_{i=0}^{t-1} \theta^i Z_{t-i} * W_{t+h} + \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} * W_t]$$

$$= 0 + 0 + 0 + E[\sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} * \sum_{i=0}^{t-1} \theta^i Z_{t-i}]$$

1.2 b

1.3 c

2 Question 2

2.1 a

2.2 b

2.3 c

3 Question 3

3.1 a

3.2 b

3.3 c