Assignment 3

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1 Question 1

1.1 a

$$\begin{aligned} X_t &= \theta X_{t-1} + Z_t \\ &= \theta^2 X_{t-2} + Z_t + \theta^1 Z_{t-1} \\ &= \theta^3 X_{t-3} + Z_t + \theta^1 Z_{t-1} + \theta^2 Z_{t-2} \\ &= \theta^t X_0 + \sum_{i=0}^{t-1} \theta^i Z_{t-i} \end{aligned}$$

$$\begin{aligned} E[X_t] &= E[\theta^t X_0 + \sum_{i=0}^{t-1} \theta^i Z_{t-i}] \\ &= E[\theta^t X_0] + E[\sum_{i=0}^{t-1} \theta^i Z_{t-i}] \\ &= 0 + 0 = 0 \end{aligned}$$

$$\begin{aligned} \gamma_y(t+h,t) &= E[(\theta^t X_0 + \sum_{i=0}^{t-1} \theta^i Z_{t-i} - 0) * (\theta^{t+h} X_0 + \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} - 0)] \\ &= E[(\theta^{2t+h} X_0^2 + \theta^t X_0 * \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} + \theta^{t+h} X_0 * \sum_{i=0}^{t-1} \theta^i Z_{t-i} + \sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} * \sum_{i=0}^{t-1} \theta^i Z_{t-i}] \\ &= 0 + 0 + 0 + E[\sum_{i=0}^{t+h-1} \theta^i Z_{t+h-i} * \sum_{i=0}^{t-1} \theta^i Z_{t-i}] \end{aligned}$$