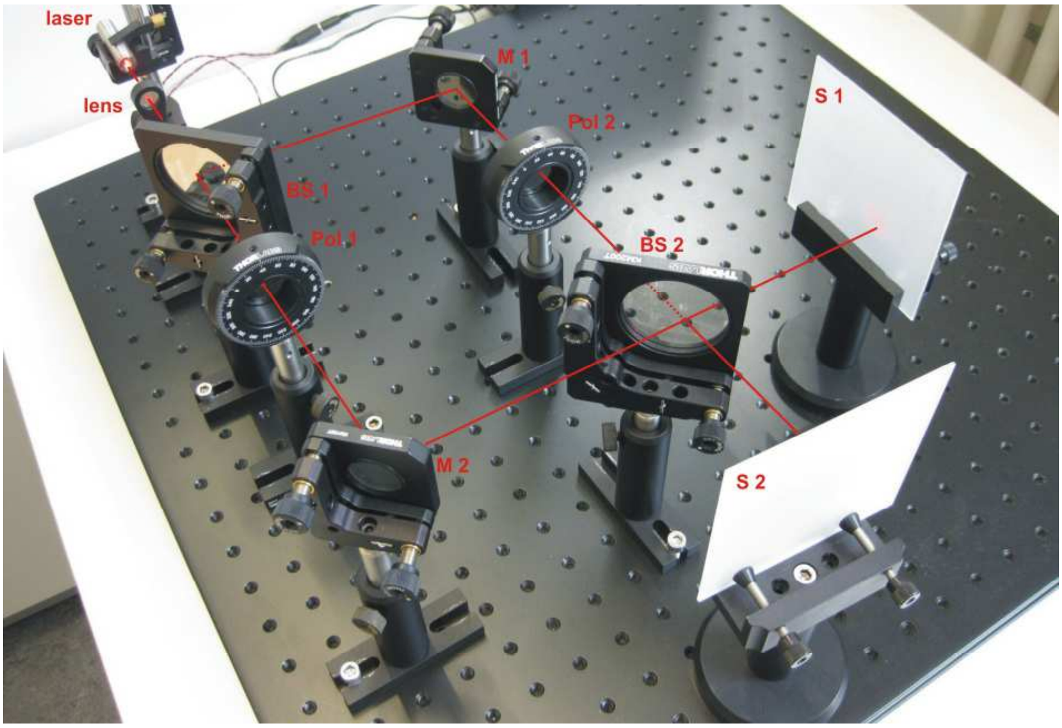
# Quantum Eraser
# KSOP Optics & Photonics Lab



# Contents

# 1 Exercises

**Exercise 1**
Set up the Mach–Zehnder interferometer. Align the optical components as described in section 4.

**Exercise 2**
Place polarisers in the two legs of the interferometer. Observe the interference pattern when changing the orientation of the two polarisers from parallel to perpendicular. Explain your observations both classically and in a quantum-mechanical picture (‘Which-Way-Information’).

**Exercise 3**
Keep the orientation of the two polarisers perpendicular and place a third polariser, the so-called quantum eraser, in front of one observation screen. Adjust the third polariser such that it is oriented $\pm 45°$ with respect to the polarisers in the two legs of the interferometer, respectively. What happens to the interference pattern behind the ‘quantum eraser’ compared to the pattern on the second screen? Explain your observation classically and in a quantum-mechanical picture (‘Which-Way-Information’).

**Exercise 4**
Assuming the light source is not a laser but a source of frequently emitted single photons. How will the time-averaged interference pattern change when only single photons pass the interferometer instead of a continuous laser beam?

# 2 Introduction: Analogue of Light and Quantum Interference

The aim of this experiment is to demonstrate quantum-mechanical effects with simple optical elements. All observations made can be explained by classical wave optics, but can also be interpreted quantum-mechanically. This section will emphazise the analogue of these two points of view.

**Wave optics** In wave optics a light beam can be described by a superposition of plane waves as they are solutions for the electric and magnetic fields of Maxwell's equations. All discussions can therefore be made based on a single plane wave $\vec{E}(\vec{x},t) = \vec{E}_0 \mathrm{e}^{\mathrm{i}(\vec{k}\vec{x}-\omega t)}$. The direction of the vector $\vec{E}_0$ represents the polarisation of light and the exponent describes its propagation. The critical part in analogy to quantum mechanics is the calculation of the photon density. For monochromatic light the photon density $n_{\mathrm{ph}}(\vec{x},t)$ is proportional to the light intensity $I(\vec{x},t) = \vec{E}(\vec{x},t)\vec{E}^*(\vec{x},t)$:

$$n_{\mathrm{ph}}(\vec{x},t) = \frac{I(\vec{x},t)}{\hbar\omega} = \frac{|\vec{E}_0|^2}{\hbar\omega}$$

**Light interference** Assuming we divide a plane wave $\vec{E}(z,t) = \vec{E}_0 \mathrm{e}^{\mathrm{i}(k_z z-\omega t)}$ into two rays with a beam splitter, delay one of the rays by a distance $\Delta z$ and recombine them with a second beam splitter (Fig. 1), the electric field of the light beam at the position $z$ after the second beam splitter would be given by the superposition of the two rays

$$\vec{E}_{\mathrm{ges}}(z,t) = \frac{1}{4}\vec{E}_0 \left(\mathrm{e}^{\mathrm{i}k_z z} + \mathrm{e}^{\mathrm{i}k_z(z+\Delta z)}\right) \mathrm{e}^{\mathrm{i}\omega t}$$

with the resulting photon density

$$\begin{aligned} n_{\mathrm{ph}}(z,t) &= \frac{|\vec{E}_0|^2}{4\hbar\omega}(\mathrm{e}^{\mathrm{i}k_z z} + \mathrm{e}^{\mathrm{i}k_z(z+\Delta z)})(\mathrm{e}^{-\mathrm{i}k_z z} + \mathrm{e}^{-\mathrm{i}k_z(z+\Delta z)}) \\ &= \frac{|\vec{E}|^2}{2\hbar\omega}(1+\cos(k_z\Delta z)). \end{aligned}$$

Treating photons as classical particles would result in a photon density only determined by the sum of the two intensities:

$$n_{\mathrm{ph}}(z,t) = \frac{|\vec{E}|^2}{2\hbar\omega}$$

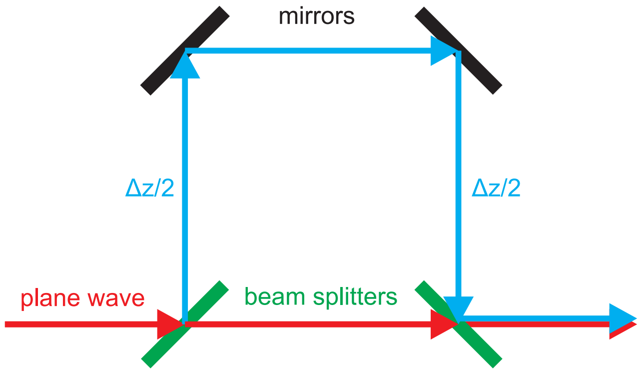



**Figure 1:** Sketch of a simple interferometer.

Therefore, the term $|\vec{E}_0|^2 \cos(k_z z)$ is called the 'interference term', the argument $(k_z z)$ 'phase shift'. The interference term contains the difference between waves and the classical description of particles, in which an initial probability density determines the temporal development of a system completely. The appearance of the interference term could never be understood when treating photons as classical particles.

**Interference in quantum mechanics** The idea of quantum mechanics is to treat single particles like electrons or atoms in equivalence to electro-magnetic waves. In this theory the electric field of a light wave gets the role of a density amplitude or wave function $\Psi(\vec{r}, t)$ of a probability density $\rho(\vec{r}, t) = \Psi(\vec{r}, t)\Psi^*(\vec{r}, t)$. The development of the density amplitude is given by Schrödinger's equation instead of the Helmholtz equation for electro-magnetic waves. The density amplitude is complex in general, in contrast to the plane wave electric fields we mentioned previously, where we use complex values only for convenience.

To demonstrate the equivalent nature of quantum particles and waves an experiment was carried out by C. Jönsson in 1961 [1, 2]. He performed the well-known double-slit experiment with electrons instead of coherent light. As in an interferometer the two rays of the slits A and B interfere on a detection screen with an angle-dependent phase shift. The probability density is then given by

$$\begin{aligned} \rho_{\mathrm{AB}}(\vec{r}, t) &= (\Psi_{\mathrm{A}}(\vec{r}, t) + \Psi_{\mathrm{B}}(\vec{r}, t))(\Psi_{\mathrm{A}}{}^*(\vec{r}, t) + \Psi_{\mathrm{B}}{}^*(\vec{r}, t)) \\ &= |\Psi_{\mathrm{A}}|^2 + |\Psi_{\mathrm{B}}|^2 + \Psi_{\mathrm{A}}(\vec{r}, t)\Psi_{\mathrm{B}}{}^*(\vec{r}, t) + \Psi_{\mathrm{B}}(\vec{r}, t)\Psi_{\mathrm{A}}{}^*(\vec{r}, t) \end{aligned}$$

Again an interference term $\Psi_{\mathrm{A}}(\vec{r}, t)\Psi_{\mathrm{B}}{}^*(\vec{r}, t) + \Psi_{\mathrm{B}}(\vec{r}, t)\Psi_{\mathrm{A}}{}^*(\vec{r}, t)$ appears in the equation. The mystic property of quantum particles is, that, similar to waves, this is not an interference of many particles but an interference of probability amplitudes, which means it appears also for a single particle. In consequence the possibilities of the electron to pass either slit A or B interfere at the detection screen.

# 3 The Quantum Eraser

In the previous section we only discussed the interference pattern appearing after a certain phase shift of the particle wave function $\Psi(\vec{r},t)$. In principle we can also manipulate the wave function by changing properties of the particle, that can lead to more complex and interesting observations. Because it is much more intuitive to understand most quantum effects when discussing light waves, we will describe what happens to the interference term when changing the polarisation of light. But remember that the general results are the same when manipulating arbitrary properties of quantum particles as, e.g., the spin of an electron.

**Manipulating the wave function** Let us return to the propagating plane wave $E(z,t) = \vec{E}_0 \mathrm{e}^{\mathrm{i}(k_z z - \omega t)}$. Since it propagates along the z-direction the wave must be polarised in the $x$-$y$ plane. We send the wave in the two arms of a so-called Mach–Zehnder interferometer (Fig. 2), in which we can change the polarisation of light in the two legs independently by two polarisers. We assume the initial wave is linearly polarised in x-direction ($\vec{E}_0 = E_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$). When the axis of the polariser is tilted by an angle $\Theta$ with respect to the polarisation of the light beam, it 'rotates' the polarisation according to the matrix:

$$\hat{R}(\Theta) = \cos(\Theta) \begin{pmatrix} \cos(\Theta) & -\sin(\Theta) \\ \sin(\Theta) & \cos(\Theta) \end{pmatrix}$$

Now we align the polarisers to 45° and -45°, respectively. This results in two beams:

$$\begin{aligned} \vec{E}_A(z,t) &= \frac{E_0}{2\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathrm{e}^{\mathrm{i}(k_z z - \omega t)} \\ \vec{E}_B(z,t) &= \frac{E_0}{2\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathrm{e}^{\mathrm{i}(k_z (z+\Delta z) - \omega t)}. \end{aligned}$$

Since $\vec{E}_A$ and $\vec{E}_B$ are obviously orthogonal the interference term $\vec{E}_A \vec{E}_B^* + \vec{E}_B \vec{E}_A^*$ vanishes when calculating the photon density $n_{\mathrm{ph}}(\vec{x},t)$ after the second beam splitter. The photon density $n_{\mathrm{ph}}(\vec{x},t) = \frac{E_0^2}{2\hbar\omega}$ is equal to the result, when photons are treated as 'classical' particles.

Of course, the question should arise where the phase shift $\Delta z$ originates from. In the real experiment we use a lens to make the beam slightly divergent. The interference of the partial beams creates a concentric intensity pattern on the screens. In our theoretical discussion we can simply add a delay line in one leg of the interferometer.

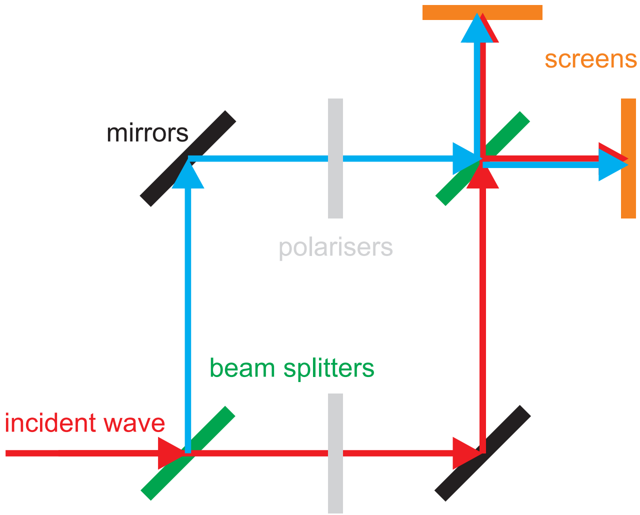



**Figure 2:** Sketch of a Mach–Zehnder interferometer.

**The quantum eraser** The situation will change when we add a third polariser, the 'quantum eraser', in front of the first screen. If we align it to 45° or -45° with respect to the initial wave we will see the intensities of beam A or B, respectively. But when we align it to 0°, surprisingly, the interference pattern will reappear. in this case, the electric fields of the two rays after the third polariser will be given by:

$$\begin{aligned}\vec{E}_A(z,t) &= \frac{E_0}{4}\begin{pmatrix}1\\0\end{pmatrix}\mathrm{e}^{\mathrm{i}(k_z z-\omega t)}\\ \vec{E}_B(z,t) &= \frac{E_0}{4}\begin{pmatrix}1\\0\end{pmatrix}\mathrm{e}^{\mathrm{i}(k_z(z+\Delta z)-\omega t)}\end{aligned}$$

The two electric fields are no longer perpendicular and the photon density is determined by

$$n_{\mathrm{ph}}(z) = \frac{E_0}{8\hbar\omega}(1+\cos(k_z\Delta z)).$$

If $\Delta z = \pi/k_z$ the photon density on the first screen changes from $E_0/(4\hbar\omega)$ to 0 when inserting the third polariser.

**The Which-Way-Information** As a consequence of the effects discussed in this section, one can conclude that the interference term only occurs when we cannot determine which way a photon has gone. When we polarise the two beams perpendicularly, a measurement of the polarisation in front of the screens gives us information about the paths the photons chose, but the interference pattern disappears. When we insert the eraser the interference reappears, since we cannot decide which path the photons went. Using a different point of view, the quantum interference disappears as soon as we gain a ‘Which-Way-Information’. This is the origin of the nomenclature ‘Quantum Eraser’.

# 4 Alignment of the Mach–Zehnder Interferometer

**Description of symbols:**

| | |
|---|---|
| **BS 1/2:** | Beam splitter 1/2 |
| **M 1/2:** | Mirror 1/2 |
| **S 1/2:** | Screen 1/2 |
| **Pol 1/2:** | Polariser 1/2 |

**Align the laser** Fix the laser to one corner of the optical breadboard by using the Kinematic VGroove mount. Align the laser beam to be parallel to the breadboard. Attach a piece of paper to one of the screens and mark the height of the laser beam immediately behind the laser. Move the screen to the end of the breadboard and align the vertical tilt of the laser until the laser spot is at the same height as your mark. From now on, do not touch the laser again.

**Align the lens** Move the screen until the laser beam hits exactly the mark on your paper. Insert the lens into the beam path and adjust its height and position until the mark is centered in the laser spot again. Flip the lens off the beam path.

**Align the first beam splitter** Insert the first beam splitter close to the lens as can be seen in Fig. 3. The laser should hit the beam splitter centrally. Tilt the beam splitter horizontally until the deflected beam is parallel to the breadboard. Use your mark to align the beam splitter vertically until the deflected laser beam is on the right height.

**Align the mirrors** Insert the mirrors as in Fig. 3. The laser should hit the mirrors in their center. Align the mirrors horizontally until the beams are parallel to the breadboard. Use your mark on the screen to align the mirrors vertically.

**Alignment of beam splitter BS2** Insert the second beam splitter such that its center is as close to the crossing of the two beams as possible (Fig. 4). Align the beam splitter horizontally until the deflected beams are parallel to the breadboard. Use your mark to align the deflected beams vertically. Remove your mark and place the two screens in the output beams.

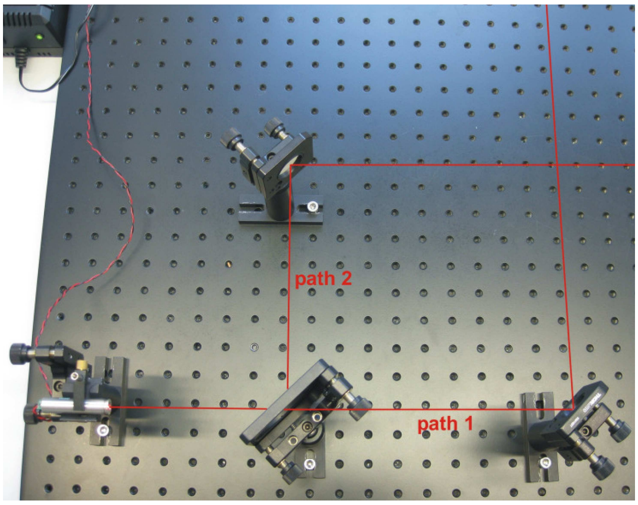



**Figure 3:** Alignment of BS1, M1 and M2.

Tilt the beam splitter vertically and horizontally until the spots on S1 and S2 seem to be mirrored at the plane of beam splitter BS2 (Fig. 5a). E.g., if the spot of beam 2 on S1 is 1 cm above and 0.5 cm on the right of the spot of beam 1, it should be 1 cm above and 0.5 cm on the left on screen S2. Please keep in mind that when you tilt the beam splitter, beam 2 will move on screen S1 and beam 1 will move on screen S2.

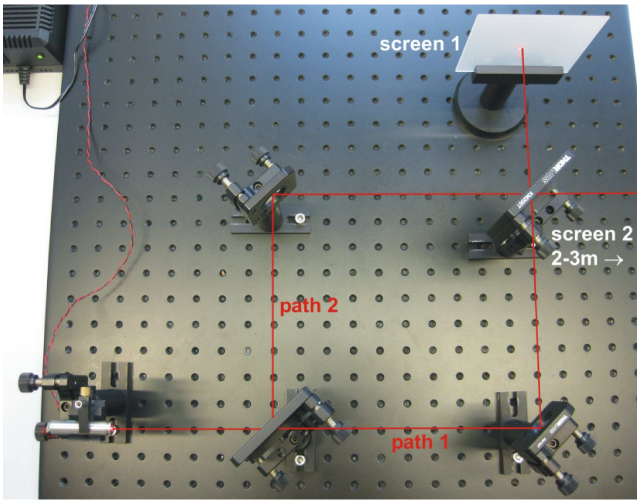



**Figure 4:** Alignment of BS2.

**Applying a beam walk** A beam walk is an iterative adjustment algorithm to deflect an initial beam into an arbitrary direction. It is very basic for the adjustment of optical beam paths. With two mirrors it is in principle possible to deflect an initial beam in every desired direction although the polarisation can change under certain conditions. After the two mirrors have roughly been adjusted, one has to choose two points along the desired beam path. Everything required afterwards is to repeat the following two steps until the beam hits both points:

1. Adjust the first mirror until the beam hits the first point.
2. Adjust the second mirror until the beam hits the second point.

After a few iterations the beam should be aligned perfectly.

**Alignment of the two paths** Apply the beam walk algorithm to adjust BS1 and M1. The two points that beam 2 should hit are the spots of beam 1 on BS2 and S2 (Fig. 5b). Adjust BS1 until beam 2 hits beam 1 at the beam splitter BS2, adjust M1 until beam 2 hits beam 1 on the screen S2 and repeat this action. The spots on S1 should fit automatically, if they do not, return to 'Alignment of beam splitter BS2'.

**Fine adjustment** Flip the lens into the beam and the interference pattern should appear on both screens (Fig. 6). Maybe you cannot observe concentric interference rings, but stripes. In this case, the interference occurs due to tilted beams. If you observe vertical stripes, try to adjust M1 and M2 horizontally in the same way. If you see horizontal lines, try to adjust M1 and M2 vertically in the same way, until the ring pattern occurs.

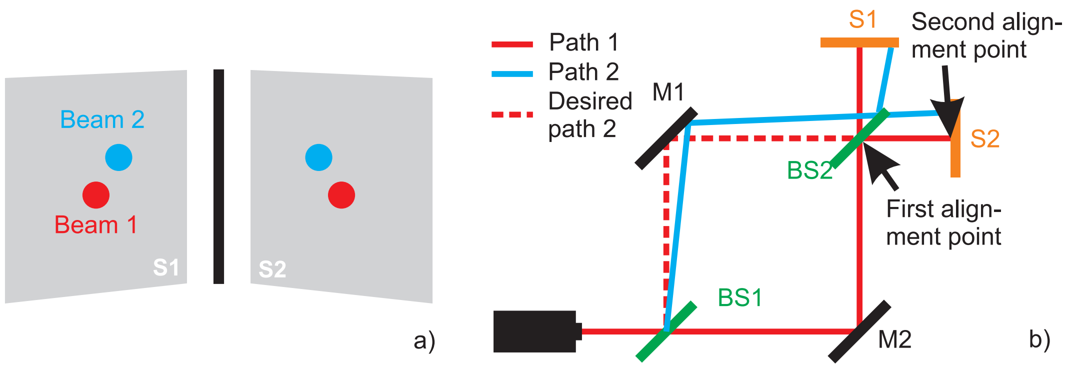



**Figure 5:** Alignment of the two paths.

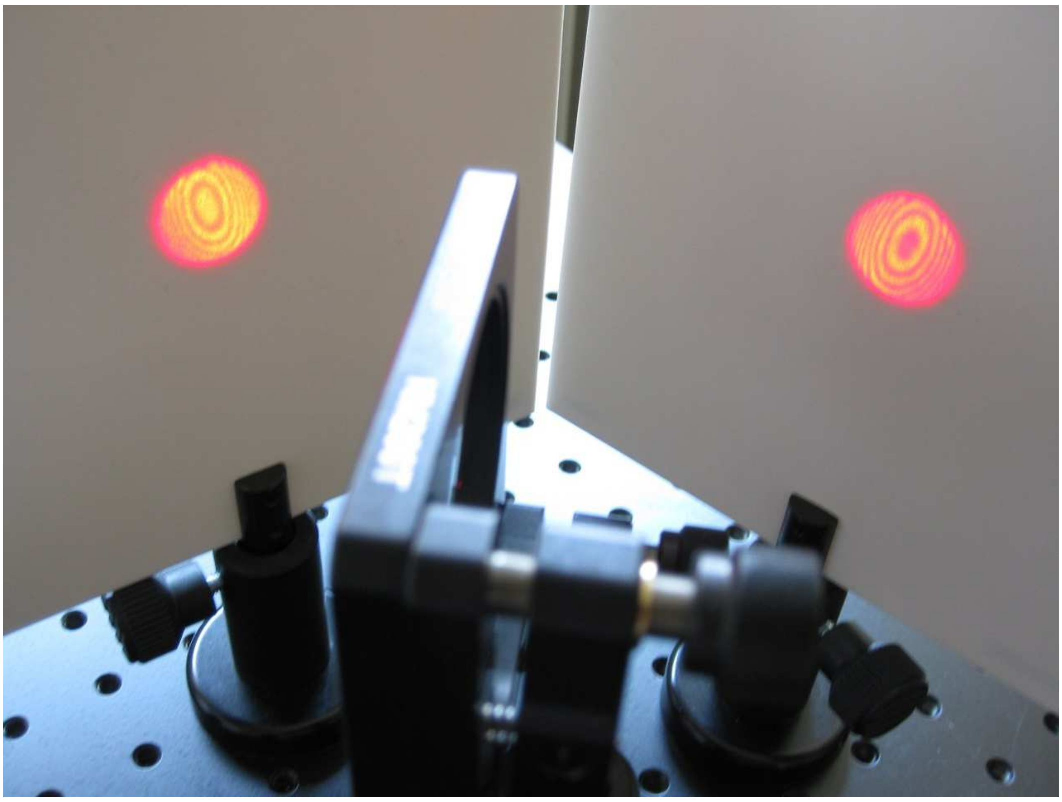


**Figure 6:** Interference pattern.

# References

[1] C. Jönsson: Elektroneninterferenzen an mehreren künstlich hergestellten Feinspalten. *Zeitschrift für Physik A Hadrons and Nuclei*, **161**:4, 1961.

[2] C. Jönsson: Electron Diffraction at Multiple Slits. *AAPT*, **42**:1, 1974.