Week 5

Problem 1

Prove that e is irrational.

Sol. assume e ~~exi~~ is rational

It exists $e = \frac{a}{b}$, $b \neq 0$

a, b pos integers.

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

$$x = b!\left(\frac{a}{b} - \sum_{n=0}^{b} \frac{1}{n!}\right) =$$

$$= b!\left(\frac{a}{b}\right) - \sum_{n=0}^{b} \frac{b!}{n!}$$

$$x = \underbrace{a(b-1)!}\ - \sum_{n=0}^{b} \frac{b!}{n!}$$

$$x = b!\Big(\underbrace{\sum_{n=0}^{\infty}\frac{1}{n!}}^{\text{ind}} - \underbrace{\sum_{n=0}^{b}\frac{1}{n!}}^{\text{int}}\Big)$$

$$= \underbrace{\sum_{n=b+1}^{\infty}\frac{b!}{n}}_{\text{Positive}}$$

$$\frac{b!}{n!} = \frac{b\cdot(b-1)(b-2)\cdots}{n(n-1)(n-2)\cdots}$$

$$= \frac{1}{n(n-1)(n-2)\cdots(b+1)} \leq \frac{\cancel{b}}{(b+1)}$$

geom.

$$0 < x = \sum_{n=b+1}^{\infty}\frac{b!}{n!} < \sum$$

Proof $1+1=2$

$\mathbb{N} = \{1, 2, 3, \dots\}$

$0 \in \mathbb{N}$

~~If $n \in \mathbb{N}$~~

Def

If $b=0$: $a+b=a$

If $b \neq 0$: $a+b = S(a+c)$ wit $b = S(c)$

$a=1$, $b=1$

$\Rightarrow b = S(0)$

$1+1 = S(1+0) = S(1) = 2$

Problem 2

Use a direct proof to show that every odd int is the difference of two squares.

of two sq...

Sol let's $a^2 - b^2$

$a^2 - b^2 = (a+b)(a-b)$

Since n is odd we can write

$n = 2k+1$ for some k.

Then $(k+1)^2$

if $n = 2k+1$, we let $a = k+1$, $b = k$

$(k+1)^2 - k^2 = k^2 + 2k + 1 - \cancel{k^2} = 2k+1$

$= n$

Problem 3

Use a proof by contradiction to proove that the sum of an irrational number and a ration. number is irrational

Sol: If r is rat and i irr.
then $s = r + i$ is irr.

We ass. $s = r + i$ is rat.
Sum of rat numbers, so
$-r$ and s must be rational.

($s = \frac{a}{b}$, $r = \frac{c}{d}$ where $d \neq 0$, $b \neq 0$
all integer

So $s + (-r) = (ad - bc)/bd$

So $s + (-r)$ is rat.
But $s + (-r) = r + i - r = i$
$\Rightarrow$ i is irrat.

So assumption was not corr

Problem 4

Problem 4

Prove or disprove that the product of two irrational numbers is irrational.

Let a, b – irrational

$a \cdot b \rightarrow$ irrational.

We know $\sqrt{2}$ is irrational

$\sqrt{2} \cdot \sqrt{2} = 2$. So 2 is rational.

Problem 5

Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

Sol: Let $x + y \geq 2 \leftarrow p$

$x \geq 1$ $\quad y \geq 1$

$q \quad r$

$p \to (q \vee r) \qquad \neg(q \vee r) \to \bar{p}$

if $x < 1$, and $y < 1$, then $x+y < 2$

So it is negation of $x+y \geq 2$

and our proof complete

Problem 6

Show that if n is an int. and $n^3 + 5$ is odd, then n is even using

a) proof by contraposition

b) proof by contradiction.

a) n is odd, n^3+5 is even

If n is odd $\Rightarrow$ $n = 2k+1$ for k.

Then $n^3 + 5 = (2k+1)^3 + 5$

Then $n^3+5 = (\ldots$

$= 8k^3+12k^2+6k+6 = 2(4k^3+6k^2+3k+3)$

Because n^3+5 is two times of some integer, so it is even

b) n^3+5 is odd. and n is odd

Since n is odd, the prod of odd nums is odd.

Problem 7

The barber is the one who shaves all those men who do not shave themselves. The question is, does the barber shave himself?

~~1. barber shaves himself~~

$\boxed{p \to q \equiv \bar{p} \vee q} \equiv \bar{q} \to \bar{p}$

$p = 3n+2$ odd

$q = n$ is odd

$\neg p = 3n+2$ even

$\neg q = n$ is even

If n is even, then $3n+2$ is even.

$n = 2k$ k is int.

$3n+2 = 6k+2 = 2 \cdot (3k+1)$

$r = 3k+1$

$2 \cdot r$

Contraposition.

Contradiction.

$p \to q$

p	q	$p \to q$	$p \wedge \bar{q}$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

$$\overline{(p \to q)} = \neg(\bar{p} \vee q)$$

$$= p \wedge \bar{q}$$

Prove that there is no largest prime number.

Assume that there is a largest prime

p_n

$2 \cdot 3 \cdot 5 \cdot 7$

$p_1 = 2$

$p = 3$

$p_2 \cdot$

$p_3 = 5$

$\ldots$

p_n = largest prime number

$1 + p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot \ldots p_n = x$

x is a new prime number

$x > p_n$

p - largest prime num

$\overline{p}$ - доказать there is no larg. prime number

Problem 7

p → barber shaves those who doesn't shave themselves

q → barber shaves himself

q ?

$\overline{}$ ... does ...

$p \wedge q$

barber doesn't shave himself

Assume $\bar{q}$

barber shaves himself.

$y \in$ barbers

$x \in U$

~~P = doesn't shave themselves~~

~~Q = barber shaves him~~

~~$\forall x \; P(x) \to Q(x)$~~

$\forall y$

$B(x) =$

barber shaves himself.

Assume q

Он не может себя стричь

according to p

barber can't shave thems

who shaves himself

Sol: There is any set like this

Рассела парадокс.

irrational numbers,

$\sqrt{2} \neq \frac{a}{b}$

$\pi \neq \frac{a}{b}$

irrat. numbers - нет конеца.

If number is irrat

then this num. has infinite

number of digits.

rational → finite
→ infinite

infinite

$p \to q$

irr $\to \boxed{\text{infinite}}$