Algebra Homework 2

1. $S = 5 + 9 + 13 + \ldots + 89$

$a_1 = 5$ 	diff $= 4$

$a_2 = 9$

$$a_n = a_1 + (n-1)d$$

$$89 = 5 + (n-1)4$$

$$89 = 5 + 4n - 4$$

$$4n = -1 + 89$$

$$4n = 88$$

$$n = 22$$

$$S = \sum_{k=1}^{22} 5 + (k-1) \cdot 4$$

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② $\sum_{k=3} (2k+1)$ $\quad k = 1$

let $j = k - 2$

if $k = 3$, $j = 1$

$$\sum_{k=3}^{15} (2k+1) = \sum_{j=1}^{13} [2(j+2)+1]$$

$$= \sum_{j=1}^{13} 2j+5$$

③ $a_1 = 12$

$a_n = a_{n-1} + d$

[illegible]

$+ a_{10} = 5 +$

$d - ?$ $a_{25}\,?$

$a_{10} = a_9 + d$

$a_n = a_1 + (n-1)d$

$a_{10} = 12 + (9)d = 57$

$9d = 45$

$d = 5$

$a_{25} = 12 + (24) \cdot 5 = 13$

(4) Sum of all multiples of 7 between 100 and 1000

$$\sum_{k=100}^{1000} 7^k$$

$a_1 = 7 \cdot 15 = 105$

$a_n = 7 \cdot 142 = 994$

$n = \frac{a_n - a_1}{d} + 1 = \frac{994 - 105}{7} + 1 = 128$

$S = \frac{n}{2}(a_1 + a_n) = \frac{128}{2}(105 + 994)$

$= 70336.$

(5) $S = \sum_{k=1}^{n} (3k+2)$ find value of n

such that $S = 2650$

$a_1 = 3 \cdot 1 + 2 = 5$

$a_n = 3n + 2$

$$S = \frac{}{2}\left[u_1 + u_n\right]$$

$$2650 = \frac{5n}{2} + \frac{n \cdot (3n+2)}{2} = \frac{5n + 3n^2 + 2n}{2}$$

$$= \frac{3n^2 + 7n}{2} = \frac{n}{2} \cdot \frac{(3n+7)}{1}$$

$$2650 = \frac{n\,(3n+7)}{2}$$

$$5300 = n\,(3n+7)$$

$$5300 = 3n^2 + 7n$$

$$3n^2 + 7n - 5300 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-7 \pm \sqrt{14 + 4 \cdot 3 \cdot 5300}}{6} =$$

$$= \frac{-7 \pm \sqrt{}}{6} =$$

...

(6) 5th term 20

15th term 60

Show 10th term arith mean of 5 and 15

$$\frac{5+15}{2} = 10 \checkmark$$

$$a_{10} = a_1 + 9d$$

a_5 : $20 = a_1 + 4d$ (1)

$$60 = a_1 + 14d$$

$$-40 = -10d$$

$$d = 4$$

$$a_5 = 20 = a_1 + 16$$

$$a = 20 - 16 = 4$$

$a_{10} = \cancel{\ } = 4 + 9 \cdot 4 = 40$

Mean $\dfrac{20 + 60}{2} = 40$

$a_{10} = 40$ ✓

7

$a_1 = 5$

$d = 0,5$

$n = 20$

$$S_n = \frac{n}{2}\left(2a + (n-1) \cdot d\right)$$

$$S_{20} = 10\left(10 + 19 \cdot 0,5\right) = 195$$

⑧ first 11

$d = 3$

Smallest value of n so that S_n exceeds 1000

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_n = \frac{n}{2}\left(2 \cdot 11 + (n-1)3\right)$$

$$= \frac{n}{2}\left(22 + 3n - 3\right) = \frac{n}{2}(3n+19)$$

$$= \frac{3n^2}{2} + \frac{19n}{2} = \frac{n(3n+19)}{2} > 1000$$

$$3n^2 + 19n > 2000$$

$$3n^2 + 19n - 2000 = 0$$

$$n = \frac{-19 \pm \sqrt{19^2 + 4 \cdot 3 \cdot 2000}}{6}$$

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(9) $\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k$ rewrite with $k=0$

$j=0$

let $j=k-3$, when $k=3$, $j=0$

when $k=12$, $j=9$

$$\sum_{j=0}^{9} 4\left(\frac{1}{2}\right)^{j+3} = \sum_{j=0}^{9} 4\cdot\left(\frac{1}{8}\right)\cdot\left(\frac{1}{2}\right)^j$$

$$= \sum_{j=0}^{9} \frac{1}{2}\left(\frac{1}{2}\right)^j = \sum\left(\frac{1}{2}\right)^{j+1}$$

$$\Rightarrow \sum_{k=0}^{9} \frac{1}{2}\left(\frac{1}{2}\right)^k$$

(10) Find 10th term of geom. seq.

if $a_2 = -6$ and $a_5 = 48$

$$a_n = a_1 r^{n-1}$$

a_2 : $-6 = a_1 r^{1}$

a_5 : $48 = a_1 r^4$ $/ \; a_2$

$$-\frac{48}{6} = r^3$$

$$r = \sqrt[3]{-8}$$

$$r = -2$$

$$-6 = a_1 (-2)$$

$$a_1 = 3$$

$$a_{10} = 3 \cdot r^9 = 3 \cdot (-2)^9 = -1536.$$

(11) $a_1 = 54$ $a_n = 1458$

Find r

$$a_n = a_1 r^{n-1}$$

$$\frac{a_7}{a_4} = \frac{a_1 r^6}{a_1 r^3} = r^3 = \frac{1458}{54}$$

$$r = \sqrt[3]{\frac{1458}{54}} = 3$$

12 Sum of first 15 terms where $a_1 = 8$ $r = \frac{3}{4}$

$$S_n = a_1 \frac{1-r^n}{1-r} = 8 \frac{1-r^n}{1-r}$$

$$= 8 \frac{1-\left(\frac{3}{4}\right)^n}{1-\frac{3}{4}}$$

$$r^{15} = \left(\frac{3}{4}\right)^{15}$$

$$1 - \left(\frac{3}{4}\right)^{15}$$

$S_{15} = 8 \frac{\quad}{1-\frac{3}{4}} = 3(1-r)$

(13) $P(x) = x^5 - 4x^3 + x^2 - 7$

Degree 5
terms 4

(14) $(2x^4 - 3x^3 + x - 5) + (x^3 - 2x^2 + 4x + 7)$

$2x^4 - 2x^3 - 2x^2 + 5x + 2$

(15) $(x^2 - x + 2)(x^2 + x + 1)$

$x^4 - \cancel{x^3} + 2x^2 + \cancel{x^3} - \cancel{x^2} + 2x +$
$\cancel{x^2} - x + 2$

$= x^4 + 2x^2 + x + 2$

(16) Find GCD and LCM
of $24x^3y^2z^5$ and $36x^5y^3z^2$

$12 \quad 3$

$12 \cdot 2 \quad x \quad x \quad x$

GCD: $12x^3y^2z^2$

LCM: $12 \cdot 3 \cdot 2\, x^5 y^3 z^5$

17 Factor $x^4 - 13x^2 + 36$

Let $a = x^2$

$a^2 - 13a + 36$

$(a-9)(a-4) = (x^2-9)(x^2-4)$

$= (x-3)(x+3)(x-2)(x+2)$

18 Expand $(2x+3y)^5$ use Bin Th.