Algebra homework 3

① $\log_2\left(\frac{8\sqrt{2}}{16}\right) + \log_2(32) - 2\log_2(4)$

$$\frac{8\sqrt{2}}{16} = \frac{8}{16}\sqrt{2} = \frac{1}{2}\,2^{\frac{1}{2}} = 2^{-1}\,2^{\frac{1}{2}} = 2^{-\frac{1}{2}}$$

$$\log_2\left(2^{-\frac{1}{2}}\right) + 5 - 4 = -\frac{1}{2} + 1 = \frac{1}{2}$$

② $\log_3(x-1) + \log_3(x+1) = 2$

$$\log_3((x-1)\cdot(x+1)) = 2$$

$$\log_3(x^2-1) = 2$$

$$3^2 = x^2 - 1$$

$$x^2 = 3^2 + 1$$

$$x^2 = 10$$

$$x = \sqrt{10}$$

③ inv. 10000$

6% quarterly $f(t)$ < 20000

$10000 \cdot \left(\left(1+0{,}06\right)/4\right)^{4t} \leq 20000$

$\cancel{\log_{4{,}75 \cdot 10000} 20000 = t}$

$\left(1+\frac{0{,}06}{4}\right)^{4t} = 2$

$\left(\frac{4{,}06}{4}\right)^{4t} = 2$

$\left(1{,}06\right)^{4t} = 2$

$\log_{1{,}06} 2 = 4t$

④ ⑤ ?

⑥ A(1, 2, 3) to B(4, 6, 9)

$$\overline{AB} = (3, 4, 6)$$

$$|AB| = \sqrt{3^2 + 4^2 + 6^2} = \sqrt{9 + 16 = 36} = \sqrt{61} \approx 7{,}8$$

$$\bar{u} = \frac{AB}{|AB|} = \frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}}, \frac{9}{\sqrt{61}}$$

(7)

$$\bar{v} = 7\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\bar{v} \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$$

$$|v| = \sqrt{7^2 + (-2)^2 + 4^2} = \sqrt{49 + 4 + 16} = \sqrt{69}$$

(8) $\bar{a} = (2, -1, 3)$

$\bar{b} = (-1, 4, 2)$

Comb a ... b

$3a - 2b$

$3\bar{a} = (6, -3, 9)$

$2b = (-2, 8, 4)$

$3\bar{a} - 2\bar{b} = (8 \quad -11 \quad 5)$

(9) $\bar{p} = (1, 2, 3) \qquad \bar{q} = (4, -5, 6)$

Angle Betw vekt.

Dot $\bar{p}\,\bar{q} = (4 \quad -10 \quad 18) = 12$

$\bar{p} = \sqrt{1+4+9} = \sqrt{14}$

$q = \sqrt{16+25+36} = \sqrt{77}$

$\cos\theta = \frac{pq}{|p||q|} = \frac{12}{\sqrt{14}\sqrt{77}}$

(10) $u = (2, -1, 4)$

$v = (-8, 4, -16)$

$\Rightarrow$ orthogonal.

$u \cdot v = (-16 \quad -4 \quad -64) = -84$

Wenn $u \cdot v \neq 0$ vectors are not orthog.

(11) $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix}$

$2A - 3B$

$= \begin{bmatrix} 4 & -2 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 12 & 15 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} -8 & -17 \\ 6 & 3 \end{bmatrix}$

(12) $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$E = CD = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

(13) $x + y + z = 6$

$2x - y + 3z = 14$

$-3x + 2y - 2z = -10$

$-1 - (2R = -1$

$$\left.\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ -3 & 2 & -2 & -10 \end{array}\right.$$

$R_2 = R_2 - 2R_1$

$$\left.\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & \frac{8}{3} & \frac{34}{3} \end{array}\right.$$

⋮

14

$$B = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & \cancel{10} & 3 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_1 = R_1 + R_3 \times 1$

15 $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ Find A^{-1}

$[A \mid I]$

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$$

$a_{11} = 1$

$R_1 = R_1 : 2$

$$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$$

$a_{21} = 1$ $R_2 = R_2 - 5R_1$

$$\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}$$

a_{22}

$R_2 = R_2 \cdot 2$

$$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -5 & 2 \end{array}\right]$$

a_{12}

$R_1 = R_1 - \left(\frac{1}{2} R_2\right)$

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$