Discrete Math
HW I

1) a) $y = x^2 + 1$ $\quad y = f(x)$ If $x = 2$

$y = 2^2 + 1 = 5$ ✓

b) $y^2 = x + 1$

$y^2 = f(x)$

If $x = 2$

$y^2 = 2 + 1 = 3$

$y = \pm\sqrt{3}$ It is no funktion,

because f can have more than one output.

2) a) $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = 3n$

$f(x) = y$

$y = 3n$

If y is multiple of 3 than exists $n = \frac{y}{3}$

- for example $y = 6 \Rightarrow n = 2$, $f(2) = 3 \cdot 2 = 6$
  $y = -9 \Rightarrow n = -3$
  $y = 0 \Rightarrow n = 0$

- kein $n \in \mathbb{Z}$ mit $3n = y$
  Because $\frac{y}{3}$ keine ganze Zahl

It is not surjektive onto $\mathbb{Z}$, since not every integer is reached.

3) $h : \{1, 2, 3\} \to \{1, 2, 3\}$
   It is surjektive.

2) $g : \{1, 2, 3\} \to \{a, b, c\}$

$$g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$$

It is not surjektive. Not every output will be reached

③ $g : \{1, 2, 3\} \to \{a, b, c\}$

$g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$ It is injektive

④ If $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{1}{x}$ - 2

is $g = f^{-1}$

1) $f(x) = \frac{1}{x+2}$

$y = \frac{1}{x+2}$

$x = \frac{1}{y+2}$

$y + 2 = \frac{1}{x}$

$y = \frac{1}{x} - 2 = \frac{1-2x}{x}$

$$\boxed{f^{-1}(y) = \frac{1-2x}{x}}$$

⑤ $f(x) = 2 + \sqrt{x-4}$

$$y = 2 + \sqrt{x-4}$$

$$x = 2 + \sqrt{y-4}$$

$$\sqrt{y-4} = x - 2$$

1 $$y - 4 = (x-2)^2$$

$$y = (x-2)^2 + 4$$

$$\boxed{\begin{aligned} f^{-1}(y) &= (y-2)^2 + 4 = y^2 - (2y\cdot(-2)) + 4 \\ &= y^2 + 4y + 4 + 4 \\ &= y^2 + 4y + 8 \end{aligned}}$$

⑥ $$C = \frac{5}{9}(F - 32)$$

$$C = \frac{5F}{9} - \frac{32\cdot 5}{9} = \frac{5F - 160}{9}$$

$$C = \frac{5F - 160}{9} \Rightarrow 9C = 5F - 160$$

$$5F = 9C + 160$$

$$F = \frac{9C + 160}{5} = \frac{9C}{5} + 32$$

⑦ $$g(x) = 2\sqrt{x-4}$$

$x-4 \geq 0$

$x \geq 4 \quad D: [4; \infty]$

(8) $h(x) = -2x^2 + 4x - 9$

$D: (-\infty; \infty)$

9) $f(x) = \dfrac{x-4}{x^2-2x-15}$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x^2 - 5x + 3x - 15$

$x = 5$

$x = -3$

$D \in \mathbb{R} \setminus \{-3, 5\}$

(10) $f(x) = \begin{cases} -2x+1 & -2 \leq x < 0 \\ x^2+2 & 0 \leq x \leq 2 \end{cases}$

x^2+1

$f(x) = -2x + 1 = 0$

$+2x = 1$

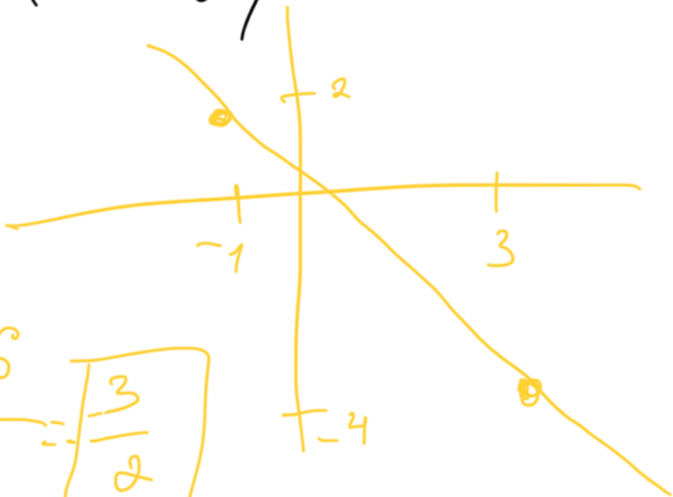
$x = \frac{1}{2}$ $\left(-2 \leq \frac{1}{2}\right)$

$x^2 = -2$

$x = \sqrt{-2}$

11 $(-1; 2)$

$(3, -4)$



$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6}{4} = \boxed{-\frac{3}{2}}$

negative slope falling from left to right

12 $\begin{array}{cc} x_1 & y_1 \\ (1, & -1) \end{array}$

$m = \frac{3}{4} = \frac{Rise}{Run}$

slope $= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{3}{4} = \frac{y_2 + 1}{x_2 - 1}$

$y = mx + b$

(13) Avg. rate of change

$[-1, 2]$

1) $(2, 1)$

2) $(-1, 4)$

$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3}{-3} = -1$

(14) $f(x) = x^2 - \frac{1}{x}$ inter $[2, 4]$

$f(2) = 2^2 - \frac{1}{2}$ $f(4) = 4^2 - \frac{1}{4}$

$= 4 - \frac{7}{2}$ $= 16 - \frac{1}{4}$

from where 7/2? $= \frac{63}{4}$?

avg r. ch

$\frac{f(4) - f(2)}{4 - 2} = \frac{\frac{63}{4} - \frac{7}{2}}{2} = \frac{\frac{63 - 14}{4}}{2} = \frac{\frac{49}{4}}{2}$

$= \frac{49}{4} \cdot \frac{1}{2} = \frac{49}{8}$

15 $f(t) = t^2 - t$

$h(x) = 3x + 2$

eval $f(h(1)) = (f \circ h)(x)$

$h(1) = 3 + 2 = 5$

$f(5) = 5^2 - 5 = 20$

16 $(f \circ g)\frac{x}{5}$ where

$f(x) = \frac{5}{x-1}$ and $g(x) = \frac{4}{3x-2}$

$3x - 2 = 0$

$x = \frac{2}{3}$

$\frac{4}{3x-2} = 1$

$4 = 3x - 2$

$3x = 6$

$x = 2$

$x \neq \frac{2}{3}, x \neq 2$

$\left(-\infty ; \frac{2}{3}\right) \left(\frac{2}{3} ; 2\right) (2; \infty)$

17

$(g - f)(x)$, $\left(\frac{g}{f}\right) x$

$f(x) = x - 1$

$g(x) = x^2 - 1$

$(g - f)(x) = g(x) - f(x) =$

$x^2 - 1 - x + 1 = x^2 - x = x(x-1)$

$\left(\frac{g}{f}\right)(x) = \frac{x^2-1}{x-1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = x+1$

$x(x-1) \neq x+1$

18 $h(x) = f(x-1) + 2$

$h(x) = \sqrt{x-1} + 2$

19 $f(x) = x^3 + 2x$

it is an odd fkt

20 $f(s) = s^4 + 3s^2 + 7$ it is even

$f(-s) = (-s)^4 + 3\cdot(-s)^2 + 7$

$= s^4 + 3s^2 + 7$

(21) point - slope form

$(5,1) \quad (8,7)$

Rewrite in the slope - intersept

$y = mx + b$

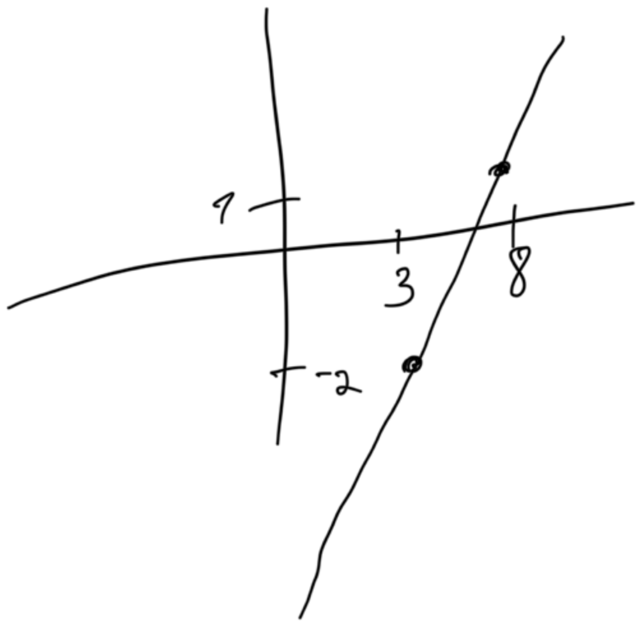
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{8-5} = \frac{6}{3} = 2$

$y = mx + b$

$(y - y_1) = 2 \cdot (x - 5)$

$y - 1 = 2x - 10$

$\boxed{y = 2x - 9}$



(22) $f(x)$

$(3, -2)$

$(8, 1)$

$m = \frac{1-(-2)}{8-3} = \frac{3}{5}$

f(x) is increasing

$m > 0 \Rightarrow$

(23) Abs max in $(-2; 16)$
$(2; 16)$

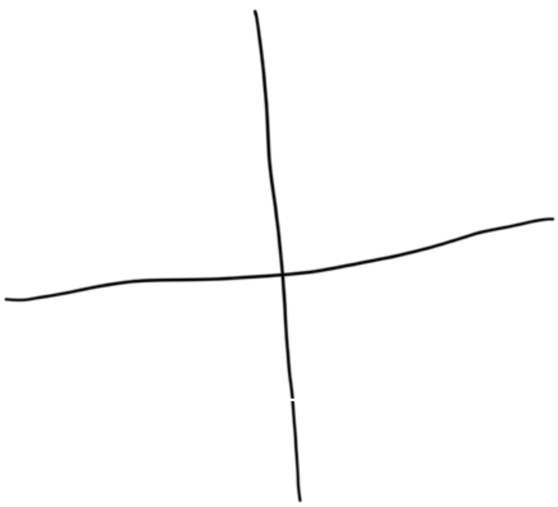
abs. min $(3; -10)$

(24)

$f(x) = 2x + 3$ $\quad$ Perp $\quad$ $h(x) = -2x + 2$

Par

$g(x) = \frac{1}{2}x - 4$ $\quad$ $j(x) = 2x - 6$



Parallel has the same slope of $\underline{2}$

(25) $2x + y = 7 \Rightarrow y = 7 - 2x$

$x - 2y = 6 \Rightarrow$

$x - 2(7-2x) = 6$

$x - 14 + 4x = 6$

$5x = 20$

$x = \frac{20}{5} = 4$

$2 \cdot x + y = 7$

$2 \cdot 4 + y = 7$

$8 + y = 7$

$y = -1 \Rightarrow \boxed{(4; -1)}$

26

$4x + 2y = 4 \Rightarrow 2y = 4 - 4x$

$6x - y = 8$

$\cancel{y = 2 - 2x}$

$y = \frac{4-4x}{2} = \frac{4(1-x)}{\cancel{2}} = \boxed{2 - 2x}$

$6x - (2 - 2x) = 8$

$6x - 2 + 2x = 8$

$8x = 10$

$x = \frac{10}{8} = \frac{5}{4}$ ✓ (

$4 \cdot \frac{5}{4} + 2y = 4$

$5 + 2y = 4$

$2y = -1$

$y = -\frac{1}{2}$

$4 \cdot \frac{5}{4} \cancel{+} 2 \cdot \frac{1}{2} = 2$ ✓

$6 \cdot \frac{5}{4} + \frac{1}{2} = 8$

$\frac{30}{4} + \frac{1}{2} = \frac{30+2}{4} = 8$ ✓

$\left(\frac{5}{4}; -\frac{1}{2}\right)$

(27) Quadratic Fkt.

$f(x) = 2x^2 - 6x + 7 \Rightarrow$ in standard form.

$h = \frac{6}{2 \cdot 2} = \frac{6}{4} = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = 2 \cdot \left(\frac{3}{2}\right)^2 - 6\,\frac{3}{2} + 7$

$= 2 \cdot \frac{9}{4} - 9 + 7 = \frac{9}{2} - 2 = \frac{9-4}{2} = \frac{5}{2}$

$f(x) = ax^2 + bx + c$

28 Domain and range

$f(x) = -5x^2 + 9x - 1$

$h = -\frac{b}{2a} = -\frac{9}{-2 \cdot 5} = \frac{9}{10}$

$f\left(\frac{9}{10}\right) = -5\frac{9}{10} + 9\frac{9}{10} - 1$

$= -\frac{9}{2} + \frac{81}{10} - 1$

$\boxed{\text{Is falsch in Solu?}}$

$= \frac{-45 + 81}{10} - 1 = \frac{36}{10} - \frac{1}{1}$

$= \frac{36 - 10}{10} = \frac{26}{10} = \frac{13}{5}$

$f(4) \leq \frac{12}{5}$

(29) $f(x) = 3x^2 + 5x - 2$

$f(0) = -2$ so $\Rightarrow$ y at $(0; -2)$

$(3x - 1)(x + 2) = 0$

$3x = 1$

$x = \frac{1}{3}$; $x = -2$

x inters at $(\frac{1}{3}; 0)$ $(-2; 0)$

(30) a) $-1 \leq 2x - 5 < 7$

$5 - 1 \leq 2x < 12$

$4 \leq 2x < 12 \;/\; 2$

$2 \leq x < 6$

a) $[2;6)$

b) $x^2+7x+10<0$

$(x+5)(x+2)<0$

$x+5>0$ $\quad$ $x+2<0$

$x>-5$ $\quad$ $x<-2$

$-5<x<-2$

$b \Rightarrow (-5;-2)$

c) $-6<x-2<4$

$-6+2<x<4+2$

$-4<x<6$

$c \Rightarrow (-4;6)$

31 $10-(2y+1) \leq -4(3y+2)-3$

$-2y+9 \leq -12y-8-3$

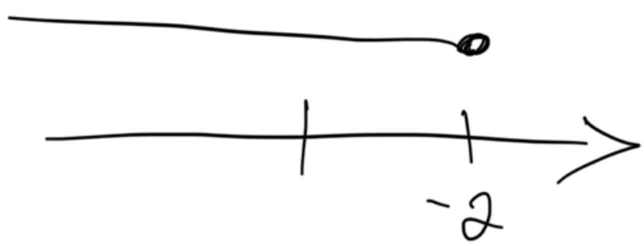
$-2y \leq -12y-11-9$

$-2y \leq -12y-20$

$10y \leq -20$

$y \leq -\frac{20}{10} = -2$

$(-\infty; -2]$

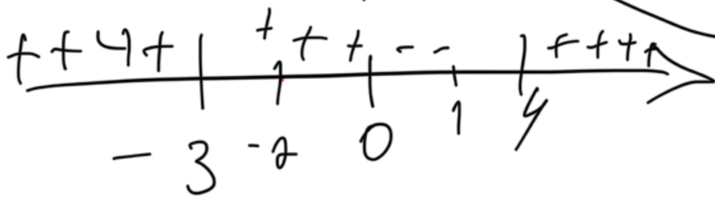


32 $x(x+3)^2(x-4) < 0$

$x = -3$ $f(-2) = +$ $f(-4) = +$

$x = 4$ $\quad f(1) = -$ $\quad f(5) = +$ $\quad (-3; 4)$

$x = 0$



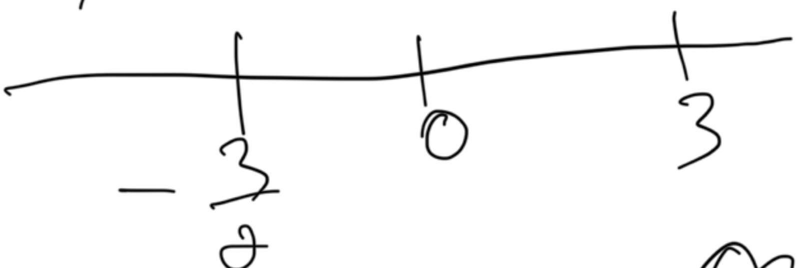
33) $2x^4 > 3x^3 + 9x^2$

$2x^4 - 3x^3 - 9x^2 > 0$

$x^2(2x^2 - 3x - 9) > 0$

$x^2((2x+3)(x-3)) = 0$

$0, 3, -\frac{3}{2}$



34) $f(x) = -\frac{1}{2}|4x - 5| + 3$

$$-\frac{1}{2}|4x-5|+3<0 \quad /\cdot 2$$

$$-\frac{1}{2}|4x-5|+3<0$$

$$|4x-5|<-6$$

$$|4x-5|>6$$

$$4x-5=6$$

$$4x=11$$

$$x=\frac{11}{4}$$

35 $\quad 13-2 \quad x-7|\leq 3$

$$-2|4x-7|\leq 3-13 \quad /:-2$$

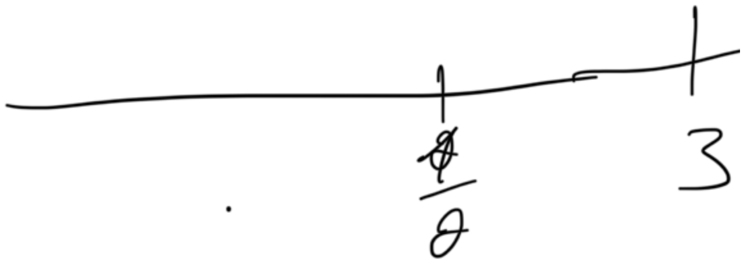
$$|4x-7|\leq \frac{10}{} = 5$$

$\ldots \frac{\ldots}{2} \geq 5 \quad 4x-7$

$4x-7=-5 \quad \geq 05 \quad x \geq 3$

$4x = 2$

$x =$



$\leq \frac{1}{2}$

$\left(-\infty; \frac{1}{2}\right] \cup [3; \infty)$