Week 6 HW

(1)

a) $\mathcal{P}(\{a, b, \{a, b\}\}) \Rightarrow 2^3 = 8$

b) $\mathcal{P}(\{\emptyset, a, \{a\}, \{\{a\}\}\}) \Rightarrow 2^4 = 16$

c) $\mathcal{P}(\mathcal{P}(\emptyset)) \Rightarrow 2^0 = 1$

(2) m^2

(3) a) $P(x): x^2 < 3$

$\{x \in \mathbb{Z} \mid x^2 < 3\} = \{-1, 0, 1\}$

b) $Q(x): x^2 > x$

$\{x \in \mathbb{Z} \mid x^2 > x\} = \{\ldots, -2, -1, 2, \ldots\}$

c) $R(x): 2x + 1 = 0$

$\{x \in \mathbb{Z} \mid 2x + 1 = 0\} = \{-\frac{1}{2}\}$

④ S: { C B A }

∅		0 0 0	Alle Variant
{A}		0 0 1	
{B}		0 1 0	
{C}		0 1 1	
		1 0 0	
		1 0 1	
		1 1 0	
		1 1 1	

⑤ $A \wedge B \wedge C = \overline{A} \vee \overline{B} \vee \overline{C}$

$A \wedge B \wedge C = A$

A	B	C
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	1
0	0	0

$A \wedge B \wedge C$
0
0
0
0
0
1
0

$A \wedge B \wedge C$
1
1
1
1
1
0
1

$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A} \vee \overline{B} \vee \overline{C}$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

⑥ a) $A \wedge (B - C)$

b) $(A \cap B) \cup (A \cap C)$

c) $(A \cap \overline{B}) \cup (A \cap \overline{C})$

(7) a) $A \cup B = A \Rightarrow B \subseteq A$

b) $A \cap B = \emptyset \Rightarrow B \subseteq \emptyset$

c) $A - B = A \Rightarrow$ ~~$B = A \cap B$~~ $A \cap B$

d) $A \cap B = B \cap A$? V

e) $A - B = B - A \Rightarrow A = B$

(8)

(9) a) $A_i = \{-i, -i+1, \dots, -1, 0, 1, \dots, i-1, i\}$

$$\bigcap_{i=1}^{\infty} A_i = A_1 = \{-1, 0, 1\}$$

$$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}$$

b) $A_i = \{-i, i\}$

$$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z} - \{0\}$$

$$\bigcup_{i=1}^{\infty} A_i \qquad \bigcap_{i=1}^{\infty} A_i$$

1) $A_i = \{-i, -i+1, -i+2, \dots -1, 0, 1, \dots, i-1, i\}$

$$\{x \mid x \in \mathbb{Z} \wedge -i \leq x \leq i$$

$A_i \subset A_{i+1}$

$A_i \cup A_{i+1} = A_{i+1}$

$\bigcup_{i=1}^{\infty} A_i = A_\infty = \mathbb{Z}$

$A_i \cap A_{i+1} = A_i$

$\bigcap_{i=1}^{\infty} A_i = A_1 = \{-1, 0, 1\}$

2) $A_i = \{-i, i\}$

$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z} - \{0\}$

$\bigcap_{i=1}^{\infty} A_i = \emptyset$

3) $A_i = \{x \mid x \in \mathbb{R} \text{ and } -i \leq x \leq i\}$

$\bigcup^{\infty} A_i = \mathbb{R}$

$$\bigcap_{i=1}^{\infty} A_i = A_1 = \{x \mid x \in R \text{ and } -1 \le x \le 1\}$$

4) $A_i = \{x \mid x \in R \text{ and } i \le x\}$

$$\bigcup_{i=1}^{\infty} A_i = A_1 = \{x \mid x \in R \text{ and } i \le x\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_9 = \{x \mid x \in R \text{ and } 9 \le x\}$$