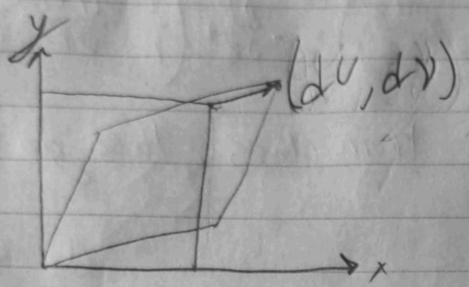
FK

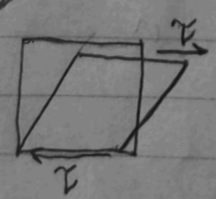


We use $\frac{\partial u}{\partial x}$ & $\frac{\partial v}{\partial y}$ to describe the displacement of the corner of the fluid element. This means they can be used to for strain measurement assuming strain rate is defined as a normalized deformation i.e. $\frac{\Delta L}{L} = \delta$. In the x & y direntions this becomes:

$\delta_x = \frac{\partial u}{\partial x}\Delta x \Delta t$, $\delta_y = \frac{\partial v}{\partial y}\Delta y \Delta t$. However, because fluids deform continuously when subjected to stress, it is more useful to talk about strain rates. This also has the useful advantage of simplifying the above expressions to:

$\varepsilon_x = \frac{\partial u}{\partial x}$, $\varepsilon_y = \frac{\partial v}{\partial x}$, it is easy to imagine this fluid element in 3-dimensions with the addition of

$\varepsilon_z = \frac{\partial w}{\partial z}$. If the deforming corner moves away from the origin, then the volume of the fluid element will increase. This means we can define the change in volume as $\frac{\Delta V}{V} = \sum_{xyz} \varepsilon$. This gets the definition "Volumetric strain rate" and directly follows from linear strain rate. If the fluid element deforms but does not change volume then there must be 2 forces acting on the fluid element



τ = shear force on fluid element

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

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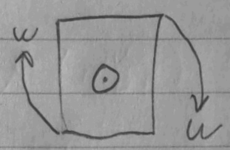
from lecture, we know that vorticity, $\vec{\omega} = \vec{\nabla} \times \vec{u}$ or
$\vec{\omega} = \mathrm{curl}(\vec{u})$

This is simply the nabla vector crossed with the velocity vector and represents a rotation of the fluid about some axis.



CM1

$P_{steam} = \text{Constant}$

hydrostatic pressure =

1) $\rho g h = 1000 \cdot 9.8 \cdot 10\,m = 98000\,Pa$

$\rightarrow P = P_h + P_{amb} = 199325\,Pa$

At ambient T

boiling point from lookup table: $T_b = 393.3K$

$T = T_b + 80 = 473.3k$, $\boxed{P = \rho R_{steam} T = \text{constant} = 199325\,Pa}$

$\rho = .9108\,kg/m^3$.

$V_{exit} = 15\,\frac{m}{s} = \text{constant}$, $U_{steam} = V_e \frac{x}{L}$

where U_s is the boundry between the water and steam, this creates the ODE $\dot{x} = V_e \frac{x}{L}$.

* See matlab *

note how if ODE starts at $x = 0$, It will always remain, but it is an unstable equalibrium, if Pressure increases slightly more, the gyser will go off. I model this with a small but positive x initial

Mass and Volume flow rates are given by $\dot{V} = \dot{x} A$, where $A = \pi R^2 = \pi \cdot .05^2$.

$\dot{m} = \dot{V} \rho_{water} = 1000\,\frac{kg}{m^3}\,\pi .05^2 m^2\,\dot{x}$

where $\dot{x}$ is given from above ode

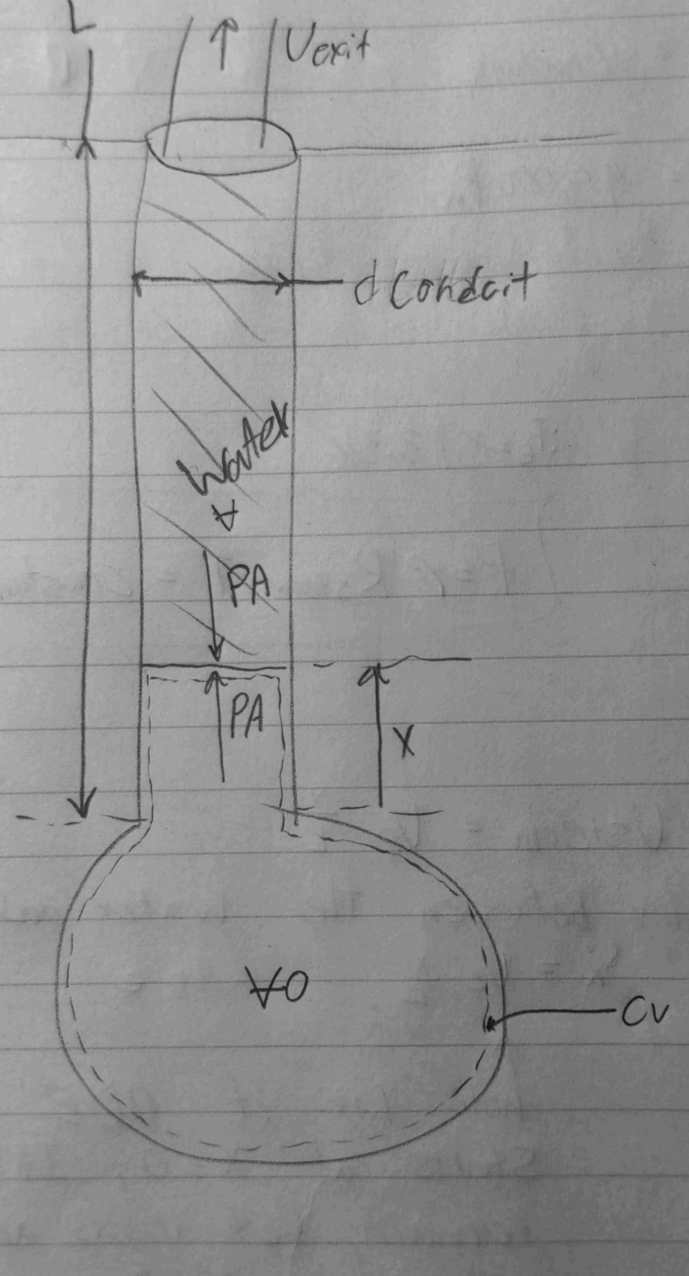
* Plot in matlab *

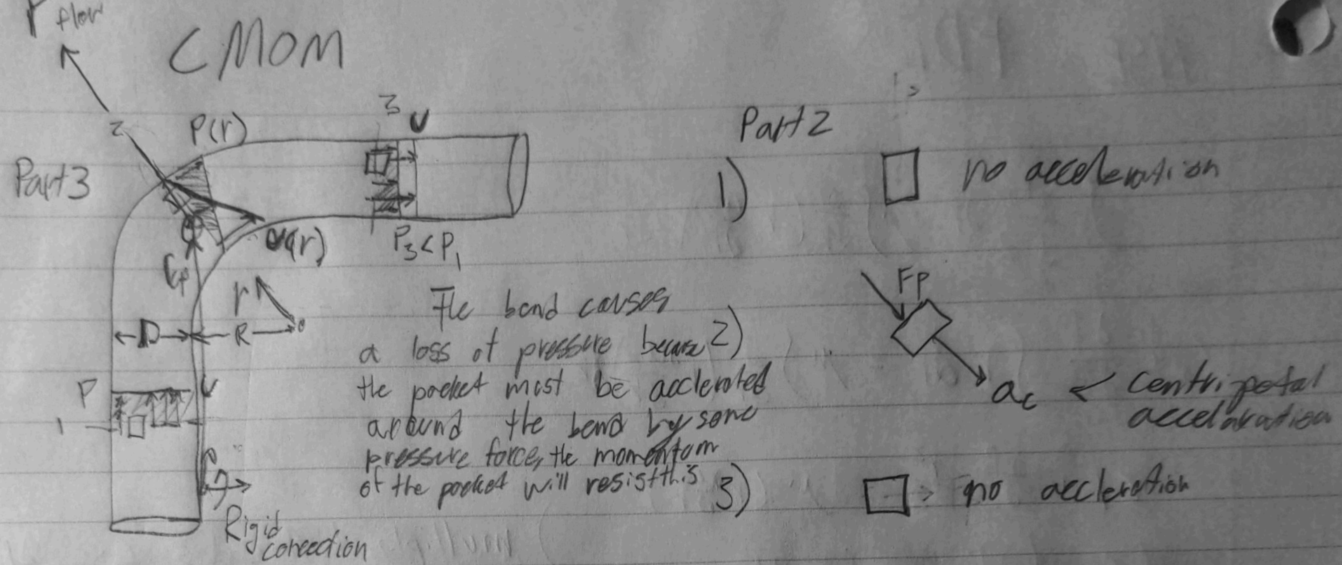
ρ of .75s is $\approx .023\,\frac{kg}{m^3}$

$x(.75s) = 9.2407 \approx 10\,m$

so the show is almost over after .75s, this is much shorter than a full old faithfl show but maybe close to one spurt.

CM 1 Page 2

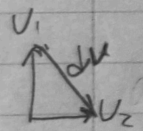




part 2 We are making the assumptions that there is no boundry condition on velocity, ie. this is an inviscid flow and there is a slip condition. This means there will be linear velocity functions.

$$\text{Bend:}\quad F_p = m\,a_c = dP\,\cancel{dA} = \frac{\rho\, d\cancel{A}\, dr\, U(r)^2}{r}$$

$$\int dP = \int \frac{\rho}{r}\, U(r)^2 dr = P(R) = \int \frac{\rho}{r}\, U(r)^2 dr = \rho \ln r\, U(r)^2$$



$$F_f = \frac{d}{dt}(mU), \quad F_x = \frac{d}{dt}(m\bar{U}\hat{x}), \quad F_y = \frac{d}{dt}(m\bar{U}\hat{y})$$

m $\quad F_f = \frac{d\vec{U}}{dt} Q\rho \quad \frac{d}{dt} m\bar{U} = \rho Q \bar{U} \quad F_x = \rho Q \bar{U}\hat{x}, \quad F_y = \rho Q \bar{U}\hat{y}$

$$\left.\begin{aligned} F_{ball\ bearing} &= (-F_x, -F_y) \\ M_{ball\ bearing} &= r_{centepressure} \times F_f \end{aligned}\right\}$$ these forces are created by exchange of momentum between gas and pipe

Part 4) assuming viscous compressible flow, P_e T_e and therefore U_e, should be solved for using isentropic relations.

where $\frac{T_i}{T_e} = 1 + Ma^2 \frac{\gamma - 1}{2}$; $\frac{P_i}{P_e} = \frac{T_i}{T_e}^{\left(\frac{\gamma}{\gamma - 1}\right)}$

NSE 2

$$\rho\left(\cancel{\frac{\partial u_z}{\partial t}}^{0} + \cancel{u_r \frac{\partial u_z}{\partial r}}^{0} + \frac{u_\theta}{r}\cancel{\frac{\partial u_z}{\partial \theta}}^{0} + \cancel{u_z \frac{\partial u_z}{\partial z}}^{0}\right) = -\frac{\partial P}{\partial z} + \left(\frac{1}{r}\frac{\partial (r\tau_{rz})}{\partial r} + \frac{1}{r}\cancel{\frac{\partial \tau_{\theta z}}{\partial \theta}}^{0} + \cancel{\frac{\partial \tau_{zz}}{\partial z}}^{0}\right) + \cancel{\rho g_z}^{0}$$

From class, these go to zero

$$\Rightarrow \frac{\partial P}{\partial z} = \frac{1}{r}\frac{\partial (r\tau_{rz})}{\partial r},$$

$$\tau_{rz} = \mu \frac{\partial u_z}{\partial r}$$

See page 2 of NSE

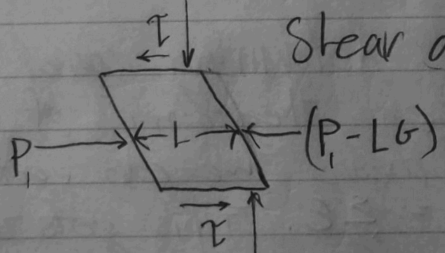
$$\frac{\partial P}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial u_z}{\partial r}\right) = G \quad , \quad \frac{\partial P}{\partial z} = G = \text{pressure gradient}$$

Solve for u using PDE

$$u_z = \frac{G}{4\mu}\left(R^2 - r^2\right)$$

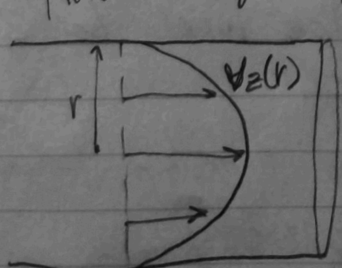
where R is radius of pipe, r is position in pipe

Using "no slip" boundry condition, i.e. $u_z(R) = 0$ and the assumption that the profile of velocity is not changing with z i.e. $\frac{\partial u_z}{\partial z} = 0$



Shear acting on a fluid element and pressure acting such that there is a net pressure moveing the element in the (+)z direction.

flow velocity in pipe



$$Q = \bar{u} A = \pi R^2 \tfrac{1}{2} u_{max} = \pi R^2 \tfrac{1}{2} \frac{GR^2}{4\mu}$$
$$= \pi R^4 \tfrac{1}{8} G \mu$$

$$Q\rho = \dot{m}$$

NSE PDE

$$\int r \frac{\partial P}{\partial z} \partial r = \mu \int \frac{\partial}{\partial r}\left(r \frac{u_z}{\partial r}\right) \partial r$$

$$\frac{\partial P}{\partial z} \int r \, dr = \mu \int \frac{d}{dr}\left(r \frac{\partial u_z}{dr}\right) dr$$

$$C_1 + \frac{\partial P}{\partial z} \frac{r^2}{2} = \mu \left(r \frac{\partial u_z}{\partial r}\right)$$

) multiply both sides by ∂r

$$\int \frac{C_1}{r} dr + \int \frac{\partial P}{\partial z} \frac{r}{2} dr = \int \mu \, \partial u_z$$

$$C_1 \int \frac{1}{r} dr + G \int \frac{r}{2} dr = \mu \int d u_z$$

$$C_1 \ln(r) + G r^2 \frac{1}{4} = \mu u_z + C_2$$

from this integral solve C_1 & C_2 using boundry conditions