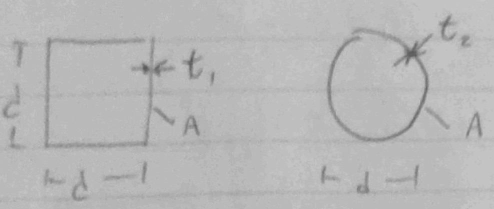
HW 6

1.1)



$\bar{A}$ is area enclosed by perimeter
A is cross sectional area
t is wall thickness
s is perimeter
$A_1 = A_2 = A = .01d^2$
$G_1 = G_2 = G$

$s_1 = 4d$ $\quad$ $s_2 = \pi d$

$$A = s_1 t_1 = s_2 t_2, \quad t_1 = \frac{A}{s_1}, \quad t_2 = \frac{A}{s_2}$$

$$\bar{A}_1 = d^2, \quad \bar{A}_2 = \pi\left(\frac{d}{2}\right)^2$$

$$q_1 = \frac{M_x}{2\bar{A}_1}, \quad q_2 = \frac{M_x}{2\bar{A}_2},$$

$$\theta = \frac{1}{2\bar{A}} \oint_s \frac{q}{Gt} ds \rightarrow \text{for constant } G, t \text{ \& } q$$

$$\theta = \frac{1}{2\bar{A}} \frac{1}{Gt} q s = \frac{1}{4\bar{A}^2} \frac{M_x}{Gt} s$$

$$\theta_1 = \frac{1}{4d^4} \frac{M_x}{GA} s_1^2 \qquad \theta_2 = \frac{1}{4\pi^2\left(\frac{d}{2}\right)^4} \frac{M_x}{AG} s_2^2$$

$$\theta_1 = \frac{1}{4d^4} \frac{M_x}{GA} 16d^4 \qquad \theta_2 = \frac{16}{4\pi^2 d^4} \frac{M_x}{AG} \pi^2 d^4$$

$$\theta_1 = 4 \frac{M_x}{GA} \qquad \theta_2 = 4 \frac{M_x}{AG}, \quad \theta_1 = \theta_2 = \theta$$

$$TR = \frac{M_x}{\theta} = \frac{M_x GA}{4M_x} = \frac{GA}{4} = \boxed{\frac{.01d^2 G}{4} \text{ for both sections}}$$

1.2) Same cross-sections but

$A_1 \neq A_2$ & $t_1 = t_2 = t$ $G_1 = G_2 = G$

$S_1 = 4d$, $S_2 = \pi d$, $\bar{A}_1 = d^2$, $\bar{A}_2 = \pi \left(\frac{d}{2}\right)^2$

$q_1 = \frac{M_x}{2\bar{A}_1}$, $q_2 = \frac{M_x}{2\bar{A}_2}$

$\theta = \frac{1}{4\bar{A}^2} \frac{M_x}{Gt} S$

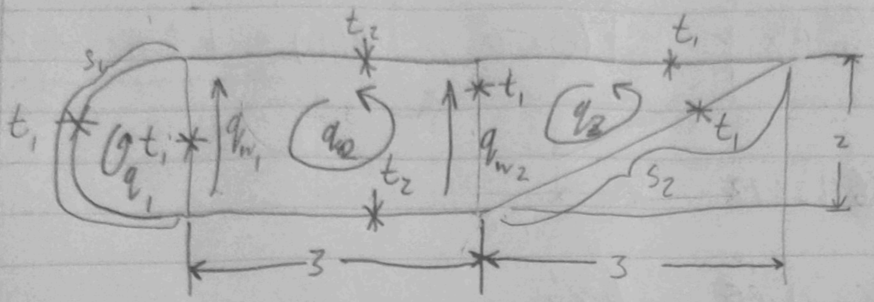
$\theta_1 = \frac{1}{4d^4} \frac{M_x}{Gt} 4d$ $\theta_2 = \frac{16}{4\pi^2 d^4} \frac{M_x}{Gt} \pi d$

$\theta_1 = \frac{1}{d^3} \frac{M_x}{Gt}$ $\theta_2 = \frac{4}{\pi d^3} \frac{M_x}{Gt}$

$\frac{4}{\pi} > 1$ so $\theta_2 > \theta_1$

$TR = \frac{M_x}{\theta}$, since $\theta_2 > \theta_1$, $\boxed{TR_1 > TR_2}$

2)



$t_1 = .05$

$t_2 = .1$

$G = 4 \times 10^6$ psi

a)

① $\frac{M_x}{2} = q_1 \bar{A}_1 + q_2 \bar{A}_2 + q_3 \bar{A}_3$

$A_1 = \frac{\pi}{2}$, $A_2 = 6$, $A_3 = 3$, $s_1 = \frac{\pi d}{2} = \pi$, $s_2 = \sqrt{13}$

$q_{w_1} = q_1 - q_2$, $q_{w_2} = q_2 - q_3$

$$\frac{1}{2\bar{A}_1}\oint_{s_1} \frac{q}{Gt} ds = \frac{1}{2\bar{A}_2}\oint_{s_2} \frac{q}{Gt} ds = \frac{1}{2\bar{A}_3}\oint_{s_3} \frac{q}{Gt} ds$$

② $\frac{1}{\bar{A}_1}\left[\frac{q_1}{t_1} s_1 + \frac{q_{w_1}}{t_1} 2\right] = \frac{1}{\bar{A}_2}\left[-\frac{q_{w_1}}{t_1} 2 + 2\left(\frac{q_2}{t_2} 3\right) + \frac{q_{w_2}}{t_1} 2\right] = \frac{1}{\bar{A}_3}\left[-\frac{q_{w_2}}{t_1} 2 + \frac{q_3}{t_1}(s_2 \dots\right.$

from ① & ② in matlab,

$$\boxed{q_1 = .036994\, M_x, \quad q_2 = .058753\, M_x, \quad q_3 = .029779\, M_x}$$

b) $\sigma_{xs} = \frac{q}{t}$, $\sigma_{xs_1} = \frac{q_1}{t_1}$, $\sigma_{xs_2} = \frac{q_2}{t_2}$, $\sigma_{xs_3} = \frac{q_3}{t_1}$

$\sigma_{xsw_1} = \frac{q_{w_1}}{t_1}$, $\sigma_{xsw_2} = \frac{q_{w_2}}{t_1}$

gives:

$$\boxed{\begin{aligned} \sigma_{xs_1} &= .739885\, M_x \\ \sigma_{xs_2} &= .587533\, M_x \\ \sigma_{xs_3} &= .595788\, M_x \\ \sigma_{xsw_1} &= -.4351803\, M_x \\ \sigma_{xsw_2} &= .5792669\, M_x \end{aligned}}$$

$$\theta = \frac{1}{2\bar{A}_1} \oint_{S_1} \frac{q}{Gt}\, ds = \frac{1}{2\bar{A}_1 G}\left[\frac{q_1}{t_1} S_1 + \frac{q_{w_1}}{t_1} 2 \right]$$

$L = 100$ in

c) $\boxed{\theta = 1.1571e{-7}\ \frac{\text{rad}}{\text{in}} M_x}$ $\quad \phi(0) = 0$, $\boxed{\phi(L) = 1.571e{-5}\ \text{rad}\ M_x}$

d) $TR = \frac{M_x}{\theta} = 8642277$

e) TR from class was 6e6

the ratio is : $\frac{TR_{HW}}{TR_{Class}} = 1.44038$

so expectibly, with more cells the overall ridgidity ~~increased~~ partially. I feel good about this ~~/~~ especially because it added aproximately 1/3 the ridgidity ~~/~~ and this came from the ~~3rd~~ cell