HW 4

Problem 1 a

a) $\underline{C}_{21} = C_z(\underline{\alpha})\, C_y(\underline{\beta})\, C_z(\underline{\gamma})$

$$= \begin{bmatrix} c\gamma c\beta c\alpha - s\gamma s\alpha, & c\gamma s\alpha + c\beta c\alpha s\gamma, & -c\alpha s\beta \\ -c\alpha s\gamma - c\gamma c\beta s\alpha, & c\gamma c\alpha - c\beta s\gamma s\alpha, & s\beta\, s\alpha \\ c\gamma s\beta, & s\gamma s\beta, & c\beta \end{bmatrix}$$

b) Since $\underline{C}_{21}(3,3) = c\beta$ and is only spot with 1 angle, check β for singularities

$$\beta = \pi \Rightarrow \underline{C}_{21} = \begin{bmatrix} c(\gamma-\alpha) & -s(\gamma-\alpha) & 0 \\ -s(\gamma-\alpha) & c(\gamma-\alpha) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$ cannot be solved

$$\beta = -\pi \Rightarrow \underline{C}_{21} = \begin{bmatrix} -c(\gamma-\alpha) & -s(\gamma-\alpha) & 0 \\ -s(\gamma-\alpha) & c(\gamma-\alpha) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$ cannot be solved

Singularities @ $\beta = \pm\pi$ & $\beta = 0$

$$\beta = 0 \Rightarrow \underline{C}_{21} = \begin{bmatrix} c(\gamma+\alpha) & s(\gamma+\alpha) & 0 \\ -s(\gamma+\alpha) & c(\gamma+\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

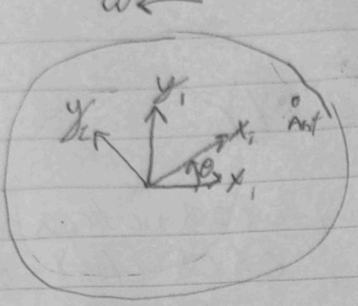
c) $$\omega_{21} = \begin{pmatrix} 0 \\ 0 \\ \dot\alpha \end{pmatrix} + C_z(\alpha)\begin{pmatrix} 0 \\ \dot\beta \\ 0 \end{pmatrix} + C_z(\alpha)\,C_y(\beta)\begin{pmatrix} 0 \\ 0 \\ \dot\gamma \end{pmatrix} = \begin{pmatrix} \dot\beta s\alpha - \dot\gamma c\alpha s\beta \\ \dot\beta c\alpha + \dot\gamma s\beta\, s\alpha \\ \dot\alpha + \dot\gamma c\beta \end{pmatrix}.$$

$$R = \begin{bmatrix} 0, & s\alpha, & -c\alpha s\beta \\ 0, & c\alpha, & s\beta\, s\alpha \\ 1, & 0, & c\beta \end{bmatrix}$$ Such that $\omega_{21} = R\begin{pmatrix} \dot\alpha \\ \dot\beta \\ \dot\gamma \end{pmatrix}$

d) if we assume small angle aproximation, $$R = \begin{bmatrix} 0 & \alpha & -\beta \\ 0 & 1 & \alpha\beta \\ 1 & 0 & 1 \end{bmatrix}$$

HW 4

Problem 3



1) $\vec{r} = \vec{\mathcal{F}}_2^{T}\, \underline{r}_2$, $\underline{r}_2 = \begin{pmatrix}1\\0\\0\end{pmatrix}\frac{1}{2}a_0 t^2$

2) $\dot{\vec{r}} = \vec{\mathcal{F}}_2^{T}\, \dot{\underline{r}}_2$, $\dot{\underline{r}}_2 = \frac{d}{dt}\underline{r}_2 = \begin{pmatrix}1\\0\\0\end{pmatrix}a_0 t$

$\ddot{\vec{r}} = \vec{\mathcal{F}}_2^{T}\, \ddot{\underline{r}}_2$, $\ddot{\underline{r}}_2 = \frac{d^2}{dt^2}\underline{r}_2 = \begin{pmatrix}1\\0\\0\end{pmatrix}a_0$

3) $\vec{\omega}_{21} = \vec{\mathcal{F}}_2^{T}\begin{pmatrix}0\\0\\1\end{pmatrix}\omega$

4) $\vec{r} = \vec{\mathcal{F}}_1^{T}\, \underline{r}_1$, $\underline{r}_1 = \underline{C}_{12}\, \underline{r}_2$, $\underline{C}_{12} = C_z(-\theta) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\underline{r}_1 = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{pmatrix}\frac{1}{2}a_0 t^2\\0\\0\end{pmatrix} = \begin{pmatrix}c\theta\\s\theta\\0\end{pmatrix}\frac{1}{2}a_0 t^2$$

5) $\dot{\vec{r}} = \vec{\mathcal{F}}_1^{T}\, \dot{\underline{r}}_1$, $\ddot{\vec{r}} = \vec{\mathcal{F}}_1^{T}\, \ddot{\underline{r}}_1$

$$\dot{\underline{r}}_1 = \begin{pmatrix}c\theta\\s\theta\\0\end{pmatrix}a_0 t + \begin{pmatrix}-s\theta\\c\theta\\0\end{pmatrix}\frac{1}{2}a_0 t^2\omega,\ \ddot{\underline{r}}_1 = \begin{pmatrix}c\theta\\s\theta\\0\end{pmatrix}a_0 + \begin{pmatrix}-s\theta\\c\theta\\0\end{pmatrix}a_0 t\omega + \begin{pmatrix}-c\theta\\s\theta\\0\end{pmatrix}\frac{1}{2}a_0 t^2\omega^2$$

6) Since it is for verification and not a derivation or proof, this is done in Matlab with symbolic toolbox