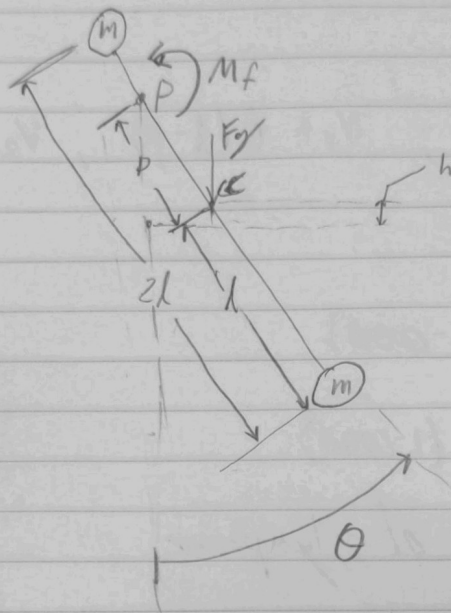


ICGEE3

1) a)



$T_P \equiv$ torque about P , $J_P \equiv$ Moment of Inertia about P

$M_f \equiv$ friction, $T_g \equiv$ Torque from gravity

5) $T_P = M_f + T_g$, $T_g = -2Pmg \sin(\theta)$, $M_f = -B\dot{\theta}$

EOM

$$\rightarrow T_P = J_P \ddot{\theta}, \quad \ddot{\theta} = \frac{-Pmg \sin \theta}{(l^2 + p^2)} - \frac{B \dot{\theta}}{2m(l^2 + p^2)}$$

$$2) \quad X = \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix}, \quad \dot{X} = \begin{pmatrix} \dot{\theta} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} \dot{\theta} \\ \frac{-Pmg \sin \theta}{l^2 + p^2} - \frac{B \dot{\theta}}{2m(l^2 + p^2)} \end{pmatrix} = f(x)$$

$$3) \quad X^* \text{ means } \dot{X}^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \dot{\theta} \\ \frac{-Pmg \sin \theta}{l^2 + p^2} - \frac{B \dot{\theta}}{2m(l^2 + p^2)} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{Pmg \sin \theta}{l^2 + p^2} \end{pmatrix}$$

$$X^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 2\pi \end{pmatrix}$$

4) $V = PE + KE,$

$$KE = \frac{1}{2} m V_1^2 + \frac{1}{2} m V_2^2, \quad V_1 = \dot{\theta}(l-p), \quad V_2 = \dot{\theta}(l+p)$$

$$KE = \frac{1}{2} m \dot{\theta}^2 (l^2 + p^2)$$

$$PE = 2mgh = 2mgp(1 - \cos\theta)$$

$$V = \frac{1}{2} m \dot{\theta}^2 (l^2 + p^2) + 2mgp(1 - \cos\theta)$$

$V(Q) = 0$, first part of Lyapunov

$$\nabla V = \begin{pmatrix} 2mpg \sin\theta \\ 2m\dot{\theta}(l^2 + p^2) \end{pmatrix}, \quad \dot{V} = (\nabla V)^T \dot{x}$$

$$\dot{V} = \begin{pmatrix} 2mpg \sin\theta \\ 2m\dot{\theta}(l^2 + p^2) \end{pmatrix} \begin{bmatrix} -p \sin\theta \\ \dot{\theta} \end{bmatrix} = 2mpg \dot{\theta} \sin\theta - \frac{B \dot{\theta}}{2m} (l^2 + p^2)$$

$\dot{V} = 0$ for any $\dot{\theta} = 0$, so there are $x \neq Q$ which

\dot{V} is not less than zero, so we cannot conclude global asymptotic stability.

5)
$$\underline{A} = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \frac{\partial \dot{x}_1}{\partial x_2} \\ \frac{\partial \dot{x}_2}{\partial x_1} & \frac{\partial \dot{x}_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-pg \cos\theta}{l^2 + p^2} & \frac{-B}{2m(l^2 + p^2)} \end{bmatrix}$$

$$\underline{A} = \begin{bmatrix} 0 & 1 \\ \frac{-pg}{l^2 + p^2} & \frac{-B}{2m(l^2 + p^2)} \end{bmatrix}$$

eigenval(A^*) \rightarrow

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ \frac{-Pg}{l^2+p^2} & \frac{-B}{2M(l^2+p^2)} - \lambda \end{vmatrix} = 0$$

$$\frac{\lambda B}{2M(l^2+p^2)} + \lambda^2 + \frac{Pg}{l^2+p^2} = 0$$

$$\lambda = \frac{-\frac{B}{2M(l^2+p^2)} \pm \sqrt{\frac{B^2}{4M^2(l^2+p^2)^2} - 4\frac{Pg}{l^2+p^2}}}{2}$$

$$0 \text{ if } \frac{B}{2M(l^2+p^2)} - 4\frac{Pg}{l^2+p^2} < 0$$

$$\lambda = -\frac{1}{2} \left[\frac{B}{2M(l^2+p^2)} \right] + bi$$

$$0 \text{ if } \frac{B}{2M(l^2+p^2)} - 4\frac{Pg}{l^2+p^2} = 0$$

$$\lambda = -\frac{1}{2} \left[\frac{B}{2M(l^2+p^2)} \right]$$

$$0 \text{ if } \frac{B}{2M(l^2+p^2)} - 4\frac{Pg}{l^2+p^2} \geq 0$$

$$\frac{B}{2M(l^2+p^2)} - 4\frac{Pg}{l^2+p^2} < \frac{B}{2M(l^2+p^2)}$$

$$\text{So } \lambda < 0$$

\hookrightarrow no matter what $\text{real}(\lambda) < 0$,
So system is locally, asymptotically stable at $X^* = 0$

housecleaning

```
clear all
close all
clc

%%single p
X0 = [.5;0];
g = 9.8;
l = 1;
p = .25;
m = 1;
B = .5;
tspan = [0,40];

options = odeset('RelTol', 1e-8, 'AbsTol',1e-8);
[t,X]=ode45(@eom,tspan,X0,options,l,p,B,m,g);

% state phase
figure
plot(X(:,1),X(:,2))
xlabel("theta")
ylabel("thetadot")
title("state phase plot, p = .25")
% state time 3d
% figure
% plot3(X(:,1),X(:,2),t)
% xlabel("theta")
% ylabel("thetadot [1/s]")
% zlabel("time [s]")

% state time 2d
figure
plot(t,X(:,1),t,X(:,2))
legend("theta","thetadot [1/s]")
xlabel("time [s]")
title("state time plot, p = .25")

%%multiple ps
ps = -.25:.125:.25;
n = length(ps);

X0 = [.5;0];
g = 9.8;
l = 1;
m = 1;
B = .5;
tspan = [0,40];
options = odeset('RelTol', 1e-8, 'AbsTol',1e-8);

%calculate X at each p to test
runs = {};
```

```

for i = 1:n
    p = ps(i);
    [t,X]=ode45(@eom,tspan,X0,options,l,p,B,m,g);
    runs{i,1} = X;
    runs{i,2} = t;
end

```

```

%%%%%%%%%% phase portrait
figure
hold on
for i = 1:n
    %     plot(runs{i,1}(:,1),runs{i,1}(:,2))
    plot3(runs{i,1}(:,1),runs{i,1}(:,2),runs{i,2})
end
xlabel("theta")
ylabel("thetadot [1/s]")
legend("p = "+string(ps))
title("phase portrait, p = var. vals")

```

```

%%%%%%%%%% all theta
temp1 = " theta";
temp2 = [];
figure
hold on
for i = 1:n
    plot(runs{i,2},runs{i,1}(:,1));
    temp2 = [temp2,"p = "+string(ps(i))+temp1];
end
legend(temp2)
xlabel("time [s]")
title("theta time plot, p = var. vals")
clear temp2 temp1

```

```

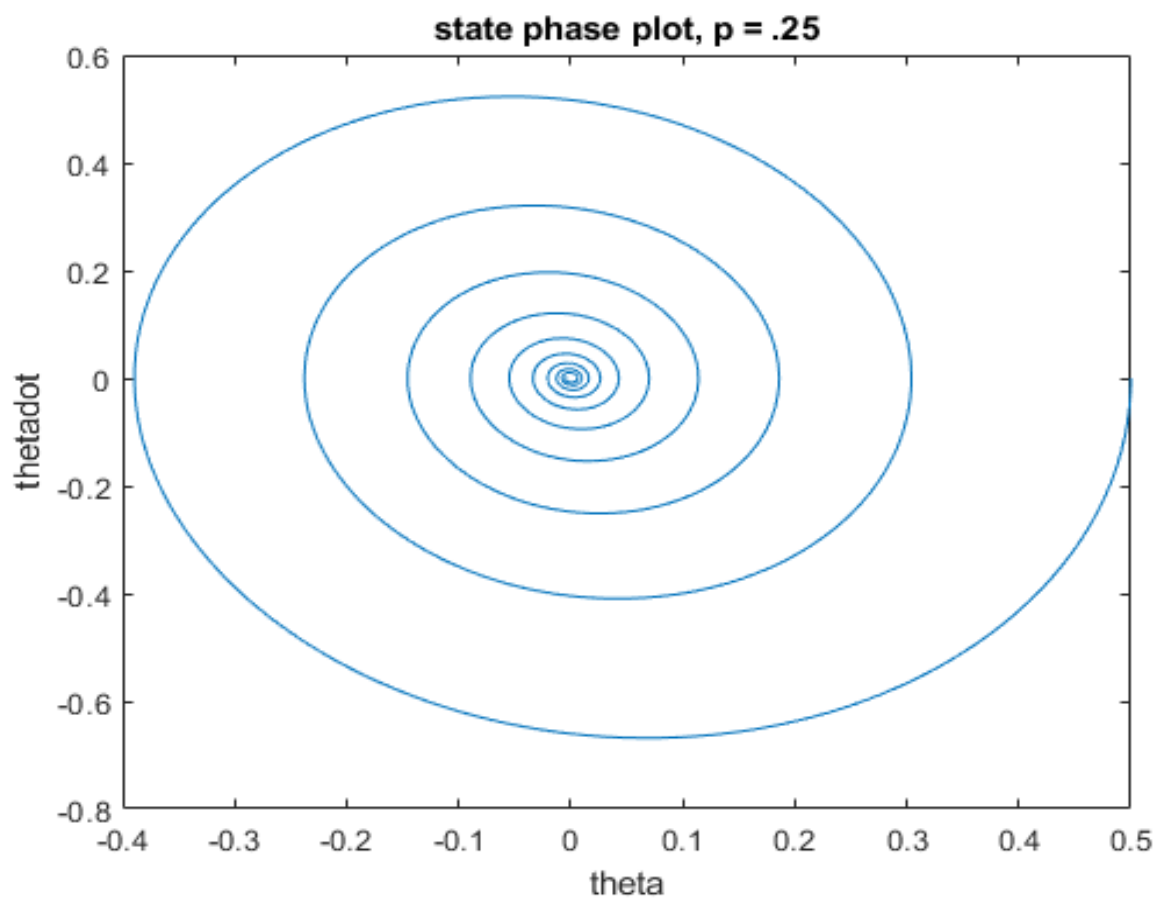
%%%%%%%%%% all thetadot
temp1 = " thetadot";
temp2 = [];
figure
hold on
for i = 1:n
    plot(runs{i,2},runs{i,1}(:,2));
    temp2 = [temp2,"p = "+string(ps(i))+temp1];
end
legend(temp2)
xlabel("time [s]")
ylabel("[1/s]")
title("thetadot time plot, p = var. vals")
clear temp2 temp1

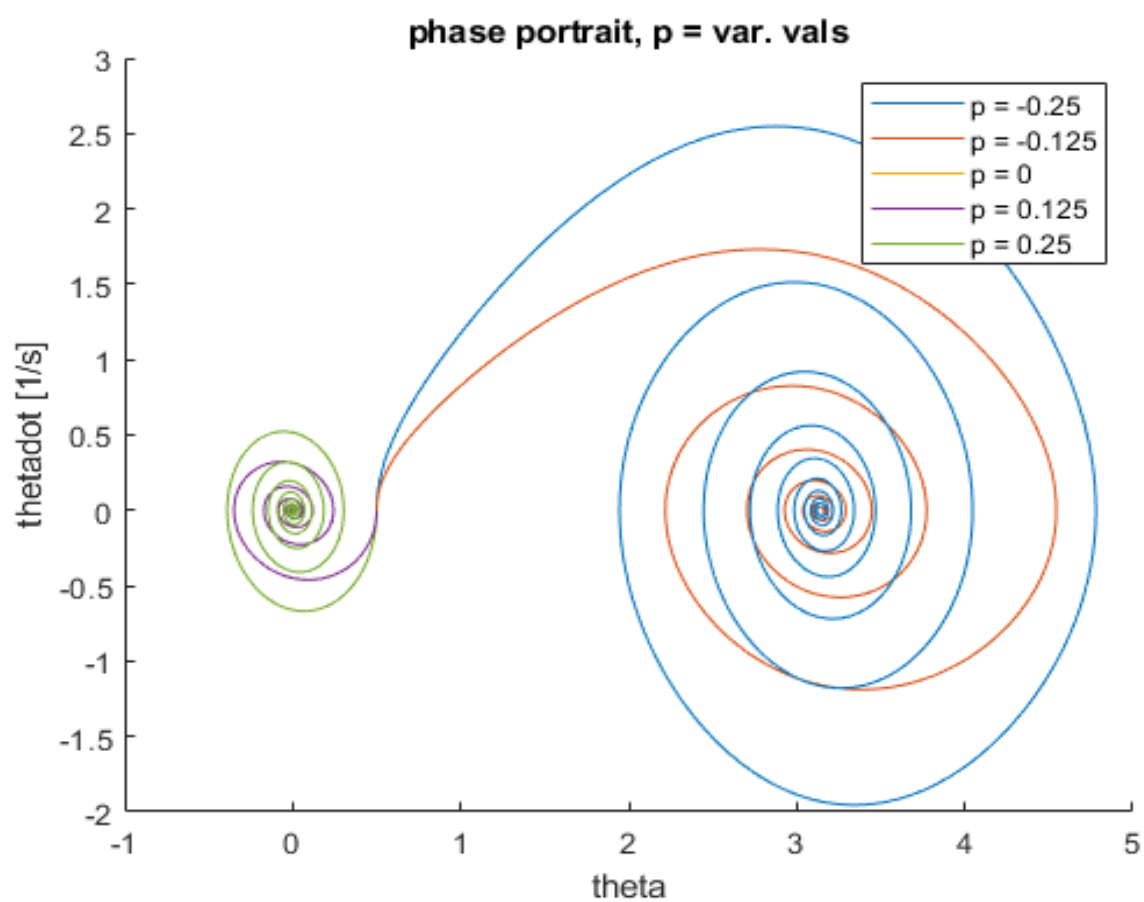
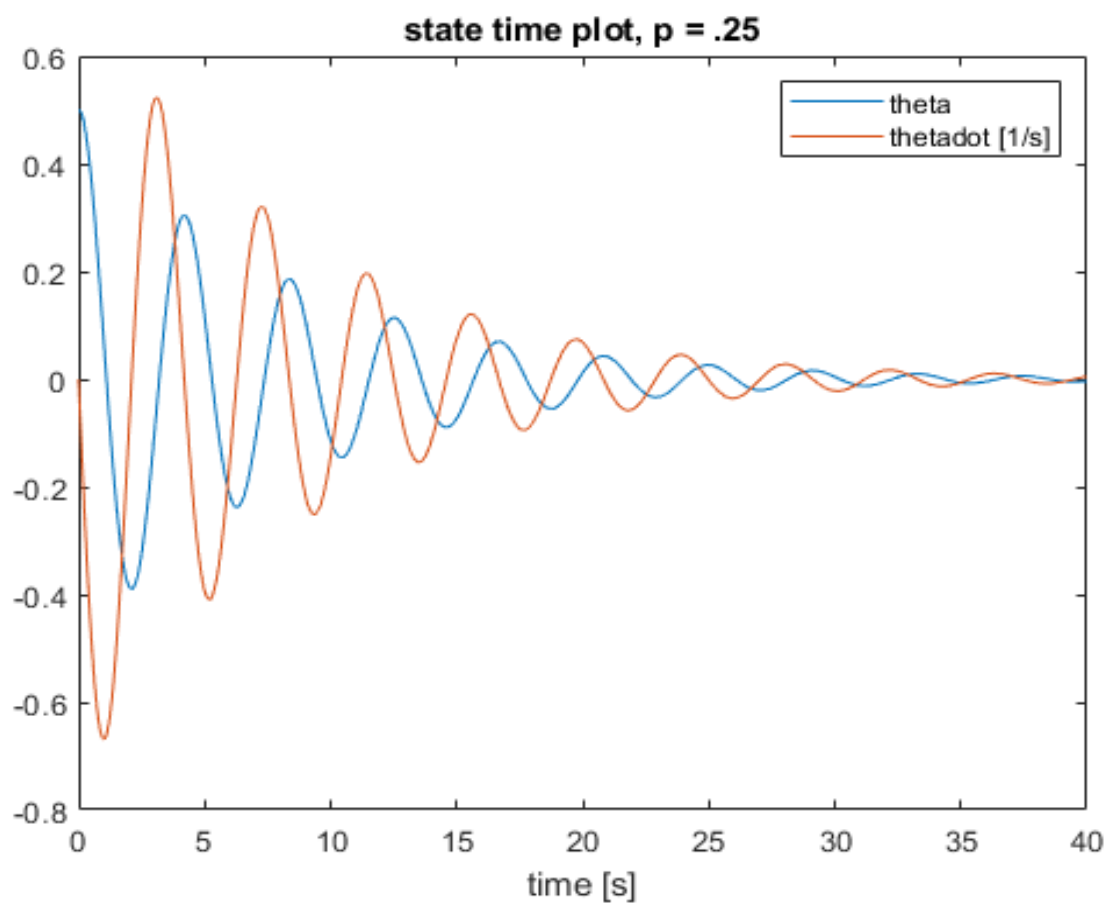
```

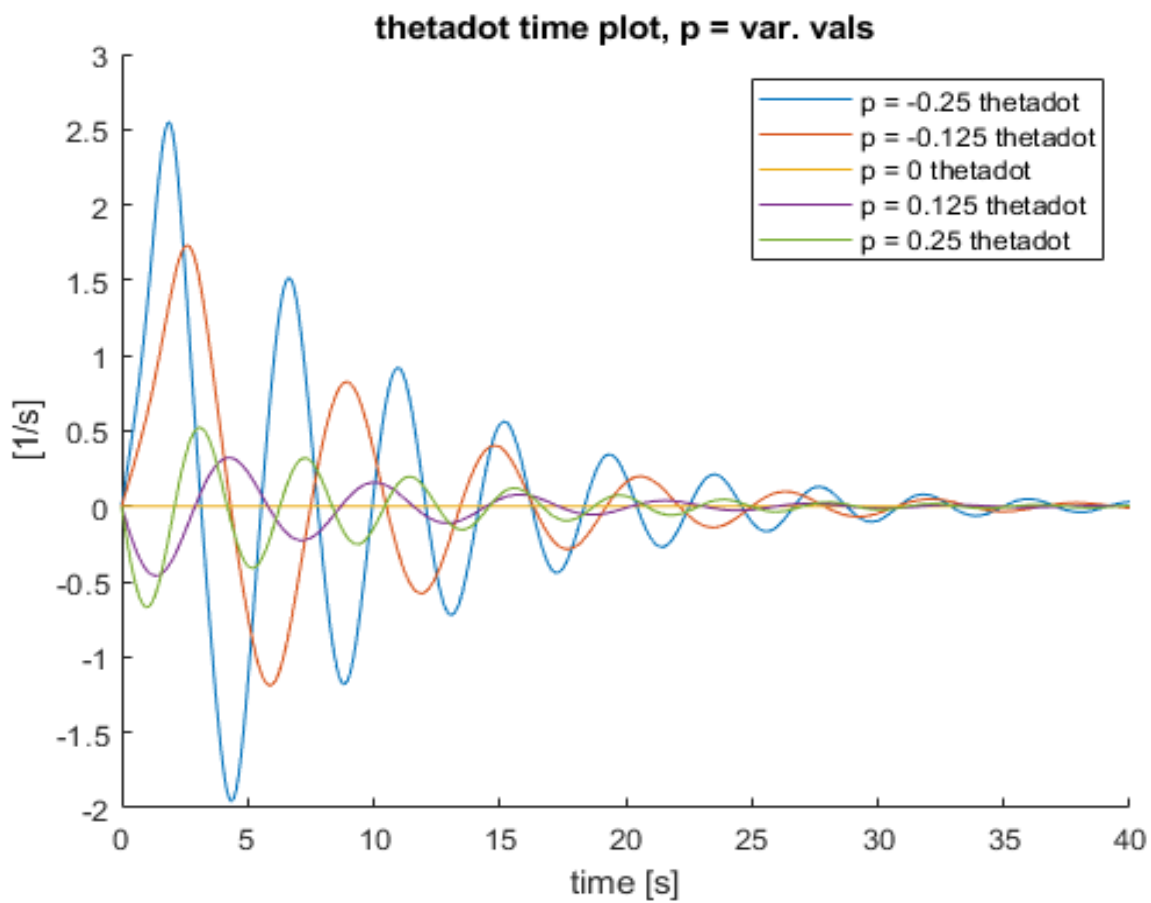
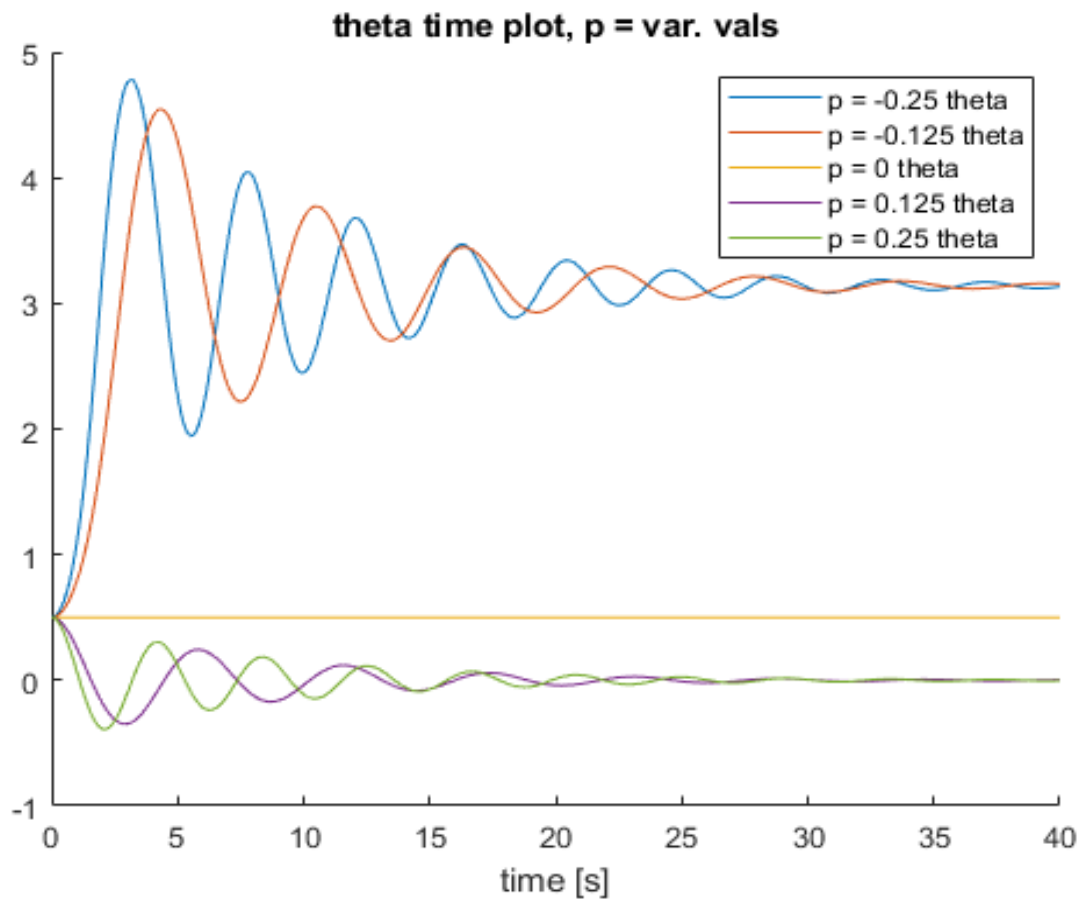
```

%%functions
function Xdot = eom(t,X,l,p,B,m,g);
theta = X(1);
thetadot = X(2);
Xdot = [
    thetadot
    -p*g*(sin(theta))/(l^2+p^2) - B*thetadot/(2*m*(l^2+p^2));
];
end

```







Further Analysis

- ① The steady-state response oscillates symmetrically around zero over time. The transient response lacks this pattern and continues to change over time before ultimately converging.
- ② The pendulum's response time remains relatively the same despite changing the parameter P for different cases of the system.
- ③ For all cases of P for the pendulum, the system locally converges to a point over time. However, without the initial conditions the stability cannot be applied globally. This concludes that the system is locally asymptotically stable, which agrees with the solution to question five.