Pre Lab 4

1. for 4×4 matrix $\underline{A}$

$$\underline{A}\,\underline{x} = \underline{b}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{23} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$$\Rightarrow \quad \begin{aligned} x_1 a_{11} + x_2 a_{12} + x_3 a_{13} + x_4 a_{14} &= b_1 \\ x_1 a_{21} + x_2 a_{22} + x_3 a_{23} + x_4 a_{24} &= b_2 \\ x_1 a_{31} + x_2 a_{32} + x_3 a_{33} + x_4 a_{34} &= b_3 \\ x_1 a_{41} + x_2 a_{42} + x_3 a_{43} + x_5 a_{44} &= b_4 \end{aligned}$$

$$x_1 = \frac{1}{a_{11}} \left(b_1 - x_2 a_{12} - x_3 a_{13} - x_4 a_{14} \right)$$

$$x_2 = \frac{1}{a_{22}} \left(b_2 - x_1 a_{21} - x_3 a_{23} - x_4 a_{24} \right)$$

$$x_3 = \frac{1}{a_{33}} \left(b_3 - x_1 a_{31} - x_2 a_{32} - x_4 a_{34} \right)$$

$$x_4 = \frac{1}{a_{44}} \left(b_4 - x_1 a_{41} - x_2 a_{42} - x_3 a_{43} \right)$$

$$\begin{matrix} 0 & & 2 & 3 \\ 1 & 0 & 4 & 5 \\ 2 & 3 & 0 & 6 \\ 4 & 5 & 6 & 0 \end{matrix}$$

2 $\quad \underline{x}^{k+1} = D^{-1}\left(\underline{b} - L\underline{x}^{k+1} - U\underline{x}^{k}\right)$

$$\begin{pmatrix} x_1^{k+1} \\ x_2^{k+1} \\ \vdots \\ x_3^{k+1} \end{pmatrix} = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & a_{nn} \end{bmatrix}^{-1} \left(\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} - \begin{bmatrix} 0 & 0 & \cdots & 0 \\ a_{21} & 0 & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{n1} & \cdots & & 0 \end{bmatrix} \begin{pmatrix} x_1^{k+1} \\ x_2^{k+1} \\ \vdots \\ x_n^{k+1} \end{pmatrix} - \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} \\ 0 & 0 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & 0 \end{bmatrix} \begin{pmatrix} x_1^{k} \\ x_2^{k} \\ \vdots \\ x_n^{k} \end{pmatrix} \right)$$

$$\underline{x}^{k+1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & \cdots & 0 \\ 0 & \frac{1}{a_{22}} & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & \frac{1}{a_{nn}} \end{bmatrix} \left(\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} - \begin{pmatrix} 0 \\ a_{21}x_1^{k+1} + \\ \vdots \\ a_{n1}x_1^{k+1} + a_{n2}x_2^{k+1} + \cdots a_{n\,n-1}x_{n-1}^{k+1} \end{pmatrix} - \begin{pmatrix} a_{12}x_2^{k} + a_{13}x_3^{k} + \cdots + a_{1n}x_n^{k} \\ a_{23}x_3^{k} + \cdots + a_{2n}x_n^{k} \\ \vdots \\ 0 \end{pmatrix} \right)$$

$$\underline{x}^{k+1} = D^{-1} \begin{pmatrix} b_1 - \overline{0} - \overline{a_{12}x_2^{k} - a_{13}x_3^{k} - a_{14}x_4^{k} - \cdots - a_{1n}x_n^{k}} \\ b_2 - \overline{a_{21}x_1^{k+1}} - \overline{0 - a_{23}x_3^{k} - a_{24}x_4^{k} - \cdots - a_{2n}x_n^{k}} \\ b_3 - \overline{a_{31}x_1^{k+1} - a_{32}x_2^{k+1}} - \overline{0 - a_{34}x_4^{k} - \cdots - a_{3n}x_n^{k}} \\ \vdots \\ b_{n-1} - a_{1}^{n-1}x_1^{k+1} - a_{2}^{n-1}x_2^{k+1} \cdots a_{n-2}^{n-1}x_{n-2}^{k+1} - a_{n}^{n-1}x_n^{k} \\ b_n - \overline{a_{1}^{n}x_1^{k+1} - a_{2}^{n}x_2^{k+1} \cdots a_{n-1}^{n}x_{n-1}^{k+1}} - \overline{0} \end{pmatrix} = D^{-1}\left(\underline{b} - M\right)$$

$$M_i = \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} + \sum_{j=i+1}^{n} a_{ij}x_j^{k}$$

$$\underline{x}^{k+1} = D^{-1}\left(\underline{b} - M\right) \quad \Rightarrow \quad x_i^{k+1} = \frac{1}{a_{ii}}\left(b_i - M_i\right)$$