331
HW2

Problem 1

$$\underline{x} = \begin{pmatrix}1\\2\\3\end{pmatrix}, \quad \underline{\sigma}(\underline{x}) = \begin{bmatrix}1 & 0 & 2y\\ 0 & 1 & 4x\\ 2y & 4x & 1\end{bmatrix} \text{ksi} = \begin{bmatrix}1 & 0 & 4\\ 0 & 1 & 4\\ 4 & 4 & 1\end{bmatrix} \text{ksi}$$

plane we study $x+y+z = 6\text{in}$, $\underline{n} = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$

a) $$\underline{T}(\underline{n}) = \underline{\sigma}\,\underline{n} = \frac{1}{\sqrt{3}}\left(\begin{bmatrix}1 & 0 & 4\\ 0 & 1 & 4\\ 4 & 4 & 1\end{bmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}\right) = \boxed{\frac{1}{\sqrt{3}}\begin{pmatrix}5\\5\\9\end{pmatrix} \text{ksi}}$$

b) $$\underline{T}_n(\underline{n}) = \underline{n}^T\underline{T}\,\underline{n} = \left(\frac{1}{\sqrt{3}}\right)^3\left[(1\ 1\ 1)\begin{pmatrix}5\\5\\9\end{pmatrix}\right]\begin{pmatrix}1\\1\\1\end{pmatrix} = 3^{-\frac{3}{2}}\left[19\begin{pmatrix}1\\1\\1\end{pmatrix}\right] = \boxed{3^{-\frac{3}{2}}19\begin{pmatrix}1\\1\\1\end{pmatrix} \text{ksi}}$$

$$\approx 3.66\begin{pmatrix}1\\1\\1\end{pmatrix}\text{ksi}$$

$$\underline{T} = \underline{T}_n + \underline{T}_t, \quad \underline{T}_t = \underline{T} - \underline{T}_n = 3^{-\frac{1}{2}}\begin{pmatrix}5\\5\\9\end{pmatrix} - 3^{-\frac{3}{2}}19\begin{pmatrix}1\\1\\1\end{pmatrix}$$

c) $$= \boxed{\begin{pmatrix}(5)3^{-\frac{1}{2}} - (19)3^{-\frac{3}{2}}\\ (5)3^{-1/2} - (19)3^{-\frac{3}{2}}\\ (9)3^{-1/2} - (19)3^{-\frac{3}{2}}\end{pmatrix}\text{ksi} \approx \begin{pmatrix}-.77\\-.77\\1.54\end{pmatrix}\text{ksi}}$$

$$|\sigma - \lambda \mathbb{1}| = 0 = \begin{vmatrix}1-\lambda & 0 & 4\\ 0 & 1-\lambda & 4\\ 4 & 4 & 1-\lambda\end{vmatrix} = -\lambda^3 + 3\lambda^2 + 29\lambda - 31$$

from the cubic equation, $\lambda_1 = 1 + 4\sqrt{2}$
$\lambda_2 = 1$
$\lambda_3 = 1 - 4\sqrt{2}$

331
HW2

$(\underline{\sigma} - \lambda \underline{\mathbb{1}})\underline{s} = 0$

$4z - x\lambda + x = 0$

$4z - y\lambda + \lambda = 0$

$4x + 4y - z\lambda + z = 0$

Plugging in gives

$\sigma_1 = 1 + 4\sqrt{2}$ ksi $\quad s_1 = \frac{1}{2}\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}$ in

$\sigma_2 = 1$ ksi $\quad s_2 = \frac{\sqrt{2}}{2}\begin{pmatrix}-1\\1\\0\end{pmatrix}$ in

$\sigma_3 = 1 - 4\sqrt{2}$ ksi $\quad s_3 = \frac{1}{2}\begin{pmatrix}-1\\-1\\\sqrt{2}\end{pmatrix}$ in

$\sigma_n = \frac{1}{3}\mathrm{tr}\,\underline{\sigma} = \boxed{1 \text{ ksi}}$

$\underline{\sigma}_{dev} = \underline{\sigma} - \sigma\,\mathbb{1} = \boxed{\begin{bmatrix}0 & 0 & 4\\0 & 0 & 4\\4 & 4 & 0\end{bmatrix} \text{ksi}}$

$\sigma_e = \left(\frac{3}{2}\sum_{\beta=x}^{z}\sum_{\alpha=x}^{z}\left(\sigma_{dev_{\alpha\beta}}^{2}\right)\right)^{1/2} = \sqrt{\frac{3}{2}(4)(4^2)} = \boxed{\sqrt{96} \text{ ksi}}$

$\sigma_y = 15.6$ ksi, $\quad F = 1.5$

tresca, yeild if $F\tau_{max} \geq \frac{\sigma_y}{2}$

$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) = 4\sqrt{2}$

$F\tau_{max} = 6\sqrt{2} \approx 8.4853$ ksi $\qquad \frac{\sigma_y}{2} = 7.8$

So tresca predicts yeilding

$\boxed{\begin{array}{c}331 \\ HW2\end{array}}$

von mises yeild if $\sigma_e > \sigma_y$

$\sigma_e \approx 9.798$ ksi

$\boxed{\text{So von mises predicts there will not be yeild}}$

Problem 2 - see matlab

housecleaning

```
clear all
close all
clc

% sympref('FloatingPointOutput',true)
% sympref('FloatingPointOutput',false)
addpath('C:\joshFunctionsMatlab\')
```

Problem 2

```
clear

% Torque
syms T
assume(1<T)

% givens
p = 100/1000; % ksi
r = 25;
t = .1;
A = pi*r^2;

% stress state
sigHoop = p*r/t;
sigAxial = p*r/(2*t);
tau = T/(2*t*A);

sig0 = [[sigAxial,tau,0];[tau,sigHoop,0];[0,0,0]];

% stress state in principle reference frame
[S,sig0] = eig(sig0);
temp = [sig0(1,1),sig0(2,2),sig0(3,3)];
[temp,I] = sort(temp,'descend');
sig0 = [[temp(1),0,0];[0,temp(2),0];[0,0,temp(3)]];

S = [S(:,I(1)),S(:,I(2)),S(:,I(3))];
clear temp I

% hydrostatic, deviatoric, max shear, effective stress
sig_h = (1/3)*trace(sig0);
sig_dev = sig0 - sig_h*eye(3);

sig_e = ((3/2)*sum(sum(sig_dev.^2)))^(1/2);
tau_max = (1/2)*(sig0(1,1)-sig0(3,3));

% solve for T
sig_y = 30; % ksi
eqn1 = tau_max == sig_y/2;
eqn2 = sig_e == sig_y;
```

```
sol1 = solve(eqn1,T);
sol2 = solve(eqn2,T);


temp = [sig0(1,1),sig0(2,2),sig0(3,3)];
disp('The principle stresses are in ksi are: ')
disp(temp')
disp('While using the tresca yeild criterion, T can be as high as
 '+string(sol1)+' = '+string(vpa(sol1,5))+' kip-in.')
disp('While using the von mises yeild criterion, T can be as high as
 '+string(sol2)+' = '+string(vpa(sol2,5))+' kip-in.')
disp('We can conclude that tresca yeild condition is more conservative than
 von mises yeild condition because ')
```

```
The principle stresses are in ksi are:
(9375*pi + (16*T^2 + 9765625*pi^2)^(1/2))/(500*pi)
(9375*pi - (16*T^2 + 9765625*pi^2)^(1/2))/(500*pi)
                                                  0

While using the tresca yeild criterion, T can be as high as
 (625*14^(1/2)*pi)/2 = 3673.4 kip-in.
While using the von mises yeild criterion, T can be as high as
 (625*23^(1/2)*pi)/2 = 4708.3 kip-in.
We can conclude that tresca yeild condition is more conservative than von
 mises yeild condition because the yield torque is lower for the tresca critirion than
The von mises
```

Published with MATLAB® R2022a