EA1

1

$$u_r = \frac{1}{r}\frac{\partial}{\partial\theta}\left[r - \frac{R^2}{r}\right] U_\infty \sin\theta$$

$$= \frac{1}{r}\left[r - \frac{R^2}{r}\right] U_\infty \cos\theta$$

$$u_\theta = -\frac{\partial}{\partial r}\left[r - \frac{R^2}{r}\right] U_\infty \sin\theta$$

$$= -U_\infty \sin\theta \left[1 + \frac{R^2}{r^2}\right]$$

2 $$\phi = \int u_r \, \partial r = U_\infty \cos\theta \int \frac{1}{r}\left[r - \frac{R^2}{r}\right] dr = \frac{1}{r} U_\infty \cos(\theta) \frac{R^2 + r^2}{r}$$

3

$$\frac{\partial(r u_r)}{\partial r} + \frac{\partial u_\theta}{\partial\theta} = U_\infty\left[\left[1 + \frac{R^2}{r^2}\right]\frac{\partial}{\partial\theta}\sin\theta + \cos\theta\frac{\partial}{\partial r}\cancel{r}\frac{1}{r}\left[r - \frac{R^2}{r}\right]\right]$$

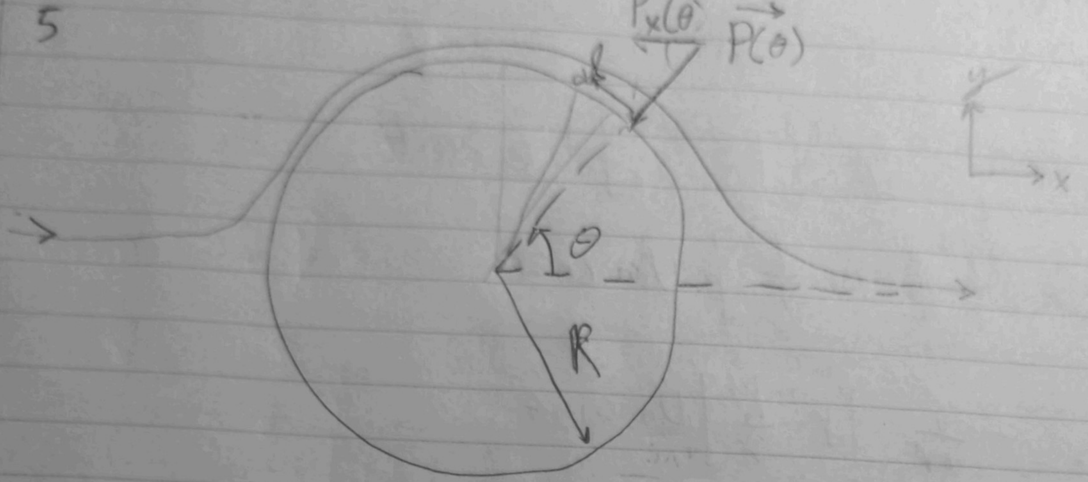
$$= U_\infty\left[-\cos\theta\left[1 + \frac{R^2}{r^2}\right] + \cos(\theta)\left[1 + \frac{R^2}{r^2}\right]\right] = 0$$

4 $$P = P_\infty + \frac{1}{2}\rho U_\infty^2 - \frac{1}{2}\rho(-2 U_\infty \sin\theta)^2$$

$$\boxed{P = P_\infty + \frac{1}{2}\rho U_\infty^2 - \frac{1}{2}\rho(4 U_\infty^2 \sin^2\theta)} = P_\infty + \frac{1}{2}\rho U_\infty^2\left[1 - 4\sin\right.$$

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = \frac{\cancel{P_\infty} + \cancel{\frac{1}{2}}\cancel{\rho} \cancel{U_\infty^2}[1 - 4\sin^2\theta] - \cancel{P_\infty}}{\cancel{\frac{1}{2}}\cancel{\rho_\infty}\cancel{U_\infty^2}} = 1 - 4\sin^2$$

5



$$P_x = -\cos(\theta)\ P(\theta)$$

$$d\vec{F} = \overrightarrow{P(\theta)}\ dl$$

$$dF_x = P_x(\theta)\,dl \quad , \ dl = R\,d\theta$$

$$\oint dF_x = F_x = \oint P_x(\theta)\,R\,d\theta = -R\int_0^{2\pi} \cos(\theta)\,P(\theta)\ d\theta$$

$$= -R\int_0^{2\pi} \cos(\theta)\left[P_\infty + \tfrac{1}{2}\rho U_\infty^2\left(1-4\sin^2\theta\right)\right] d\theta = 0$$

BLT 1

laminar : $C_{f_{x\ell}} = \dfrac{.664}{Re_x^{1/2}}$, turbulent : $C_{f_{xt}} = \dfrac{.059}{Re_x^{1/5}}$

1) laminar = $C_{f_{d\ell}} = 2d\, C_{f_{x\ell}} = \boxed{d \dfrac{1.3280}{Re_x^{1/2}}}$

2) turbulent $C_{f_{dt}} = 2d\, C_{f_{xd}} = \boxed{d \dfrac{.1180}{Re_x^{1/5}}}$

3) 30% laminar 70% turbulent = $C_{f_d} = 30\%\, C_{f_{d\ell}} + 70\%\, C_{f_{dt}}$

$$= \boxed{d\left[\frac{.3984}{Re_x^{1/2}} + \frac{.0826}{Re_x^{1/5}}\right]}$$

4) 1. transition instantly from laminar to turbulent @ trip

2. trip on top and bottom

3. frisbee as infinite flat plate

EA2

Show $F = 6\pi\mu a V_0$, $Re < 1$

$V_r = \frac{1}{r^2 \sin\theta} \frac{\partial \psi}{\partial \theta}$, $V_\theta = \frac{-1}{r \sin\theta} \frac{\partial \psi}{\partial r}$

from $\nabla \cdot \vec{V} = 0$ @ wall , ie $div(\vec{v})|_{r=a} = 0$

let $Q\psi = \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial \theta} \frac{\sin\theta}{r^2} \frac{\partial \frac{1}{\sin\theta}}{\partial \theta}$

$\frac{\partial P}{\partial r} = \frac{\mu}{r^2 \sin\theta} \frac{\partial \left[\frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial \theta} \frac{\sin\theta}{r^2} \frac{\partial \frac{1}{\sin\theta}}{\partial \theta} \right]}{\partial \theta}$, $\frac{\partial P}{\partial \theta} = -\frac{\mu}{\sin\theta} \frac{\partial \left[\frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial \theta} \frac{\sin\theta}{r^2} \frac{\partial \frac{1}{\sin\theta}}{\partial \theta} \right]}{\partial r}$

where $Q = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2} \frac{\partial}{\partial \theta} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \right]$

Sin p is in both eqs, divice by p to get

$Q^2 \psi = 0$, let $\psi = f(r) \sin^2\theta$

So $r^4 f^4 - 4r^2 f'' + 8rf' - 8f = 0$
whose general solution is

$f(r) = \frac{A}{r} + Br + Cr^2 + Dr^4$ since the roots are $-1, 1, 2, 4$
respectively

$\psi(r, \theta) = V_0 \left[\frac{a^3}{4r} - \frac{3ar}{4} + \frac{r^2}{2} \right] \sin^2\theta$

2) $P = -\left[\frac{3a\mu V_0}{2r^2} \right] \cos\theta$, $V_r = U_0 \left[\frac{a^3}{2r^3} - \frac{3a}{2r} + 1 \right] \cos\theta$, $V_\theta = V_0 \left[\frac{a^3}{4r^3} + \frac{3a}{4r} - 1 \right] \sin\theta$

$\tau = -\mu \left[r \frac{\partial}{\partial r} \left(\frac{V_\theta}{r} \right) + \frac{1}{r} \left(\frac{\partial V_r}{\partial \theta} \right) \right]$

1) $F_{pressure} = 2\pi a^2 \int_0^{\pi} p \sin\theta \cos\theta \, d\theta = 2\pi a \mu V_0$

$F_{skin} = 2\pi a^2 \int_0^{\pi} -\mu \left[r \frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right) + \frac{1}{r}\left(\frac{\partial V_r}{\partial \theta}\right)\right] \sin^2\theta \, d\theta = 4\pi a \mu V_0$

By linear combination,

$$V_0 = \frac{2ga^2}{9\mu}\left[\rho_{stereo} - \rho_{fluid}\right]$$

3) $C_p = \frac{2P}{\rho V_\infty^2} = \frac{-6a\mu V_0 \cos\theta}{r^2 \rho V_0^2} = \frac{-6a\mu\cos\theta}{\rho r^2 V_0}$

$D = 6\pi a \mu V_0 = F_p - F_s$

$C_d = \frac{2D}{\rho V_\infty^2} = \frac{12\pi a \mu V_\infty}{Re} = \frac{24}{Re}$

$Re = \frac{\rho V_\infty}{\mu} \quad \frac{\mu}{\rho V_\infty}$

Table of Contents

clean up

```
clear all
close all
clc
```

BLT1

SP1

```
clear all
close all
syms t x y
% U = @(t) [10*cos(10*t)*exp(-t);0];
Ux = 10*cos(10*t)*exp(-t);
Uy = 0;
psiy = int(Ux,y);
psix = -int(0,x);

yfun = matlabFunction(psiy);
yfun2 =@(t) .1.*cos(t.*1.0e+1).*exp(-t).*1.0e+1;
% xfun = matlabFunction(psix);
xfun = @(t) 0;
figure
fplot(xfun,yfun2,[0,10])
title("steamlines X Y")
xlabel("x [m]")
ylabel("y [m]")

figure
fplot(xfun,yfun2,[0,10])
title("pathlines X Y")
xlabel("x [m]")
ylabel("y [m]")

t = @(t) t;
figure
fplot3(xfun,yfun2,t,[0,10])
title("pathlines X Y t")
xlabel("x [m]")
ylabel("y [m]")
```

```
zlabel("t [s]")

disp("the flows are unsteady beacuse e pathlines change with respect to time.
 It looks like it aproaching a steady flow around t = 8s.")
```

Warning: Function behaves unexpectedly on array inputs. To improve performance,
properly vectorize your function to return an output with the same size and shape as the input arguments.
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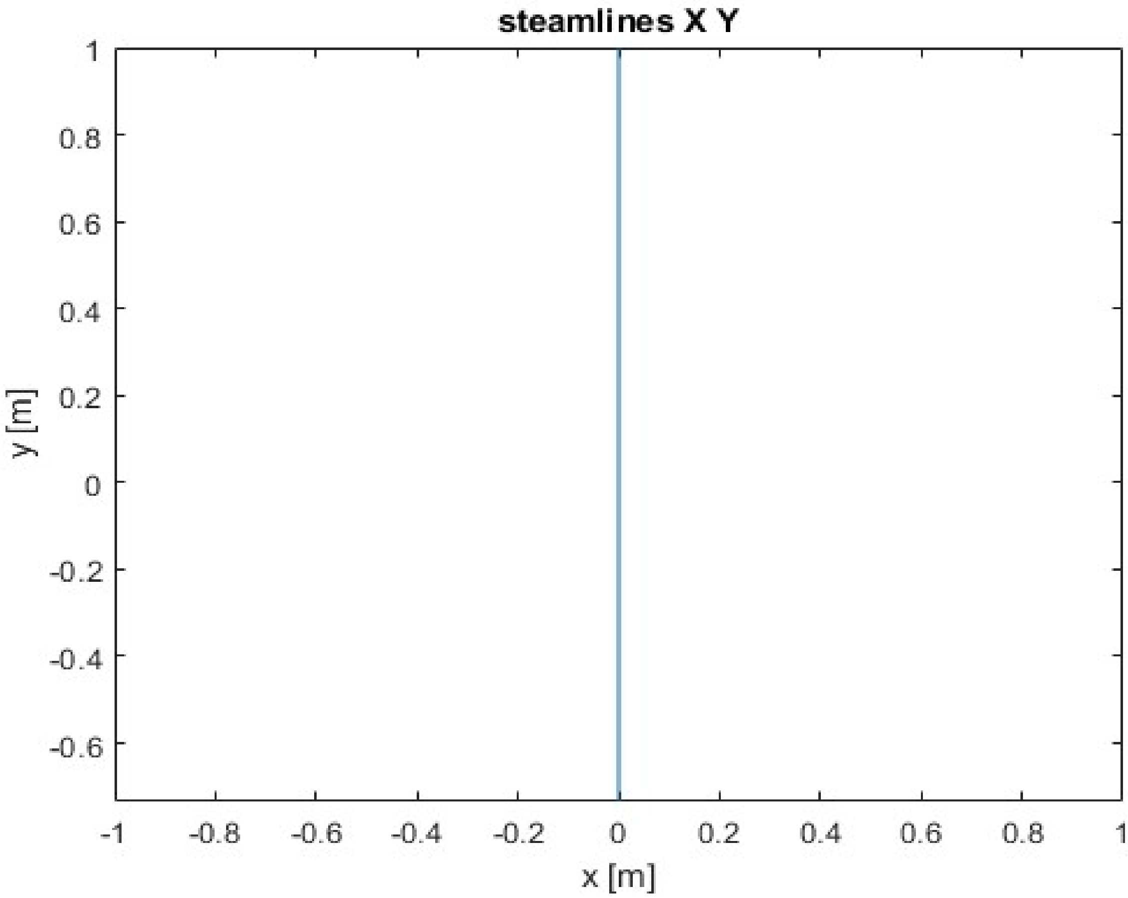
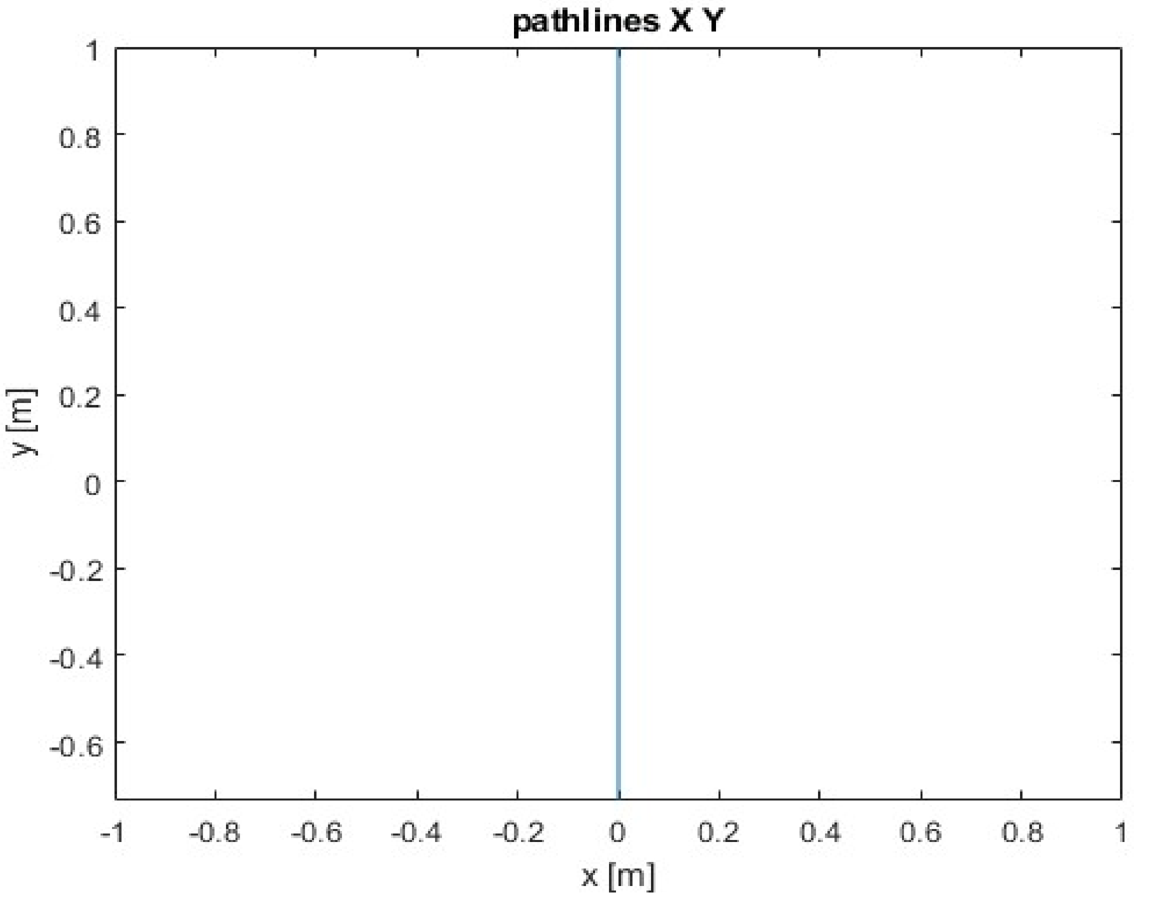
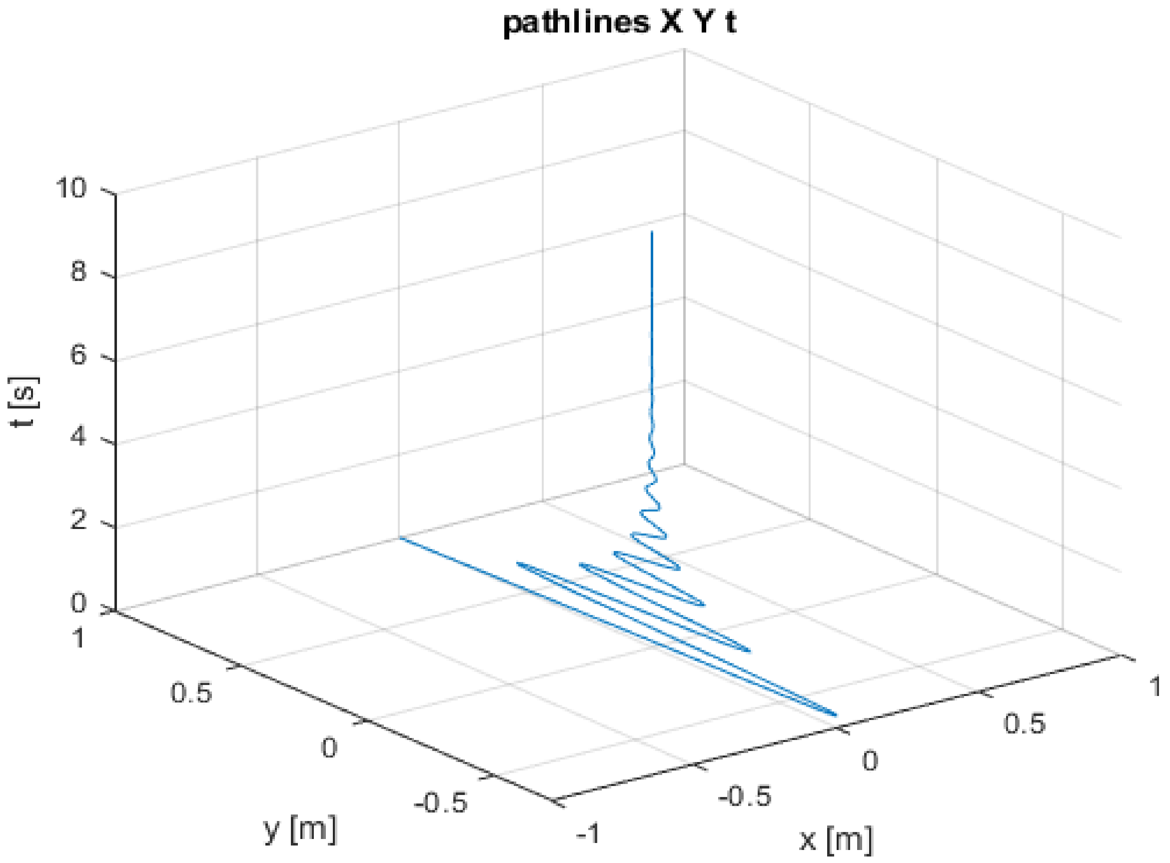



pathlines X Y
y [m]
x [m]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


pathlines X Y t
t [s]
10
8
6
4
2
0
y [m]
1
0.5
0
-0.5
x [m]
-1
-0.5
0
0.5
1

EA1

```
syms t r R Uinf
psi = (r-R^2/r)*Uinf*sin(t);
ur = diff((1/r)*psi,t);
ut = -diff(psi,r);

phir = int(ur,r);
phir = simplify(rewrite(phir,'sincos'));
phit = int(ut*r,t);
isAlways(phir==phit);

phi = phir;
clear phit phir

int(cos(t),t,[0,2*pi])
int(sin(t)^2,t,[0,2*pi])
int(sin(t)^2*cos(t),t)

syms Pinf rho
P = Pinf + .5*rho*Uinf^2-.5*rho*(-2*Uinf*sin(t))^2;
P2 = Pinf+.5*rho*Uinf^2*(1-4*sin(t)^2);
isAlways(P==P2)
eq = -R*cos(t)*P;
int(eq,t,[0,2*pi]);
```

```
ans =

0


ans =

pi


ans =

sin(t)^3/3


ans =

  logical

   1
```

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