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Josh O

```
----- HW2 - Josh Oates -----
```

4.15

```
-----P4.15-----
My calculations have the following results:
Velocity in Perifocal Frame [km/s]:
        0 12.2156
Position in Perifocal Frame [km]:
       6678
                     0
Velocity in ECI Frame [km/s]:
 -10.3559 -5.7627
                    2.9611
Velocity in Perifocal Frame [km]:
  1.0e+03 *
  -1.9838 5.3488 3.4715
H/C: norms of r and v in either frame should be equal:
Norm veci: 12.2156 Norm vp: 12.2156
Norm reci: 6678 Norm rp: 6678
```

-4.8864 6.0226 3.0479

V2 in km/s:

-6.9168 1.2549 -1.3988

H/C: We should be using the short side and theta1 < theta2 so this makes sense

6.8

-----P6.8-----

My calculations have the following results:

dv [km/s]: 1.1977

transit time [s]: 59.6542

H/C: transfer time is halff of the tranfer period. This period makes sense for something LEO MEOish.

6.23

-----P6.23-----

My calculations have the following results:

dv [km/s]: 3.4054

H/C: The orbital period used in the calculation makes sense for a MEO orbit.

6.25

-----P6.25-----

My calculations have the following results:

Delta gamma [degrees]: -8.1813

Delta v [km/s]: 0.91545

 ${\it H/C:}$ We would imagine a moderate delta v for a manuver like this. This seems to make sense for a small apseline rotation.

6.31

on paper

6.44

-----P6.44-----

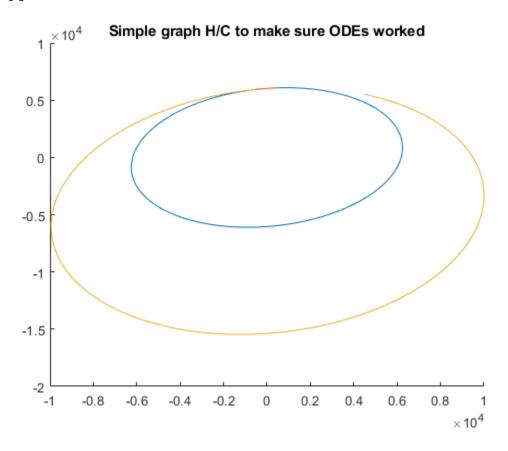
My calculations have the following results:

A) dv [km/s]: 2.7927

B) dv [km/s]: 2.6951

C) dv [km/s]: 2.7835

H/C: We would expect the lowest delta v to be a combined manuver at a greater altitude, this way there is less velocity to change so to speak and a small dv will create a larger inc change.



dependancies

```
My code uses the following functions:
    {'C:\AERO351\A351HW3\HW3.m'}
    {'C:\joshFunctionsMatlab\joshAnomalyCalculator.m'}
    {'C:\joshFunctionsMatlab\joshAxisRotation.m'}
    {'C:\joshFunctionsMatlab\joshCOE.m'}
    {'C:\joshFunctionsMatlab\joshHomann.m'}
    {'C:\joshFunctionsMatlab\joshIsOnes.m'}
    {'C:\joshFunctionsMatlab\joshLawCos.m'}
    {'C:\joshFunctionsMatlab\joshStumpffCoeffs.m'}
    {'C:\joshFunctionsMatlab\joshStumpffCoeffs.m'}
    {'C:\joshFunctionsMatlab\joshStumpffZ.m'}
    {'C:\joshFunctionsMatlab\joshStumpffZ.m'}
    {'C:\joshFunctionsMatlab\joshStumpffZ.m'}
}
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```

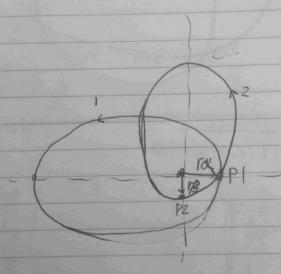
functions

 ${\it H/C:}$ the time to get to apogee seems to be within range for 2ish orbits in LEO which is what the included graph seems to predict.

HW3

6.31

ecci - eccz



$$\Gamma_{2}(0^{\circ}) = \Gamma \beta$$

$$\Gamma_{2}(90^{\circ}) = \Gamma \alpha$$

$$\Gamma_{1}(0^{\circ}) = \Gamma \alpha$$

r= h2 (THELL COSA)

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Josh O

```
[Cx,Cy,Cz]=joshAxisRotation();
mu_e = 398600;
r_e = 6378; % km
ecc = 1.5;
zpapse = 300; % km
inc = 35; % degrees
raan = 130; % degrees
aop = 115; % degrees
```

```
inc = deg2rad(inc);
raan = deg2rad(raan);
aop = deg2rad(aop);
rpapse = zpapse + r e;
theta = 0;
h = sqrt(mu e*rpapse*(1+ecc*cos(theta)));
vpapse = h/rpapse;
rp = [1,0,0]*rpapse;
vp = [0,1,0]*vpapse;
clear vpapse rpapse zpapse
Q = Cz (raan) *Cx (inc) *Cz (aop);
Q = Q'; % Perifocal -> ECI
reci = rp*Q;
veci = vp*Q;
disp("-----")
disp("My calculations have the following results:")
disp("Velocity in Perifocal Frame [km/s]:")
disp(vp)
disp("Position in Perifocal Frame [km]:")
disp(rp)
disp("Velocity in ECI Frame [km/s]:")
disp(veci)
disp("Velocity in Perifocal Frame [km]:")
disp(reci)
disp("H/C: norms of r and v in either frame should be equal:")
disp("Norm veci: "+string(norm(veci))+" Norm vp: "+string(norm(vp)));
disp("Norm reci: "+string(norm(reci))+" Norm rp: "+string(norm(rp)));
5.6
clear all
coefs = 15;
[Cc,Sc]=joshStumpffCoeffs(coefs);
C = @(z) sum(Cc.*joshStumpffZ(z,coefs));
S = @(z) sum(Sc.*joshStumpffZ(z,coefs));
mu = 398600;
r1 = [5644, 2830, 4170]; % km
r2 = [-2240, 7320, 4980]; % km
dt = 20; % min
dt = dt*60; % sec
z = cross(r1, r2);
z = z(3);
```

```
clear all;
mu = 398600;
r e = 6378; % km
z1 = 300; % km
z3 = 3000; % km
r1 = r e + z1;
r3 = r_e + z3;
v1 = sqrt(mu e/r1);
v3 = sqrt(mu e/r3);
ecc2 = (r3-r1)/(r3+r1);
a2 = (r3+r1)/2;
h2 = sqrt(a2*mu e*(1-ecc2^2));
vp2 = h2/r1;
va2 = h2/r3;
dv = abs(va2-v3) + abs(vp2-v1);
P2 = (2*pi/sqrt(mu e))*a2^1.5; % sec
tranfertime = P2/2;
tranfertime = tranfertime/60;
disp("----")
disp("My calculations have the following results:")
disp("dv [km/s]: "+string(dv))
disp("transit time [s]: "+string(tranfertime))
disp("H/C: transfer time is halff of the tranfer period. This period makes
sense for something LEO MEOish.")
```

```
clear all
mu = 398600;
r e = 6378; % km
theta1 = 150; % degrees
theta1 = deg2rad(theta1);
theta2 = 45;% degrees
theta2 = deg2rad(theta2);
ra1 = 18900; %km
rp1 = 8100; %km
a1 = (ra1+rp1)/2;
P1 = (2*pi/sqrt(mu_e))*a1^1.5; % sec
ecc1 = (ra1-rp1)/(ra1+rp1);
Me1 = joshAnomalyCalculator(ecc1, theta1);
n1 = sqrt(mu e/a1^3);
t1 = Me1/n1; % time since periapse orbit 1 object B
Me2 = joshAnomalyCalculator(ecc1, theta2);
t2 = Me2/n1; % time since periapse orbit 1 object C
P2 = (P1-t1); % time till periapse object c
P2 = P2+t2; % time till rendezvous
h1 = sqrt(a1*mu e*(1-ecc1^2));
r1 45 = (h1^2/mu e)/(1+ecc1*cos(theta2)); % at theta 45 what is the radius of
both orbits
rp2 = r1 45; % at a theta of 45, satalite B makes its location its new
periapse
a2 = (P2*(sqrt(mu e)/(2*pi)))^(2/3);
ra2 = a2*2-rp2;
ecc2 = (ra2-rp2)/(ra2+rp2);
h2 = sqrt(a2*mu e*(1-ecc2^2));
vp2 = h2/rp2;
[vaz1 45, vr1 45] = joshVazVr(theta2,ecc1,h1,mu e); % orignal vaz and vr
vr2 = 0;
vaz2 = vp2;
% v1 45 = sqrt(vaz1 45^2+vr1 45^2)
vp2 = [vaz2, vr2];
v1 	45 = [vaz1 	45, vr1 	45];
dv = norm(v1 45-vp2)*2;
disp("----")
disp("My calculations have the following results:")
disp("dv [km/s]: "+string(dv));
```

```
disp("H/C: The orbital period used in the calculation makes sense for a MEO
    orbit.")
```

```
clear all
mu = 398600;
r e = 6378; % km
theta1 = deg2rad(100);
rp1 = 1270 + r e;
vp1 = 9;
[a1,ecc1,~,~,~,h1,T1,E1] = joshCOE(rp1,vp1,mu e,"magnitude");
[vaz1, vr1, gamma1] = joshVazVr(theta1, ecc1, h1, mu e);
r100 = h1^2/(mu e^*(1+ecc1*cos(theta1)));
ecc2 = .4;
a2 = r100*(1+ecc2*cos(theta1))/(1-ecc2^2);
h2 = sqrt(a2*mu e*(1-ecc2^2));
[vaz2, vr2, gamma2] = joshVazVr(theta1, ecc2, h2, mu e);
dgamma = rad2deg(gamma2-gamma1);
dv = norm([vaz2, vr2]-[vaz1, vr1]);
disp("----")
disp("My calculations have the following results:")
disp("Delta gamma [degrees]: "+string(dgamma))
disp("Delta v [km/s]: "+string(dv))
disp("H/C: We would imagine a moderate delta v for a manuver like this. This
seems to make sense for a small apseline rotation.")
```

6.31

on paper

```
clear all
mu_e = 398600;
r_e = 6378; % km

r1 = 300+r_e;
r2 = 600+r_e;

v1 = sqrt(mu_e/r1);
v2 = sqrt(mu_e/r2);
inc = deg2rad(20);
```

```
clear all
mu = 398600;
r e = 6378; % km
r0 = [436;6083;2529];
v0 = [-7.340; -.5125; 2.497];
m0 = 1000;
X0 = [r0; v0; m0];
options = odeset('RelTol', 1e-8, 'AbsTol', 1e-8);
tspan = [0,89]*60;%s
[t1,X1] = ode45(@XdotFunCoast,tspan,X0,options);
tspan = [0, 120];
X0 = X1 (end,:);
[t2, X2] = ode45(@XdotFunBurn, tspan, X0, options);
X = [X1; X2];
t = [t1; t2+t1 (end)];
v3 = X(end, 4:6);
r3 = X(end, 1:3);
% [a,ecc,theta,inc,raan,aop,h,T,E] = joshCOE(r3,v3)
X0 = X2 (end,:);
tspan = [0,200]*60;%s
```

```
[t3,X3] = ode45(@XdotFunCoast,tspan,X0,options);
X = [X1; X2; X3];
t = [t1; t2+t1 (end); t3+t2 (end)+t1 (end)];
close all
figure
hold on
plot3(X1(:,1),X1(:,2),X1(:,3))
plot3(X2(:,1),X2(:,2),X2(:,3))
plot3(X3(:,1),X3(:,2),X3(:,3))
title("Simple graph H/C to make sure ODEs worked")
n = length(X);
rmag = zeros(n,1);
for i = 1:n
    rmag(i) = norm([X(i,1),X(i,2),X(i,3)]);
end
[rmax, i] = max(rmag);
zmax = rmax - r e;
tmax = t(i);
tmax = tmax/60;
disp("----")
disp("My calculations have the following results:")
disp("Max altitude [km]: "+string(zmax))
disp("Max altitude time since t0 [min]: "+string(tmax))
disp("H/C: the time to get to apogee seems to be within range for 2ish orbits
in LEO which is what the included graph seems to predict.")
```

dependancies

```
disp("-----Dependancies-----")
disp("My code uses the following functions: ")
depends = matlab.codetools.requiredFilesAndProducts('C:\AERO351\A351HW3\HW3');
disp(depends')
```

functions

```
% 6.47 dX
function dX = XdotFunBurn (t,X)
    mu_e = 398600;
    isp = 300; % s
    g0 = 9.80665; % m/s^2
    g0 = g0/1000; % km/s^2
    T = 10000; % kg.m/s^2
    T = T/1000; % kg.km/s^2
    r = X(1:3);
    v = X(4:6);
    m = X(7);
    a = (-mu_e*r)/norm(r)^3 + (T/m)*(v/norm(v));
```

```
dm = (-T/(isp*g0));
    dX = [v;a;dm];
end
function dX = XdotFunCoast (t, X)
    mu = 398600;
    isp = 300; % s
    g0 = 9.80665; % m/s^2
    g0 = g0/1000; % km/s^2
   T = 0; % kg.m/s^2
    T = T/1000; % kg.km/s^2
   r = X(1:3);
   v = X(4:6);
   m = X(7);
    a = (-mu e*r)/norm(r)^3 + (T/m)*(v/norm(v));
    dm = (-T/(isp*g0));
   dX = [v;a;dm];
end
% for lamberts
function out = fp_zcheck(z,fp,fpz0)
    if z == 0
         out = fpz0;
    else
        out = fp;
    end
end
```

```
function [M,E] = joshAnomalyCalculator(ecc,theta)
% M will be Me, Mp or Mh depending on ecc
% E will be Eccentric Anomoly when applicable or F: hyperbolic Ecctric
% anomoly. E will be set to
% values in Rads
arguments
    ecc (1,1) double {mustBeReal}
    theta (1,1) double {mustBeReal}
end
if ecc <1 % Me & E
    E = 2*atan(sqrt((1-ecc)/(1+ecc))*tan(theta/2)); % definintion of E,
rewriten to solve {\tt E}
    M = E - ecc * sin(E); % definition of M
elseif ecc > 1 % Mh & F
    E = log((sqrt(ecc+1) + sqrt(ecc-1) *tan(theta/2)) / (sqrt(ecc+1) -
sqrt(ecc-1)*tan(theta/2)));
    M = ecc(sinh(F) - F);
else % ecc == 1 Mp
    E = nan; % This is a rare case and E doesnt have a definition for ecc == 1
    M = .5*tan(theta/2) + (1/6)*tan(theta/2)^3;
end
end
```

```
function [Cx,Cy,Cz] = joshAxisRotation(opt)
arguments
        opt {mustBeMember(opt, {'degree', 'radian'})} = 'radian'
end
if strcmp(opt,'degree')
Cx = 0 \text{ (theta)} \dots
    [[1 0 0];...
    [0 cosd(theta) sind(theta)];...
    [0 -sind(theta) cosd(theta)]];
Cy = @(theta)...
    [[cosd(theta) 0 -sind(theta)];...
    [ 0 1 0];...
    [sind(theta) 0 cosd(theta)]];
Cz = 0 \text{ (theta)} \dots
    [[cosd(theta) sind(theta) 0];...
    [-sind(theta) cosd(theta) 0];...
    [0 0 1]];
else
Cx = @(theta)...
    [[1 0 0];...
    [0 cos(theta) sin(theta)];...
    [0 -sin(theta) cos(theta)]];
Cy = @(theta)...
    [[cos(theta) 0 -sin(theta)];...
    [ 0 1 0];...
    [sin(theta) 0 cos(theta)]];
Cz = @(theta)...
    [[cos(theta) sin(theta) 0];...
    [-sin(theta) cos(theta) 0];...
    [0 0 1]];
end
end
```

```
function [a,ecc,theta,inc,raan,aop,h,T,E] = joshCOE(R,V,u,magOrVec)
888888888888888888888
% Revamped to do rads and fit new naming convention %
88888888888888888888
%COESOATESJOSHUA Takes postion and velocity vector and returns COES, all in
%ECI frame of reffrenece, km and seconds as units and degrees
% a = semi major axis
% ecc = eccentricity
% i = inclination
% raan = right accention acending node
% aop = argument of periapsis
% theta = true anomaly
% will return T in s as a period
% and E (sometimes epsilon) in km^2/s^2 as specific mechanical energy
% h is agular momentum
% magOrVec is a parameter that can be set for vector inputs to return
% vector h and ecc
% scalar entry should only be used if the spacecraft is at apoapse or
% periapse
arguments
    R {mustBeNumeric, mustBeReal}
    V {mustBeNumeric, mustBeReal}
    u (1,1) {mustBeNumeric, mustBeReal, mustBePositive} = 3.986004418 *
 (10^5) %km^3/s^2
    magOrVec {mustBeMember(magOrVec, {'magnitude', 'vector'})} = 'magnitude'
end
[m1,n1] = size(R);
[m2,n2] = size(V);
if joshIsOnes([m1 n1 m2 n2])
    magOrVec = 'magnitude';
    R = [0 \ 0 \ R];
    V = [V \ 0 \ 0];
    warning("joshCOE will assume that R and V are normal if the inputs are
 scalar ie: the craft is in a circular orbit or is at periapse or apoapse")
elseif (\simjoshIsOnes([m1 n1] == [m2 n2]))|\sim((n1==1&m1==3)|(n1==3&m1==1))
    throw (MException ("COEsOatesJoshua:invalidInput", "R and V must be either
1x3 vectors or scalars"))
end
% uearth = 3.986004418(8) x 10^14 m^3/s^2
ihat=[1,0,0];
khat = [0, 0, 1];
```

```
Rm = norm(R);
Vm = norm(V);
%calculate orbital constants
h = cross(R,V); %angular momentum vector
hm = norm(h);
E = ((Vm^2) / 2) - (u / Rm); %specfic mechanical energy
%calculate COEs
a = -u / (2 * E); %semi major axis in km
T = 2*pi*sqrt((a^3)/u); %period in s
e = (1/u) * (((Vm^2) - (u/Rm))) * R - (dot(R,V) * V)); %eccentricity vector
em = norm (e); %magnitude of e
inc = acos((dot(khat,h))/hm); %inclination
n = cross(khat,h); %node vector
nm = norm(n); %magnitude n
%raan
raan = acos(dot(ihat,n)/nm);
if n(2) < 0 %checks the vector relative to j to see if angle is positive or
negative
    raan = 360 - raan;
end
%aop
aop = acos(dot(n,e)/(nm*em));
if e(3) < 0
    aop = 360 - aop;
end
%theta
theta = acos(dot(e,R)/(em*Rm));
if(dot(R,V) < 0) % cehck flight path angle to see if it is postive or negative
    theta = 360 -theta;
end
if strcmp(magOrVec, 'magnitude') %for magnitude mode, vectors will not be
 returned
   h = hm;
    e = em;
    if joshIsOnes([m1 n1 m2 n2]) % for scalar inputs it is not possible to
 calculate these values
        theta = NaN;
        inc = NaN;
        raan = NaN;
        aop = NaN;
    end
end
ecc = e;
end
```



```
function mx = joshCross(m)
% takes a column vector and returns the associated 'cross' matrix such that
% mx*b == cross(m,b)
arguments
   m(3,1)
end
if isa(m, "double") % overloaded to handle either symbolic or double type
vectors
   mx = zeros(3);
elseif isa(m,"sym")
    syms mx [3 3]
    throw(MException("joshCross:invalidInput","m must be type sym or double"))
end
    for i = 1:3
       mx(i,i) = 0;
    end
    mx(1,2) = -m(3);
   mx(1,3) = m(2);
   mx(2,3) = -m(1);
   mx(2,1) = m(3);
    mx(3,1) = -m(2);
    mx(3,2) = m(1);
end
```

```
function [isOnes] = joshIsOnes(M)
% takes a value (persumably a logical type matrix) and returns true iff all
% entries are true
[m,n] = size(M);
isOnes = true;
for i = 1:m
    for j = 1:n
        if M(i,j) ~= 1
            isOnes = false;
        end
    end
end
end
```

```
function [C,S] = joshStumpffCoeffs(n)
% AERO 351 code
\mbox{\%} Generates the first n terms of the stumpff coeffcients for \mbox{@S(z)} and \mbox{@C(z)}
in a vector
% these coeffs are used for the universal variable approach to orbital
mechanics
% for use as companion function with joshStrumpffZ
% coeffs should be saved to workspace and reused to save compute time
% @S(z) == sum(S.*Z) == polyval(flip(S),z) : where Z = [z^0 z^1 ... z^n]
% C(z) == sum(C.*z) == polyval(flip(C),z) : where Z = [z^0 z^1 ... z^n]
arguments
    n (1,1) {mustBePositive, mustBeInteger} = 15;
end
C = zeros(1,n);
S = C;
for i = 1:n
    k = i-1;
    C(i) = (-1)^k*(1/factorial(2*k+2));
    S(i) = (-1)^k* (1/factorial(2*k+3));
end
end
```

```
function [Z] = joshStumpffZ(z,n)
% AERO 351 code
\mbox{\%} Generates the first n terms of the stumpff coeffcients for \mbox{@S(z)} and \mbox{@C(z)}
in a vector
% for use as companion function with joshStrumpffCoeffs
% @S(z) == sum(S.*Z) == polyval(flip(S),z) : where S given by <math>(-1)^k*(1/2)
factorial(2*k+2))
% C(z) == sum(C.*z) == polyval(flip(C),z): where C given by (-1)^k*(1/z)
factorial(2*k+3))
arguments
    z(1,1)
    n (1,1) {mustBePositive, mustBeInteger} = 15;
end
Z = ones(1,n);
for i = 2:n
    Z(i) = z*Z(i-1);
end
end
```

```
function [Cx,Cy,Cz] = joshAxisRotation(opt)
arguments
        opt {mustBeMember(opt,{'degree','radian'})} = 'radian'
end
if strcmp(opt,'degree')
Cx = @(theta)...
   [[1 0 0];...
    [0 cosd(theta) sind(theta)];...
    [0 -sind(theta) cosd(theta)]];
Cy = @(theta)...
    [[cosd(theta) 0 -sind(theta)];...
    [ 0 1 0];...
    [sind(theta) 0 cosd(theta)]];
Cz = @(theta)...
   [[cosd(theta) sind(theta) 0];...
    [-sind(theta) cosd(theta) 0];...
    [0 0 1]];
else
Cx = @(theta)...
    [[1 0 0];...
    [0 cos(theta) sin(theta)];...
   [0 -sin(theta) cos(theta)];
Cy = @(theta)...
   [[cos(theta) 0 -sin(theta)];...
    [ 0 1 0];...
    [sin(theta) 0 cos(theta)]];
Cz = @(theta)...
    [[cos(theta) sin(theta) 0];...
    [-sin(theta) cos(theta) 0];...
    [0 0 1]];
end
end
```