```
% Josh Oates
% HW 6
clear all
J = [...
    [2 -1 0];...
    [-1 3 0];...
    [0 0 1]];

[E,Lam] = eig(J);

disp("Problem 5")
disp("The principle moments of intertia are:")
disp(Lam)
disp("Problem 6")
disp("The components of basis vectors for the principle axes in the refrence
 fram Fc are:")
disp("X:")
disp(E(:,1))
disp("Y:")
disp(E(:,2))
disp("Z:")
disp(E(:,3))
```

```
Problem 5
The principle moments of intertia are:
    1.0000         0         0
         0    1.3820         0
         0         0    3.6180

Problem 6
The components of basis vectors for the principle axes in the refrence fram Fc
 are:
X:
     0
     0
     1

Y:
   -0.8507
   -0.5257
         0

Z:
   -0.5257
    0.8507
         0
```

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HW6

Problem 1

1) $$J = \begin{bmatrix} m\frac{R^2}{4} + m\frac{h^2}{3} & 0 & 0 \\ 0 & m\frac{R^2}{4} + m\frac{h^2}{3} & 0 \\ 0 & 0 & m\frac{R^2}{2} \end{bmatrix}$$

$$r = \begin{pmatrix} 0 \\ 0 \\ h/2 \end{pmatrix}$$

$$I = J + m r^{\times} r^{\times} = J + \begin{bmatrix} -\frac{h^2}{4}m & 0 & 0 \\ 0 & -\frac{h^2}{4}m & 0 \\ 0 & 0 & m\frac{R^2}{2} \end{bmatrix}$$

$$I = \begin{bmatrix} \frac{1}{12}m(3R^2+h^2) & 0 & 0 \\ 0 & \frac{1}{12}m(3R^2+h^2) & 0 \\ 0 & 0 & m\frac{R^2}{2} \end{bmatrix} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

$$I_y = I_x$$

2) $$\underline{T_c} = \begin{pmatrix} T_x \\ T_y \\ 0 \end{pmatrix}, \quad \underline{w_0} = \begin{pmatrix} w_{x0} \\ w_{y0} \\ w_{z0} \end{pmatrix} \neq 0$$

$$\dot{\underline{w}} = -\underline{I}^{-1}(\underline{w}^{\times}\underline{I}\,\underline{w} - \underline{T_c})$$

$$I_x \dot{w}_x + (I_z - I_x) w_z w_y = T_x$$
$$I_y \dot{w}_y + (I_x - I_z) w_x w_z = T_y$$
$$I_z \dot{w}_z + \cancelto{0}{(I_y - I_x)} w_y w_x = T_z = 0$$
$$\dot{w}_z = 0$$

Redefine problem to be simpler

$$I_z - I_x = \beta, \quad I_x - I_z = -\beta, \quad I = I_x = I_y$$

$$I\dot{w}_x + \beta w_z w_y = T_x, \quad \dot{w}_x = \frac{T_x}{I} - w_y \frac{\beta w_z}{I}$$
$$I\dot{w}_y - \beta w_x w_z = T_y, \quad \dot{w}_y = \frac{T_y}{I} - w_x \frac{-\beta w_z}{I}$$

$$\alpha = \frac{\beta w_z}{I}$$

$\dot{\omega}_x = \frac{T_x}{I} - \alpha\omega_y$, $\ddot{\omega}_x = -\alpha\dot{\omega}_y = -\alpha\frac{T_y}{I} - \alpha^2\omega_x$

$\dot{\omega}_y = \frac{T_y}{I} + \alpha\omega_x$, $\ddot{\omega}_y = \alpha\dot{\omega}_x = \alpha\frac{T_x}{I} + \alpha^2\omega_x$

$\omega_x = A_1\cos(\alpha t) + B_1\sin(\alpha t) + \alpha\frac{T_x}{I}$ $\xrightarrow{@t=0}$ $A_1 = \omega_{x0} - \alpha\frac{T_y}{I}$

$\omega_y = A_2\cos(\alpha t) + B_2\sin(\alpha t) - \alpha\frac{T_x}{I}$ $\qquad A_2 = \omega_{y0} + \alpha\frac{T_x}{I}$

$\omega_x = \left(\omega_{x0} - \alpha\frac{T_y}{I}\right)\cos(\alpha t) + B_1\sin(\alpha t) + \alpha\frac{T_x}{I}$

$\omega_y = \left(\omega_{y0} + \alpha\frac{T_x}{I}\right)\cos(\alpha t) + B_2\sin(\alpha t) - \alpha\frac{T_x}{I}$

↓ derivative

$\dot{\omega}_x = -\left(\omega_{x0} - \alpha\frac{T_y}{I}\right)\alpha\sin(\alpha t) - B_1\alpha\sin(\alpha t)$

$\dot{\omega}_y = -\left(\omega_{y0} + \alpha\frac{T_x}{I}\right)\alpha\sin(\alpha t) - B_2\alpha\sin(\alpha t)$

↓ Plug in $\dot{\omega}_x = \frac{T_x}{I} - \alpha\omega_y$, $\dot{\omega}_y = \frac{T_y}{I} + \alpha\omega_x$, solve for $\frac{T_x}{I}$

$\left(-\omega_{x0}\alpha + \frac{T_y}{I}\alpha^2 + B_2\alpha\right)\sin(\alpha t) + \left(B_1\alpha + \omega_{y0}\alpha + \alpha^2\frac{T_x}{I}\right)\cos(\alpha t) - \alpha^2\frac{T_x}{I} = \frac{T_x}{I}$

@ $t=0$ this will simplify $\alpha(B_1 + \omega_{y0}) = \frac{T_x}{I}$, $B_1 = \frac{T_x}{\alpha I} - \omega_{y0}$

; $\alpha(B_2 - \omega_{x0}) = \frac{T_y}{I}$, $B_2 = \frac{T_y}{\alpha I} + \omega_{x0}$

$$\boxed{\begin{aligned} \omega_x &= \cos(\alpha t)\,\omega_{x0} + \alpha\left(-\frac{T_y}{I}\cos(\alpha t) + \alpha\frac{T_y}{I}\right) + \sin(\alpha t)\left(\frac{T_x}{\alpha I} + \omega_{y0}\right) \\ \omega_y &= \cos(\alpha t)\,\omega_{y0} + \alpha\left(\frac{T_x}{I}\cos(\alpha t) - \alpha\frac{T_x}{I}\right) + \sin(\alpha t)\left(\frac{T_y}{\alpha I} + \omega_{x0}\right) \end{aligned}}$$

Problem 2

1. $\underline{J}\,\underline{x} = \lambda\,\underline{x}$

$\bar{x}^T J x = \bar{x}^T \lambda x$

$(\underline{J}\ \underline{\bar{x}})^T = \bar{x}^T \bar{J}^T = \bar{x}^T J$

$(\underline{J}\,\underline{\bar{x}})^T \underline{x} = \underline{\bar{x}}^T \lambda\,\underline{x}$ Conjugate both sides

$(\underline{J}\,\underline{x})^T \underline{\bar{x}} = \underline{x}^T \bar{\lambda}\,\underline{\bar{x}}$

$(\lambda \underline{x})^T \underline{\bar{x}} = \bar{\lambda}\,\underline{x}^T \underline{\bar{x}}$

$\lambda\,\underline{x}^T \underline{\bar{x}} = \bar{\lambda}\,\underline{x}^T \underline{\bar{x}}$

$\boxed{\lambda = \bar{\lambda}}$ so λ is real

2. $\underline{x}^T \underline{J}\,\underline{x} > 0$

$\underline{x}^T \underline{J}\,\underline{x} = \underline{x}^T \lambda\,\underline{x} > 0$

$\underline{x}^T \lambda\,\underline{x} = \lambda\,\underline{x}^T \underline{x} = \lambda\,\|x\|^2 > 0 \implies \boxed{\lambda > 0}$

3. a $\lambda_1 \underline{x}_1^T \underline{x}_2 = \lambda_2 x_1^T x_2 \leftarrow$ Prove

$\lambda_1 \underline{x}_1^T \underline{x}_2 = (\lambda_1 \underline{x}_1)^T \underline{x}_2 = (\underline{J}\,\underline{x}_1)^T x_2 = \underline{x}_1^T(\underline{J}\,\underline{x}_2)$

~~4~~3. b $= \boxed{x_1^T \lambda_2 x_2}$, since $\lambda_1 x_1^T x_2 = \lambda_2 x_1^T x_2$, & $\lambda_1 \neq \lambda_2$,

$\boxed{x_1^T x_2 = 0}$

HW6

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$$\underline{J} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{x}^T \underline{J} \underline{x} = 2x_1^2 - 2x_1x_2 + 3x_2^2 + x_3^2$$

$$|2x_1x_2| < 2x_1^2 \quad \text{if} \quad x_1 > x_2 \Rightarrow 2x_1^2 - 2x_1x_2 > 0$$

$$|2x_1x_2| \leq 2x_2^2 \text{ if } \quad x_2 \geq x_1 \Rightarrow 3x_2^2 - 2x_1x_2 \geq 0$$

if $x_1 > x_2$

$$2x_1^2 - 2x_1x_2 + 3x_2^2 + x_3^2 \quad > \quad 0 + 3x_2^2 + x_3^2 > 0$$

if $x_2 \geq x_1$

$$2x_1^2 - 2x_1x_2 + 3x_2^2 + x_3^2 \quad \geq \quad 2x_1^2 + 0 + x_3^2 > 0$$