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HW5 joshua oates

```
clear all;
close all;
clc

addpath("C:\joshFunctionsMatlab\")
```

Cylinder

```
clear all
syms z r t R h m
% Integral of J(1,1) on paper included

x = r*cos(t) % to convert cartesien to polar for integration
y = r*sin(t)

rho = [x;y;z] % cartesian rho

rhox = joshCross(rho)
rhoxx = simplify(rhox*rhox)
J1 = simplify(int(int(int(-rhoxx*r,r,[0,R]),t,[-pi,pi]),z,[-h,0])) % triple
 integral over area, inertia matrix Cylinder
J1 = subs(J1,R^2*pi*h,m);% where density is the constant 1

J2 = limit(J1,h,0) % as h approaches 0, disk
J3 = limit(J1,R,0) % as R approaches 0, rod
disp("------------P1------------")
disp("My work for this problem have the following results: ")
disp("J for cylinder: ")
disp(J1)
disp("J for disk: ")
disp(J2)
disp("J for thin rod: ")
disp(J3)
disp("Computation of J11 can be found in included hand work.")
disp("Due to previous comments from the graders, all symbolic steps will show
 their outputs.")
```

```
x =

r*cos(t)


y =

r*sin(t)


rho =

r*cos(t)
r*sin(t)
       z


rhox =

[         0,        -z,  r*sin(t)]
[         z,         0, -r*cos(t)]
[-r*sin(t), r*cos(t),         0]


rhoxx =

[- r^2*sin(t)^2 - z^2,      (r^2*sin(2*t))/2, r*z*cos(t)]
[    (r^2*sin(2*t))/2, - r^2*cos(t)^2 - z^2, r*z*sin(t)]
[          r*z*cos(t),           r*z*sin(t),      -r^2]


J1 =

[(pi*R^2*h*(3*R^2 + 4*h^2))/12,                             0,             0]
[                            0, (pi*R^2*h*(3*R^2 + 4*h^2))/12,             0]
[                            0,                             0, (pi*R^4*h)/2]


J2 =

[(R^2*m)/4,          0,          0]
[        0, (R^2*m)/4,          0]
[        0,          0, (R^2*m)/2]


J3 =

[(h^2*m)/3,          0, 0]
[        0, (h^2*m)/3, 0]
[        0,          0, 0]
```

------------P1------------
My work for this problem have the following results:
J for cylinder:

```
[(m*(3*R^2 + 4*h^2))/12,                      0,          0]
[                     0, (m*(3*R^2 + 4*h^2))/12,          0]
[                     0,                      0, (R^2*m)/2]

J for disk:
[(R^2*m)/4,         0,         0]
[        0, (R^2*m)/4,         0]
[        0,         0, (R^2*m)/2]

J for thin rod:
[(h^2*m)/3,         0, 0]
[        0, (h^2*m)/3, 0]
[        0,         0, 0]

Computation of J11 can be found in included hand work.
Due to previous comments from the graders, all symbolic steps will show their
 outputs.
```

cone

```
clear all
syms r t z m h R0

rc = [0;0;-(3/4)*h] % from hand calcs

x = r*cos(t) % to convert cartesien to polar for integration
y = r*sin(t)

rho = [x;y;z] % cartesian rho
rhox = joshCross(rho)
rhoxx = simplify(rhox*rhox)

R = -R0*z/h
J = simplify(int(int(int(-rhoxx*r,r,[0,R]),t,[-pi,pi]),z,[-h,0])) % triple
 integral over area, inertia matrix cone

w = [1;t;sin(t)]
dw = diff(w)
wx = joshCross(w)

rcx = joshCross(rc)
I = J + m*rcx*rcx

Tc = I*dw + wx*I*w
Tc = subs(Tc,m,1)
Tc = subs(Tc,h,1)
Tc = subs(Tc,R0,1)
Tc = simplify(Tc)

disp("------------P2------------")
disp("My work for this problem have the following results: ")
disp("Hand calculations are included which find the center of mass of a cone
 along with an initial guess.")
```

```
disp("J cone: ")
disp(J)
disp("I cone: ")
disp(I)
disp("Net torque: ")
disp(Tc)

disp("Due to previous comments from the graders, all symbolic steps will show
 their outputs.")
```

```
rc =

       0
       0
-(3*h)/4


x =

r*cos(t)


y =

r*sin(t)


rho =

r*cos(t)
r*sin(t)
       z


rhox =

[        0,        -z,  r*sin(t)]
[        z,         0, -r*cos(t)]
[-r*sin(t), r*cos(t),         0]


rhoxx =

[- r^2*sin(t)^2 - z^2,      (r^2*sin(2*t))/2, r*z*cos(t)]
[    (r^2*sin(2*t))/2, - r^2*cos(t)^2 - z^2, r*z*sin(t)]
[          r*z*cos(t),           r*z*sin(t),       -r^2]


R =

-(R0*z)/h
```

```
J =

[(pi*R0^2*h*(R0^2 + 4*h^2))/20,                              0,            0]
[                             0, (pi*R0^2*h*(R0^2 + 4*h^2))/20,            0]
[                             0,                              0, (pi*R0^4*h)/10]


w =

     1
     t
sin(t)


dw =

     0
     1
cos(t)


wx =

[     0, -sin(t),  t]
[sin(t),       0, -1]
[    -t,       1,  0]


rcx =

[       0, (3*h)/4, 0]
[-(3*h)/4,       0, 0]
[       0,       0, 0]


I =

[(pi*R0^2*h*(R0^2 + 4*h^2))/20 - (9*h^2*m)/16,
            0,              0]
[                                            0, (pi*R0^2*h*(R0^2 + 4*h^2))/20 -
 (9*h^2*m)/16,              0]
[                                            0,
            0, (pi*R0^4*h)/10]


Tc =

                                           t*sin(t)*((9*h^2*m)/16 -
 (pi*R0^2*h*(R0^2 + 4*h^2))/20) + (pi*R0^4*h*t*sin(t))/10
(pi*R0^2*h*(R0^2 + 4*h^2))/20 - (9*h^2*m)/16 - sin(t)*((9*h^2*m)/16 -
 (pi*R0^2*h*(R0^2 + 4*h^2))/20) - (pi*R0^4*h*sin(t))/10

                         (pi*R0^4*h*cos(t))/10
```

```
Tc =

                                        t*sin(t)*((9*h^2)/16 -
 (pi*R0^2*h*(R0^2 + 4*h^2))/20) + (pi*R0^4*h*t*sin(t))/10
(pi*R0^2*h*(R0^2 + 4*h^2))/20 - sin(t)*((9*h^2)/16 - (pi*R0^2*h*(R0^2 +
 4*h^2))/20) - (9*h^2)/16 - (pi*R0^4*h*sin(t))/10

                     (pi*R0^4*h*cos(t))/10


Tc =

                             (pi*R0^4*t*sin(t))/10 - t*sin(t)*((pi*R0^2*(R0^2
 + 4))/20 - 9/16)
sin(t)*((pi*R0^2*(R0^2 + 4))/20 - 9/16) - (pi*R0^4*sin(t))/10 + (pi*R0^2*(R0^2
 + 4))/20 - 9/16

 (pi*R0^4*cos(t))/10


Tc =

          (pi*t*sin(t))/10 - t*sin(t)*(pi/4 - 9/16)
pi/4 + sin(t)*(pi/4 - 9/16) - (pi*sin(t))/10 - 9/16
                                       (pi*cos(t))/10


Tc =

                  -(3*t*sin(t)*(4*pi - 15))/80
pi/4 - (9*sin(t))/16 + (3*pi*sin(t))/20 - 9/16
                                  (pi*cos(t))/10

------------P2------------
My work for this problem have the following results:
Hand calculations are included which find the center of mass of a cone along
 with an initial guess.
J cone:
[(pi*R0^2*h*(R0^2 + 4*h^2))/20,                              0,              0]
[                            0, (pi*R0^2*h*(R0^2 + 4*h^2))/20,              0]
[                            0,                              0, (pi*R0^4*h)/10]

I cone:
[(pi*R0^2*h*(R0^2 + 4*h^2))/20 - (9*h^2*m)/16,
           0,              0]
[                                            0, (pi*R0^2*h*(R0^2 + 4*h^2))/20 -
 (9*h^2*m)/16,              0]
[                                            0,
           0, (pi*R0^4*h)/10]

Net torque:
                  -(3*t*sin(t)*(4*pi - 15))/80
pi/4 - (9*sin(t))/16 + (3*pi*sin(t))/20 - 9/16
```

*(pi*cos(t))/10*

Due to previous comments from the graders, all symbolic steps will show their outputs.

ODE

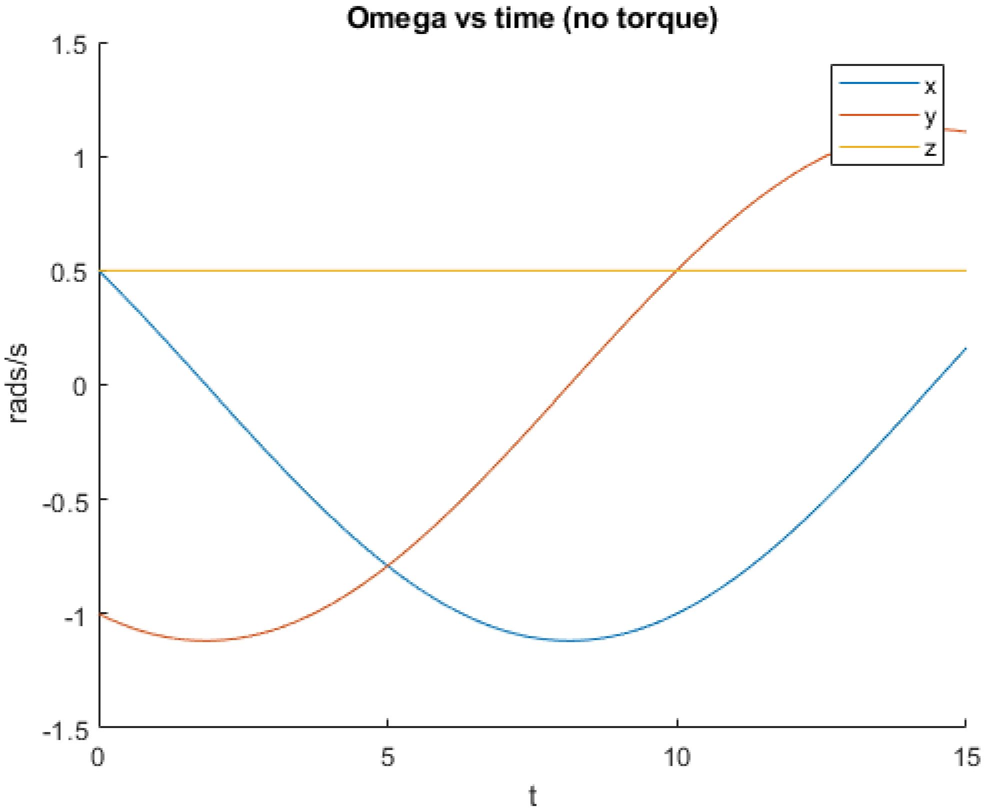
```
clear all
close all

m = 1;
h = 3;
r = 1;
w0 = [.5;-1;.5];
E0 = [0;0;0];
C0 = eye(3);
[eta0,eps0] = joshRotM2Quat(C0);


I = [[1 0 0];...
    [0 1 0];...
    [0 0 0]]*(1/12)*m*(3*r^2+h^2);
I(3,3) = .5*m*r^2;

tspan=[0,15];

X0 = [w0;E0;eps0;eta0];
options = odeset('RelTol', 1e-8,'AbsTol',1e-8);
[tC,XC] = ode45(@odefunCoast,tspan,X0,options);
[tT,XT] = ode45(@odefunTorque,tspan,X0,options);

%%%%%%%%%%%%%%%%%%%%

figure
hold on
plot(tC,XC(:,1),tC,XC(:,2),tC,XC(:,3))
title("Omega vs time (no torque)")
xlabel("t")
ylabel("rads/s")
legend("x","y","z")

figure
hold on
plot(tC,XC(:,4),tC,XC(:,5),tC,XC(:,6))
title("Euler angles vs time (no torque)")
xlabel("t")
ylabel("rads")
legend("x","y","z")

figure
hold on
plot(tC,XC(:,7),tC,XC(:,8),tC,XC(:,9))
title("Epsilon vs time (no torque)")
```

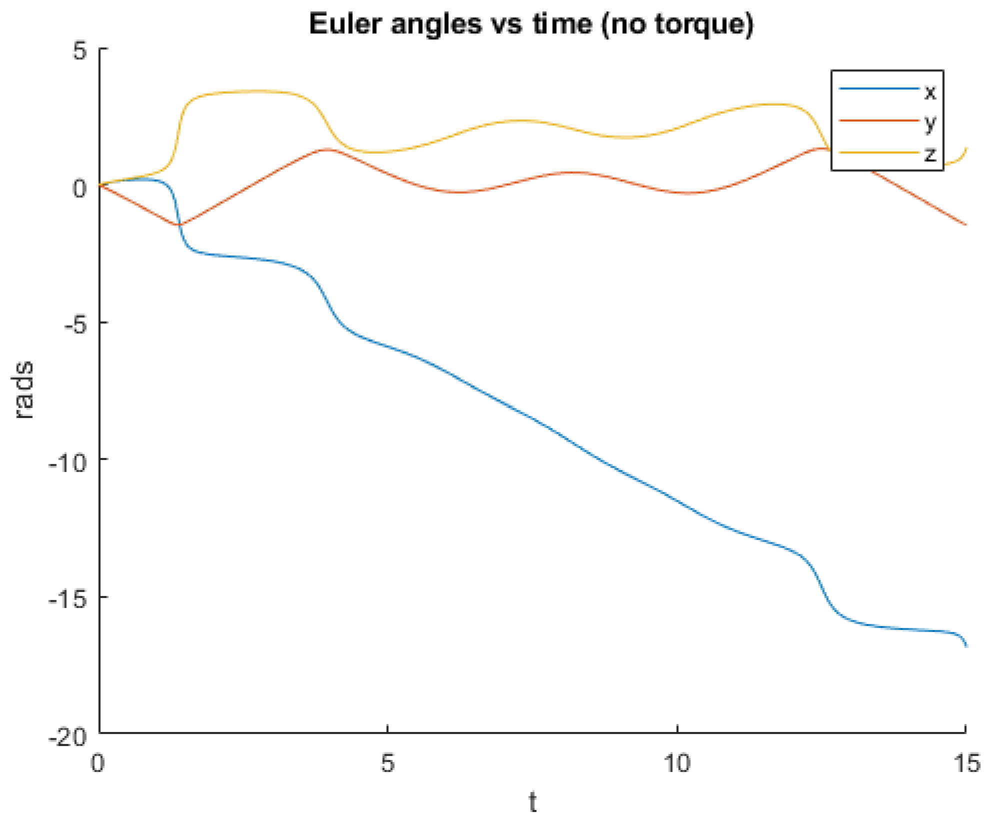
```
xlabel("t")
legend("x","y","z")

figure
plot(tC,XC(:,10))
title("Eta vs time (no torque)")
xlabel("t")

%%%%%%%%%%%%%%%%%%%%%

figure
hold on
plot(tT,XT(:,1),tT,XT(:,2),tT,XT(:,3))
title("Omega vs time (torque)")
xlabel("t")
ylabel("rads/s")
legend("x","y","z")

figure
hold on
plot(tT,XT(:,4),tT,XT(:,5),tT,XT(:,6))
title("Euler angles vs time (torque)")
xlabel("t")
ylabel("rads")
legend("x","y","z")

figure
hold on
plot(tT,XT(:,7),tT,XT(:,8),tT,XT(:,9))
title("Epsilon vs time (torque)")
xlabel("t")
legend("x","y","z")

figure
plot(tT,XT(:,10))
title("Eta vs time (torque)")
xlabel("t")

disp("------------P3------------")
disp("My work for this problem have the following results: ")
disp("See included hand calculations for equivalent cuboid.")
disp("See the 8 included plots.")
```

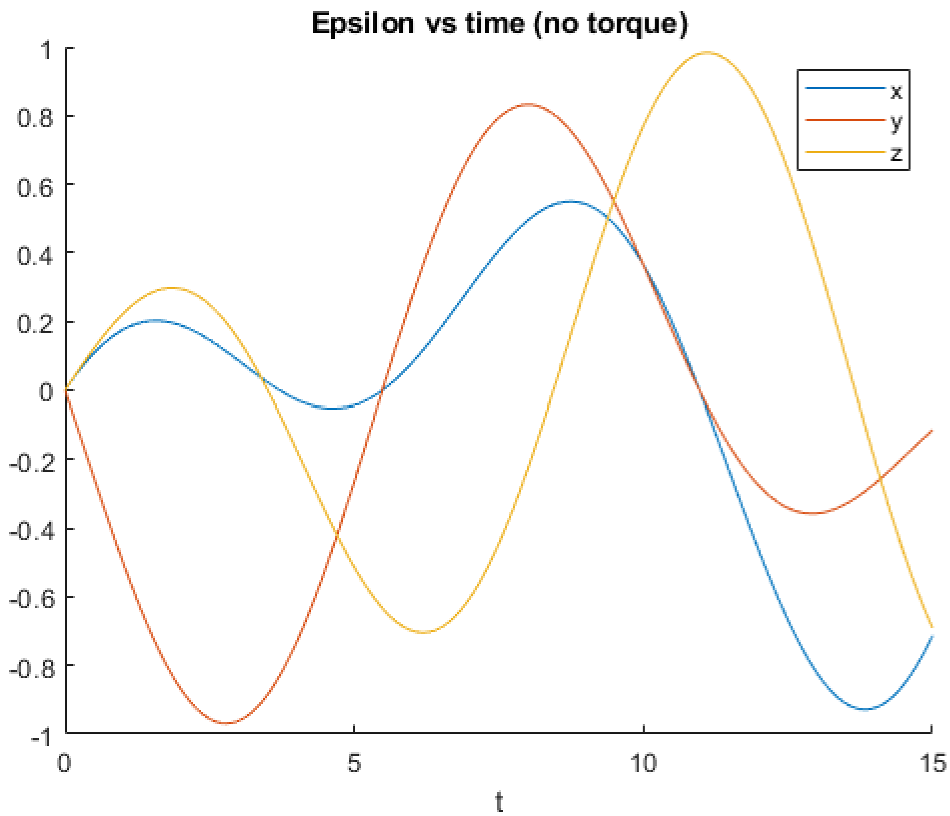
functions

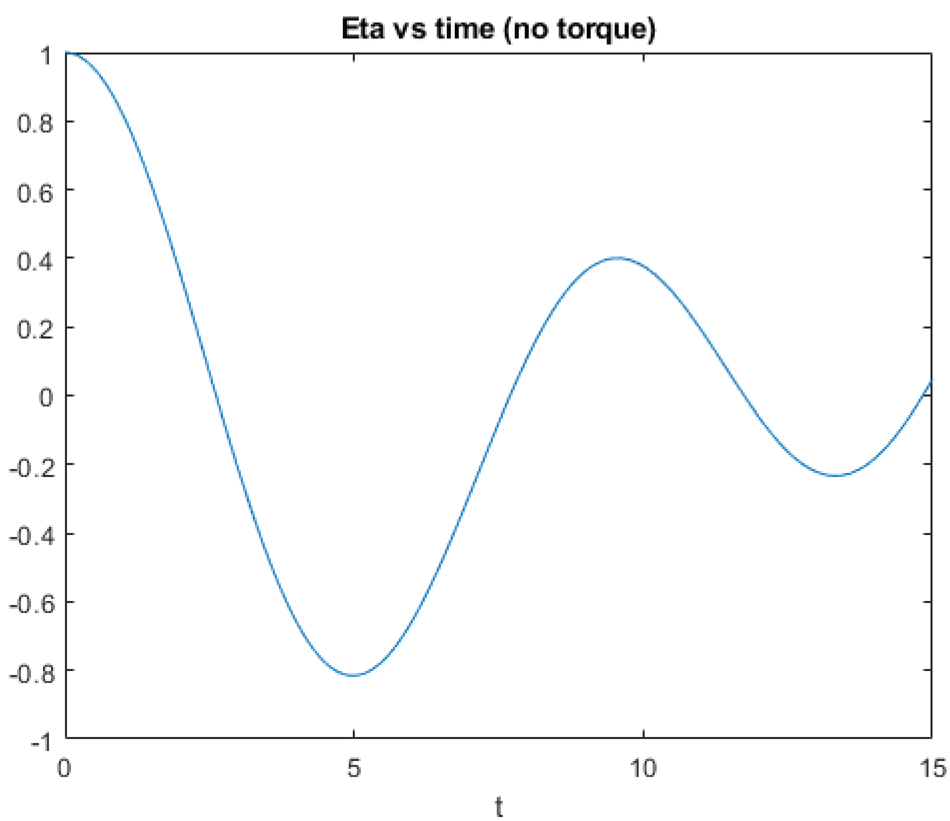
```
function Xdot = odefunCoast(t,X)
w = X(1:3);
E = X(4:6);
eps = X(7:9);
eta = X(10);
T = [0;0;0];

I = [[1 0 0];[0,1,0];[0,0,.5]];
```

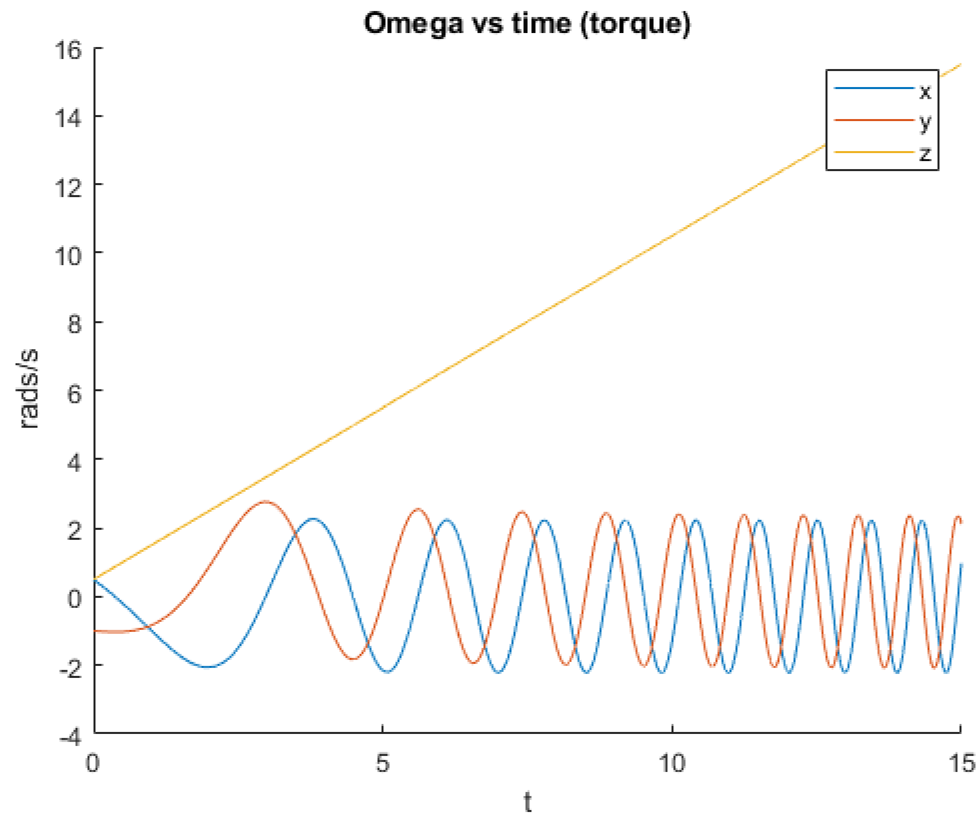
```
mat = [[cos(E(2)),       sin(E(2))*sin(E(1)),
 sin(E(2))*cos(E(1))];...
    [0,                 cos(E(2))*cos(E(1)),            -
cos(E(2))*sin(E(1))];...
    [0,                 sin(E(1)),                       cos(E(1))
  ]]*(1/cos(E(2)));

Edot = mat*w;

epsx = joshCross(eps);
epsdot = .5*(eta*eye(3)+epsx)*w;
etadot = -.5*eps'*w;

wx = joshCross(w);
wdot = -inv(I)*(wx*I*w-T);


Xdot = [wdot;Edot;epsdot;etadot];
end

function Xdot = odefunTorque(t,X)
w = X(1:3);
E = X(4:6);
eps = X(7:9);
eta = X(10);
T = [-1;0;.5];

I = [[1 0 0];[0,1,0];[0,0,.5]];

mat = [[cos(E(2)),       sin(E(2))*sin(E(1)),
 sin(E(2))*cos(E(1))];...
    [0,                 cos(E(2))*cos(E(1)),            -
cos(E(2))*sin(E(1))];...
    [0,                 sin(E(1)),                       cos(E(1))
  ]]*(1/cos(E(2)));

Edot = mat*w;

epsx = joshCross(eps);
epsdot = .5*(eta*eye(3)+epsx)*w;
etadot = -.5*eps'*w;

wx = joshCross(w);
wdot = -inv(I)*(wx*I*w-T);


Xdot = [wdot;Edot;epsdot;etadot];
end

------------P3------------
My work for this problem have the following results:
See included hand calculations for equivalent cuboid.
See the 8 included plots.
```

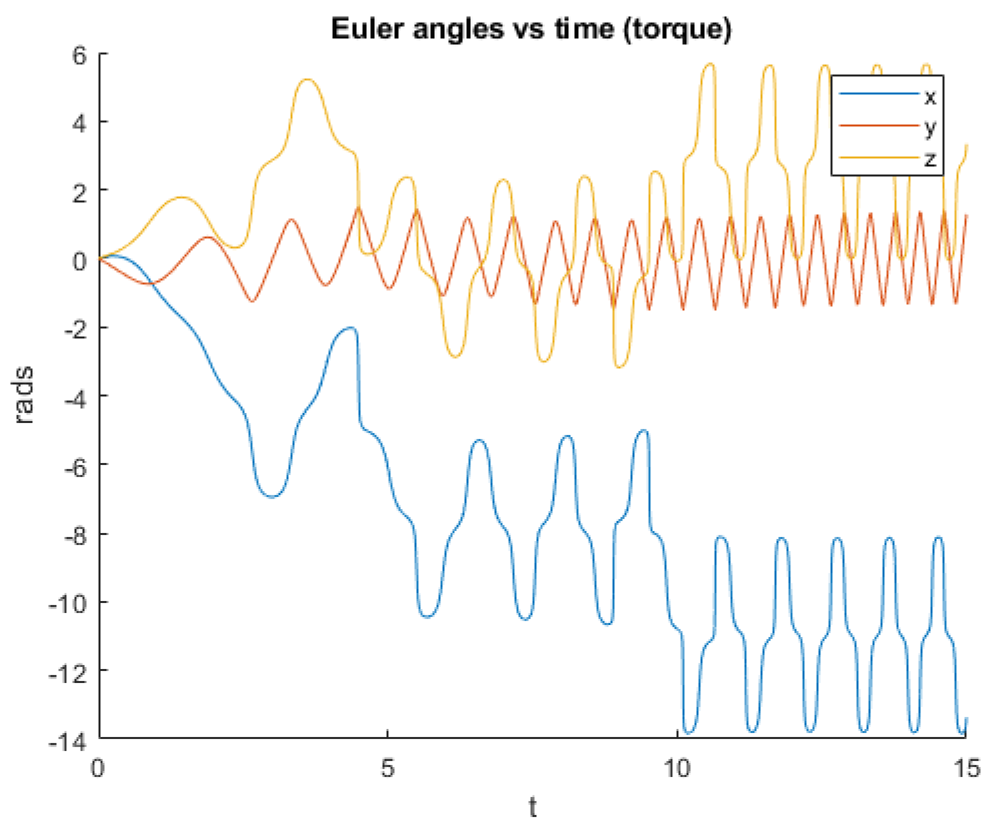

Omega vs time (no torque)
1.5
1
0.5
0
-0.5
-1
-1.5
rads/s
0
5
10
15
t
x
y
z

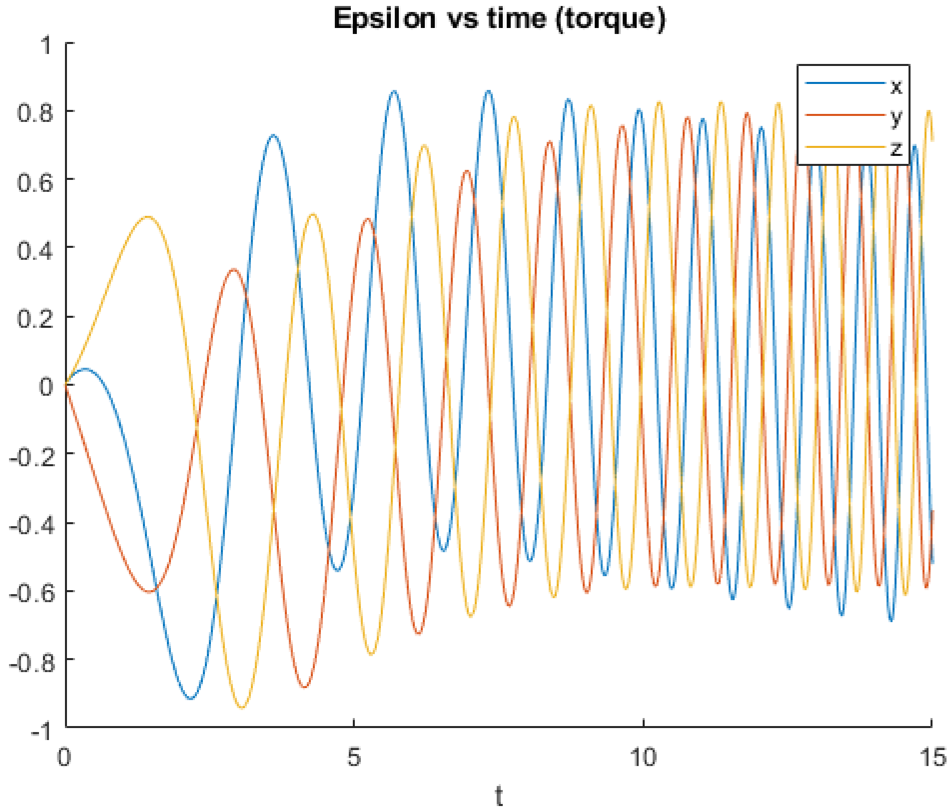

Euler angles vs time (no torque)
5
0
-5
-10
-15
-20
rads
0
5
10
15
t
x
y
z

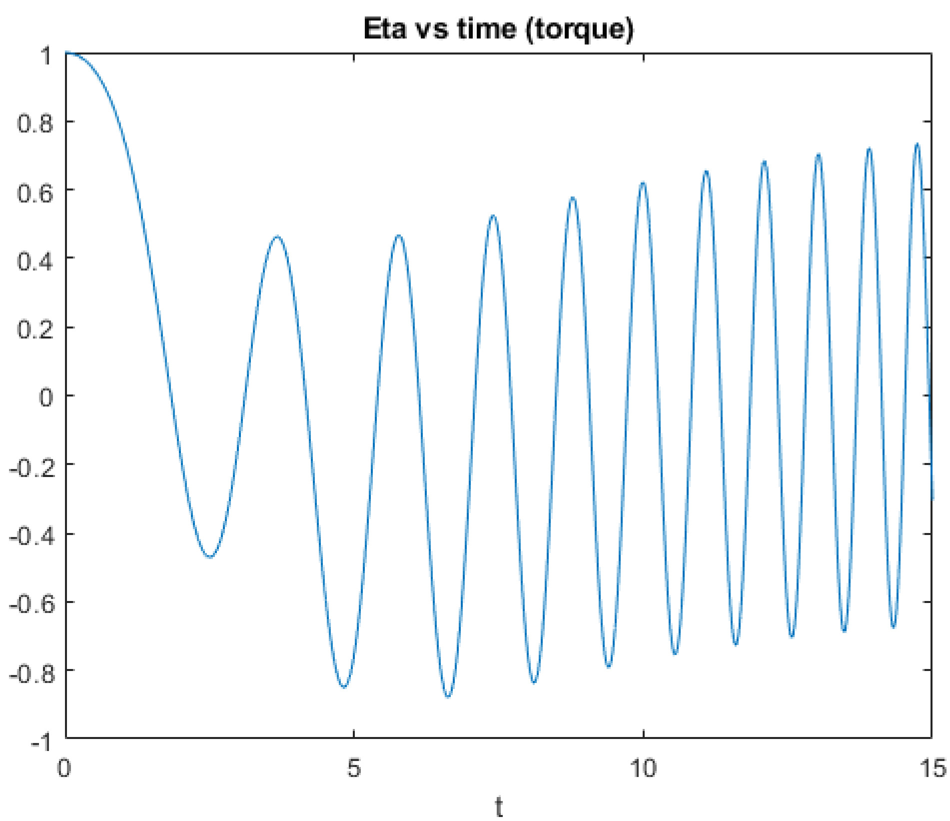

Epsilon vs time (no torque)
x
y
z
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
t


Eta vs time (no torque)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
t


Omega vs time (torque)
16
14
12
10
8
6
4
2
0
-2
-4
rads/s
0
5
10
15
t
x
y
z


Euler angles vs time (torque)
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
rads
0
5
10
15
t
x
y
z


Epsilon vs time (torque)
x
y
z
t


Eta vs time (torque)
t

cone with imports

clear all

syms t m h R w = [1;t;sin(t)]; dw = diff(w); wx = joshCross(w);

I= zeros(3,3);

I(1,1) = 1; I(2,2) = 1; I = I * (-9*h^2*m/16+(3/20)*m*(4*h^2+R^2)); I(3,3) = (3*m*R^2)/10;

Tc = I*dw + wx*I*w;

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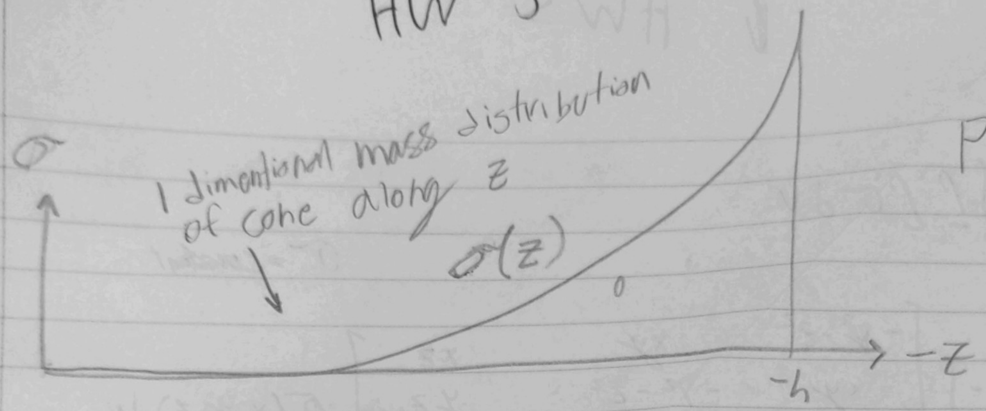
HW 5



P2, Center of mass

$$\sigma(z) = \beta A$$

$$A = \pi R^2$$

$$R = -R_0 \frac{z}{h}$$

β = constant area density $\left(\frac{kg}{m^2}\right)$

$$\sigma(z) = \beta \pi \left(R_0 \frac{z}{h}\right)^2$$

center mass

$$\bar{z} = \frac{\int_0^{-h} z\,\sigma(z)\,dz}{\int_0^{-h} \sigma(z)\,dz}$$

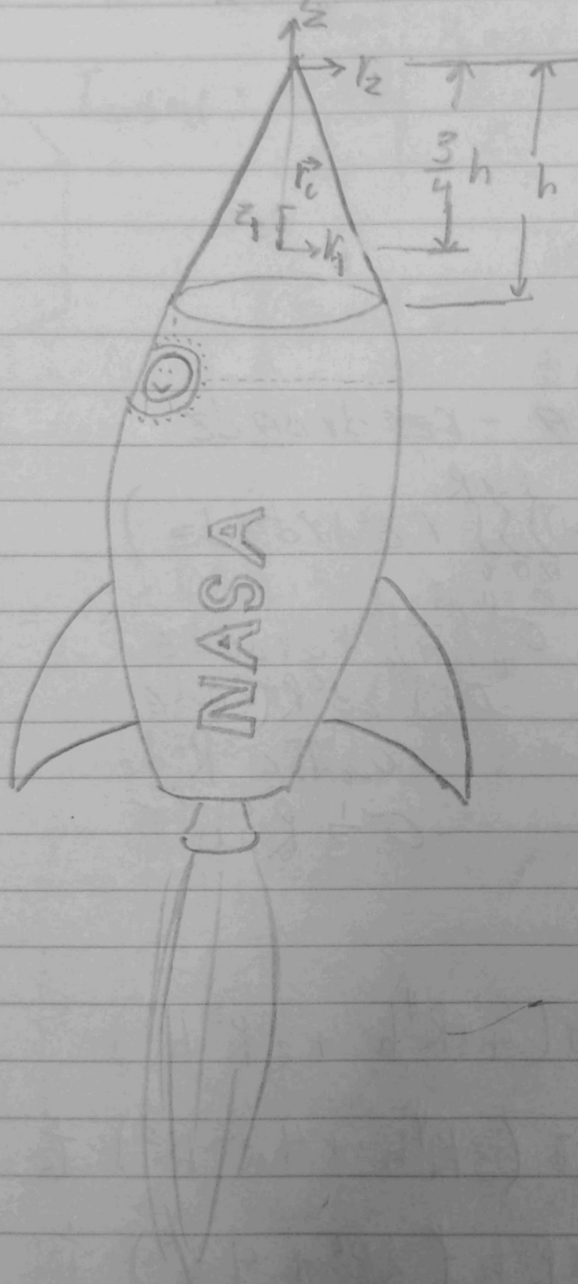
$$\frac{\int_0^{-h} z \beta \pi \left(R_0 \frac{z}{h}\right)^2 dz}{\int_0^{-h} \beta \pi \left(R_0 \frac{z}{h}\right)^2 dz} = \frac{\cancel{\beta \pi \frac{R_0^2}{h^2}} \int_0^{-h} z^3\,dz}{\cancel{\beta \pi \frac{R_0^2}{h^2}} \int_0^{-h} z^2\,dz}$$

$$\frac{\frac{(-h)^4}{4}}{\frac{(-h)^3}{3}} = \boxed{-h\frac{3}{4}}$$

HW5

3

P 2, diagram



HW5

P2, J_{11} entry

$$(\rho^\times \rho^\times)_{11} = -r^2 \sin^2(\theta) - z^2$$

$$J = -\int_{-h}^{0}\int_0^{2\pi}\int_0^R \sigma \rho^\times \rho^\times r \, dr \, d\theta \, dz$$

$$J_{11} = \sigma \int_0^h \int_0^{2\pi}\int_0^R (\rho^\times \rho^\times)_{11} r \, dr \, d\theta \, dz$$

$$= \sigma \int_0^h \int_0^{2\pi} \int_0^R -r^3 \sin^2\theta - r z^2 \, dr \, d\theta \, dz$$

$$= -\sigma \left(\int_0^h \int_0^{2\pi} \int_0^R r^3 \sin^2\theta \, dr \, d\theta \, dz + \sigma \int_0^h \int_0^{2\pi} \int_0^R r z^2 \, dr \, d\theta \, dz \right)$$

$$\sigma \int_0^h \int_0^{2\pi} \sin^2\theta \int_0^R r^3 \, dr \, d\theta \, dz \qquad \sigma \int_0^h \int_0^{2\pi} z^2 \int_0^R r \, dr \, d\theta \, dz$$

$$\sigma \int_0^h \int_0^{2\pi} \tfrac{1}{2} z^2 R^2 \, d\theta \, dz$$

$$-\sigma \int_0^h \frac{1}{4} \int_0^{2\pi} \sin^2\theta R^4 \, d\theta \, dz \qquad \sigma \int_0^h \pi z^2 R^2 \, dz$$

$$-\sigma \tfrac{1}{3} R^2 h^3 \pi$$

$$-\sigma \int_0^h \tfrac{1}{4} \pi R^4 \, dz$$

$$-\sigma \tfrac{1}{4} \pi R^4 h$$

$$+\sigma\pi \left(\tfrac{1}{4} R^4 h + \tfrac{1}{3} R^2 h^3 \right)$$

$$= +\sigma \pi \left(3R^4 h + 4R^2 h^3 \right) \tfrac{1}{12}$$

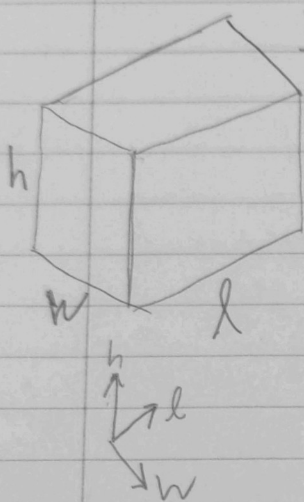
$$+\sigma \pi R^2 h \left(3R^2 + 4h^2 \right) \tfrac{1}{12}$$

$$= m \left(3R^2 + 4h^2 \right) \tfrac{1}{12}$$

P3

from ch58

$$I_{cuboid} = \frac{1}{12} m \begin{bmatrix} h^2+w^2 & 0 & 0 \\ 0 & l^2+w^2 & 0 \\ 0 & 0 & h^2+l^2 \end{bmatrix}$$



$$= \begin{bmatrix} \frac{1}{12} m(3r^2+H^2) & 0 & 0 \\ 0 & \frac{1}{12} m(3r^2+H^2) & 0 \\ 0 & 0 & \frac{1}{2} m r^2 \end{bmatrix}$$

$$h^2+w^2 = 3r^2+H^2 = l^2+w^2$$

$$l^2 = h^2, \quad \boxed{l = h}$$

$$\frac{1}{12} m (h^2+h^2) = \frac{1}{2} m r^2$$

$$\frac{1}{6} 2(h^2) = r^2$$

$$\frac{1}{3} h^2 = r^2$$

$$\boxed{h = l = \sqrt{3r^2}}$$

$$3r^2 + w^2 = 3r^2 + H^2$$

$$\boxed{w = H}$$

```
function [eta,epsilon] = joshRotM2Quat(C)

if ~joshIsRotM(C)
    throw(MException("joshRotM2Quat:invalidInput":"C must be a rotational
 matrix"))
end

eta =.5*sqrt(1+trace(C));
epsilon =...
    [(C(2,3)-C(3,2))/(4*eta);...
    (C(3,1)-C(1,3))/(4*eta);...
    (C(1,2)-C(2,1))/(4*eta)];
end
```

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```
function mx = joshCross(m)
% takes a column vector and returns the associated 'cross' matrix such that
% mx*b == cross(m,b)
arguments
    m (3,1)
end
if isa(m,"double") % overloaded to handle either symbolic or double type
 vectors
    mx = zeros(3);
elseif isa(m,"sym")
    syms mx [3 3]
else
    throw(MException("joshCross:invalidInput","m must be type sym or double"))
end
    for i = 1:3
        mx(i,i) = 0;
    end
    mx(1,2) = -m(3);
    mx(1,3) =  m(2);
    mx(2,3) = -m(1);

    mx(2,1) =  m(3);
    mx(3,1) = -m(2);
    mx(3,2) =  m(1);
end
```

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```
function [isRotM] = joshIsRotM(M)
    isRotM = (round(M*M',14) == eye(3) & round(M'*M,14) == eye(3) &
 round(det(M),14) == 1);
end
```

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