EAI

1

$$u_r = \frac{1}{r}\frac{\partial}{\partial\theta}\left[r - \frac{R^2}{r}\right]U_\infty \sin\theta$$

$$= \frac{1}{r}\left[r - \frac{R^2}{r}\right]U_\infty \cos\theta$$

$$u_\theta = -\frac{\partial}{\partial r}\left[r - \frac{R^2}{r}\right]U_\infty \sin\theta$$

$$= -U_\infty \sin\theta\left[1 + \frac{R^2}{r^2}\right]$$

2 $\phi = \int u_r\, \partial r = U_\infty \cos\theta \int \frac{1}{r}\left[r - \frac{R^2}{r}\right] dr = \frac{1}{r}U_\infty \cos(\theta)\,\frac{R^2+r^2}{r}$

3

$$\frac{\partial(rU_r)}{\partial r} + \frac{\partial U_\theta}{\partial\theta} = U_\infty\left[\left[1 + \frac{R^2}{r^2}\right]\frac{\partial}{\partial\theta}\sin\theta + \cos(\theta)\frac{\partial}{\partial r}\cancel{r}\frac{1}{r}\left[r - \frac{R^2}{r}\right]\right]$$

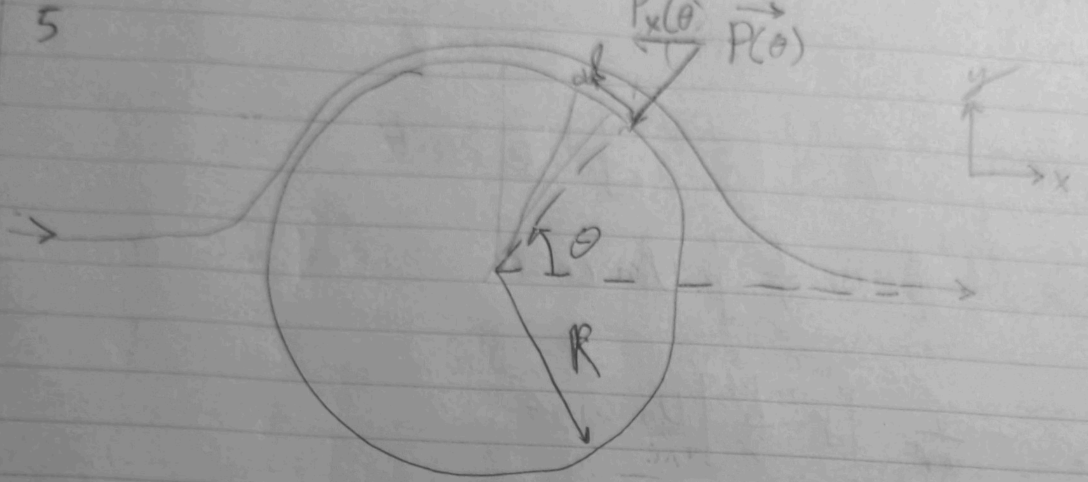
$$= U_\infty\left[-\cos\theta\left[1 + \frac{R^2}{r^2}\right] + \cos(\theta)\left[1 + \frac{R^2}{r^2}\right]\right] = 0$$

4 $P = P_\infty + \frac{1}{2}\rho U_\infty^2 - \frac{1}{2}\rho\left(-2U_\infty \sin\theta\right)^2$

$$\boxed{P = P_\infty + \frac{1}{2}\rho U_\infty^2 - \frac{1}{2}\rho\left(4U_\infty^2 \sin^2\theta\right)} = P_\infty + \frac{1}{2}\rho U_\infty^2\left[1 - 4\sin\right.$$

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = \frac{\cancel{P_\infty} + \cancel{\tfrac{1}{2}\rho U_\infty^2}\left[1 - 4\sin^2\theta\right] - \cancel{P_\infty}}{\cancel{\tfrac{1}{2}\rho_\infty U_\infty^2}} = 1 - 4\sin^2$$

5



$$P_x = -\cos(\theta)\ P(\theta)$$

$$d\vec{F} = \vec{P(\theta)}\ dl$$

$$dF_x = P_x(\theta)\, dl \quad , \ dl = R\, d\theta$$

$$\oint dF_x = F_x = \oint P_x(\theta)\, R\, d\theta = -R\int_0^{2\pi} \cos(\theta)\, P(\theta)\ d\theta$$

$$= -R\int_0^{2\pi} \cos(\theta) \left[P_\infty + \tfrac{1}{2}\rho\, U_\infty^2 \left(1 - 4\sin^2\theta\right)\right] d\theta = 0$$

BLT 1

laminar : $Cf_{xl} = \frac{.664}{Re_x^{1/2}}$, turbulent : $Cf_{xt} = \frac{.059}{Re_x^{1/5}}$

1) laminar - $Cf_{dl} = 2d\, Cf_{xl} = \boxed{d\ \frac{1.3280}{Re_x^{1/2}}}$

2) turbulent $Cf_{dt} = 2d\, Cf_{xd} = \boxed{d\ \frac{.1180}{Re_x^{1/5}}}$

3) 30% laminar 70% turbulent = $Cf_d = 30\%\, Cf_{dl} + 70\%\, Cf_{dt}$

$$= \boxed{d\left[\frac{.3984}{Re_x^{1/2}} + \frac{.0826}{Re_x^{1/5}}\right]}$$

4) 1. transition instantly from laminar to turbulent @ trip
2. trip on top and bottom
3. frisbee as infinite flat plate

EA2

Show $F = 6\pi \mu a V_0$, $Re < 1$

$V_r = \frac{1}{r^2 \sin\theta} \frac{\partial \psi}{\partial \theta}$, $V_\theta = \frac{-1}{r \sin\theta} \frac{\partial \psi}{\partial r}$

from $\nabla \cdot \vec{V} = 0$ @ wall , ie $div(\vec{v})|_{r=a} = 0$

let $Q\psi = \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial \theta} \frac{\sin\theta}{r^2} \frac{\partial \frac{1}{\sin\theta}}{\partial \theta}$

$\frac{\partial P}{\partial r} = \frac{\mu}{r^2 \sin\theta} \frac{\partial \left[\frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial \theta} \frac{\sin\theta}{r^2} \frac{\partial \frac{1}{\sin\theta}}{\partial \theta} \right]}{\partial \theta}$, $\frac{\partial P}{\partial \theta} = -\frac{\mu}{\sin\theta} \frac{\partial \left[\frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial \theta} \frac{\sin\theta}{r^2} \frac{\partial \frac{1}{\sin\theta}}{\partial \theta} \right]}{\partial r}$

where $Q = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2} \frac{\partial}{\partial \theta} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \right]$

Sin p is in both eqs, divice by p to get

$Q^2 \psi = 0$, let $\psi = f(r) \sin^2\theta$

So $r^4 f'''' - 4r^2 f'' + 8rf' - 8f = 0$

whose general solution is

$f(r) = \frac{A}{r} + Br + Cr^2 + Dr^4$ since the roots are $-1, 1, 2, 4$ respectively

$\psi(r, \theta) = V_0 \left[\frac{a^3}{4r} - \frac{3ar}{4} + \frac{r^2}{2} \right] \sin^2\theta$

2) $P = -\left[\frac{3a\mu V_0}{2r^2} \right] \cos\theta$, $V_r = U_0 \left[\frac{a^3}{2r^3} - \frac{3a}{2r} + 1 \right] \cos\theta$, $V_\theta = V_0 \left[\frac{a^3}{4r^3} + \frac{3a}{4r} - 1 \right] \sin\theta$

$\tau = -\mu \left[r \frac{\partial}{\partial r} \left(\frac{V_\theta}{r} \right) + \frac{1}{r} \left(\frac{\partial V_r}{\partial \theta} \right) \right]$

1) $F_{pressure} = 2\pi a^2 \int_0^{\pi} p \sin\theta \cos\theta \, d\theta = 2\pi a \mu V_0$

$F_{skin} = 2\pi a^2 \int_0^{\pi} -\mu \left[r \frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right) + \frac{1}{r}\left(\frac{\partial V_r}{\partial \theta}\right)\right] \sin^2\theta \, d\theta = 4\pi a \mu V_0$

By linear combination,

$$V_0 = \frac{2ga^2}{9\mu}\left[\rho_{stero} - \rho_{fluid}\right]$$

3) $C_p = \frac{2P}{\rho V_\infty^2} = \frac{-6a\mu V_0 \cos\theta}{r^2 \rho V_0^2} = \frac{-6a\mu \cos\theta}{\rho r^2 V_0}$

$D = 6\pi a \mu V_0 = F_p - F_s$

$C_d = \frac{2D}{\rho V_\infty^2} = \frac{12\pi a \mu V_\infty}{Re} = \frac{24}{Re}$

$Re = \frac{\rho V_\infty}{\mu} \quad \frac{\mu}{\rho V_\infty}$