AERO 331: AEROSPACE STRUCTURAL ANALYSIS I

Winter 2023

Cal Poly San Luis Obispo

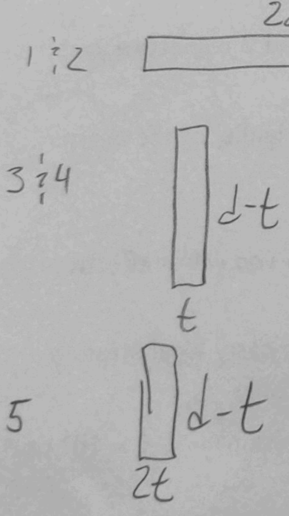
Exam 3

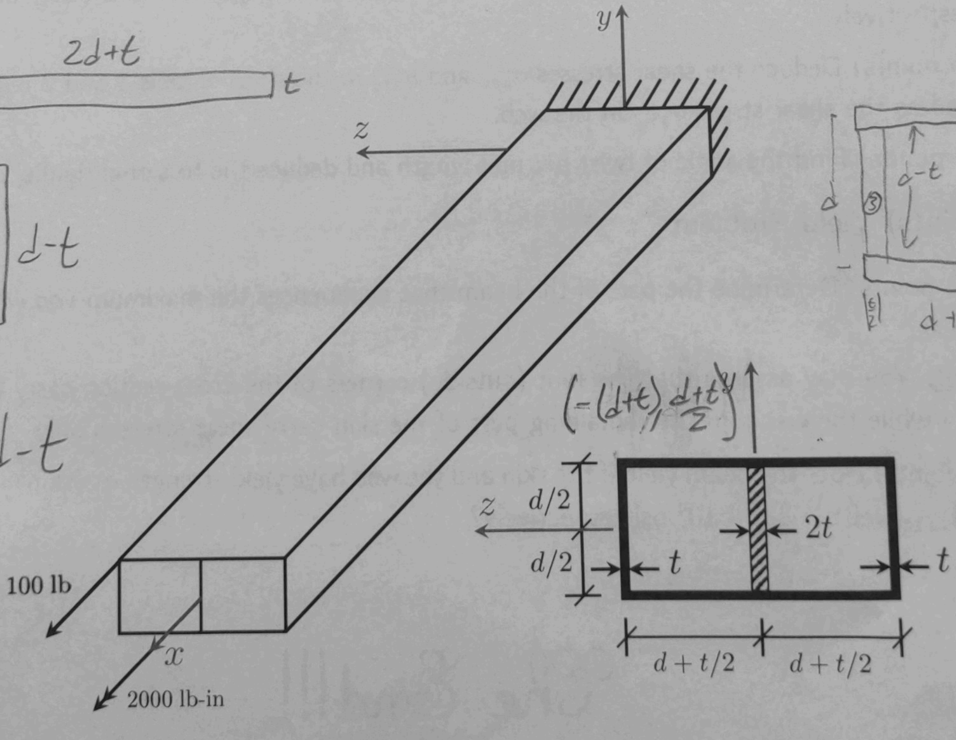
Problem 1 (100 points)

Consider a heterogeneous beam with a thin-walled closed section subjected to applied loads as shown in Figure 1. The outer skin has uniform thickness and the *Young's modulus* and *Poisson's ratio* for each portion is as given below:

$$\text{(Outer) skin:} \quad E = 30 \times 10^6 \text{ psi}\,, \qquad \nu = 0.32$$
$$\text{(Middle) web:} \quad E = 10 \times 10^6 \text{ psi}\,, \qquad \nu = 0.33\,.$$

The beam is cantilevered at $x = 0$ and has length $L = 100$ in. Cross sectional dimensions are measured with respect to the median line.





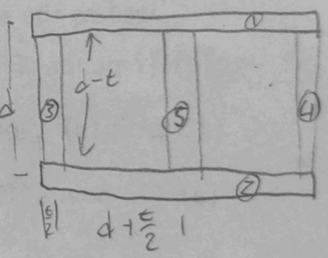
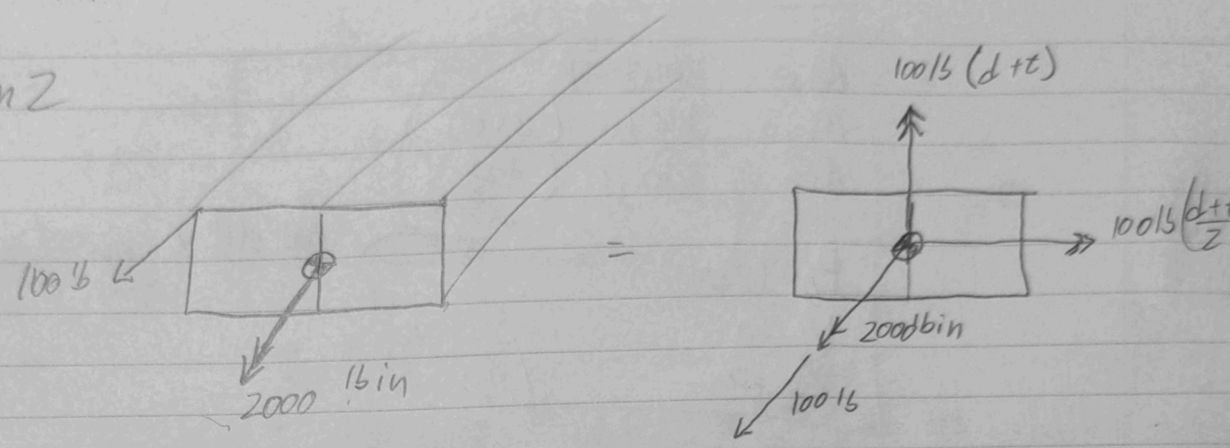


Figure 1: Schematic of the beam described in Problem 1.

Exam 3 Joshua Oates

problem 2



no running loads are applied to this beam, so

$$\boxed{p_x = p_y = p_z = m_x = m_y = m_z = 0}$$

$$P'(x) = -p_x, \quad V_y'(x) = -p_y, \quad V_z'(x) = p_z$$

$$\boxed{P'(x) = V_y'(x) = V_z'(x) = 0}$$

$$\boxed{M_x'(x) = m_x = 0}$$

$$M_y'(x) = -\cancelto{0}{m_y(x)} + V_z(x)$$

$$M_z'(x) = -\cancelto{0}{m_z(x)} - V_y(x)$$

Since no loads are appied in the lateral directions and there are no running loads, $V_z = V_y = 0$ so

$$\boxed{M_y'(x) = M_z'(x) = 0}$$

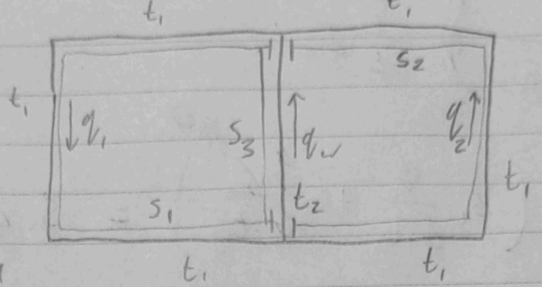
ie, all stresses in beam are constant. looking at centroid resolved figure above, we get ... continued

$M_x = 200\,lb\,in$
$M_y = 100\,lb\,(d+t)$
$M_z = 100\,lb\,(\frac{d+t}{2})$

$P_x = 100\,lb$
$V_y = 0$
$V_z = 0$

for all x on bem

problem 3



$t_1 = .1$
$t_2 = .2$

$S_1 = S_2 = 2d + t + d$
$= 3d + t = 30.1$

$S_3 = d = 10$

$q_w = q_1 - q_2$

$\bar{A}_1 = \bar{A}_2 = \bar{A} = d\left(d + \frac{t}{2}\right) = d^2 + \frac{dt}{2} = 100 + \frac{1}{2} = 100.5$

$$\frac{1}{2A_1} \oint_{cell_1} \frac{q}{Gt} ds = \frac{1}{2\bar{A}_2} \oint_{cell_2} \frac{q}{Gt} ds$$

$G = \frac{E}{2(1+\nu)}$, $G_1 = \frac{E_1}{2(1+\nu_1)} = 1.1364e7$, $G_2 = \frac{E_2}{2(1+\nu_2)} = 3.759$

$$\frac{1}{2\bar{A}_1} \oint_{cell_1} \frac{q}{Gt} ds = \frac{1}{2\bar{A}_1}\left(S_1 \frac{q_1}{G_1 t_1} + S_3 \frac{q_w}{G_2 t_2}\right)$$

$$\frac{1}{2\bar{A}_2} \oint_{cell_2} \frac{q}{Gt} ds = \frac{1}{2\bar{A}_2}\left(S_1 \frac{q_2}{G_1 t_1} - S_3 \frac{q_w}{G_2 t_2}\right)$$

$$M_x = 2\sum_{i=1}^{n} q_i \bar{A}_i = 2q_1\bar{A}_1 + 2q_2\bar{A}_2 = 2\bar{A}(q_1 + q_2)$$

$M_x = 200 \text{ lbin}$

this system of eq is solved in Matlab

b) $\sigma_{sx} = \frac{q}{t}$ gives $\boxed{\sigma_{sx,skin} = \frac{q_s}{t_1} = 2.4876, \ \sigma_{sx,web} = \frac{q_w}{t_2} = 0}$

Psi

$$\theta_1 = \theta_2 = \theta = \frac{1}{2\bar{A}} \oint_s \frac{q}{Gt} ds$$

$$\theta_1 = \frac{1}{2\bar{A}} \left(s_1 \frac{q_1}{G_1 t_1} + s_3 \frac{q_{1w}}{G_2 t_2} \right) = 3.2781e-8$$

$$TR = 3.0505e4$$

4) because of the problem assumptions we get stress ~~states~~ that have only one component, either shear $= \sigma_{xs}$, or axial $= \sigma_{xx}$

we know on candidate ~~stress~~ state is

$$\underline{\sigma} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

the other 2 of shear are

$$\underline{\sigma} = \begin{bmatrix} 0 & \sigma_{sx} & 0 \\ \sigma_{sx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; \& \; \underline{\sigma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma_{sx} \\ 0 & \sigma_{sx} & 0 \end{bmatrix}$$

in the case of the shear, it works out to the same τ_{max} & σ_e, so I will only analyze the first.