Further Analysis

1. The steady-state response oscillates symmetrically around zero over time. The transient response lacks this pattern and continues to change over time before ultimately converging.

2. The pendulum's response time remains relatively the same despite changing the parameter P for different cases of the system

3. For all cases of P for the pendulum, the system locally converges to a point over time. However, without the initial conditions the stability cannot be applied globally. This concludes that the system is locally asymptotically stable, which agrees with the solution to question five.

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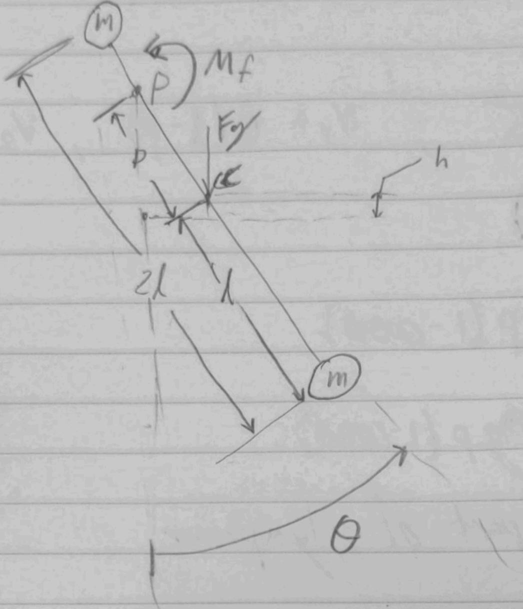
ICGE3

1) a)



$T_p \equiv$ torque about P, $J_p \equiv$ Moment of Inertia about P

$M_f \equiv$ friction, $T_g \equiv$ Torque from gravity

b) EOM

$T_p = M_f + T_g$, $T_g = -2Pmg\sin(\theta)$, $M_f = -B\dot{\theta}$

$\to T_p = J_p\ddot{\theta}$, $\ddot{\theta} = \frac{-Pg\sin\theta}{(l^2+p^2)} - \frac{B\dot{\theta}}{2m(l^2+p^2)}$

2) $\underline{x} = \begin{pmatrix}\theta \\ \dot{\theta}\end{pmatrix}$, $\dot{\underline{x}} = \begin{pmatrix}\dot{\theta} \\ \ddot{\theta}\end{pmatrix} = \begin{pmatrix}\dot{\theta} \\ \frac{-Pg\sin\theta}{l^2+p^2} - \frac{B\dot{\theta}}{2m(l^2+p^2)}\end{pmatrix} = f(x)$

3) $\underline{x}^*$ means $\dot{\underline{x}}^* = \begin{pmatrix}0 \\ 0\end{pmatrix} = \begin{pmatrix}\dot{\theta} \\ \frac{-Pg\sin\theta}{l^2+p^2} - \frac{B\dot{\theta}}{2m(l^2+p^2)}\end{pmatrix} = \begin{pmatrix}0 \\ -\frac{Pg\sin\theta}{l^2+p^2}\end{pmatrix}$

$x^* = \begin{pmatrix}0 \\ 0\end{pmatrix}$ or $\begin{pmatrix}0 \\ 2\pi\end{pmatrix}$

4) $V = PE + KE$,

$KE = \frac{1}{2} m V_1^2 + \frac{1}{2} m V_2^2$, $V_1 = \dot{\theta}(l-p)$, $V_2 = \dot{\theta}(l+p)$

$KE = \frac{1}{2} m \dot{\theta}^2 (l^2 + p^2)$

$PE = 2mgh = 2mg\,p(1-\cos\theta)$

$V = \frac{1}{2} m \dot{\theta}^2 (l^2+p^2) + 2mg\,p(1-\cos\theta)$

$V(\underline{0}) = 0$, first part of lyapunov

$$\nabla V = \begin{pmatrix} 2mpg\sin\theta \\ 2m\dot{\theta}(l^2+p^2) \end{pmatrix}, \quad \dot{V} = (\nabla V)^T \dot{x}$$

$$\dot{V} = \left(\begin{matrix} 2mpg\dot{\theta}\sin(\theta) \\ 2m\dot{\theta}\left[\frac{-pg\sin\theta}{l^2+p^2} - \frac{B\dot{\theta}}{2m(l^2+p^2)}\right](l^2+p^2) \end{matrix} \right) = \left(\begin{matrix} 2mpg\dot{\theta}\sin\theta \\ 2m\dot{\theta}\left[pg\sin\theta - \frac{B\dot{\theta}}{2m}\right] \end{matrix} \right)$$

$\dot{V} = 0$ for any $\dot{\theta} = 0$, so there are $\underline{x} \neq \underline{0}$ which $\dot{V}$ is not less than zero, so we cannot conclude global asymptotic stability.

5) $$\underline{A} = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \frac{\partial \dot{x}_1}{\partial x_2} \\ \frac{\partial \dot{x}_2}{\partial x_1} & \frac{\partial \dot{x}_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-pg\cos\theta}{l^2+p} & \frac{-B}{2m(l^2+p^2)} \end{bmatrix}$$

$$\underline{A} = \begin{bmatrix} 0 & 1 \\ \frac{-pg}{l^2+p^2} & \frac{-B}{2m(l^2+p^2)} \end{bmatrix}$$

eigenval(A^*) →

$$\det(A - \lambda \mathbb{1}) = \begin{vmatrix} -\lambda & 1 \\ \frac{-Pg}{l^2+p^2} & \frac{-B}{2m(l^2+p^2)} - \lambda \end{vmatrix} = 0$$

$$\frac{\lambda B}{2m(l^2+p^2)} + \lambda^2 + \frac{Pg}{l^2+p^2} = 0$$

$$\lambda = \frac{-\frac{B}{2m(l^2+p^2)} \pm \sqrt{\frac{B}{2m(l^2+p^2)} - 4\frac{Pg}{l^2+p^2}}}{2}$$

① if $\frac{B}{2m(l^2+p^2)} - 4\frac{Pg}{l^2+p^2} < 0$

$$\lambda = -\frac{1}{2}\left[\frac{B}{2m(l^2+p^2)}\right] + bi$$

② if $\frac{B}{2m(l^2+p^2)} - 4\frac{Pg}{l^2+p^2} = 0$

$$\lambda = -\frac{1}{2}\left[\frac{B}{2m(l^2+p^2)}\right]$$

③ if $\frac{B}{2m(l^2+p^2)} - 4\frac{Pg}{l^2+p^2} > 0$

$$\frac{B}{2m(l^2+p^2)} - 4\frac{Pg}{l^2+p^2} < \frac{B}{2m(l^2+p^2)}$$

So $\lambda < 0$

↳ no matter what real$(\lambda) < 0$,
So system is locally, asymptotically stable at $\underline{x}^* = \underline{0}$