1) a) $\left[\begin{array}{cc|c} -1 & 1 & 4 \\ 3 & -1 & 0 \end{array}\right] \xrightarrow{r_2 = r_2 - (-3)r_1} \left[\begin{array}{cc|c} -1 & 1 & 4 \\ 0 & 2 & 12 \end{array}\right] \Rightarrow$

$2y = 12,\ y = 6 \Rightarrow -x + 6 = 4 \Rightarrow x = 2$

---

b) $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 2 & -1 & 1 & 2 \end{array}\right] \xrightarrow{r_3 = r_3 - (2)r_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & -5 & 3 & -2 \end{array}\right] \xrightarrow{r_3 = r_3 - \left(-\frac{5}{3}\right)r_2}$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 3+\frac{5}{3} & \frac{20}{3}-2 \end{array}\right] \Rightarrow \left(3+\frac{5}{3}\right)z = \left(\frac{20}{3}-2\right) \Rightarrow 3y + 1 = 4 \Rightarrow x + 2 - 1 = 2$

$z = 1$ $\quad y = 1 \quad x = 1$

2) a) $\underline{A} = \begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} \xrightarrow{r_3 - (1)r_1} \begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 1^* & 0^* & 3 \end{bmatrix} \xrightarrow{r_2 - (2)r_1} \begin{bmatrix} 3 & 1 & 2 \\ 2^* & 1 & 0 \\ 1^* & 0^* & 3 \end{bmatrix}$

$\underline{L} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad \underline{U} = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

b) $\underline{A} = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0^* & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix} \xrightarrow{r_3 - (1)r_1} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0^* & 2 & 1 & 0 \\ 1^* & 4 & 3 & 2 \\ 0^* & 2 & 1 & -1 \end{bmatrix} \xrightarrow{r_3 - (2)r_2} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0^* & 2 & 1 & 0 \\ 1^* & 2^* & 1 & 2 \\ 0^* & 2 & 1 & -1 \end{bmatrix} \xrightarrow{r_4 - (1)r_2}$

$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0^* & 2 & 1 & 0 \\ 1^* & 2^* & 1 & 2 \\ 0^* & 1^* & 0^* & -1 \end{bmatrix} \Rightarrow \underline{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad \underline{U} = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

3) a)

$$\underline{L}\,\underline{y}=\underline{b} = \begin{pmatrix}0\\1\\3\end{pmatrix} = \begin{bmatrix}1&&\\2&1&\\1&0&1\end{bmatrix}\underline{y} = \begin{pmatrix}y_1\\2y_1+y_2\\y_1+y_3\end{pmatrix} \Rightarrow \underline{y} = \begin{pmatrix}0\\1\\3\end{pmatrix}$$

$$\underline{U}x=\underline{y} = \begin{pmatrix}0\\1\\3\end{pmatrix} = \begin{bmatrix}3&1&2\\0&1&0\\0&0&3\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}3x_1+x_2+2x_3\\+x_2\\+3x_3\end{pmatrix} = \begin{pmatrix}0\\1\\3\end{pmatrix} \Rightarrow \underline{x} = \begin{pmatrix}-1\\1\\1\end{pmatrix}$$

b.)

$$\underline{L}\,\underline{y}=\underline{b} = \begin{pmatrix}5\\0\\9\\-1\end{pmatrix} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\1&2&1&0\\0&1&0&1\end{bmatrix}\underline{y} = \begin{pmatrix}y_1\\y_2\\y_1+2y_2+y_3\\y_2+y_4\end{pmatrix} \Rightarrow \underline{y} = \begin{pmatrix}5\\0\\4\\-1\end{pmatrix}$$

$$\underline{U}\,\underline{x}=\underline{y} = \begin{pmatrix}5\\0\\4\\-1\end{pmatrix} = \begin{bmatrix}1&-1&1&2\\0&2&1&0\\0&0&1&2\\0&0&0&-1\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = \begin{pmatrix}2x_4+x_3-x_2+x_1\\2x_2+x_3\\x_3+2x_4\\-x_4\end{pmatrix} \Rightarrow \begin{pmatrix}0\\-1\\2\\1\end{pmatrix} = \underline{x}$$

Problem 3 $\|X\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p} = \left(|x_1|^p + |x_2|^p + \dots + |x_n|^p\right)^{1/p}$

$\|x\|_\infty = \lim_{p\to\infty} \|x\|_p$

Because the largest term will be infinitely larger than the next largest, when the square root is taken, only the largest will matter.

Proof

$\|x\|_p = x_n \left(\sum_{k=1}^{n} \left(\frac{x_k}{x_n}\right)^p\right)^{\frac{1}{p}} = x_n \left(1 + \sum_{k=1}^{n-1} \left(\frac{x_k}{x_n}\right)^p\right)^{\frac{1}{p}}$

$0 \leq \frac{x_k}{x_n} \leq 1$

$x_n = x_n \cdot 1^{1/p} \leq \|x\|_p \leq x_n (n \cdot 1)^{1/p} = x_n \cdot n^{1/p}$

as $p \to \infty$

by squeeze theorem, $\lim_{p\to\infty} \|x\|_p = x_n$

---

imagine $V = [1, 2, 3, 3]$

$\|V\|_1 = (1^1 + 2^1 + 3^1 + 3^1)^1 = (9)^1 = 9$

$\|V\|_2 = (1^2 + 2^2 + 3^2 + 3^2)^{\frac{1}{2}} = (29)^{1/2} = 4.7958$

$\|V\|_{10} = (1^{10} + 2^{10} + 3^{10} + 3^{10})^{1/10} = (1025 + 118098)^{\frac{1}{10}} = 3.2181$

$\|V\|_{100} = (1.3e^{30} + 1.02e^{48})^{\frac{1}{100}} = (1.03e^{48})^{\frac{1}{100}} = 3.0209$

$(2.06e^{48})^{\frac{1}{100}} = 3.0419$

$(5e^{47})^{\frac{1}{100}} = 2.999$

any small factors the largest value (3) is multiplied by are brought to negligible amounts by around $p = 100$. they are further reduced as $p \to \infty$

---

2) a) $X = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\|X\|_2 = \sqrt{1^2 + (-2)^2 + 2^2} = 3$

$\|X\|_1 = 1 + |-2| + 2 = 5$

$\|X\|_\infty = 2$

b) $X = \begin{pmatrix} 5 \\ 0 \\ 3 \\ -1 \end{pmatrix}$ $\|X\|_2 = 5.9161$

$\|X\|_1 = 9$

$\|X\|_\infty = 5$

```matlab
% this is my matlab HW 3 for Aero 299
% Joshua Oates
```

Section 0 - cleanup

```matlab
clear all;
close all;
clc;
```

Section 1 - Use Newtons on function

```matlab
% create p of lambda for use in this lab
f =@(l) l^2-(2^.5+.5)*l+.5*2^.5-1;

% set up other vars for newtons and secant methods
TOL = 10e-6;
fp = @(l) 2*l-(2^.5+.5);

x0 = .1;
x1 = .2;
[r1N,c1N] = JoshNewtons(f,fp,x0);
[r1S,c1S] = JoshSecant(f,x0,x1);

x0 = 2.6;
x1 = 2.7;
[r2N,c2N] = JoshNewtons(f,fp,x0);
[r2S,c2S] = JoshSecant(f,x0,x1);

format long
disp("Newtons found roots at")
disp(r1N)
disp(r2N)
disp("in ")
disp(c1N)
disp(c2N)
disp("iterations.")
disp("Secant found roots at")
disp(r1S)
disp(r2S)
disp("in ")
disp(c1S)
disp(c2S)
disp("iterations.")

disp("in both cases, the functions found the roots in the same number of
 iterations")
```

```
Newtons found roots at
  -0.142414300651696

   2.056627863037191
```

```
in
     4

     4

iterations.
Secant found roots at
  -0.142414300565768

   2.056627867651300

in
     5

     5

iterations.
in both cases, the functions found the roots in the same number of iterations
```

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