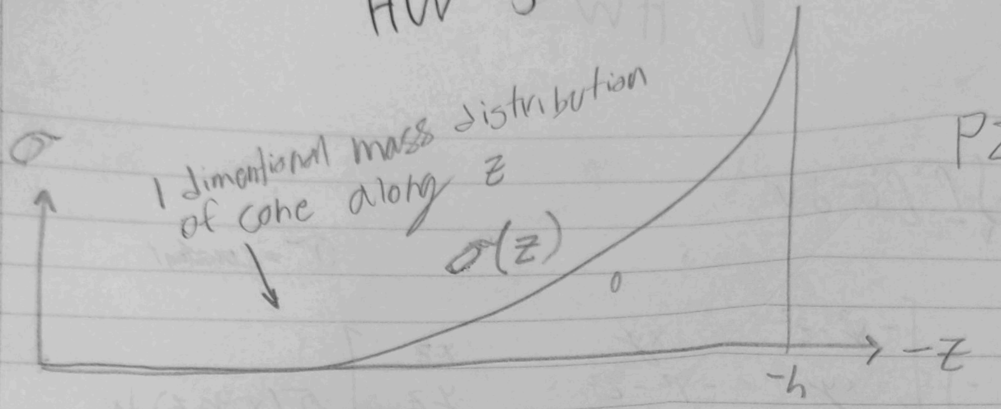
HW 5

P2, Center of mass



$\sigma(z) = \beta A$ $\qquad \beta$ = constant area density $\left(\frac{kg}{m^2}\right)$

$A = \pi R^2$

$R = -R_0 \frac{z}{h}$

$\sigma(z) = \beta \pi \left(R_0 \frac{z}{h}\right)^2$

center mass

$$\bar{z} = \frac{\int_0^{-h} z\,\sigma(z)\,dz}{\int_0^{-h} \sigma(z)\,dz}$$

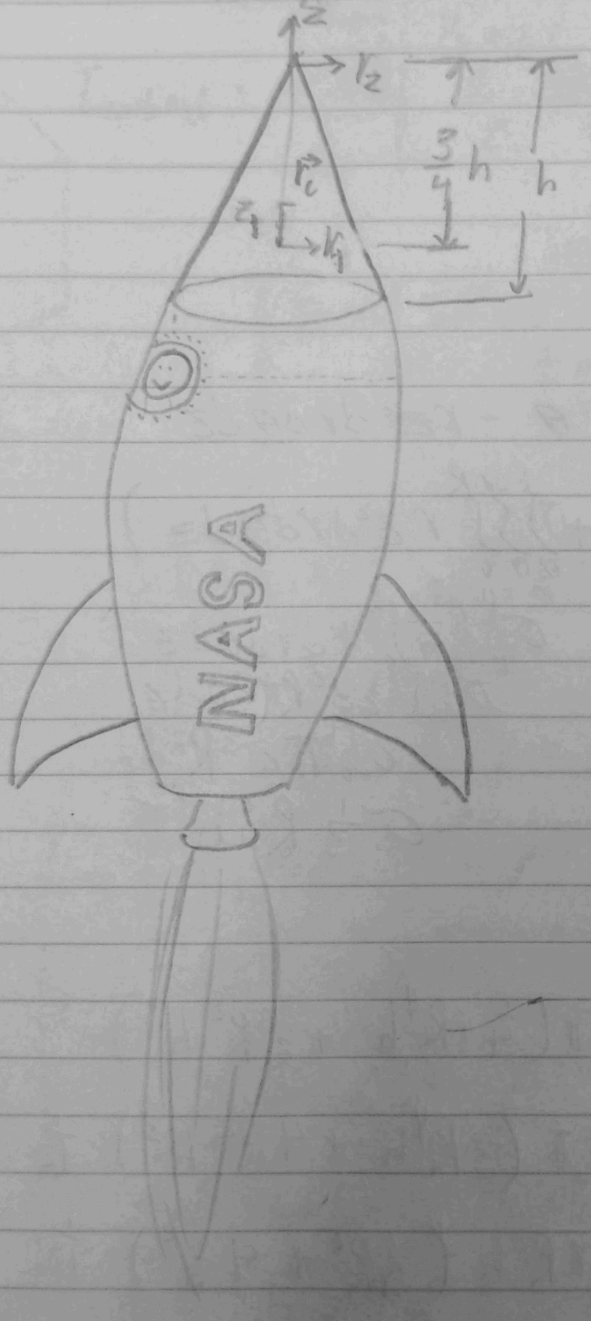
$$\frac{\int_0^{-h} z\beta\pi\left(R_0 \frac{z}{h}\right)^2 dz}{\int_0^{-h} \beta\pi\left(R_0 \frac{z}{h}\right)^2 dz} = \frac{\cancel{\beta\pi \frac{R_0^2}{h^2}} \int_0^{-h} z^3\,dz}{\cancel{\beta\pi \frac{R_0^2}{h^2}} \int_0^{-h} z^2\,dz}$$

$$\frac{\frac{(-h)^4}{4}}{\frac{(-h)^3}{3}} = \boxed{-h\frac{3}{4}}$$

HW5

3

P2, diagram



HW5

P2, J_{11} entry

$$(\rho^\times \rho^\times)_{11} = -r^2 \sin^2(\theta) - z^2$$

$$J = -\int_{-h}^{0}\int_0^{2\pi}\int_0^R \sigma \rho^\times \rho^\times r \, dr \, d\theta \, dz$$

$$J_{11} = \sigma \int_0^h \int_0^{2\pi}\int_0^R (\rho^\times \rho^\times)_{11} r \, dr \, d\theta \, dz$$

$$= \sigma \int_0^h \int_0^{2\pi} \int_0^R -r^3 \sin^2\theta - r z^2 \, dr \, d\theta \, dz$$

$$= -\sigma \left(\int_0^h \int_0^{2\pi}\int_0^R r^3 \sin^2\theta \, dr \, d\theta \, dz + \sigma \int_0^h\int_0^{2\pi}\int_0^R r z^2 \, dr \, d\theta \, dz \right)$$

$$\sigma \int_0^h \int_0^{2\pi} \sin^2\theta \int_0^R r^3 \, dr \, d\theta \, dz \qquad \sigma \int_0^h \int_0^{2\pi} z^2 \int_0^R r \, dr \, d\theta \, dz$$

$$\sigma \int_0^h \int_0^{2\pi} \tfrac{1}{2} z^2 R^2 \, d\theta \, dz$$

$$-\sigma \int_0^h \frac{1}{4} \int_0^{2\pi} \sin^2\theta R^4 \, d\theta \, dz \qquad \sigma \int_0^h \pi z^2 R^2 \, dz$$

$$-\sigma \frac{1}{3} R^2 h^3 \pi$$

$$-\sigma \int_0^h \frac{1}{4}\pi R^4 \, dz$$

$$-\sigma \frac{1}{4} \pi R^4 h$$

$$+\sigma\pi \left(\frac{1}{4} R^4 h + \frac{1}{3} R^2 h^3 \right)$$

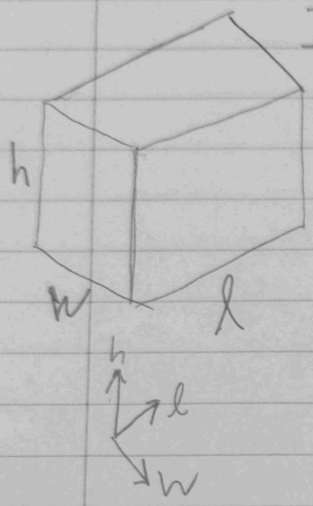
$$= +\sigma \pi (3R^4 h + 4R^2 h^3) \frac{1}{12}$$

$$+\sigma \pi R^2 h (3R^2 + 4h^2) \frac{1}{12}$$

$$= m(3R^2 + 4h^2)\frac{1}{12}$$

P3

from ch8



$$I_{cuboid} = \frac{1}{12} m \begin{bmatrix} h^2+w^2 & 0 & 0 \\ 0 & l^2+w^2 & 0 \\ 0 & 0 & h^2+l^2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{12}m(3r^2+H^2) & 0 & 0 \\ 0 & \frac{1}{12}m(3r^2+H^2) & 0 \\ 0 & 0 & \frac{1}{2}mr^2 \end{bmatrix}$$

$$h^2+w^2 = 3r^2+H^2 = l^2+w^2$$

$$l^2 = h^2, \quad \boxed{l = h}$$

$$\frac{1}{12}m(h^2+h^2) = \frac{1}{2}mr^2$$

$$\frac{1}{6}\,2(h^2) = r^2$$

$$\frac{1}{3}h^2 = r^2$$

$$\boxed{h = l = \sqrt{3r^2}}$$

$$3r^2 + w^2 = 3r^2 + H^2$$

$$\boxed{w = H}$$