APPENDIX

COMPUTING THE DIFFERENCE BETWEEN NEARLY EQUAL NUMBERS

F

Let **a**, **b**, and **c** be vectors such that **c** = **b** − **a** and $a \ll b$. Clearly, $c \approx b$. To calculate

$$F \equiv 1 - c^3/b^3 \tag{F.1}$$

we may first define

$$q \equiv 1 - c^2/b^2 \tag{F.2}$$

It follows that

$$F = 1 - \left(c^2/b^2\right)^{3/2} = 1 - (1-q)^{3/2} = \left[1 - (1-q)^{3/2}\right]\frac{1 + (1-q)^{3/2}}{1 + (1-q)^{3/2}} = \frac{1 - (1-q)^3}{1 + \left(\sqrt{1-q}\right)^3}$$

or

$$F(q) = \frac{q^2 - 3q + 3}{1 + (1-q)^{3/2}}q \tag{F.3}$$

Using this formula to compute F does not require finding the difference between nearly equal numbers, as in Eq. (F.1). However, that problem persists when using Eq. (F.2) to calculate q. We can work around that issue by observing that

$$q = \frac{b^2 - c^2}{b^2} = \frac{(\mathbf{b} - \mathbf{c}) \cdot (\mathbf{b} + \mathbf{c})}{b^2}$$

or, since **c** = **b** − **a**,

$$q = \frac{\mathbf{a} \cdot (2\mathbf{b} - \mathbf{a})}{b^2} \tag{F.4}$$

Computing q by means of this formula and substituting the result into Eq. (F.3) avoids roundoff error that may occur by calculating F using Eq. (F.1) when $c/b \approx 1$ (Battin, 1987).

REFERENCE

Battin, R.H., 1987. An Introduction to the Mathematics and Methods of Astrodynamics. AIAA Education Series, New York.