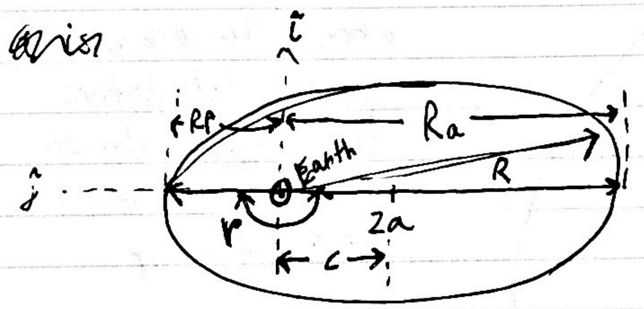
COEs Write up

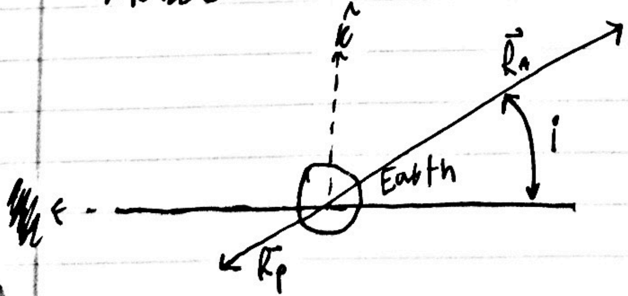
There are 6 Clasical orbital Elements for any given orbit:

a = semi major axis
e = eccentricity
i = inclination
raan = Ω = right accention of accending node
ν = true anomoly = nu
ω = argument of perigee

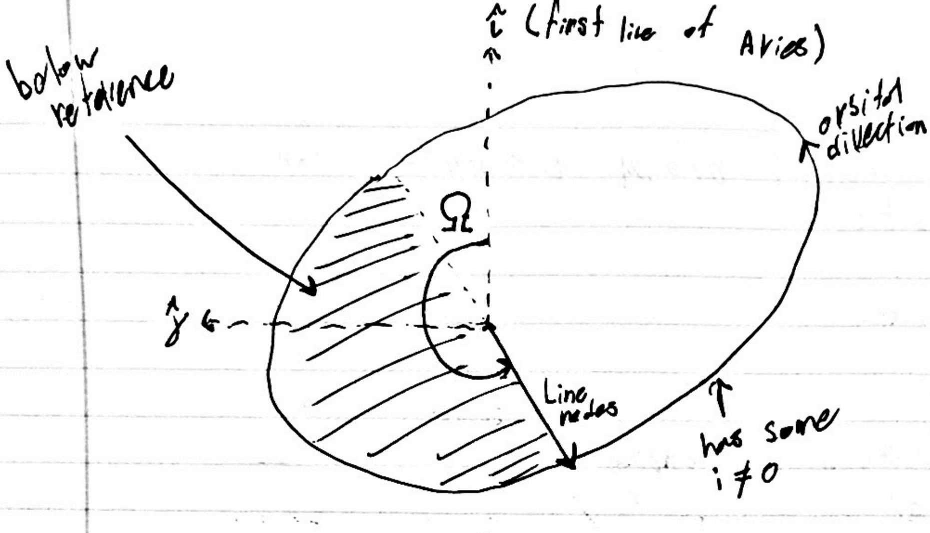


$$e = \frac{2c}{2a} = \frac{R_a - R_p}{R_a + R_p}$$

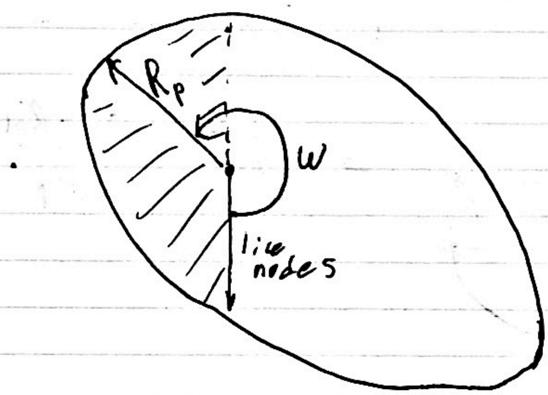
the above shows how a and e define the shape of the orbit and how ν defines position within it. a is half the distance between apoapsis and perapsis $(R_a+R_p)/2$. c is the distance between the foci and center. e defines the ratio between a & c. This is a # that makes the orbit more or less circular



i is the angle between the reference, equitorial, plane and orbital plane, seen here from "side"



here seen from above, Ω is the angle between $\hat{i}$ and the line of nodes. the line of nodes is the intersection between the reference and orbital planes.



here seen from above the orbital plane, ω defines the angle between the line of nodes and ~~the~~ R_p

any given $\vec{V}$ & $\vec{R}$ will generate unique COEs for a specific body.

Solve COEs for given $\vec{V}$ & $\vec{R}$

$\vec{R} = [-2315.9, 2168.6, 6314.5]$ km, $R = 7066.8$ km

$\vec{V} = [-3.0599, 6.0645, -3.2044]$ km/s, $V = 7.5106$ km/s

a) $\mathcal{E} = \frac{V^2}{2} - \frac{\mu}{R}$, $\mu = 398600.0 \frac{km^3}{s^2}$ from google

$= -28.2 \frac{km^2}{s^2} = -\frac{\mu}{2a} \Rightarrow a = -\frac{\mu}{2\mathcal{E}} \Rightarrow \underline{a = 7067.3 \text{ km}}$

e) $\vec{e}$ is an important vector for later calculations e is the COE

$\vec{e} = \frac{1}{\mu}\left[\left(V^2 - \frac{\mu}{R}\right)\vec{R} - (\vec{R} \cdot \vec{V})\vec{V}\right]$ Plug into vector calculator to find that

$\vec{e} = [3.128E{-6}, -3.273E{-5}, 9.905E{-5}]$ $\vec{e}$ is unitless

$\underline{e = |\vec{e}| = 1.044E{-4}}$

i) i is solved using $\vec{h} = \vec{R} \times \vec{V}$ = angular momentum

Plug into cross product calculator and find $\vec{h} = [-45243.0, -26743.0, -74091$

use $i = \arccos\left[\frac{\hat{k} \cdot \vec{h}}{h}\right]$ in vector calculator to find

$\underline{i = 98°}$

raan) raan similarly needs line of nodes $\vec{n} = \hat{k} \times \vec{h} = [26743.0, -45243.0, 0]$

raan given by $raan = \arccos\left[\frac{\hat{i} \cdot \vec{n}}{n}\right]$, adjust answer if it should be made "(+)",

$\underline{raan = 301°}$

ω) ω is solved similar to raan, $\omega = \arccos\left[\frac{\vec{n} \cdot \vec{e}}{n(e)}\right]$

$$\underline{\omega = 73.4^\circ}$$

ν) true anomoly is solved similarly, $\nu = \arccos\left[\frac{\vec{e} \cdot \vec{R}}{e(R)}\right]$

this needs to be adjusted if flight path is negative to $\nu = 360 - \arccos\left[\frac{\vec{e} \cdot \vec{R}}{e(R)}\right]$

$\underline{\nu = 42.1^\circ}$

Write up

In this assignment I saw how COE are used and calculated. I created a solid conceptual understanding and then learned the equations associated with these concepts before coding and executing a verision in MatLab.