Homework 5

Problem 1 1) $f(x) = x^2$

find) $f'(.5)$

$h = .1$ $f'(x) = \frac{f(x+h) - f(x-h)}{2h} = \frac{.6^2 - .4^2}{.2} = 1$

$h = .01$ $f'(x) = \frac{.51^2 - .49^2}{.02} = 1$

2) find) $f'(x)$

a) $f'(x) = \frac{f(x+h) - f(x)}{h} + O(h)$

b) $f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2)$

c) $f(x) = \left[\sum_{n=0}^{4} \frac{1}{n!} f^{(n)}(x) (x - x_0)^n\right] + O(h^5)$

$f(x-2h) = f(x) - 2hf'(x) + 2h^2 f''(x) - \frac{4}{3}h^3 f'''(x) + \frac{2}{3}h^4 f''''(x) + O(h^5)$

$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(x) + \frac{h^4}{24} f''''(x) + O(h^5)$

$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(x) + \frac{h^4}{24} f''''(x) + O(h^5)$

$f(x+2h) = f(x) + 2hf'(x) + 2h^2 f''(x) + \frac{4}{3}h^3 f'''(x) + \frac{2}{3}h^4 f''''(x) + O(h^5)$

$\Rightarrow$ ① $f(x+h) - f(x-h) = \frac{h^3}{3} f'''(x) + 2hf'(x) + O(h^5)$

② $f(x+2h) - f(x-2h) = \frac{8}{3}h^3 f'''(x) + 4hf'(x) + O(h^5)$

② $- 2$ ① $= f(x+2h) - f(x-2h) - 2[f(x+h) - f(x-h)] = \frac{8}{3}h^3 f'''(x) - \frac{2}{3}h^3 f'''(x) + O(h^5)$

$= \frac{8}{3} f'''(x) - \frac{2}{3} f'''(x) + O(h^3)$

$\Rightarrow f'''(x) = \frac{f(x+2h) - f(x-2h) - 2[f(x+h) - f(x-h)]}{2h^3} + O(h^3)$

Problem 2

$$\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx \quad , \qquad \begin{aligned} u &= x^2+9 \\ du &= 2x\,dx \\ \tfrac{1}{2}\,du &= x\,dx \end{aligned}$$

$$= \int_9^{25} \frac{du}{2\sqrt{u}}$$

$$= \frac{1}{2}\int_9^{25} u^{-.5}\,du = \frac{1}{2}\left[\frac{1}{.5}\,u^{.5}\right]\Bigg|_9^{25} = u^{.5}\Bigg|_9^{25}$$

$$= \sqrt{25} - \sqrt{9} = 5 - 3 = 2$$

$$\int uv\,dx = u\int v\,dx - \int u'\left(\int v\,dx\right)dx$$

$$\int_0^{2\pi} \cos(x)e^x\,dx \quad , \quad v = e^x,\ u = \cos(x)$$

(1) $\int \cos(x)e^x\,dx = \cos(x)\int e^x dx - \int -\sin(x)\left(\int e^x dx\right)dx = \cos(x)e^x + \int \sin(x)e^x\,dx$

$W = e^x,\ \omega = \sin x$

(1) $= \cos(x)e^x + \sin(x)e^x - \int \cos(x)\left(\int e^x dx\right)dx = \int \cos(x)e^x dx$

(2) $2\int \cos(x)e^x\,dx = \cos(x)e^x + \sin(x)e^x + C \Rightarrow (1) = \frac{\cos(x)e^x + \sin(x)e^x}{2} + C$

$$\int_0^{2\pi} \cos(x)e^x\,dx = \frac{\cos(2\pi)e^{2\pi} - \sin(2\pi)e^{2\pi}}{2} - \frac{1}{2} = \frac{e^{2\pi}}{2} - \frac{1}{2} = 267.246$$

b) $f(x) = \cos(x)e^x$, use trap

$f(0) = 1$

$f(\pi) = -e^\pi = -23.1407$

$f(2\pi) = e^{2\pi} = 535.4917$

$$\int_0^{2\pi} f(x) \approx \pi\left(\frac{f(0)+f(\pi)}{2}\right) + \pi\left(\frac{f(\pi)+f(2\pi)}{2}\right) = \frac{\pi}{2}\left(f(0) + 2f(\pi) + f(2\pi)\right)$$

$$= \frac{\pi}{2}\left(1 - 2e^\pi + e^{2\pi}\right) = 770.02$$

$f(x) = \frac{x}{\sqrt{x^2+9}}$, use trap

$f(0) = 0$

$f(2) = \frac{2}{\sqrt{13}} = .5547$

$f(4) = \frac{4}{\sqrt{25}} = .8$

$$\int_0^4 f(x) \approx \left(f(0) + 2f(2) + f(4)\right) = \left(0 + 2(.5547) + .8\right) = 1.9094$$

c) $f(x) = \cos(x)e^x$, use simp

$f(0) = 1$ $\quad f(\frac{\pi}{2}) = 0$

$f(\pi) = -23.1407$ $\quad f(\frac{3\pi}{2}) = 0$ $\qquad h = \frac{2\pi - 0}{4} = \frac{\pi}{2}$

$f(2\pi) = 535.4917$

$$\int_0^{2\pi} f(x) \approx \frac{h}{3}\left(f(0) + f(2\pi) + 4\left(f(0) + f(\pi)\right) + 2\left(f(\tfrac{\pi}{2}) + f(\tfrac{3\pi}{2})\right)\right)$$

$$= \frac{1}{3}\left(1 + 535.4917 + 2(1 - 23.1407)\right) = 257$$

$f(x) = \frac{x}{\sqrt{x^2+9}}$, $f(0) = 0$ $\quad f(1) = .3162$ $\quad h = 1$

$f(2) = .5547$ $\quad f(3) = .7071$

$f(4) = .8$

$$\int_0^4 f(x) \approx \frac{h}{3}\left(0 + .8 + 4(0 + .5547) + 2(.7071 + .8)\right) = 2.00$$

Table of Contents

section 0 - clean up

```
clear all;
close all;
clc

myFig = figure;
myFig.Visible = "off";
subplot(2,1,1)
hold on
title("function 1 error")
xlabel('n')
ylabel('error')
a = gca;
a.YScale = 'log';

subplot(2,1,2)
hold on
title("function 2 error")
xlabel('n')
ylabel('error')
a = gca;
a.YScale = 'log';
```

section 1 - use composite trap for f1

```
%clear all;

f1 = @(x) x/((x^2+9)^.5); % this is the given f1
F1 = @(x1,x2) (x2^2+9)^.5 - (x1^2+9)^.5; % this is the true definite integral
 of f1

a = 0; % set bounds
b = 4;
realSol = F1(a,b); % find real solution

n = 2; % use method to aproximate integral at different n values
I02 = CompositeTrapezoid(f1,a,b,n);
n = 4;
I04 = CompositeTrapezoid(f1,a,b,n);
```

```
n = 8;
I08 = CompositeTrapezoid(f1,a,b,n);
n = 16;
I16 = CompositeTrapezoid(f1,a,b,n);
n = 32;
I32 = CompositeTrapezoid(f1,a,b,n);

e(1) = I02 - realSol; % find the errors for each approximation
e(2) = I04 - realSol;
e(3) = I08 - realSol;
e(4) = I16 - realSol;
e(5) = I32 - realSol;

e = abs(e);
subplot(2,1,1)
plot([2,4,8,16,32],e)
```

section 2 - use composite trap for f2

```
%clear all;

f2 = @(x) cos(x)*exp(x);% this is the given f2
F2 = @(x1,x2) (exp(x2)/2)*(sin(x2)+cos(x2)) - (exp(x1)/2)*(sin(x1)+cos(x1));%
 this is the true definite integral of f2

a = 0; % set bounds
b = 2*pi;
realSol = F2(a,b); % find real solution

n = 2; % use method to aproximate integral at different n values
I02 = CompositeTrapezoid(f2,a,b,n);
n = 4;
I04 = CompositeTrapezoid(f2,a,b,n);
n = 8;
I08 = CompositeTrapezoid(f2,a,b,n);
n = 16;
I16 = CompositeTrapezoid(f2,a,b,n);
n = 32;
I32 = CompositeTrapezoid(f2,a,b,n);

e(1) = I02 - realSol; % find the errors for each approximation
e(2) = I04 - realSol;
e(3) = I08 - realSol;
e(4) = I16 - realSol;
e(5) = I32 - realSol;

e = abs(e);
subplot(2,1,2)
plot([2,4,8,16,32],e)
```

section 3 - use composite simpsons for f1

```
%clear all;
```

```
f1 = @(x) x/((x^2+9)^.5);% this is the given f1
F1 = @(x1,x2) (x2^2+9)^.5 - (x1^2+9)^.5;% this is the true definite integral
 of f1

a = 0; % set bounds
b = 4;
realSol = F1(a,b); % find real solution

n = 2; % use method to aproximate integral at different n values
I02 = CompositeSimpson(f1,a,b,n);
n = 4;
I04 = CompositeSimpson(f1,a,b,n);
n = 8;
I08 = CompositeSimpson(f1,a,b,n);
n = 16;
I16 = CompositeSimpson(f1,a,b,n);
n = 32;
I32 = CompositeSimpson(f1,a,b,n);

e(1) = I02 - realSol; % find the errors for each approximation
e(2) = I04 - realSol;
e(3) = I08 - realSol;
e(4) = I16 - realSol;
e(5) = I32 - realSol;

e = abs(e);
subplot(2,1,1)
plot([2,4,8,16,32],e)
```

section 4 - use composite simpsons for f2

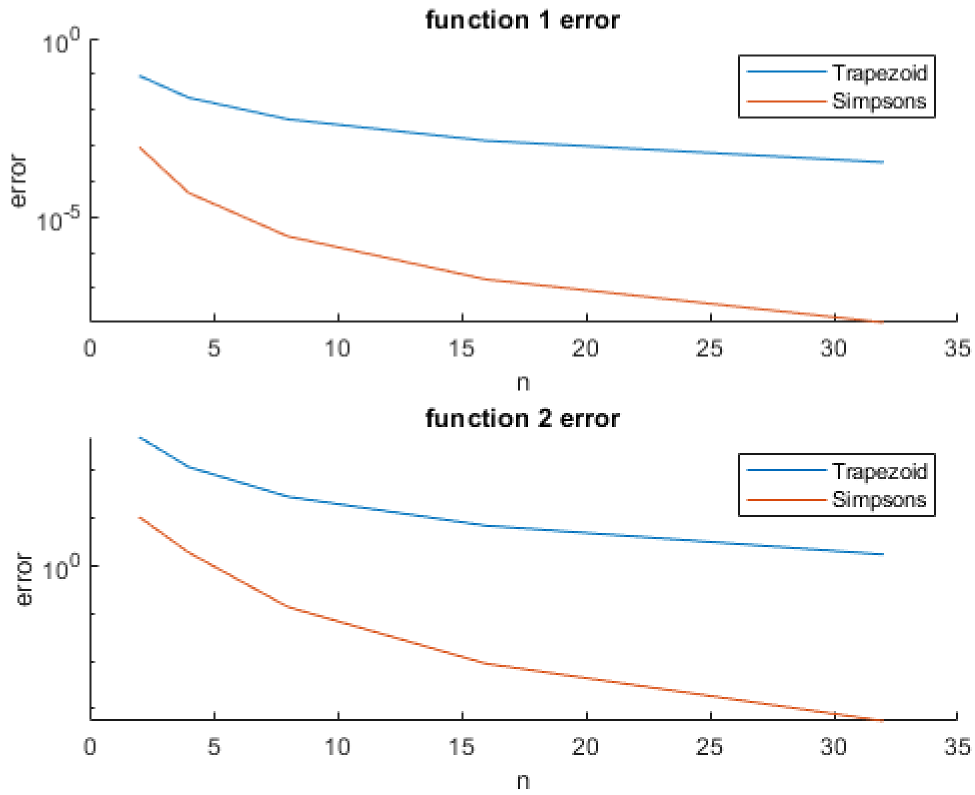
```
%clear all;

f2 = @(x) cos(x).*exp(x);% this is the given f2
F2 = @(x1,x2) (exp(x2)/2)*(sin(x2)+cos(x2)) - (exp(x1)/2)*(sin(x1)+cos(x1));%
 this is the true definite integral of f2

a = 0; % set bounds
b = 2*pi;
realSol = F2(a,b); % find real solution

n = 2; % use method to aproximate integral at different n values
I02 = CompositeSimpson(f2,a,b,n);
n = 4;
I04 = CompositeSimpson(f2,a,b,n);
n = 8;
I08 = CompositeSimpson(f2,a,b,n);
n = 16;
I16 = CompositeSimpson(f2,a,b,n);
n = 32;
I32 = CompositeSimpson(f2,a,b,n);
```

```
e(1) = I02 - realSol; % find the errors for each approximation
e(2) = I04 - realSol;
e(3) = I08 - realSol;
e(4) = I16 - realSol;
e(5) = I32 - realSol;

e = abs(e);
subplot(2,1,2)
plot([2,4,8,16,32],e)
```

section 5 - plot finishing

```
myFig.Visible = "on";
subplot(2,1,1)
legend('Trapezoid','Simpsons')
subplot(2,1,2)
legend('Trapezoid','Simpsons')

disp("f1 is f = x/((x^2+9)^.5)")
disp("f2 is f = cos(x)*exp(x)")
```

f1 is f = x/((x^2+9)^.5)
*f2 is f = cos(x)*exp(x)*



section 6 - discuss differences (Problem 2 f)

```
disp("In both cases (f1 and f2) Simpsons method converges more quickly than
 trapezoid, this makes sense since Simpsons method is known to be of higher
 order than Trapezoid.")
```

In both cases (f1 and f2) Simpsons method converges more quickly than trapezoid, this makes sense since Simpsons method is known to be of higher order than Trapezoid.

section 7 - function definition

```
function [I] = CompositeTrapezoid(f, a, b, n)
%Approximates the integral of a function using the Composite Trapezoid Rule
%   f: function to be integrated
%   a: lower bound of interval
%   b: upper bound of interval
%   n: number of panels used for the approximation
h = (b-a)/n; %width of a panel
x = linspace(a,b,n+1); %Create n+1 equally spaced points for the n panels
I = f(a) + f(b);
for i=1:n-1
        I = I + 2*f(x(i+1));
end
I = I*h/2;
end

function [I] = CompositeSimpson(f, a, b, n)
%Approximates the integral of a function using the Composite Simpson's Rule
%   f: function to be integrated
%   a: lower bound of interval
%   b: upper bound of interval
%   n: number of panels used for the approximation
h = (b-a)/(2*n); %width of a subinterval
x = linspace(a,b,2*n+1); %Create 2n+1 equally spaced points for the n panels
I = f(a) + f(b);
for i=1:n-1
        I = I + 2*f(x(2*i+1)) + 4*f(x(2*i));
end
I = (I + 4*f(x(2*n)))*h/3;
end
```

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