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0

----- HW1 - Josh Oates -----

2.23 find gamma(theta) and z(theta)

2.36 excess V

2.37 meteroid

-----P2.37-----

My calculations have the following results:

Eccentricity: 1.086

H/C: For hyperbolic ecc should be above 1

Altitude at periapse: 5087.5854 km

H/C: Altitude at closest approach is lower than initial velocity but high enough to orbit

Velocity at closest approach: 8.5158 km/s

H/C: v at closest approach is higher than v initial

3.8 time above 400

3.10

3.20

4.5

-----P4.5-----My calculations have the following results:
Semimajor axis: 9081.4773 km
Eccentricity: 0.22261

True anomaly: 134.7259 degrees Inclination: 32.445 degrees

Right ascention of ascending node: 107.5713 degrees

Argument of periapse: 72.3586 degrees

h: 58655.7755m^2/s

4.7

-----P4.7-----

My calculations have the following results: Inclination of this orbit: 43.2661 degrees H/C: r vector has a relatively large z component so this makes sense nominally

dependencies

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```
% Joshua Oates - AERO351 - HW2
% Feel free to use MATLAB (or similar product) when appropriate but please
submit a pdf/html
% of the published code.
% Chapter 2 problems: 2.23, 2.36, 2.37
% Chapter 3 problems: 3.8, 3.10, and 3.20 (check this answer with using ODE45
as well, do they
% agree?)
% Chapter 4 problems: 4.5, 4.7
0
clear all;
close all;
clc
addpath('C:\joshFunctionsMatlab\')
disp("-----")
```

2.23 find gamma(theta) and z(theta)

```
mu_e = 398600;
r_e = 6378; % km
zp = 500; % km
rp=zp+r_e; % km
v = 10; % km/s
theta = 120; % degrees
theta = deg2rad(theta); % rad
```

2.36 excess V

```
clear all;
mu = 398600;
r e = 6378; % km
zp = 250; % km
rp=zp+r e; % km
v = 11; % km/s
theta = 100; %degrees
theta = deg2rad(theta); % rad
[a,ecc,~,~,~,h,T,E]=joshCOE(rp,v,mu e);
v inf = sqrt(mu e/-a);
r100 = (h^2/mu e)/(1+ecc*cos(theta));
z100 = r100-r e;
[vaz, vr, gamma] = joshVazVr(theta, ecc, h, mu e);
disp("----")
disp("My calculations have the following results:")
disp("Excess escape velocity: "+string(v inf)+" km/s")
disp("H/C: Hyperbolic tragector should have positive excess escape velocity")
disp("r at theta = 100 degrees: "+string(r100)+" km")
disp("H/C: z > 250")
disp("Azmuthal velocity at theta = 100 degrees: "+string(vaz)+" km/s")
disp("H/C: Azmuthal velocity should be positive")
disp("Radial velocity at theta = 100 degrees: "+string(vr)+" km/s")
disp("H/C: At theta = 100 object is on departure so radial velocity should be
positive")
```

2.37 meteroid

```
clear all;
mu_e = 398600;
r_e = 6378; % km
r0=402000;%km
theta=150;%deg
theta=deg2rad(theta);
v0 = 2.23;%km/s
```

```
E=(v0^2/2) - (mu e/r0); % Energy
a = mu e/(2*E);
syms h % set up h for symbolic solving
assume(h, 'real')
assumeAlso(h>0)
eqn = sym(r0== (h^2/mu e)*(1+(2*E*(h^2/mu e^2)+1)^.5*cos(theta))^-1); %
symbolic equation to solve for h
h = solve(eqn,h);
h = double(h);
ecc = (1/(r0*(mu e/h^2))-1)/cos(theta); % solve for ecc
rp = a*(ecc-1);
zp = rp-r e;
v = h/rp;
disp("----")
disp("My calculations have the following results:")
disp("Eccentricity: "+string(ecc))
disp("H/C: For hyperbolic ecc should be above 1")
disp("Altitude at periapse: "+string(zp)+" km")
disp("H/C: Altitude at closest approach is lower than initial velocity but
high enough to orbit")
disp("Velocity at closest approach: "+string(v)+" km/s")
disp("H/C: v at closest approach is higher than v initial")
% [a,ecc,~,~,~,h,T,E]=joshCOE(ri,vi,mu e);
```

3.8 time above 400

```
clear all;
mu = 398600;
r e = 6378; % km
ra = r e + 600;
rp = r e + 200;
a = (rp+ra)/2;
ecc = (ra-rp)/(rp+ra); % definition of ecc
h = sqrt(a*(1-ecc^2)*mu e); % solve for h
r = 400 + r e; % solve r
theta = acos((1/(r*(mu e/h^2))-1)/ecc); % solve theta
[Me, Ean] = joshAnomalyCalculator(ecc, theta); % convert TA to Me
P=((2*pi)/sqrt(mu e))*a^1.5;
n=sqrt(mu e/a^3); % definition n
tsp = Me/n;
t400k = P-2*tsp; % time above 400 km
t400k = t400k/60;
disp("----")
```

```
disp("My calculations have the following results:") disp("Time spent over 400 km: "+string(t400k)+" min") disp("H/C: time over 400 km is less than period") disp("Intermediate H/C used:") disp(" ecc < 1") disp(" period resonable for LEO")
```

3.10

```
clear all;
mu = 398600;
rp = 10000; % km
P = 14*60*60;
a = ((sqrt(mu e)/(2*pi))*P)^(2/3); % equation for a
ra = a*2-rp;
ecc = (ra-rp)/(rp+ra);
h = sqrt(a*(1-ecc^2)*mu e);
v0= h/rp; % this is because v0 is at periapse and is perpendicular to r at
periapse
r0 = rp;
pr = 0; % dot product of v0 and r0 as vectors, 0 b/c they are perpendicular
dt = 10*60*60;
coefs = 15; % 15 terms in stumpff functions
[Cc,Sc]=joshStumpffCoeffs(coefs); % generate stumpff coeffecients
C = Q(z) \quad sum(Cc.*joshStumpffZ(z,coefs)); % generate stumpff functions
S = @(z) sum(Sc.*joshStumpffZ(z,coefs));
[fX,fpX]=joshfChi(r0,v0,mu e,a,dt,Cc,Sc,pr); % generate f and fp which can be
put into a newton solver for X
X0 = sqrt(mu e)*dt*abs(1/a); % initial quess
[X] = joshNewtons(fX, fpX, X0, 1e-14); % newton solver for X
Ean = X/sqrt(a); % Eccentric anomaly
theta = 2*atan(tan(Ean/2)/sqrt((1-ecc)/(1+ecc)));
% [Me, Ean2] = joshAnomalyCalculator(ecc, theta)
% [~,Ean2]=joshQuadrant(Ean2)
r = (h^2/mu e)/(1+ecc*cos(theta)); % radial distance
[vaz,vr] =joshVazVr(theta,ecc,h,mu e); % azmuthal and radial velocity
speed = sqrt(vaz^2 + vr^2);
disp("----")
disp("My calculations have the following results:")
disp("radial position: "+string(r)+" km")
disp("H/C: r is between rp and ra")
disp("speed: "+string(speed)+" km/s")
disp("H/C: MEO orbit resonable velocity seems beleivable")
disp("radial velocity: "+ string(vr)+" km/s")
disp("H/C: since period is 14hrs and this is 10hrs in, the orbit is past
 apoapse, so radial velocity should be negative")
```

3.20

```
clear all;
```

```
mu = 398600;
dt = 2*60*60;
r0 = [20 -105 -19]*1000; % km
v0 = [.9 -3.4 -1.5]; % km/s
[a] = joshCOE(r0, v0, mu e);
%%%%%%%% same as above %%%%%%%%%
coefs = 15;
[Cc,Sc]=joshStumpffCoeffs(coefs);
C = Q(z) sum(Cc.*joshStumpffZ(z,coefs));
S = @(z) sum(Sc.*joshStumpffZ(z,coefs));
[fX, fpX]=joshfChi(r0, v0, mu e, a, dt, Cc, Sc);
X0 = sqrt(mu e)*dt*abs(1/a);
[X] = joshNewtons(fX, fpX, X0, 1e-14);
f = 1 - (X^2/norm(r0)) *C(X^2/a); % f and g functions are possible because v0 and
r0 are vectors
g = dt - ((1/sqrt(mu e))*X^3*S(X^2/a));
r = f*r0+g*v0; % position
f dot = (sqrt(mu e)/(norm(r)*norm(r0)))*X*((X^2/a)*S(X^2/a)-1);
g dot = 1 - (X^2/norm(r)) *C(X^2/a);
v = f_dot*r0 + g_dot*v0; % velocity
shouldBe1 = f*g_dot-f_dot*g; % H/C
disp("----")
disp("My calculations have the following results:")
disp("The final position vector in km:")
disp(r)
disp("The final velocity vector in km/s:")
disp("H/C: f*g dot-f dot*g value: "+string(shouldBe1))
4.5
clear all;
mu = 398600;
R = [6.5 - 7.5 - 2.5] * 1000;
V = [4 \ 3 \ -3];
[a,ecc,theta,inc,raan,aop,h,T,E] = joshCOE(R,V,mu e); % COEs
inc = rad2deg(inc);
raan = rad2deg(raan);
aop = rad2deg(aop);
theta = rad2deg(theta);
disp("-----")
disp("My calculations have the following results:")
disp("Semimajor axis: "+string(a)+" km")
disp("Eccentricity: "+string(ecc))
disp("True anomaly: "+string(theta)+" degrees")
disp("Inclination: "+string(inc)+" degrees")
disp("Right ascention of ascending node: "+ string(raan)+" degrees")
```

```
disp("Argument of periapse: "+string(aop)+" degrees")
disp("h: "+string(h)+"m^2/s")
4.7
clear all;
r = [-6.6 -1.3 -5.2] *1000;
ecc = [-.4 -.5 -.6];
mu = 398600;
theta = a\cos(\det(r,ecc)/(norm(ecc)*norm(r))); % solve for true anomaly
h = sqrt(mu e*norm(r)*(1+norm(ecc)*cos(theta))); % use TA and get h
h = h*(cross(r,ecc)/norm(cross(r,ecc)));
inc = acos((dot([0 0 1],h))/norm(h)); % inclination degrees
inc = rad2deg(inc);
disp("----")
disp("My calculations have the following results:")
disp("Inclination of this orbit: "+string(inc)+" degrees")
disp("H/C: r vector has a relatively large z component so this makes sense
nominally")
```

dependencies

```
function [vaz, vr, gamma] = joshVazVr(theta, ecc, h, mu)
\ensuremath{\$} gives magnitude of azmuthal velocity and radial velocity
% takes theta ecc and h
% optionally takes mu for the center body
% assumes shperical body and 2 body
% angles in rad
arguments
    theta (1,1) double {mustBeReal}
    ecc (1,1) double {mustBeReal, mustBeNonnegative}
    h (1,1) double {mustBeReal, mustBePositive}
    mu (1,1) double {mustBeReal} = 3.986004418 * (10^5) {km^3/s^2 mu_earth}
end
vr = (mu/h) *ecc*sin(theta);
vaz = (mu/h) * (1+ecc*cos(theta));
gamma = atan2(vr,vaz);
end
```

```
function [isOnes] = joshIsOnes(M)
% takes a value (persumably a logical type matrix) and returns true iff all
% entries are true
[m,n] = size(M);
isOnes = true;
for i = 1:m
    for j = 1:n
        if M(i,j) ~= 1
            isOnes = false;
        end
    end
end
end
```

```
function [a,ecc,theta,inc,raan,aop,h,T,E] = joshCOE(R,V,u,magOrVec)
888888888888888888888
% Revamped to do rads and fit new naming convention %
88888888888888888888
%COESOATESJOSHUA Takes postion and velocity vector and returns COES, all in
%ECI frame of reffrenece, km and seconds as units and degrees
% a = semi major axis
% ecc = eccentricity
% i = inclination
% raan = right accention acending node
% aop = argument of periapsis
% theta = true anomaly
% will return T in s as a period
% and E (sometimes epsilon) in km^2/s^2 as specific mechanical energy
% h is agular momentum
% magOrVec is a parameter that can be set for vector inputs to return
% vector h and ecc
% scalar entry should only be used if the spacecraft is at apoapse or
% periapse
arguments
    R {mustBeNumeric, mustBeReal}
    V {mustBeNumeric, mustBeReal}
    u (1,1) {mustBeNumeric, mustBeReal, mustBePositive} = 3.986004418 *
 (10^5) %km^3/s^2
    magOrVec {mustBeMember(magOrVec, {'magnitude', 'vector'})} = 'magnitude'
end
[m1,n1] = size(R);
[m2,n2] = size(V);
if joshIsOnes([m1 n1 m2 n2])
    magOrVec = 'magnitude';
    R = [0 \ 0 \ R];
    V = [V \ 0 \ 0];
    warning("joshCOE will assume that R and V are normal if the inputs are
 scalar ie: the craft is in a circular orbit or is at periapse or apoapse")
elseif (\simjoshIsOnes([m1 n1] == [m2 n2]))|\sim((n1==1&m1==3)|(n1==3&m1==1))
    throw (MException ("COEsOatesJoshua:invalidInput", "R and V must be either
1x3 vectors or scalars"))
end
% uearth = 3.986004418(8) x 10^14 m^3/s^2
ihat=[1,0,0];
khat = [0, 0, 1];
```

```
Rm = norm(R);
Vm = norm(V);
%calculate orbital constants
h = cross(R,V); %angular momentum vector
hm = norm(h);
E = ((Vm^2) / 2) - (u / Rm); %specfic mechanical energy
%calculate COEs
a = -u / (2 * E); %semi major axis in km
T = 2*pi*sqrt((a^3)/u); %period in s
e = (1/u) * (((Vm^2) - (u/Rm))) * R - (dot(R,V) * V)); %eccentricity vector
em = norm (e); %magnitude of e
inc = acos((dot(khat,h))/hm); %inclination
n = cross(khat,h); %node vector
nm = norm(n); %magnitude n
%raan
raan = acos(dot(ihat,n)/nm);
if n(2) < 0 %checks the vector relative to j to see if angle is positive or
negative
    raan = 360 - raan;
end
%aop
aop = acos(dot(n,e)/(nm*em));
if e(3) < 0
    aop = 360 - aop;
end
%theta
theta = acos(dot(e,R)/(em*Rm));
if(dot(R,V) < 0) % cehck flight path angle to see if it is postive or negative
    theta = 360 -theta;
end
if strcmp(magOrVec, 'magnitude') %for magnitude mode, vectors will not be
 returned
   h = hm;
    e = em;
    if joshIsOnes([m1 n1 m2 n2]) % for scalar inputs it is not possible to
 calculate these values
        theta = NaN;
        inc = NaN;
        raan = NaN;
        aop = NaN;
    end
end
ecc = e;
end
```



```
function [fX,fpX] = joshfChi(r0,v0,mu,a,dt,C,S,pr)
% note fX = sqrt(mu)*dt - integral(rdX)
% fpX = -r
% it is highly recommended that C and S are passed in
arguments
    r0 double {mustBeReal}
    v0 double {mustBeReal}
    mu (1,1) double {mustBeReal}
    a (1,1) double {mustBeReal}
    dt (1,1) double {mustBePositive}
    C (1,:) double {mustBeReal} = nan
    S (1,:) double {mustBeReal} = nan
    pr (1,1) double {mustBeReal} = nan
end
if isnan(C) | isnan(S)
    [C,S] = joshStumpffCoeffs();
end
[m1,n1] = size(r0);
[m2,n2] = size(v0);
if joshIsOnes([m1 n1 m2 n2])
    if isnan(pr)
        throw (MException ("joshfChi:invalidInput", "If r0 and v0 are given as
 vectors, pr should be the dot product of them"))
    end
elseif(\sim joshIsOnes([m1 n1] == [m2 n2]))|\sim ((n1==1&m1==3)|(n1==3&m1==1))
    throw (MException ("joshfChi:invalidInput", "r0 and v0 must be either 1x3
vectors or scalars"))
else
    pr = dot(r0, v0);
end
coefs = length(C);
if length(S)~=coefs
    throw (MException ("joshfChi:invalidInput", "S and C should be the same
length"))
end
sm = sqrt(mu);
r0 = norm(r0);
% v0 = norm(v0);
% fX = Q(X) (pr/sm) *X^2*C(z) + (1-a*r0) *X^3*S(z) + r0*X - sm*dt;
% fpX = @(X) (pr/sm)*X*(1-a*X^2*S(z)) + (1-a*r0)*X^2*C(z) + r0;
% (sum(C.*joshStumpffZ(X^2/a)))
```

```
% fX = @(X) (pr/sm)*X^2*(sum(C.*joshStumpffZ(X^2*a,coefs))) + (1-
a*r0)*X^3*(sum(S.*joshStumpffZ(X^2*a,coefs))) + r0*X - sm*dt;
% fpX = @(X) (pr/sm)*X*(1-a*X^2*(sum(S.*joshStumpffZ(X^2*a,coefs)))) + (1-
a*r0)*X^2*(sum(C.*joshStumpffZ(X^2*a,coefs))) + r0;

a = 1/a;
fX = @(X) (pr/sm)*X^2*sum(C.*joshStumpffZ(X^2*a,coefs)) + (1-
a*r0)*X^3*sum(S.*joshStumpffZ(X^2*a,coefs)) + r0*X - sm*dt;
fpX = @(X) (pr/sm)*X*(1-a*X^2*sum(S.*joshStumpffZ(X^2*a,coefs)))+(1-
a*r0)*X^2*sum(C.*joshStumpffZ(X^2*a,coefs))+r0;
% X^2*(sum(S.*joshStumpffZ(((X^2)/a)))) = X^2*C(z)
% end
```

```
function [M,E] = joshAnomalyCalculator(ecc,theta)
% M will be Me, Mp or Mh depending on ecc
% E will be Eccentric Anomoly when applicable or F: hyperbolic Ecctric
% anomoly. E will be set to
% values in Rads
arguments
    ecc (1,1) double {mustBeReal}
    theta (1,1) double {mustBeReal}
end
if ecc <1 % Me & E
    E = 2*atan(sqrt((1-ecc)/(1+ecc))*tan(theta/2)); % definintion of E,
rewriten to solve {\tt E}
    M = E - ecc * sin(E); % definition of M
elseif ecc > 1 % Mh & F
    E = log((sqrt(ecc+1) + sqrt(ecc-1) *tan(theta/2)) / (sqrt(ecc+1) -
sqrt(ecc-1)*tan(theta/2)));
    M = ecc(sinh(F) - F);
else % ecc == 1 Mp
    E = nan; % This is a rare case and E doesnt have a definition for ecc == 1
    M = .5*tan(theta/2) + (1/6)*tan(theta/2)^3;
end
end
```

```
function [r, count, xVector, errorVector, errorRatioVector] = joshNewtons(f,
fp, x0, TOL, maxI)
% Aero300 code updated 10/12
% see A300 HW3
% this function will check if newtons method will converge
% this function takes a diff'able function and its derivative as well as a
% guess and a tolerance.
% this function is not gaurranted to converge and will exit early if it
% does not returning an empty array for r
% this algorithim taken from canvas has been cleaned up for readablity and
% commented for understanding
arguments
    f
    fp
    x0 (1,1) {mustBeNumeric, mustBeReal}
    TOL (1,1) {mustBeNumeric,mustBeReal} = .0001
   maxI (1,1) {mustBeNumeric,mustBeReal,mustBePositive,mustBeInteger} = 200
end
if ~(isa(f, 'function handle')&isa(fp, 'function handle'))
    throw (MException ("joshNewtons:invalidInput": "f and fp must be function
handles"))
end
x1 = x0 - f(x0)/fp(x0); % create x1 for use in algorithim
count = 1;
xVector = [x0; x1];
error = abs(x1 - x0);
errorVector = error;
errorRatioVector = [];
while (error > TOL) && (count <= maxI)</pre>
    % set x0 to x1 and recalculate x1 using function and functionPrime
    x0 = x1;
    x1 = x1 - f(x0)/fp(x0); % newtons algorithim
   count = count + 1;
   % for the sake of the error ratio compute errorSquared
    errorSquared = error^2; % this is the previous error since it hasnt been
 updated
    error = abs(x1 - x0); % update error
    errorRatio = error/errorSquared; % update errorRatio
    xVector = [xVector; x1]; % append new data to old
    errorVector = [errorVector; error];
    errorRatioVector = [errorRatioVector; errorRatio];
end
```

```
if (count > maxI)
    r = NaN; % r DNE for these inputs
    warn = "Did not converge in " + num2str(maxI) + " intervals";
    warning(warn)
elseif isnan(error)
    r = NaN; % r DNE for these inputs
    warn = "Value exploded in "+ num2str(maxI) + " intervals";
    warning(warn)
else
    r = xVector(end); % last guess is also best prediction for the rootend
```

```
function [C,S] = joshStumpffCoeffs(n)
% AERO 351 code
\mbox{\%} Generates the first n terms of the stumpff coeffcients for \mbox{@S(z)} and \mbox{@C(z)}
in a vector
% these coeffs are used for the universal variable approach to orbital
mechanics
% for use as companion function with joshStrumpffZ
% coeffs should be saved to workspace and reused to save compute time
% @S(z) == sum(S.*Z) == polyval(flip(S),z) : where Z = [z^0 z^1 ... z^n]
% C(z) == sum(C.*z) == polyval(flip(C),z) : where Z = [z^0 z^1 ... z^n]
arguments
    n (1,1) {mustBePositive, mustBeInteger} = 15;
end
C = zeros(1,n);
S = C;
for i = 1:n
    k = i-1;
    C(i) = (-1)^k*(1/factorial(2*k+2));
    S(i) = (-1)^k* (1/factorial(2*k+3));
end
end
```

```
function [Z] = joshStumpffZ(z,n)
% AERO 351 code
\mbox{\%} Generates the first n terms of the stumpff coeffcients for \mbox{@S(z)} and \mbox{@C(z)}
in a vector
% for use as companion function with joshStrumpffCoeffs
% @S(z) == sum(S.*Z) == polyval(flip(S),z) : where S given by <math>(-1)^k*(1/2)
factorial(2*k+2))
% C(z) == sum(C.*z) == polyval(flip(C),z): where C given by (-1)^k*(1/z)
factorial(2*k+3))
arguments
    z(1,1)
    n (1,1) {mustBePositive, mustBeInteger} = 15;
end
Z = ones(1,n);
for i = 2:n
    Z(i) = z*Z(i-1);
end
end
```