AERO 331: AEROSPACE STRUCTURAL ANALYSIS I

Winter 2023

Cal Poly San Luis Obispo

Exam 3

Problem 1 (100 points)

Consider a heterogeneous beam with a thin-walled closed section subjected to applied loads as shown in Figure 1. The outer skin has uniform thickness and the *Young's modulus* and *Poisson's ratio* for each portion is as given below:

$$\text{(Outer) skin:} \quad E = 30 \times 10^6 \text{ psi}, \qquad \nu = 0.32$$

$$\text{(Middle) web:} \quad E = 10 \times 10^6 \text{ psi}, \qquad \nu = 0.33\,.$$

The beam is cantilevered at $x = 0$ and has length $L = 100$ in. Cross sectional dimensions are measured with respect to the median line.

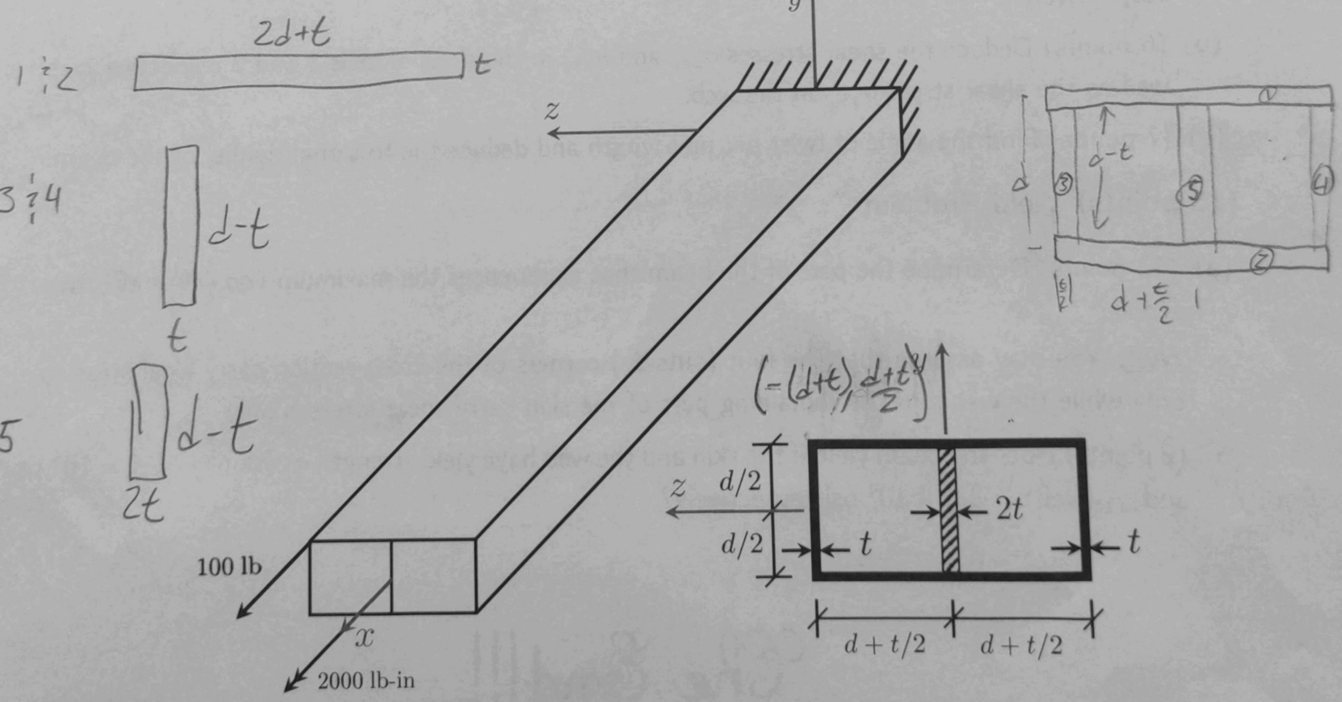
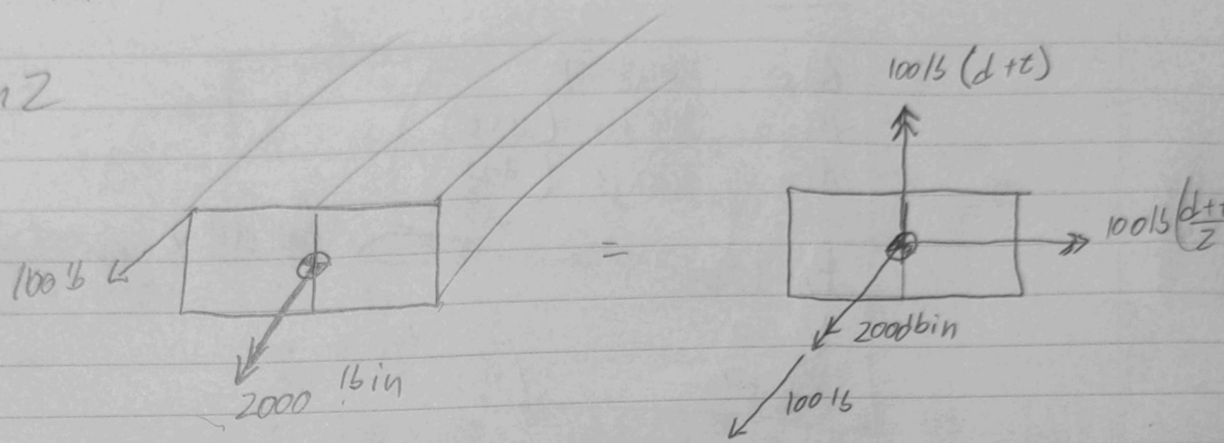


Figure 1: Schematic of the beam described in Problem 1.

Exam 3 Joshua Oates

problem 2



no running loads are applied to this beam, so

$$\boxed{p_x = p_y = p_z = m_x = m_y = m_z = 0}$$

$$P'(x) = -p_x, \quad V_y'(x) = p_y, \quad V_z'(x) = p_z$$

$$\boxed{P'(x) = V_y'(x) = V_z'(x) = 0}$$

$$\boxed{M_x'(x) = m_x = 0}$$

$$M_y'(x) = -\cancel{m_y(x)}^{0} + V_z(x)$$

$$M_z'(x) = -\cancel{m_z(x)}^{0} - V_y(x)$$

Since no loads are appied in the lateral directions and there are no running loads, $V_z = V_y = 0$ so

$$\boxed{M_y'(x) = M_z'(x) = 0}$$

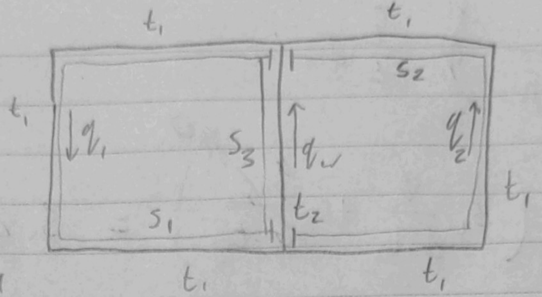
ie, all stresses in beam are constant. looking at centroid resolved figure above, we get ... continued

$M_x = 200\text{lb in}$
$M_y = 100\text{lb}\ (d+t)$
$M_z = 100\text{lb}\ (\frac{d+t}{2})$

$P_x = 100\text{lb}$
$V_y = 0$
$V_z = 0$

for all x on bem

problem 3



$t_1 = .1$
$t_2 = .2$

$S_1 = S_2 = 2d + t + d$
$= 3d + t = 30.1$

$S_3 = d = 10$

$q_w = q_1 - q_2$

$\bar{A}_1 = \bar{A}_2 = \bar{A} = d\left(d + \frac{t}{2}\right) = d^2 + \frac{dt}{2} = 100 + \frac{1}{2} = 100.5$

$$\frac{1}{2\bar{A}_1} \oint_{cell_1} \frac{q}{Gt} ds = \frac{1}{2\bar{A}_2} \oint_{cell_2} \frac{q}{Gt} ds$$

$$G = \frac{E}{2(1+\nu)}, \quad G_1 = \frac{E_1}{2(1+\nu_1)} = 1.1364e7, \quad G_2 = \frac{E_2}{2(1+\nu_2)} = 3.759$$

$$\frac{1}{2\bar{A}_1} \oint_{cell_1} \frac{q}{Gt} ds = \frac{1}{2\bar{A}_1}\left(S_1 \frac{q_1}{G_1 t_1} + S_3 \frac{q_w}{G_2 t_2}\right)$$

$$\frac{1}{2\bar{A}_2} \oint_{cell_2} \frac{q}{Gt} ds = \frac{1}{2\bar{A}_2}\left(S_1 \frac{q_2}{G_1 t_1} - S_3 \frac{q_w}{G_2 t_2}\right)$$

$$M_x = 2 \sum_{i=1}^{n} q_i \bar{A}_i = 2 q_1 \bar{A}_1 + 2 q_2 \bar{A}_2 = 2\bar{A}(q_1 + q_2)$$

$M_x = 200$ lbin

this system of eq is solved in Matlab

b) $\sigma_{sx} = \frac{q}{t}$ gives $\boxed{\sigma_{sx,skin} = \frac{q_s}{t_1} = 2.4876, \quad \sigma_{sx,web} = \frac{q_w}{t_2} = 0}$

Psi

$$\theta_1 = \theta_2 = \theta = \frac{1}{2\bar{A}} \oint_s \frac{q}{Gt} ds$$

$$\theta_1 = \frac{1}{2\bar{A}} \left(s_1 \frac{q_1}{G_1 t_1} + s_3 \frac{q_3}{G_2 t_2} \right) = 3.2781e-8$$

$$TR = 3.0505e4$$

4) because of the problem assumptions we get stress ~~states~~ that have only one component, either shear $= \sigma_{xs}$, or axial $= \sigma_{xx}$

we know on candidate stress state is

$$\underline{\sigma} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

the other 2 of shear are

$$\underline{\sigma} = \begin{bmatrix} 0 & \sigma_{sx} & 0 \\ \sigma_{sx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ \& } \underline{\sigma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma_{sx} \\ 0 & \sigma_{sx} & 0 \end{bmatrix}$$

in the case of the shear, it works out to the same τ_{max} & σ_e, so I will only analyze the first.

Table of Contents

Exam 3

```
clear all
close all
clc

addpath("C:\joshFunctionsMatlab\")


Ei = [30e6 30e6 30e6 30e6 10e6];
L = 100;

E1 = 30e6;
Ei_E1 = Ei./E1;

syms t d

% Ai = [[1 1]*(d+t)*2*t,[1 1]*(d-t)*t,2*t*(d-t)];
%
% zip = [[0 0 1 -1 0]*(d+t/2)];
% yip = [[1 -1 0 0 0]*(d/2)];
%
% Izoi = [t^3*(d+t/2)*2 t^3*(d+t/2)*2 d^3*t d^3*t d^3*t*2]./12
% Iyoi = [t*((d+t/2)*2)^3 t*((d+t/2)*2)^3 d*t^3 d*t^3 d*(t*2)^3]./12
%
% Iyzoi = [[1 1]*t*((d+t/2)*2)*(t^2+((d+t/2)*2)^2)/12, [1
 1]*t*d*(t^2+d^2)/12,t*d*2*(t^2+(t*2)^2)/12 ];

L1 = 2*d+t;
L2 = d-t;
Ai = [[1 1]*L1*t,[1 1]*L2*t,L2*2*t];

zip = [0 0 1 -1 0]*(d+t/2);
yip = [1 -1 0 0 0]*(d/2);

Izoi = [[1 1]*t^3*L1,[1 1]*t*L2^3,2*t*L2^3];
Iyoi =  [[1 1]*t*L1^3,[1 1]*t^3*L2,(2*t)^3*L2];

Iyzoi = [[1 1]*t*L1*(t^2*L1^2),[1 1]*t*L2*(t^2*L2^2),
(t*2)*L2*((t*2)^2*L2^2)]./12;


thing = joshAdvBeam(Ai,yip,zip,Iyoi,Izoi,Iyzoi,Ei_E1);
```

```
thing.Izzs = simplify(thing.Izzs);
thing.Iyzs = simplify(thing.Iyzs);
thing.Iyys = simplify(thing.Iyys);

disp("My findings for problem 1 using the diagram Ive drawn on the handwritten
 portion:")
disp(thing)

% Izzs = subs(thing.Izzs,t,.1);
% Izzs = vpa(subs(Izzs,d,10))
% Iyzs = subs(thing.Iyzs,t,.1);
% Iyzs = vpa(subs(Iyzs,d,10))
% Iyys = subs(thing.Iyys,t,.1);
% Iyys = vpa(subs(Iyys,d,10))
```

```
My findings for problem 1 using the diagram Ive drawn on the handwritten
 portion:
       y: [d/2    -d/2     0     0     0]
       z: [0     0     d + t/2     - d - t/2     0]
      As: 2*t*(d - t) + 2*t*(2*d + t) + (t*(2*d - 2*t))/3
     yps: 0
     zps: 0
    Iyys: (t*(108*d^3 + 144*d^2*t + 91*d*t^2 - 19*t^3))/6
    Izzs: (t*(22*d^3 - 45*d^2*t + 72*d*t^2 - 4*t^3))/6
    Iyzs: (7*t^3*(d - t)^3)/18 + (t^3*(2*d + t)^3)/6
```

Problem 2

```
clear

d = 10;
t=.1;
As = 6.680;
Iyys = 337.4;
Izzs = 122.6;
Iyzs = 0;
E1 = 30e6;
E2 = 10e6;

syms x y z

P = 100;
Mx = 2000;
My = 100*(d+t);
Mz = 100*(d+t)/2;

eps = P/(E1*As)- ((Mz*Iyys)+My*Iyzs)*y/(E1*(Iyys*Izzs-Iyzs^2)) +
 ((My*Izzs)+Mz*Iyzs)*z/(E1*(Iyys*Izzs-Iyzs^2));
% eps = vpa(eps)

disp("Stress as a function of y and z is given by:")
sig1 = eps*E1
```

```
sig2 = eps*E2
disp("Where sig1 applies in the skin and sig2 applies in the web.")
disp("From these euqations you can see that that the maximum stress in either
 case will be where z is maximized and y is minimized. The minimum stress can
 be found in the opposite corner. For the web, y is in the range +-(d-t)/2 and
 z is in the range +-t. For the skin, y is in the range +-(d+t)/2 and z is in
 the range +-(d+t). I will substitute these 4 possiblilities into sig1 and
 sig2 to find which point has the greatest value.")

skinMax = vpa(subs(subs(sig1,y,-(d+t)/2),z,d+t))
skinMin = vpa(subs(subs(sig1,y,(d+t)/2),z,-(d+t)))

webMax = vpa(subs(subs(sig2,y,-(d-t)/2),z,t))
webMin = vpa(subs(subs(sig2,y,(d-t)/2),z,-t))

disp("This shows that the global maximum axial stress is "+string(skinMax)+"
 psi at the location y = -(d+t)/2 and z = d+t.")

syms u(x) v(x) w(x) x
du = diff(u,x)
eqn = du == P/(E1*As)

dv = diff(v,x);
ddv = diff(dv,x);
eqn = [eqn;ddv == ((Mz*Iyys)+My*Iyzs)/(E1*(Iyys*Izzs-Iyzs^2))];


dw = diff(w,x);
ddw = diff(dw,x);

eqn = [eqn;ddw == -((My*Izzs)+Mz*Iyzs)/(E1*(Iyys*Izzs-Iyzs^2))];

eqn = [eqn;u(0) == 0];
eqn = [eqn;v(0) == 0];
eqn = [eqn;w(0) == 0];

eqn = [eqn;dv(0) == 0];
eqn = [eqn;dw(0) == 0];

sol = dsolve(eqn);
u = matlabFunction(sol.u);
v = matlabFunction(sol.v);
w = matlabFunction(sol.w);

figure
hold on
fplot(u,[0,100])
fplot(v,[0,100])
fplot(w,[0,100])
xlabel("x [in]")
ylabel("displacement [in]")
legend(["u","v","w"],"location","best")
```

Stress as a function of y and z is given by:

sig1 =

*(5050*z)/1687 - (2525*y)/613 + 1104595453515542109375/73786976294838206464*

sig2 =

*(5050*z)/5061 - (2525*y)/1839 + 368198484505180703125/73786976294838206464*

Where sig1 applies in the skin and sig2 applies in the web.
From these euqations you can see that that the maximum stress in either case will be where z is maximized and y is minimized. The minimum stress can be found in the opposite corner. For the web, y is in the range +-(d-t)/2 and z is in the range +-t. For the skin, y is in the range +-(d+t)/2 and z is in the range +-(d+t). I will substitute these 4 possiblilities into sig1 and sig2 to find which point has the greatest value.

skinMax =

66.005589953315370489359173576914

skinMin =

-36.065470192836325959156536715109

webMax =

11.886295270783545927138016212088

webMin =

-1.9062553506238644170704705914862

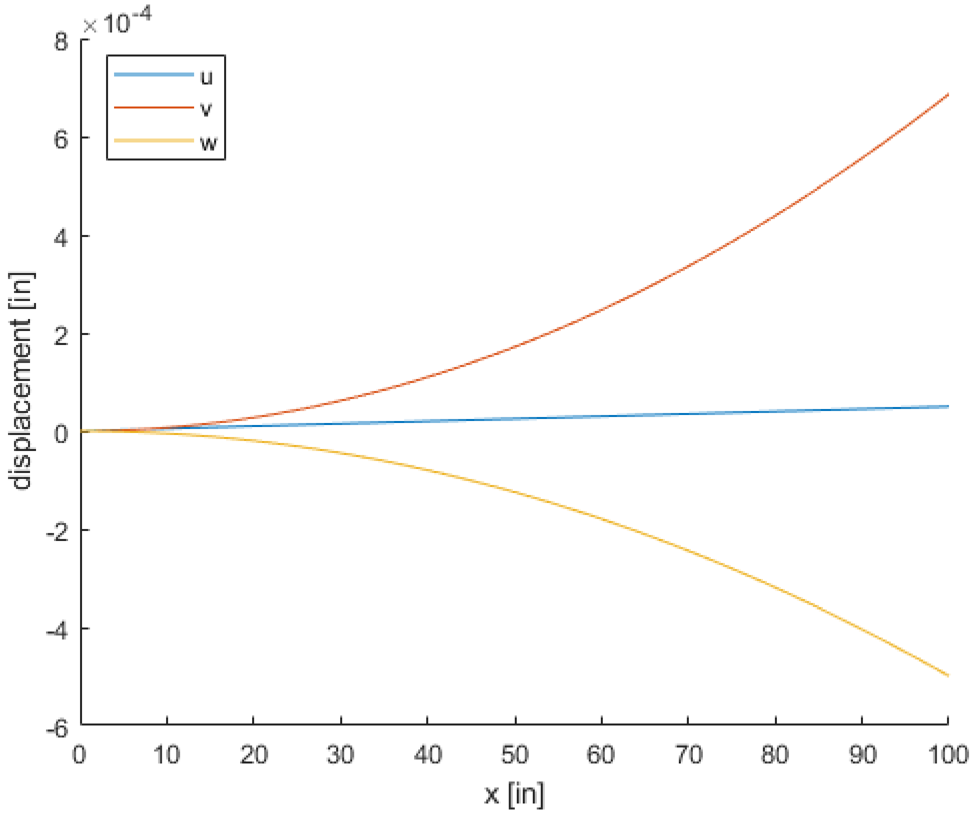
This shows that the global maximum axial stress is 66.005589953315370489359173576914 psi at the location y = -(d+t)/2 and z = d +t.

du(x) =

diff(u(x), x)

eqn(x) =

diff(u(x), x) == 4712940601666313/9444732965739290427392



Problem 3

```
E1 = 30e6;
E2 = 10e6;
nu1 = .32;
nu2 = .33;
Mx = 2000;

G1 = E1/(2*(1+nu1));
G2 = E2/(2*(1+nu2));

Abar = d^2+d*t/2;
s1 = 3*d+t;
s3 = d;

t1=t;
t2=t*2;

syms q1 q2

qw = q1-q2;

ex1 = 1/(2*Abar)*(s1*q1/(G1*t1)+s3*qw/(G2*t2));
ex2 = 1/(2*Abar)*(s1*q2/(G1*t1)-s3*qw/(G2*t2));
eqn1 = ex1 == ex2;
```

```
eqn2 = Mx ==2*Abar*(q1+q2);
sol = solve(eqn1,eqn2);

theta = double(vpa(subs(subs(ex1,q1,sol.q1),q2,sol.q2)));

q1 = vpa(sol.q1)
q2 = vpa(sol.q2)
sigsx = q1/t1
theta
TR = Mx/theta


q1 =

4.9751243781094527363184079601990


q2 =

4.9751243781094527363184079601990


sigsx =

49.751243781094527363184079601990


theta =

   6.5563e-07


TR =

   3.0505e+09
```

Problem 4

```
sig1 = skinMax*[
    [1  0  0]
    [0  0  0]
    [0  0  0]
];

sig2 = sigsx*[
    [0  1  0]
    [1  0  0]
    [0  0  0]
];

sige1 = ((3/2)*sum(sum((sig1-eye(3)*(1/3)*trace(sig1)).^2)))^.5
sige2 = ((3/2)*sum(sum((sig2-eye(3)*(1/3)*trace(sig2)).^2)))^.5
```

```
taummax1 = sig1(1,1)
[Vecs,Diag] = eig(sig2);

taumax2 = (Diag(1,1)-Diag(3,3))/2

disp("My stresses have reasonable trends but seem susplicously low. In either
 case, I find that the beam will not yield with Tresca or Von Mises yeild
 criteria by almost 2 orders of magnitude.")
```

```
sige1 =

66.0055899533153704893591735769l4


sige2 =

86.17168196860086037449981798536?


taummax1 =

66.0055899533153704893591735769l4


taumax2 =

49.7512437810945273631840796019?

My stresses have reasonable trends but seem susplicously low. In either case,
 I find that the beam will not yield with Tresca or Von Mises yeild criteria
 by almost 2 orders of magnitude.
```

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```
function [out] =
 joshAdvBeam(Ai,yi_prime,zi_prime,Iyoiyoi,Izoizoi,Iyoizoi,Ei_E1,alphai,Ei)
% this function takes several



% arguments
%     Ai {mustBeReal}
%     yi_prime {mustBeReal}
%     zi_prime {mustBeReal}
%     Iyoiyoi {mustBeReal}
%     Izoizoi {mustBeReal}
%     Iyoizoi {mustBeReal}
%     Ei_E1 {mustBeReal}
%     alphai {mustBeReal} = nan
%     Ei {mustBeReal} = nan
% end

arguments
    Ai
    yi_prime
    zi_prime
    Iyoiyoi
    Izoizoi
    Iyoizoi
    Ei_E1
    alphai = nan
    Ei   = nan
end

A = Ai;
yp = yi_prime;
zp = zi_prime;
Iz0 = Izoizoi;
Iy0 = Iyoiyoi;
Iyz0 = Iyoizoi;

% n = length(A);
% if length(yp) ~= n  | length(zp) ~= n  | length(Iz0) ~= n  | length(Iy0) ~=
 n  | length(Iyz0) ~= n | length(Ei_E1) ~= n
%     throw(MException('joshAdvBeam:invalidInput','At least one of the input
 vectors is not the correct length'))
% end


% Ai*(Ei/E1)
AE_E1 = Ei_E1.*Ai;
% A*
As = sum(AE_E1);

% A*(E/E1)*y'
AE_E1yp = AE_E1.*yp;
```

```
% y'*
yps = sum(AE_E1yp)/As;


% Ai*(Ei/E1)*zi'
AE_E1zp = AE_E1.*zp;
% z'*
zps = sum(AE_E1zp)/As;

% yy
% (Ei/E1)*(Iyoiyoi+Ai'*zi'^2)
var1 = (Ei_E1.*(Iy0+A.*zp.^2));
% I*y'y'
Iyps = sum(var1);
% I*yy = I*y'y' - A*(z'*)^2
Iys = Iyps - As.*zps.^2;

% zz
var2 = (Ei_E1.*(Iz0+A.*yp.^2));
Izps = sum(var2);
Izs = Izps - As.*yps.^2;

% yz
var3 = (Ei_E1.*(Iyz0+A.*zps.*yps));
Iyzps = sum(var3);
Iyzs = Iyzps - As.*zps.*yps;

% y and z
y = yp-yps;
z = zp-zps;

out.y = y;
out.z = z;
out.As = As;
out.yps = yps;
out.zps = zps;

% out.Iyyps = Iyps;
out.Iyys = Iys;

% out.Izzps = Izps;
out.Izzs = Izs;

% out.Iyzps = Iyzps;
out.Iyzs = Iyzs;

if (~isnan(alphai)) & (~isnan(Ei))

    if length(alphai) ~= n | length(Ei) ~= n
        throw(MException('joshAdvBeam:invalidInput','Either alphai or Ei is
 the wrong length'))
    end
    E = Ei;
```

```
    EalphaA = E.*alphai.*A;
    EalphaAy = E.*alphai.*A.*y;
    EalphaAz = E.*alphai.*A.*z;

    PT_DT = sum(EalphaA);
    Mz_DT = sum(EalphaAy);
    My_DT = sum(EalphaAz);

    out.PT_DT = PT_DT;
    out.MzT_DT = Mz_DT;
    out.MyT_DT = My_DT;
end


end
```

```
Error using joshAdvBeam
Invalid argument list. Function requires 7 more input(s).
```

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