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HW2

Problem 1

$$\underline{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \underline{\underline{\sigma}}(\underline{x}) = \begin{bmatrix} 1 & 0 & 2y \\ 0 & 1 & 4x \\ 2y & 4x & 1 \end{bmatrix} \text{ksi} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 4 \\ 4 & 4 & 1 \end{bmatrix} \text{ksi}$$

plane we study $x+y+z = 6\text{in}$, $\underline{n} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

a) $$\underline{T}(\underline{n}) = \underline{\underline{\sigma}}\,\underline{n} = \frac{1}{\sqrt{3}}\left[\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 4 \\ 4 & 4 & 1 \end{bmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right] = \boxed{\frac{1}{\sqrt{3}}\begin{pmatrix} 5 \\ 5 \\ 9 \end{pmatrix} \text{ksi}}$$

b) $$\underline{T}_n(\underline{n}) = \underline{n}^T\underline{T}\,\underline{n} = \left(\frac{1}{\sqrt{3}}\right)^3\left[\begin{bmatrix} (1 & 1 & 1) \end{bmatrix}\begin{pmatrix} 5 \\ 5 \\ 9 \end{pmatrix}\right]\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 3^{-\frac{3}{2}}\left[19\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right] = \boxed{3^{-\frac{3}{2}}19\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ksi}}$$

$$\approx 3.66\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ksi}$$

$$\underline{T} = \underline{T}_n + \underline{T}_t, \quad \underline{T}_t = \underline{T} - \underline{T}_n = 3^{-\frac{1}{2}}\begin{pmatrix} 5 \\ 5 \\ 9 \end{pmatrix} - 3^{-\frac{3}{2}}19\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

c) $$= \boxed{\begin{pmatrix} (5)3^{-\frac{1}{2}} - (19)3^{-\frac{3}{2}} \\ (5)3^{-\frac{1}{2}} - (19)3^{-\frac{3}{2}} \\ (9)3^{-\frac{1}{2}} - (19)3^{-\frac{3}{2}} \end{pmatrix} \text{ksi} \approx \begin{pmatrix} -.77 \\ -.77 \\ 1.54 \end{pmatrix} \text{ksi}}$$

$$|\underline{\underline{\sigma}} - \lambda\,\underline{\underline{1}}| = 0 = \begin{vmatrix} 1-\lambda & 0 & 4 \\ 0 & 1-\lambda & 4 \\ 4 & 4 & 1-\lambda \end{vmatrix} = -\lambda^3 + 3\lambda^2 + 29\lambda - 3$$

from the cubic equation, $\lambda_1 = 1 + 4\sqrt{2}$

$\lambda_2 = 1$

$\lambda_3 = 1 - 4\sqrt{2}$

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$$(\underline{\sigma} - \lambda \mathbb{1})\underline{s} = 0$$

$$4z - x\lambda + x = 0$$

$$4z - y\lambda + \lambda = 0$$

$$4x + 4y - z\lambda + z = 0$$

Plugging in gives

$\sigma_1 = 1 + 4\sqrt{2}$ ksi $\quad s_1 = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix}$ in

$\sigma_2 = 1$ ksi $\quad s_2 = \frac{\sqrt{2}}{2}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ in

$\sigma_3 = 1 - 4\sqrt{2}$ ksi $\quad s_3 = \frac{1}{2}\begin{pmatrix} -1 \\ -1 \\ \sqrt{2} \end{pmatrix}$ in

$$\sigma_n = \frac{1}{3}\operatorname{tr}\underline{\sigma} = 1 \text{ ksi}$$

$$\underline{\sigma}_{dev} = \underline{\sigma} - \sigma\mathbb{1} = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 4 \\ 4 & 4 & 0 \end{bmatrix} \text{ksi}$$

$$\sigma_e = \left(\frac{3}{2}\sum_{\beta=x}^{z}\sum_{\alpha=x}^{z}\left(\sigma_{dev_{\alpha\beta}}^2\right)\right)^{1/2} = \sqrt{\frac{3}{2}(4)(4^2)} = \sqrt{96} \text{ ksi}$$

$\sigma_y = 15.6$ ksi, $\quad F = 1.5$

tresca, yeild if $F\tau_{max} \geq \frac{\sigma_y}{2}$

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) = 4\sqrt{2}$$

$$F\tau_{max} = 6\sqrt{2} \approx 8.4853 \text{ ksi} \qquad \frac{\sigma_y}{2} = 7.8$$

So tresca predicts yeilding

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Von mises yeild if $\sigma_e \geq \sigma_y$

$\sigma_e \approx 9.7798$ ksi

So von mises predicts there will not be yeild

Problem 2 - see matlab