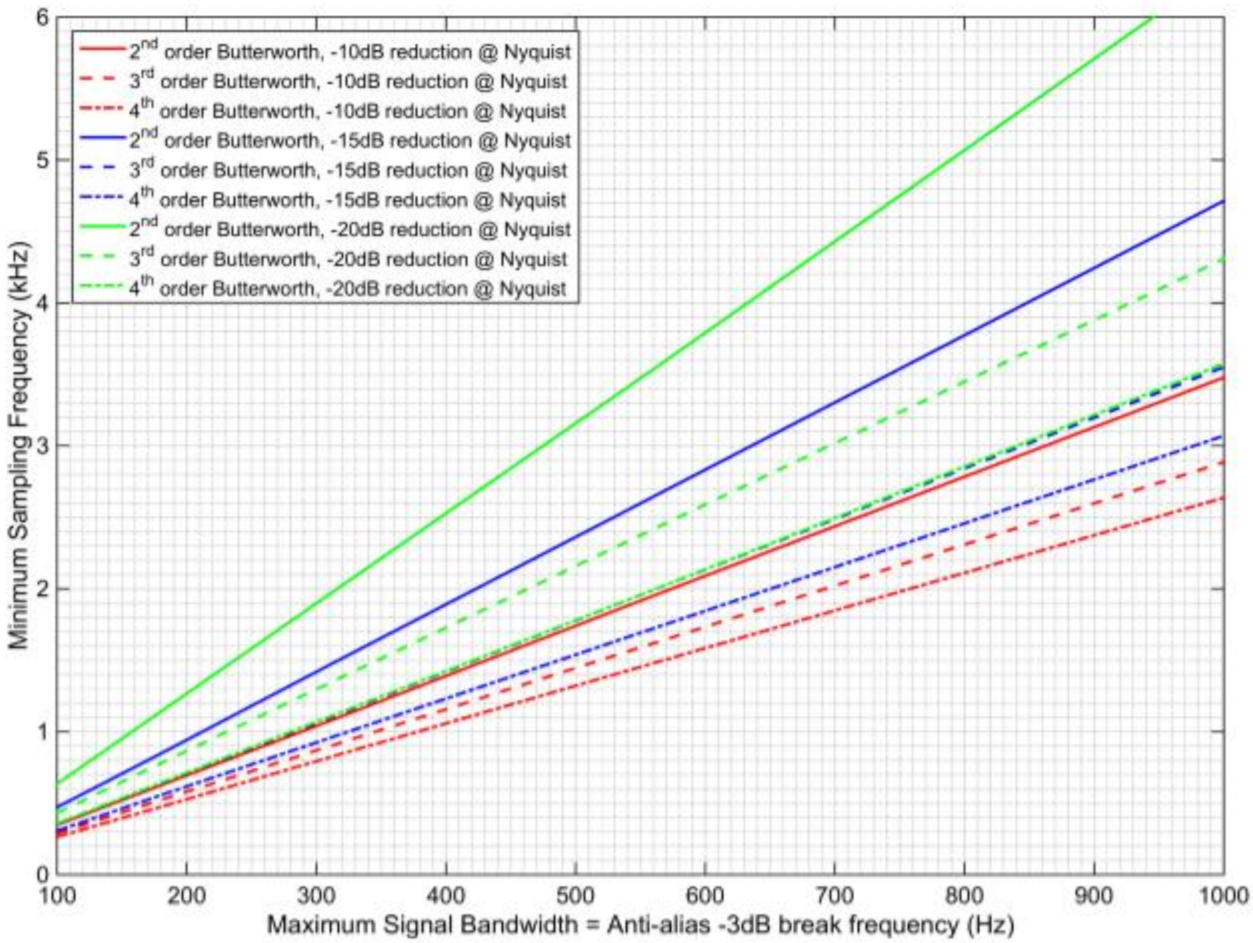
```
clc
clear all
close all
pause('off')
```

Assignment 2

Japnit Sethi



1. Using the chart above, determine the maximum anti-alias filter break frequency given the following requirements. Use Matlab to compute and plot the frequency response of the designed anti-alias filter. Draw a horizontal -3dB reference line and a vertical line at the Nyquist frequency.

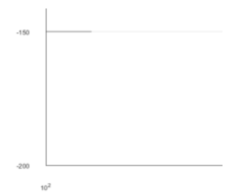
    - At least a 3rd order Butterworth anti-alias filter
    - At least -20dB reduction at the Nyquist frequency
    - Sample rate of 2kHz

```
% Problem 1

% Building the Anti Alias Filter
dB_red = -20; % 20 dB reduction
fs = 2e3;     % Hz Sample Rate
N = 3; % 3rd Order

fbreak = 460; % Break frequency from the plot
wbreak = 2*pi*fbreak;

fprintf('<strong> Maximum anti-alias filter break frequency: %3.0f Hz
</strong>', fbreak);
```

Maximum anti-alias filter break frequency: 460 Hz

```

[num, den] = butter(N, wbreak, 's');

fr = logspace(2, 4, 1000);
P1_frf = freqs(num, den, 2*pi*fr);

Magnitude = abs(P1_frf);
Phase_deg = (180/pi)*angle(P1_frf);


figure()
% Magnitude Plot
subplot(2,1,1)
semilogx(fr, 20*log10(Magnitude), 'LineWidth', 3);
grid on
hold on
% xlim([100 1500]);
ylim([-100 10]);
xlabel('Frequency (Hz)');
ylabel('Magnitude (dB)');
title('Butterworth Anti-Alias FIlter Frequency Response');

% -3dB Horizontal Reference line
yline(-3, '--r', '-3dB', 'LineWidth', 3);

% -20dB Horizontal Reference line
yline(-20, '--c', '-20dB', 'LineWidth', 3);

% Nyquist frequency Vertical Reference line
xline((2000)/2, '--y', 'Nyquist Freq', 'LineWidth', 3); % Because sample rate
is 2kHz
```

```
% Phase Plot
subplot(2,1,2)
semilogx(fr, Phase_deg, 'LineWidth', 3);
grid on
xlabel('Frequency (Hz)');
ylabel('Phase (deg)');

% In order to maximize the figure window in Windows
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
```



2. Using the chart above, determine the minimum sample rate given the following requirements. Use Matlab to compute and plot the frequency response of the designed anti-alias filter. Draw a horizontal -3dB reference line and a vertical line at the Nyquist frequency.

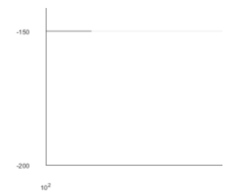
   - At least a 3th order Butterworth anti-alias filter
   - -10dB reduction at the Nyquist frequency
   - Break frequency of 690Hz

```
% Problem 2

% Building the Anti Alias Filter
dB_red = -10; % 10 dB reduction
fs = 2e3;      % Hz Sample Rate from the plot
N = 3; % 3rd Order

fbreak = 690; % Break frequency
wbreak = 2*pi*fbreak;
```

```
fprintf('<strong> Minimum anti-alias filter sample rate: %3.0f Hz </strong>',
fs);
```

```
Minimum anti-alias filter sample rate: 2000 Hz
```

```
[num, den] = butter(N, wbreak, 's');

fr = logspace(2, 4, 1000);
P2_frf = freqs(num, den, 2*pi*fr);

Magnitude = abs(P2_frf);
Phase_deg = (180/pi)*angle(P2_frf);


figure()
% Magnitude Plot
subplot(2,1,1)
semilogx(fr, 20*log10(Magnitude), 'LineWidth', 3);
grid on
hold on
% xlim([100 1500]);
ylim([-100 10]);
xlabel('Frequency (Hz)');
ylabel('Magnitude (dB)');
title('Butterworth Anti-Alias FIlter Frequency Response');

% -3dB Horizontal Reference line
yline(-3, '--r', '-3dB', 'LineWidth', 3);

% -10dB Horizontal Reference line
yline(-10, '--c', '-10dB', 'LineWidth', 3);

% Nyquist frequency Vertical Reference line
xline((2000)/2, '--y', 'Nyquist Freq', 'LineWidth', 3); % Because sample rate
is 2kHz

% Phase Plot
subplot(2,1,2)
semilogx(fr, Phase_deg, 'LineWidth', 3);
grid on
xlabel('Frequency (Hz)');
ylabel('Phase (deg)');

% In order to maximize the figure window in Windows
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
```



3. Using the chart above, determine the minimum filter order given the following requirements. Use Matlab to compute and plot the frequency response of the designed anti-alias filter. Draw a horizontal -3dB reference line and a vertical line at the Nyquist frequency.

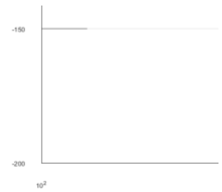
   - Sample rate of 1200 Hz
   - -15dB reduction at the Nyquist frequency
   - Break frequency of 350Hz

```
% Problem 3

% Building the Anti Alias Filter
dB_red = -15; % 15 dB reduction
fs = 1.2e3;     % Hz Sample Rate

fbreak = 350; % Break frequency
wbreak = 2*pi*fbreak;

[num2, den2] = butter(2, wbreak, 's');
[num3, den3] = butter(3, wbreak, 's');
[num4, den4] = butter(4, wbreak, 's');

fr = logspace(2, 4, 1000);
P32_frf = freqs(num2, den2, 2*pi*fr);
P33_frf = freqs(num3, den3, 2*pi*fr);
P34_frf = freqs(num4, den4, 2*pi*fr);
P3_frf = [P32_frf; P33_frf; P34_frf];

Magnitude = abs(P3_frf);
Phase_deg = (180/pi)*angle(P3_frf);
```

```
figure()
% Magnitude Plot
subplot(2,1,1)
semilogx(fr, 20*log10(Magnitude), 'LineWidth', 3);
grid on
hold on
% xlim([100 1500]);
ylim([-100 10]);
xlabel('Frequency (Hz)');
ylabel('Magnitude (dB)');
title('Butterworth Anti-Alias FIlter Frequency Response');

% -3dB Horizontal Reference line
yline(-3, '--r', '-3dB', 'LineWidth', 3);

% -15dB Horizontal Reference line
yline(-15, '--c', '-15dB', 'LineWidth', 3);

% Nyquist frequency Vertical Reference line
xline((fs)/2, '--y', 'Nyquist Freq', 'LineWidth', 3); % Because sample rate is
2kHz

% Phase Plot
subplot(2,1,2)
semilogx(fr, Phase_deg, 'LineWidth', 3);
grid on
xlabel('Frequency (Hz)');
ylabel('Phase (deg)');
legend('2nd Order', '3rd Order', '4th Order');

fprintf('<strong> Minimum filter order from graph: 3 </strong>');
```

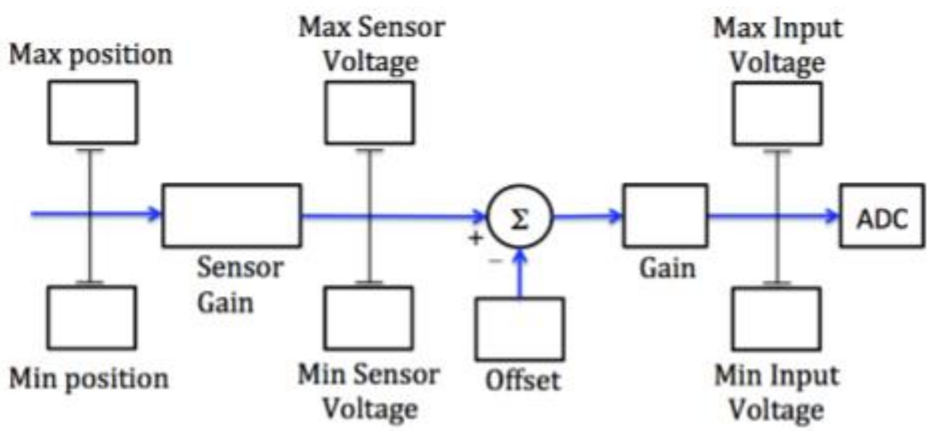
Minimum afilter order from graph: 3rd

```
% In order to maximize the figure window in Windows
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
```



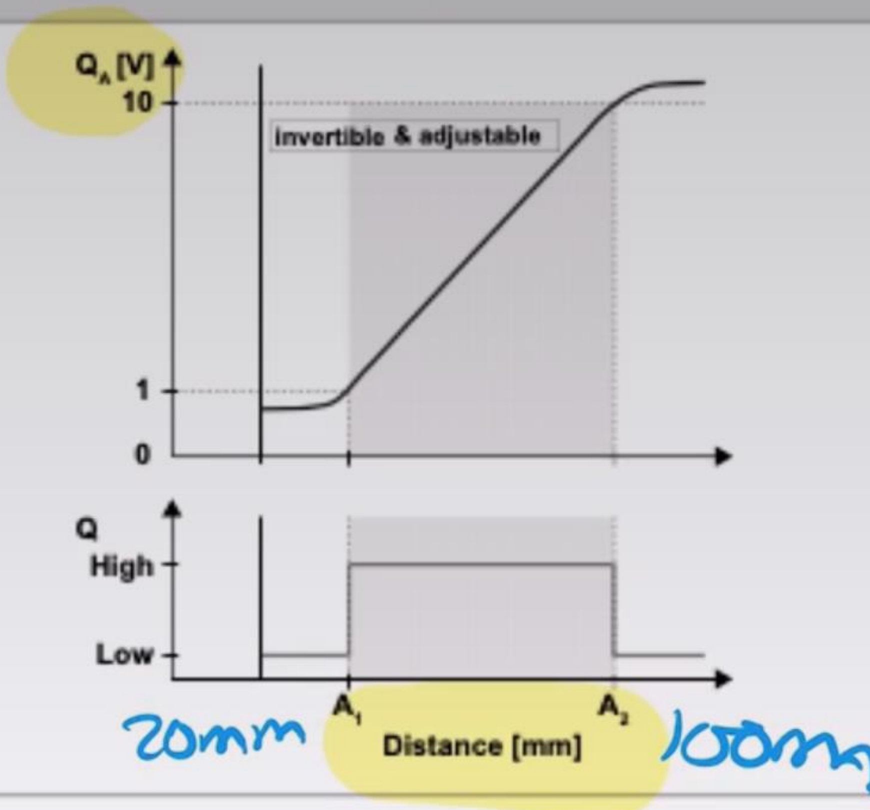
4. You are designing a position control system that requires a non-contacting position sensor. The FT 25-RLA miniature laser distance sensor (www.sensopart.com) has been chosen for this application. Although this sensor is capable of outputting a digital value, it also outputs an analog voltage which will be the input to an ADC for the control system. Download the data sheet for this sensor and determine the input position measurement range and the corresponding sensor analog output voltage range. What is the "sensitivity" gain of this sensor?

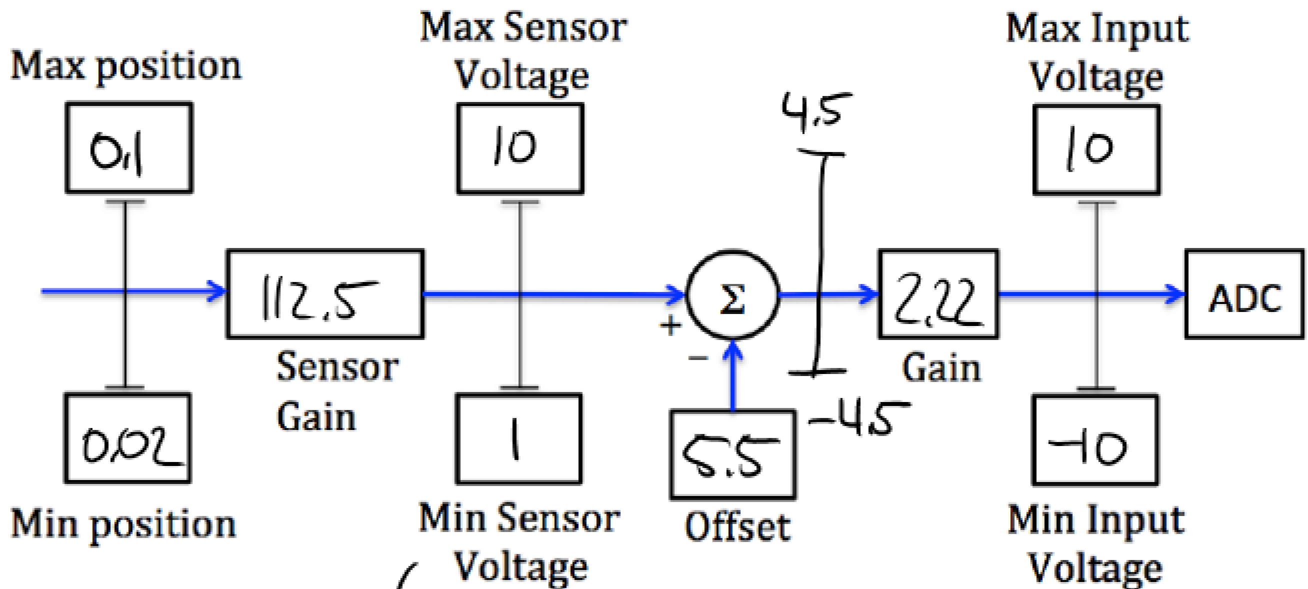
   Assuming that 100% of the sensor voltage range will be used, and the ADC is bipolar with a range of ±10V, design an appropriate analog **offset** and **gain** such that exactly 100% of the ADC range is used. In the figure below, fill in the empty boxes with the appropriate values.



Solution:

Characteristics





$\Delta V = 9\ V$

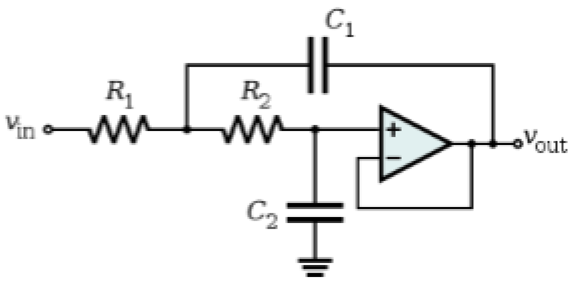
$\text{Offset} = -\frac{1}{2}(10+1) = -5.5$ (centerpoint of sensor V

With offset: $V_{max} = 10 - 5.5 = 4.5$
$V_{min} = 1 - 5.5 = -4.5$

Gain: boost 4.5 V to 10 V $\longrightarrow$ $G = 10/4.5 = 2.$

```
% Problem 4

% Datasheet link: https://www.sensopart.com/jdownloads/Produktdetails/FT_25-RLA_e.pdf
```

5. In 1955, Professors Sallen and Key introduced an active circuit topology (shown below) that implements a unity gain second-order analog low-pass filter. This particular implementation is known as a Sallen-Key topology and is widely used for anti-aliasing in many applications today.



Sallen-Key Active Low-Pass Filter

$$\frac{V_{out}(s)}{V_{in}(s)} = \left(\frac{\left(\frac{1}{R_1C_1R_2C_2}\right)}{s^2 + \left(\frac{1}{R_2C_1} + \frac{1}{R_1C_1}\right)s + \left(\frac{1}{R_1C_1R_2C_2}\right)} \right)$$

Transfer Function

Only two parameters are required to define a unity gain second order system (i.e. the natural frequency and the damping ratio); however, the Sallen-Key filter has four component parameter values that can be selected. Because the circuit is over-determined, additional constraints are usually applied depending on how this circuit is to be used in an application. There are a number of online design tools to help you select the resistance and capacitance values given a desired filter type or given a desired break frequency and damping ratio. Some of these tools are:

http://sim.okawa-denshi.jp/en/OPseikiLowkeisan.htm

http://www.daycounter.com/Filters/SallenKeyLP/Sallen-Key-LP-Filter-Design-Equations.phtml

http://www.daycounter.com/Filters/Sallen-Key-LP-Calculator.phtml

http://www.changpuak.ch/electronics/calc_08.php

http://www.pronine.ca/actlpf.htm

Use one of the online tools above to design an appropriate 2nd order Sallen-Key analog low-pass filter that can be used for anti-aliasing in a low-bandwidth control application where the sample rate is 100Hz. Your design must achieve at least 20 dB of reduction at the Nyquist frequency and it must maximize the bandwidth of the filter.

Document your design by listing the Resistance and Capacitance values as well as the resulting natural frequency (sometimes called the "cutoff" frequency), the damping ratio (or the "Quality" factor if you are familiar with those). You must also include a Bode diagram of your final anti-alias filter design (i.e. dB magnitude vs. log frequency and phase (degrees) vs. log frequency). From the Bode magnitude plot, determine and label the -3 dB Bandwidth of your design as well as the dB reduction at the Nyquist frequency.

Hint: You will probably get better results if you allow the online design tool to select standard capacitors from the E24 sequence and standard resistors from the E96 sequence. You will also get better results if you allow the tool to target a Q factor of 0.707.

Using the following parameters:

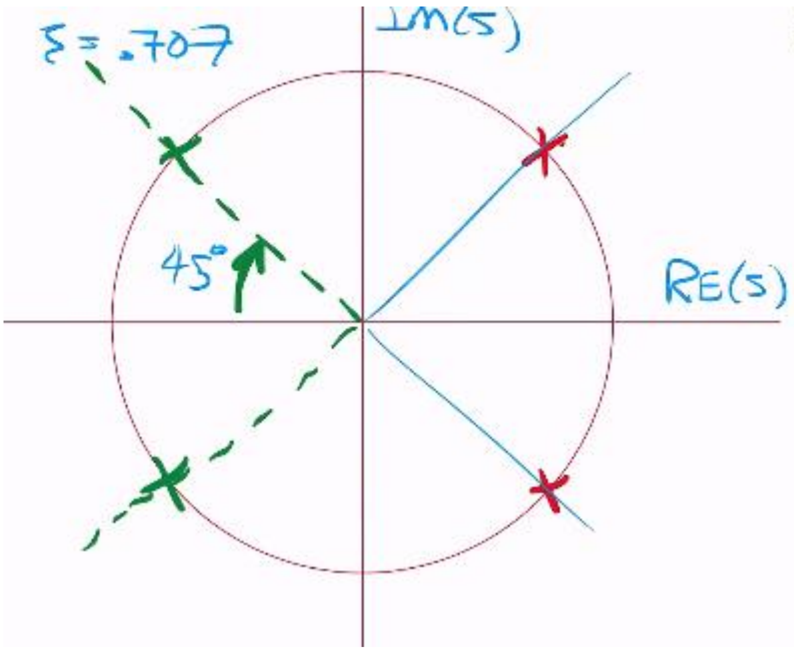
Capacitors -> E24 Sequence

Resistors -> E96 Sequence

Q factor = 0.707 $\Longrightarrow$ $Q = \dfrac{1}{2 * \zeta}$ Therefore, ζ (Damping ratio) = 0.707

Note: Cutoff Frequency = $\dfrac{1}{2 * \Pi * \sqrt{R_1 * R_2 * C_1 * C_2}}$

Desired break frequency = Cutoff Frequency = 15.83 Hz (Found Above)

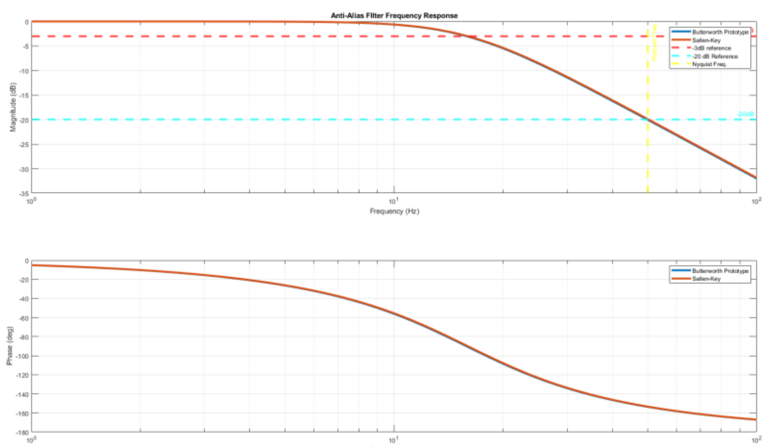


Solution:

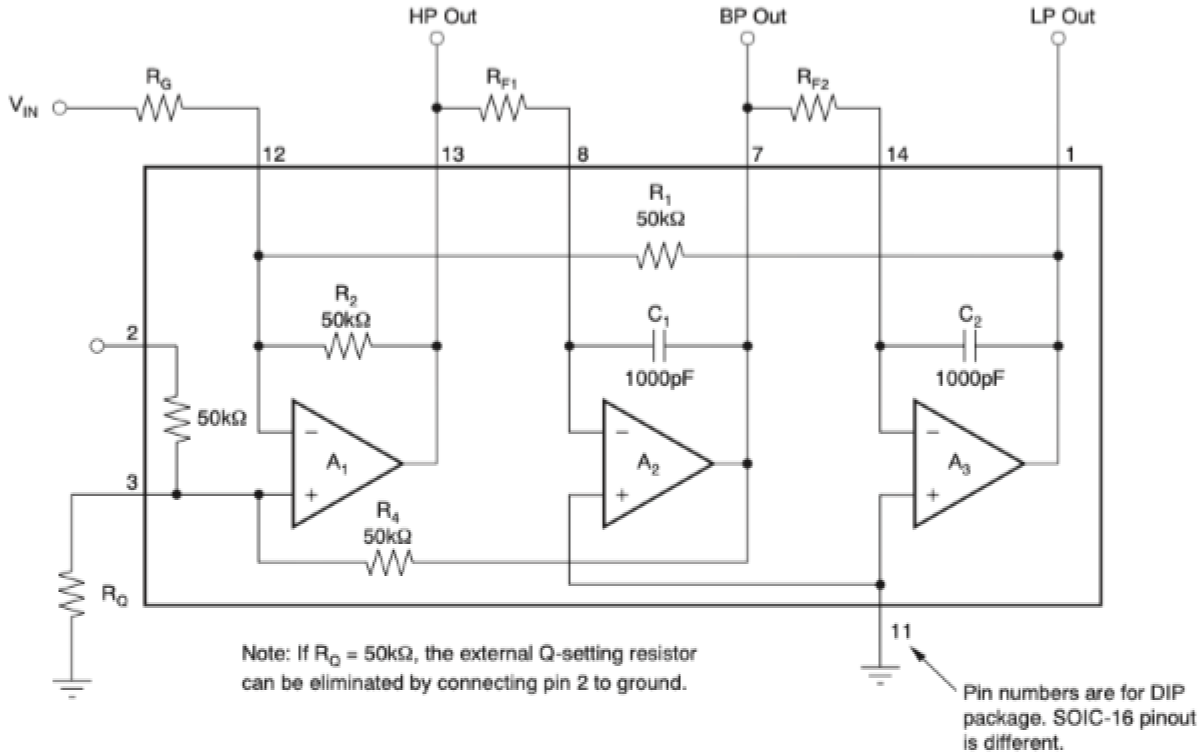
```
% Problem 5

% Prototyping Butterworth  to get
fr = logspace(0, 2, 1000);
dB_red = -20;
fbreak = 15.83;
fs = 100; % Hz

[numb, denb] = butter(2, 2*pi*fbreak, 's'); % 2nd order filter

AAb = freqs(numb, denb, 2*pi*fr);
```

```
Magnitude = abs(AAb);
Phase_deg = (180/pi)*angle(AAb);

% For Sallen-Key
R1 = 18700; % Ohms
R2 = 11300; % Ohms
C1 = 1e-6; % F
C2 = 0.47e-6; % F
zeta = 0.707; % Damping Ratio

% Building the Anti Alias Filter
s = tf('s');
cutoff_frequency5 = 1/(2*pi*sqrt(R1*R2*C1*C2));
transfer_function5 = ((2*pi*cutoff_frequency5)^2)/(s^2 +
s*2*zeta*((2*pi*cutoff_frequency5))  + ((2*pi*cutoff_frequency5)^2));
minimum_samplerate5 = 100; % Hz

% Simulating the Frequency Response of the Sallen-Key filter
magnitude6 = abs(squeeze(freqresp(transfer_function5, 2*pi*fr)));
Phase6 = (180/pi)*angle(squeeze(freqresp(transfer_function5, 2*pi*fr)));



% Plotting Both Butterworth and Sallen-Key filter
figure()
% Magnitude Plot
subplot(2,1,1)
semilogx(fr, 20*log10(Magnitude), 'LineWidth', 3);
grid on
hold on
semilogx(fr, 20*log10(magnitude6), 'LineWidth', 3);
xlabel('Frequency (Hz)');
ylabel('Magnitude (dB)');
title('Anti-Alias FIlter Frequency Response');
xlim([1 100]);

% -3dB Horizontal Reference line
yline(-3, '--r', '-3dB', 'LineWidth', 3);

% -20dB Horizontal Reference line
yline(-20, '--c', '-20dB', 'LineWidth', 3);

% Nyquist frequency Vertical Reference line
xline((fs)/2, '--y', 'Nyquist Freq', 'LineWidth', 3); % Because sample rate is
2kHz
```

```
legend('Butterworth Prototype', 'Sallen-Key', '-3dB reference', '-20 dB
Reference', 'Nyquist Freq.');


% Phase Plot
subplot(2,1,2)
semilogx(fr, Phase_deg, 'LineWidth', 3);
grid on
hold on
semilogx(fr, Phase6, 'LineWidth', 3);
xlabel('Frequency (Hz)');
ylabel('Phase (deg)');
legend('Butterworth Prototype', 'Sallen-Key');


% In order to maximize the figure window in Windows
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
```



6. More recently, Texas Instruments introduced an active circuit topology (shown below) that implements a second-order analog low-pass filter with any desired gain. This particular integrated circuit is known as a Universal Active Filter (UAF42). Unlike the Sallen-Key architecture, the UAF42 only uses external resistors to define the natural frequency, damping ratio, and gain.

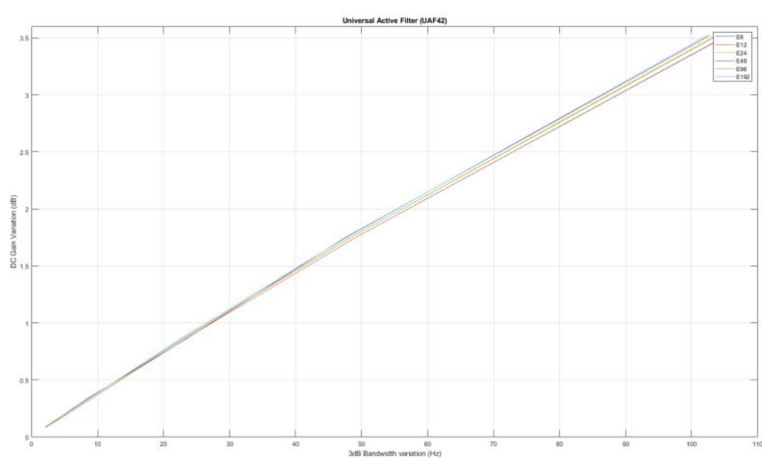


Use the **uaf42_designer.p** application in Matlab (available in Canvas) to generate the equivalent 2^{nd} order filter design as in problem 5. Only use the "inverting" architecture for this problem! Generate plots of the amount of potential variation in DC gain and the amount of potential variation in the 3dB bandwidth as a function of resistor precision (i.e. % tolerance must match the E-series selection). Review the following link to learn more about E-series resistors:

https://en.wikipedia.org/wiki/E_series_of_preferred_numbers

```
% Problem 6

% For E6
y1 = [0.0868793, 0.173763, 0.347561, 0.869511, 1.7434, 3.52262]; % dB
x1 = [2.23049, 4.66403, 9.33875, 23.5443, 48.8885, 105.368]; % Hz

y2 = [0.0868793, 0.173763, 0.347567, 0.869511, 1.7434, 3.52]; % dB
x2 = [2.23049, 4.66403, 9.33875, 23.5443, 48.888, 105.37]; % Hz

y3 = [0.086881, 0.173766, 0.347567, 0.869527, 1.74343, 3.52268]; % dB
```

```
x3 = [2.30857, 4.43984, 8.87387, 23.148, 48.1465, 103.696]; % Hz

y4 = [0.0868808, 0.173766, 0.347566, 0.869525, 1.74343, 3.52267]; % dB
x4 = [2.14872, 4.29771, 8.76867, 23.6456, 47.3779, 102.807]; % Hz

y5 = [0.08688, 0.173764, 0.347563, 0.869517, 1.74341, 3.52264]; % dB
x5 = [2.05302, 4.29293, 8.95316, 23.0865, 48.107, 103.991]; % Hz

y6 = [0.0868807, 0.173766, 0.347566, 0.869525, 1.74343, 3.52267]; % dB
x6 = [2.19676, 4.39378, 8.96469, 22.7856, 47.5101, 102.381]; % Hz

figure()
plot(x1 , y1);
hold on
grid on
plot(x2 , y2);
plot(x3 , y3);
plot(x4 , y4);
plot(x5 , y5);
plot(x6 , y6);
hold off
xlim([0 110]);
ylim([0 3.6]);
legend('E6', 'E12', 'E24', 'E48', 'E96', 'E192');
xlabel('3dB Bandwidth variation (Hz)');
ylabel('DC Gain Variation (dB)');
title('Universal Active Filter (UAF42)');

% In order to maximize the figure window in Windows
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
```



```
% uaf42_designer();
```