```
clc
clear all
close all
pause('off')
```

# Assignment 3

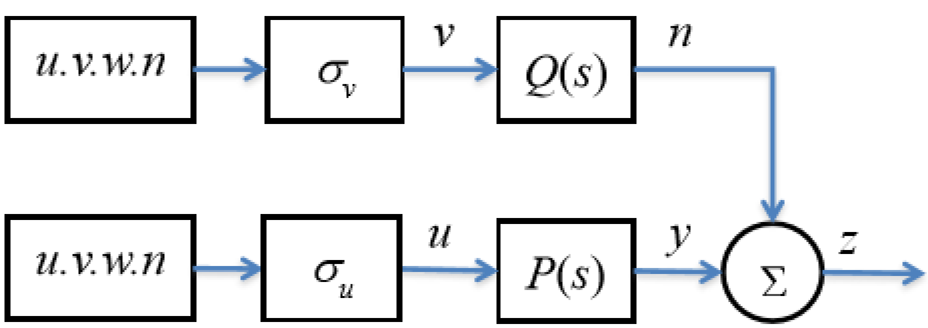
## Japnit Sethi

Two continuous time dynamic systems, $P(s)$ and $Q(s)$, are given by the following transfer functions:

$$P(s) = \frac{60.655s^2 + 1524.4s + 9.674e5}{s^3 + 955.04s^2 + 12870s + 9.674e5}$$

$$Q(s) = \frac{58.214s^2 + 43892s + 4.5964e6}{s^3 + 1105.8s^2 + 44581s + 4.5964e6}$$

As indicated in the block diagram below, $P(s)$ is excited by a normally distributed white noise signal $u(t)$ of unknown standard deviation $\sigma_u$. The output $y(t)$ is corrupted by an additive noise signal $n(t)$ to produce a measurement $z(t)$. The noise signal $n(t)$ is the output of $Q(s)$, which is excited by a different normally distributed white noise signal $v(t)$ of unknown standard deviation $\sigma_v$. Both white noise signals are assumed to be generated by independent unity-variance white noise ($u.v.w.n.$) signal generators such as what the Matlab `randn()` function produces. The output $z(t)$ will be sampled at 2000 Hz; however, both $P(s)$ and $Q(s)$ already include analog anti-alias filters so you do not need to design your own.




1. In a first experiment, no signal is applied to $P(s)$ (i.e. $u = 0$ and $n \neq 0$). From this data set, you are given that the standard deviation of $z(t)$ is estimated to be $\sigma_z = 12$ (this is NOT a dB value!). Determine the approximate standard deviation $\sigma_v$ of the excitation to the noise filter.

# Solution

We need to find standard deviation of v(t) {$\sigma_v$} given standard deviation of z(t) {$\sigma_z$}

**Note**: If for example we feed in noise v with standard deviation $\sigma_v{}^*$ = 1, we get a signal output with variance k, after that we just scale $\sigma_v{}^*$ by $\sigma_z$/k to get $\sigma_v$

This is based on the assumption that the noise standard deviation scales with magnitude as it is fed through the transfer function Q

```
% To actually find the magnitude of noise from phi_z to phi_v, we can
% assume the noise standard dev scales with magnitude as we feed it
% through the transfer function Q. If we feed noise v in with standard
% deviation phi_v_star = 1, and get a signal output with variance k,
% then the we just need to scale phi_v_star by phi_z/k to get phi_v.

fs = 2000;     ts = 1/fs;

% Defining the Plant Filter
nump = [0, 60.655, 1524.4, 9.674e+05];
denp = [1, 955.04, 12870, 9.674e+05];
P = tf(nump, denp);

% Defining the Noise Filter
numq = [0, 58.214, 43892, 4.5964e+06];
denq = [1, 1105.8, 44571, 4.5964e+06];
Q = tf(numq, denq);

Sn_measured = 12;         % measured: Sv = Sn

N = 2^22;
t = ts * [0:N-1];

Sv_guess = 1;           % Initial guess
v = Sv_guess * randn(1, N);
Sv_check = std(v)
```

```
Sv_check = 1.0004
```

```
n = lsim(Q, v, t);          % generate noise sequence
Sn_estimated = std(n)       % actual
```

```
Sn_estimated = 0.1926
```

```
Sv_estimated = Sn_measured / Sn_estimated    % Estimated Standard Deviation v
```

```
Sv_estimated = 62.3171
```

```
v = Sv_estimated * randn(1, N);
Sv_check = std(v)
```

```
Sv_check = 62.3284
```

```
n = lsim(Q, v, t);          % generate noise sequence
Sn_estimated = std(n);      % actual
Sn_measured
```

```
Sn_measured = 12
```

2. In a second experiment, an excitation signal is applied to $P(s)$ (i.e. $u \neq 0$ and $n \neq 0$ ). The noise standard deviation $\sigma_n$ is assumed to be the same in this experiment as in the first experiment. Given that the SNR measured at $z$ is +10 dB, determine the approximate standard deviation of the excitation to the plant $\sigma_u$ for this condition.

## **Solution**

Signal-to-noise ratio (SNR) = +10dB $\Longrightarrow$ 20*$\log_{10}(x) = 10$ $\Longrightarrow$3.16228

Noise standard deviation $\sigma_n$ = 12

Magnitude of Signal output y = 3.16228*12 = 37.947

Now with knowledge of standard deviation of y, we can find:

$\sigma_u$ = ?

```
% Problem 2

SNR_desired = 10;    % dB

Sy_desired = Sn_measured * 10^(SNR_desired/20)
```

```
Sy_desired = 37.9473
```

```
Su_guess = 1        % initial guess
```

```
Su_guess = 1
```

```
u = Su_guess *  randn(1, N);
```

```
y = lsim(P, u, t);
Sy_estimated = std(y)                    % actual
```

```
Sy_estimated = 0.1359
```

```
Su_guess = Sy_desired / Sy_estimated    % actual % Required Answer
```

```
Su_guess = 279.1591
```

```
u = Su_guess *  randn(1, N);

y = lsim(P, u, t);
Sy_estimated = std(y)
```

```
Sy_estimated = 38.2195
```

```
SNR_actual = 20*log10(Sy_estimated / Sn_measured)
```

```
SNR_actual = 10.0621
```

```
SNR_desired
```

```
SNR_desired = 10
```

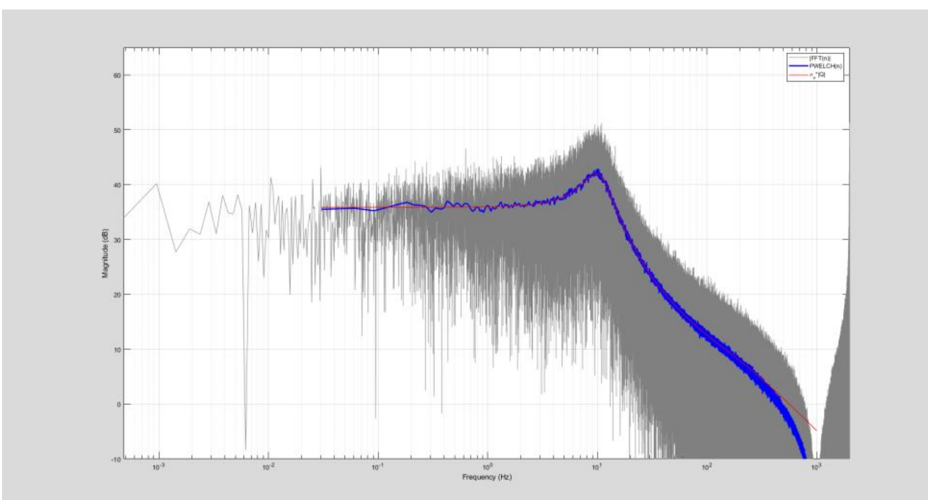
3. Use the Matlab function `fft()` to compute the spectrum of the sampled noise signal $n$ from problem 1. Also use the Matlab function `pwelch()` to compute the spectrum of the sampled noise signal $n$ from problem 1. Plot both properly

scaled spectrums on a single axes using dB magnitude and log frequency scales. On the same axes, overlay the frequency response magnitude of the filter $Q$. All three curves should overlay fairly closely. Make sure you properly label both the axes and generate an appropriate legend for this figure.

$$\text{abs}\left(\frac{\text{fft}(n)}{\sqrt{\text{Length of Signal } n}}\right)$$

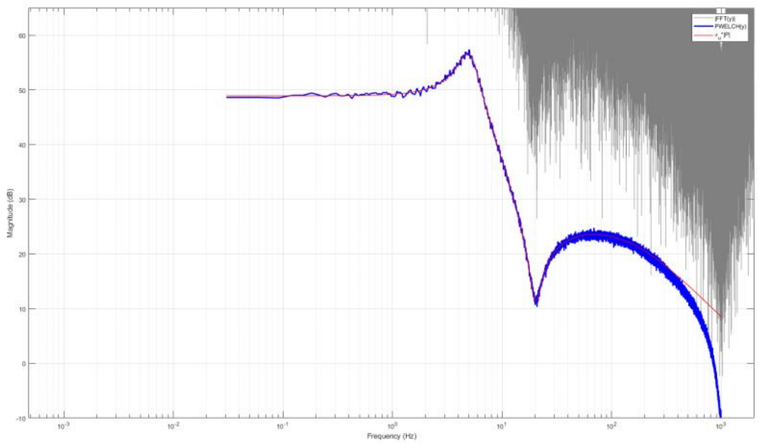
```
% Problem 3

NOISE_FFT = fft(n) / sqrt(N); % complex

F = fs/N;
fr1 = F * [0:N-1];

nfft = 2^16;  % tuning parameter
wndo = nfft;
ovlp = nfft/2;
```

```
% fr2 is 0 to fs/2
% NOISE_PWELCH is Power spectral density
[NOISE_PWELCH, fr2] = pwelch(n, wndo, ovlp, nfft, fs);

NOISE_PWELCH = NOISE_PWELCH * (fs/2);

Q_MAG = Sv_estimated * squeeze(freqresp(Q, 2*pi*fr2));

figure()
whitebg(gcf, 'w');
subplot(1,1,1);
semilogx(fr1, 20*log10(abs(NOISE_FFT)), 'color', 0.5*[1 1 1])
grid on
hold on
semilogx(fr2, 10*log10(abs(NOISE_PWELCH)), 'b', 'linewidth', 2)
semilogx(fr2, 20*log10(abs(Q_MAG)), 'r', 'linewidth', 1)
xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)')
xlim([fr1(2), max(fr1)])
ylim([-10, 65])
legend('|FFT(n)|', 'PWELCH(n)', '\sigma_v*|Q|')

% In order to maximize the figure window in Windows
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
```



4. Use the Matlab function `fft()` to compute the spectrum of the sampled output signal $y$ from problem 2. Also use the Matlab function `pwelch()` to compute the spectrum of the sampled signal $y$ from problem 2. Plot both properly scaled spectrums on a single axes using dB magnitude and log frequency scales. On the same axes, overlay the frequency response magnitude of the filter $P$. All three curves should overlay fairly closely. Make sure you properly label both the axes and generate an appropriate legend for this figure.

```
% Problem 4

Y_FFT = fft(y) / sqrt(N);

[Y_PWELCH, fr2] = pwelch(y, wndo, ovlp, nfft, fs);

Y_PWELCH = Y_PWELCH * (fs/2);

P_MAG = Su_guess * squeeze(freqresp(P, 2*pi*fr2));

figure()
subplot(1,1,1);
semilogx(fr1, 20*log10(abs(Y_FFT)), 'color', 0.5*[1 1 1])
grid on
hold on
semilogx(fr2, 10*log10(abs(Y_PWELCH)), 'b', 'linewidth', 2)
semilogx(fr2, 20*log10(abs(P_MAG)), 'r', 'linewidth', 1)
xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)')
xlim([fr1(2), max(fr1)])
ylim([-10, 65])
legend('|FFT(y)|', 'PWELCH(y)', '\sigma_u*|P|')

% In order to maximize the figure window in Windows
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
```




5a. Determine the exact autocorrelation for the following pseudo-random periodic signal:

| $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $\cdots$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | $-1$ | 1 | $-1$ | $-1$ | $x_0$ | $x_1$ | $\cdots$ |

5b. Determine the power of this signal.

5c. Sketch the approximate spectral magnitude of this signal.

## Solution:

**5a)**

$$\rho_{xx}(0) = (1\times1)+(1\times1)+(1\times1)+(-1\times-1)+(1\times1)+(-1\times-1)+(-1\times-1) = 7$$
$$\rho_{xx}(1) = (1\times1)+(1\times1)+(1\times-1)+(-1\times1)+(1\times-1)+(-1\times-1)+(-1\times1) = -1$$
$$\rho_{xx}(2) = (1\times1)+(1\times-1)+(1\times1)+(-1\times-1)+(1\times-1)+(-1\times1)+(-1\times1) = -1$$
$$\rho_{xx}(3) = (1\times-1)+(1\times1)+(1\times-1)+(-1\times-1)+(1\times1)+(-1\times1)+(-1\times1) = -1$$
$$\rho_{xx}(4) = (1\times1)+(1\times-1)+(1\times-1)+(-1\times1)+(1\times1)+(-1\times1)+(-1\times-1) = -1$$
$$\rho_{xx}(5) = (1\times-1)+(1\times-1)+(1\times1)+(-1\times1)+(1\times1)+(-1\times-1)+(-1\times1) = -1$$
$$\rho_{xx}(6) = (1\times-1)+(1\times1)+(1\times1)+(-1\times1)+(1\times-1)+(-1\times1)+(-1\times-1) = -1$$
$$\rho_{xx}(7) = (1\times1)+(1\times1)+(1\times1)+(-1\times-1)+(1\times1)+(-1\times-1)+(-1\times-1) = 7$$

original sequence    shifted sequence

**5b)**

$$\sigma_x^2 = R_{xx}(0) = E\left[x_k^2\right] = \left(\frac{\rho_{xx}(0)}{N}\right) = \left(\frac{7}{7}\right) = 1$$

**5c)**

$$N = 7 \qquad W = e^{-j2\pi/N} = (0.623 - j0.782) \qquad X(mF) = \sum_{n=0}^{6} x_n W^{mn}$$

```
% Problem 5

% Part_A
xk = [1 1 1 -1 1 -1 -1];
N = 7;
```

```
col = xk';
row = [xk(end), xk(1:end-1)];

Y = hankel(col, row)
```

```
Y = 7×7
     1     1     1    -1     1    -1    -1
     1     1    -1     1    -1    -1     1
     1    -1     1    -1    -1     1     1
    -1     1    -1    -1     1     1     1
     1    -1    -1     1     1     1    -1
    -1    -1     1     1     1    -1     1
    -1     1     1     1    -1     1    -1
```

```
Rxy = (1/N) * Y * xk'
```

```
Rxy = 7×1
    1.0000
   -0.1429
   -0.1429
   -0.1429
   -0.1429
   -0.1429
   -0.1429
```

```
% Part C

W = exp(-1i*2*pi/N)
```

```
W = 0.6235 - 0.7818i
```

```
for m = 1:N
    X(m) = 0;
    for n = 1:N
        X(m) = X(m) + xk(n)*W^((m-1)*(n-1));
    end
end
abs(X)
```

```
ans = 1×7
    1.0000    2.8284    2.8284    2.8284    2.8284    2.8284    2.8284
```