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Q1 To find maximum likelihood estimates of the parameters θ_1 (mean) and θ_2 (variance) for a normal distribution, we will use likelihood function and then maximize it.

Ans Given that $X_1, X_2, \ldots, X_n$ is a random sample from a normal distribution with mean θ_1 and variance θ_2, the likelihood function is:

$$L(\theta_1, \theta_2 \mid x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

Taking log on both sides:

$$\ln L(\theta_1, \theta_2 \mid x_1, x_2 \ldots x_n)$$

$$= -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

To find MLE, we will differentiate the log-likelihood with respect to θ_1 and θ_2, set derivative equal to zero.

(i) For θ_1:

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2 \mid x_1, x_2, \ldots x_n) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

Setting this equal to zero:

$$\frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)=0 \Rightarrow \sum_{i=1}^{n}(x_i-\hat{\theta}_1)=0$$

$$\therefore \hat{\theta}_1=\frac{1}{n}\sum_{i=1}^{n}x_i$$

(ii) For θ_2:

$$\frac{\partial \ln L(\theta_1,\theta_2|x_1,x_2,\dots x_n)}{\partial\theta_2}=\frac{-n}{2\theta_2}+\frac{1}{2\theta_2^2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Setting this equal to zero:

$$-\frac{n}{2\hat{\theta}_2}+\frac{1}{2\hat{\theta}_2^2}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2=0$$

$$\Rightarrow \frac{n}{2\hat{\theta}_2}=\frac{1}{2\hat{\theta}_2^2}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2$$

$$\Rightarrow \hat{\sigma}_2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2$$

So, MLE for θ_2 is the sample variance

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Q2 ~~To find MLE of θ for~~
Let $X_1, X_2 - - X_n$ be a random sample from $B(m, \theta)$ distri, where $\theta \in \Theta = (0,1)$ is unknown and 'm' is known +ve integer. Compute value of θ using MLE.

Ans The likelihood for this scenario is:

$$\Rightarrow L(\theta | x_1, x_2 - - x_n) = \prod_{i=1}^{n} P(X_i = x_i | \theta)$$

Since X_i follows a Bernoulli dist.

$$P(X_i = x_i | \theta)$$

$$= \theta^{x_i} (1-\theta)^{m-x_i} \text{ for each } i$$

$\Rightarrow$ Taking log on both sides:

$$\ln L(\theta | x_1, x_2, - x_n) = \sum_{i=1}^{n} \ln(\theta^{x_i} (1-\theta)^{m-x_i})$$

$$= \sum_{i=1}^{n} (x_i \ln\theta + (m - x_i) \ln(1-\theta))$$

Now differentiate w.r.t θ and set to zero.

$$\frac{d}{d\theta} (\ln L(\theta | x_1, x_2 - x_n)) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right) = 0$$