DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
MINOR TEST-II 2013-2014 SECOND SEMESTER
MAL 230 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour **Max. Marks: 25**

1a. If A= tridiag $\{1,4,1\}$, find an upper bound for $\|A^{-1}\|_\infty$. (3)

1b. Consider the linear system $Ax = b$ where

$$A = \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{5} \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} \frac{7}{12} \\ 0.45 \end{bmatrix}. \quad \text{Suppose} \quad b + \delta b = \begin{bmatrix} 0.583 \\ 0.45 \end{bmatrix}.$$

Find a bound for the estimate

$$\frac{\|\delta x\|_\infty}{\|x\|_\infty}.$$

(5)

2. Find the optimal relaxation factor ω_{opt} if the following linear system is solved by Relaxation method.

$$\begin{aligned} 4x + 0y + 2z &= 4 \\ 0x + 5y + 2z &= -3 \\ 5x + 4y + 10z &= 2 \end{aligned}$$

(4)

3. Using Given's method transform the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

to the tridiagonal form. Hence write the Strum sequence. Also find the number of eigenvalues lying in the interval $(-2,2)$ and in the interval $(5,6)$ using Sturm theorem. (5)

4a. True or false justify the statement:
The interpolating polynomial for $f(x) = x^{n+1}$ interpolating at the points $x_0, x_1, \ldots\ldots, x_n$ is given by $x^{n+1} - (x - x_0)(x - x_1)\ldots\ldots(x - x_n)$. (2)

4b. Consider the problem of finding a quadratic polynomial $p(x)$ for which $p(x_0) = y_0$, $p'(x_1) = y_1'$, $p(x_2) = y_2$ with $x_0 \neq x_2$ and $\{y_0, y_1', y_2\}$ the given data. Assuming that the nodes x_0, x_1, x_2 are real, what conditions must be satisfied for such a $p(x)$ to exist and be unique?. (3)

4c. Let $f \in C^4[a,b]$. Let $x = a$ and $x = b$ be the nodes and $H(x)$ be Hermite interpolating polynomial of f. Then prove or disprove

$$\|f - H\|_\infty \leq \frac{(b-a)^4}{384} \|f^{(4)}\|_\infty.$$

(3)