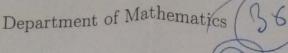
MTL 100: Calculus Major: 2017-18 Semester I



Total marks: 45 Max Time: 2 hrs

Using the linear transformation u = x - y and v = x + y, evaluate the double integral

$$\iint\limits_{R} (x-y)^2 \sin^2(x+y) dx dy,$$

where R is the parallelogram with vertices $(\pi, 0), (2\pi, \pi), (\pi, 2\pi), (0, \pi)$.

- Find the maximum of the function $f(x, y, z) = \log x + \log y + 3 \log z$ on that portion of the sphere $x^2 + y^2 + z^2 = 20$ where x > 0, y > 0, z > 0.
- 3. (a) Test the convergence of the improper integral

$$\int_0^\infty (\log x)^2 e^{-x^2} dx.$$
 (b) Show that $\int_0^1 (x - x\sqrt{x})^{\frac{1}{2}} dx = \frac{32}{105}$. [4+3]

4. (a) Let f be a continuous function on [a,b] and define a function $F:[a,b]\to\mathbb{R}$ by

$$F(x) := \int_{a}^{x} f(t)dt.$$

Then prove that F is differentiable on (a, b) and for every $x \in (a, b)$, F'(x) = f(x).

- Prove that if g is Riemann integrable on [a, b], then |g| is also Riemann integrable on [a, b].
- 5. Find the linear approximation of the function $f(x,y) = x^2 + xy + y^2$ at the point (1.1) and also estimate the approximation error in the region $R = \{(x,y) : |x-1| \le 1, |y-1| \le 1\}.$
- Show that $f(x,y) = \begin{cases} \frac{x^2y|y|^{\alpha}}{x^4 + y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) \not\equiv (0,0). \end{cases}$
 - (a) f is continuous at (0,0) for all $\alpha > 0$;
 - (b) f is differentiable at (0,0) for all $\alpha > 1$. [3+3]
- Test the uniform continuity of the function $f(x) = x \sin x$ on $[0, \infty)$. [4]

Which of the following statements are TRUE/FALSE: Justify your answer.

- (a) If a real sequence $\{x_n\}$ satisfies $|x_{n+1} x_n| < \frac{1}{n}$ for every $n \in \mathbb{N}$, then $\{x_n\}$ is a Cauchy sequence.
- (b) The sequence $x_n = \left(1 \frac{1}{n}\right) \cos \frac{n\pi}{2}, n \in \mathbb{N}$, is convergent. [3+3]