Applied Game Theory (HUL 311)

Major examination

1st May, 2015

Time limit: Two hours

Attempt all questions.

[Note: Any answer (or attempt) without companion explanation will fetch 'no credit'. Please provide adequate rationale for every notation/model/result/computation that you use or carry out.]

1. There are $n \geq 1$ shareholders (henceforth, players) who together own a firm. Each player $i$ chooses an effort level $x_i \geq 0$, resulting in total profit $g(y)$ for their firm, where $y$ is the sum of all players efforts. The profit function $g : R^+ \rightarrow R^+$ satisfies $g(0) = 0$ and it is twice differentiable with $g' > 0$, and $g'' \leq 0$. The profit is shared equally by the players, and each players effort gives her disutility. The resulting utility level for each player $i$ is $u_i(x_1, .., x_n) = \frac{1}{n}g(x_1+, \ldots, +x_n) - \frac{x_i^2}{2}$ Each player $i$ has to decide her effort $x_i$ simultaneously.

(a) Show that the game has exactly one Nash equilibrium (in pure strategies), and show that all players make the same effort, $x^*$, in equilibrium. (A precise and formal argumentation is required.) Is the individual equilibrium effort $x^*$ increasing or decreasing in $n$, or is it independent of $n$? Is the aggregate equilibrium effort, $y^* = nx^*$, increasing or decreasing in $n$, or is it independent of $n$? [2+2+2+1.5]

(b) Suppose that the players can (pre-)commit to a common effort level, the same for all. Let $\hat{x}$ be the common effort level that maximizes the sum of the players utilities. Characterize $\hat{x}$ in terms of an equation, and compare this level with the equilibrium effort $x^*$in (a), for $n = 1, 2, ....$ Are the players better off now than in the equilibrium in (a)? How does this depend on $n$? Explain. [2+1.5+1]

2. (a) Two neighbor states are engaged in a dispute. Each state may choose to take a Hard line or a Soft line. Each prefers to take a Hard line if the other

takes a Soft line, and each prefers to take a Soft line if the other takes a Hard line. However, given its own position, each state prefers the other take a Soft line. Write down a bi matrix that might represent the strategic form of this game. [2]

(b) Prove by the method of contradiction that: "any Nash equilibrium strategy profile consists of rationalizable strategies". [2]

4. Two players, 1 and 2 bargain on how to split $v$ dollars. The rules are as follows. The game begins in period 1 and player 1 makes an offer of a split to player 2. A split is any real number between $[0, v]$. Player 2 can then accept or reject the split. If she accepts, then the game ends. If she rejects it then the game continues to another round where player 2 gets to make an offer of a split which player 1 can either accept or reject. If no agreement is reached in $T$ periods, then both players get 0. Additionally, there is a discount factor $\delta \in (0, 1)$ so that a dollar received in period $t$ is worth $\delta^{t-1}$ dollars in period 1 dollars. (Note that this model has assumed that both the players are equally impatient, i.e. $\delta_1 = \delta_2 = \delta$.)

What is SPNE of this game when $T$ is odd? Compute when does the agreement take place. Also compute the pay-offs to each player when SPNE is reached. [7]

5. Consider a duopoly game between two firms who are producing similar products. The demand for the product of firm $i$ is given by $Q_i = a - bP_i + dP_j$, where $b > d > 0$, $i = 1, 2$. Firm $i$ has a per unit cost of production $c_i > 0$, $i = 1, 2$. While it is a common knowledge that for any firm $i$, $c_i$ can take either of two values: $c_i^h$ or $c_i^l$ (where $c_i^h > c_i^l$) with probabilities $p_i^h$ and $p_i^l$ respectively, only firm $i$ knows $c_i$, $i = 1, 2$. If the firms are using quantity produced as their strategic variable, then characterize the Bayes' Nash Equilibrium (BNE) in this case. You must compute the BNE as well as indicate it on a diagram with the intersection of expected best response functions. [7]