Time : 2 Hours   **MAL 382 / MTL783– Theory of Automata**   Full Marks - 40
May 10, 2016

Note: In any of the questions if you design a Turing Machine - then first state algorithmically how you intend the machine to work before giving the transitions. For each important step of the algorithms you draw figure to explain how the transitions are expected to perform.

Q1. a) Design a Turing Machine that takes as input two integers m, n in unary, and computes the quotient and residue when m is divided by n.

b) Hence (or otherwise) design a Turing Machine that takes as input strings of the form $\{a^k \mid k > 0\}$, and tests if the input belongs to the language $\{a^{n!} \mid n > 0)$

[5 + 5 = 10]

Q2. a) Write the grammar for the language $\{ww \mid w \in \{a + b)^+\}$. Explain how it generates the strings.

b) Give the scheme of a Turing Machine that accepts the above language.

[5 + 5 = 10]

Q3. a) Consider the language $L = \{a^n b^{2n} \mid n > 0\}$. Explain how a stack can be used to accept this language.

b) Write a TM that simulates the stack behaviour to accept this language.

[4 + 6 = 10]

Q4. a) What is the difference between Universal Language (L_u) and Universal Turing Machine?

b) How will you define the complement language of L_u? Is it Recursively Enumerable? Justify your answer with a proof.

[4 + 6 = 10]