

1. The Lena image is placed at the input plane of a 4F optical processor and the output images corresponding to three different filters in the Fourier transform plane are shown in Figures (a), (b), (c) respectively. Describe the form of the filter functions (with appropriate reasoning) that may have resulted in these three different outputs. (15 points)

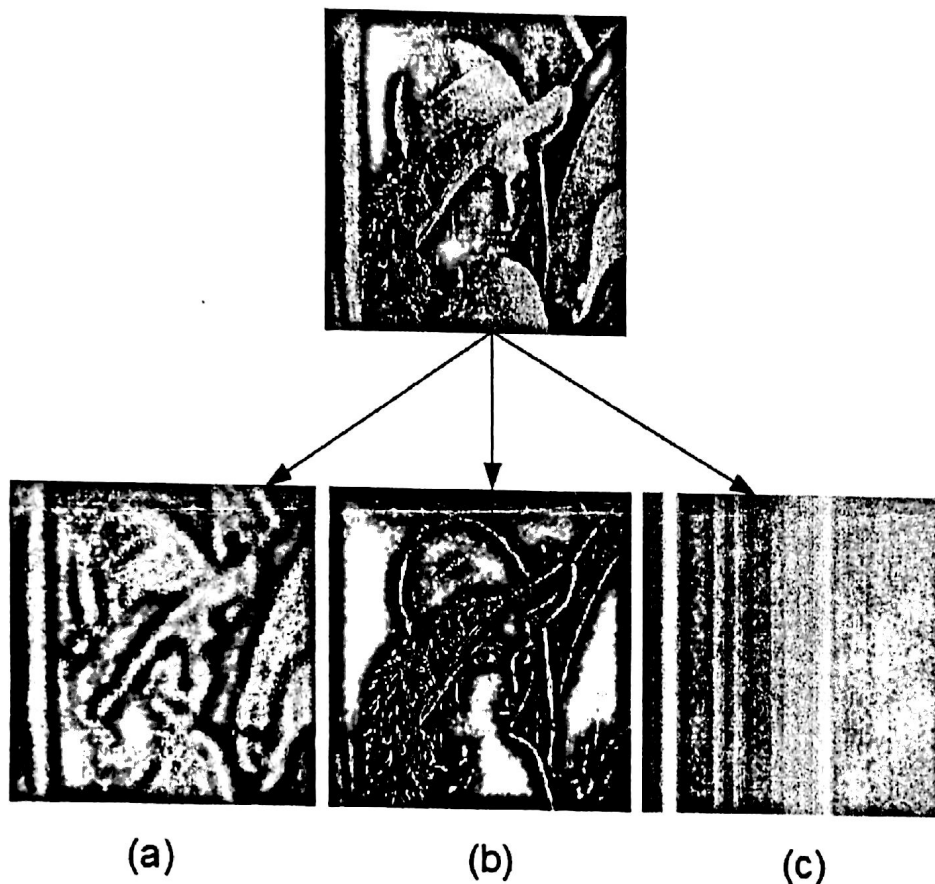


Figure 1: The effect of three different filters on the Lena object function is shown in Figures (a), (b), (c) respectively.

2. The transfer function for free-space diffraction in terms of the angular spectrum is given by:

$$H(f_x, f_y; z) = \exp(i\alpha z).$$

- ✓ (a) Starting with the Helmholtz equation  $(\nabla^2 + k^2)U(x, y; z) = 0$  find the value of  $\alpha$ . (5 points)
- ✓ (b) Describe the nature of the transfer function when  $k^2 \geq 4\pi^2(f_x^2 + f_y^2)$  and  $k^2 < 4\pi^2(f_x^2 + f_y^2)$ . (5 points)

(c) A diffraction grating with a transmission function:

$$t(x, y) = \frac{1}{2} \left[ 1 + \cos\left(2\pi \frac{x}{a}\right) \right],$$

Show that a more accurate expression for the Talbott (or self-imaging) distance corresponding to the grating is given by:

$$z_T = \frac{\lambda}{1 - \sqrt{1 - \lambda^2/a^2}}.$$

(5 points)

3. An image is printed on a transparency using a printer with resolution  $M$  dots per mm and the transparency is used as an object in a F-Lens-F Fourier transform setup. The transparency is illuminated using a plane wave of wavelength  $\lambda$  travelling along the optical axis of the system. Estimate the size of an array sensor (in terms of number of pixels) that is required to observe the highest detail in the object transparency. Assume square pixels of dimension  $p \mu\text{m}$ . (5 points)

4. An imaging system has a centrally obscured square aperture of size  $a \times a$ . The central  $a/3 \times a/3$  square part of the aperture is opaque. Sketch (approximately) the MTF curve for the system along the  $x$ -axis. (5 points)

$$\frac{1}{\sqrt{1 + (x^2 + y^2)}}$$

$$t(x, y) = \frac{1}{2} \left( 1 + \cos\left(2\pi \frac{x}{a}\right) \right)$$

1.2

$$e^{i\pi p^2 \cos^2 \theta}$$



$$g(x, y)$$



$$g_0(x, y)$$