

(Part A : Answer on these sheets only; Be Precise & Relevant)

1. State the degrees of freedom for the following :

[4]

(a) System of two miscible, non-reacting species which exists as an azeotrope in vapour-liquid equilibrium.

(b) A system prepared by partially decomposing CaCO_3 into an evacuated space. $\text{Rx}^n : \text{CaCO}_3 (\text{s}) \rightarrow \text{CaO} (\text{s}) + \text{CO}_2 (\text{g})$

2. For solid-liquid equilibria, write the exact expression to evaluate ψ_i using H_i^l and H_i^s . ψ_i is the ratio of pure component fugacities f_i^s/f_i^l .

[6]

If temperature effect on enthalpies can be ignored, then ψ_i can be written

If liquid phase is an ideal soln and in the solid phase both species are completely immiscible, then liquid mole fractions x_1 and x_2 are :

3. For the micro-canonical ensemble with its natural variables (E, V, N), do E and N fluctuate ? For this ensemble, write the expression/relation for (i) probability of state ν , P_ν (ii) entropy, S and (iii) Boltzmann factor β (= $1/k_B T$).

[6]

4. Write the natural variables for the Grand canonical ensemble. Which system does this ensemble represent and what are the two variables which fluctuate ?

[8]

For the g-canonical ensemble write the probability P_ν and the partition function, Ξ . Starting with the Gibbs entropy formula, obtain the expression for S in terms of Ξ and other variables. What is $\ln \Xi = ?$

5. What is the significance of Curie temperature ? [8]

In words state what is the mean field approximation (w.r.t. Ising magnet model) and what is its utility ?

Write the mean field $\langle H_i \rangle$ in terms of H, J, z, s_i and μ .

6. Why do surfactant molecules 'self-assemble' ? [8]

When are spherical micelles more expected ?

For a system of non-interacting α -micelles the partition function is given below :

$$Q^{id} = \prod_{\alpha=1}^N \left[(V q_{\alpha}^{int} / \Lambda_{\alpha}^3)^{n_{\alpha}} / n_{\alpha} ! \right] \quad ; \quad n_{\alpha} = N_{\alpha} / \alpha$$

From this obtain the chemical potential, μ_{α} .

1. An isomerization reaction, $A \rightarrow B$, occurs in liquid phase. A and B are miscible liquids for which

$$G^E/RT = x_A x_B$$

For the reaction $\Delta G^\circ = -1000 \text{ J/mol}$ at 25°C .

(a) what is the equilibrium composition of the mixture, if the reaction occurs at 25°C ?

(b) how much error is there if A and B are taken to form an ideal solution ?

[10]

2. (a) Starting with entropy S which is natural function of (E, V, N) , obtain the Legendre transforms of entropy that are natural functions of (i) (T, V, N) and (ii) $(1/T, V, \mu/T)$. Show all steps clearly.

(b) The canonical partition function for a gas (system of N particles each of mass m) is given below :

$$Q(N, V, \beta) = (1/N!) [(2\pi m/h^2 \beta)^{3N/2} (V - Nb)^N e^{-\beta a N^2/V}] ; \quad a, b \text{ are constants}$$

(i) Although it is not so, assuming it to be a system of non-interacting, indistinguishable particles, what is the equivalent single particle partition function 'q' ?

(ii) From the system partition function Q , obtain the equation of state of the gas.

[10]

3. Consider a system of N distinguishable, non-interacting spins, n_i (each of magnetic moment μ) in an applied external magnetic field, H . Each spin can take only two states, $+1$ or -1 , i.e. parallel or anti-parallel to the applied field H . However total magnetization M is fixed, i.e. in an ensemble all members have the same number of up spins, n_+ . $N = n_+ + n_-$. E (energy) and H, N are the natural variables of the ensemble to be used.

(a) Which ensemble should be used ? What are M and E for the above ?

(b) Obtain the entropy S and the relation for $\beta (= 1/k_B T)$

(c) For large N , find βH in terms of μ, N, n_- and H

(d) From (c) get n_+ and hence M

[12]

4. In the lattice gas model, $n_i = 0$ or 1 denotes whether a cell is empty or occupied by the particle. The interaction energy between two neighbouring particles 1 and 2 is given by

$$\epsilon_{12} = -\epsilon \quad \text{if } n_1 = n_2 = 1 ; \text{ and } = 0 \text{ otherwise}$$

If the lattice has N cells, find the total energy E_p and the particles N_p for use in the grand canonical partition function Ξ . Use also the variable transformation $s_i = 2n_i - 1$, i.e. s_i takes the values ± 1 , as in the Ising magnet model.

Write Ξ as Q with E_{eff} to correspond to the Ising model and find the corresponding lattice gas J and H .

State also for the lattice gas what is (i) the order parameter (ii) the equivalent of Curie point.

[8]