

# CSL 101 Introduction to Computers and Programming

Major, Sem I 2007-08, Max 70, Time 2 hr

Name \_\_\_\_\_ Entry No. \_\_\_\_\_ Group \_\_\_\_\_

Note (i) This question paper has 5 pages

(ii) Write only in the space provided below each question. You can use the extra sheets for rough work.

(iii) Write your answers neatly and precisely. You won't get a second chance to explain what you have written. Partial marks will be hard to come by.

1. Suppose Java did not support two dimensional arrays. Then a matrix  $A^{m,n}$  with  $m$  rows and  $n$  columns can be allocated space in a one dimensional array  $B$  of size  $m \times n$ . A well-defined simple mapping from  $A^{m,n}[i, j]$  to  $B[k]$  is based on *row major* indexing scheme where all the elements of row 1 appears before row 2 and so on. For a fixed row  $i$ , the elements of the columns are in consecutive locations.

(i) Which location of  $B$  stores  $A^{m,n}[i, j]$  ? (All indices begin from 0). (2)

(ii) The transpose of a matrix is defined as redistributing data such that the elements  $A^{m,n}[i, j]$  and  $A^{m,n}[j, i]$  are exchanged. For example, the transpose of the matrix  $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$  is  $\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$ .

Complete the following, given that  $m = n$  that transposes the matrix  $A^{m,n}$  stored in the 1 dimensional array  $B$  without using any extra arrays - this is called in-place transpose. After transposing, the contents of  $B$  must correspond to the transpose of  $A^{m,n}$  according to the row-major map. (10)

```
public void transpose ( int [] B , int m , int n) {

    int temp; //no other variables can be used

    for (          ;          ;          ) {

        for (          ;          ;          ) {

            // inner for

        } //outer for

    } //end of function
```

2. For many functions, the indefinite integrals are very difficult to compute or not even known. To compute the area under these functions, a common computational method is the *trapezoidal rule*. For a given function  $f$ , and an interval  $[a, b]$  over which we want to compute the area, we first split the interval into  $N$  (to be determined later) equal sized sub-intervals each of length  $(b - a)/N$ . We obtain the area by summing the area within each sub-interval and the area within each sub-interval  $[x_1, x_2]$  is approximated by  $\left( \frac{f(x_1) + f(x_2)}{2} \cdot (x_2 - x_1) \right)$ . It is known that the total error in computing area by this method is  $\frac{1}{12} f''(\zeta) \frac{(b-a)^3}{N^2}$  for some  $\zeta \in [a, b]$ . Given a function  $f$ , we want to approximate the area in a given interval  $[a, b]$  with error bounded by  $D$ . Assume that we are also given a bound  $M$  on  $f''(\zeta)$  as part of input. Write a function in either Ocaml that computes the area under  $f$ . (10)

3. Consider an array A of integers of length N. We would like to think of this as a "circular array", i.e., after the last element A[N-1], we go to A[0] (see the picture below for a circular array of length 6). We would like to implement a list of numbers using this array. We maintain two variables start and last – start is equal to the first element of the list of numbers and last is the last element in the list. For example, suppose the array is storing the list of numbers {3,6,5}. If start = 2, last = 4, then A[2]=3, A[3] = 6, A[4] = 5. Note that the variable last could be less than start. For example, if start = 5, last = 1, then A[5] = 3, A[0] = 6, A[1] = 5 (see the figure below).



Complete the following JAVA code which maintains a list of numbers and adds or removes elements from either the beginning or the end of the list. Note some important points :

- If the list is empty, the values of start, last = -1.
- The array can not store a list of size more than N.

In the code below, assume that N is some fixed number.

(3 × 4)

```
class ListofNumbers {

    int[] A;
    int start, last;

    public ListofNumbers() {
        A = new int[N];
        start = -1;
        last = -1;
    }

    public void InsertBeginning(int x) { // insert x at the beginning of the list

    }

    public void InsertEnd(int x) { // insert x at the end of the list
```

```

    }
    public void RemoveEnd(int x) { // remove from the end of the list

}

}

```

The methods InsertBeginning and InsertEnd should print the message "Array is Full" if the array is already storing N numbers. Similarly, the method RemoveEnd should print the message "Array is empty" if the array is not storing any number.

4. An *expression* is defined as below.

```

type expression =
  Const of float
| Var of string
| Sum of expression*expression
| Diff of expression*expression
| Prod of expression*expression
;;

```

For example Prod ( Const 3.5 , Var "x" ) is a valid expression.

- (a) What are the valid expressions corresponding to (2 × 2)
- (i)  $5x + 7y$
- (ii)  $(xy + yz) \cdot x$
- (b) You can verify that the derivative of such an expression can be calculated using the rules of differentiation, and is also an expression. For example, deriv Var "x" "x" should be Const 1.0 where deriv exp var returns the derivative of the expression exp with respect to the variable var. Complete the following Ocaml program that returns derivative expression of a given input expression. (8)

```

let rec deriv exp dv =

  match exp with

    Const c -> Const 0.0
  | Var v -> if v = dv then Const 1.0 else -----

  | Sum (f,g) -> -----

  | Diff (f,g) -> -----

  | Prod(f,g) -> -----

```

5. Mergesort is described as a recursive function that sorts the first half of the elements (recursively), then sorts the other half recursively and subsequently merges the two sorted sets. The following program implements mergesort in an array WITHOUT using recursive calls. Since every element of the initial array can be regarded as 1 element sorted sequence, we can merge them into  $n/2$  sorted sequences of length 2. Subsequently the 2 element sequences can be merged into 4 element sequences and so on till there is one sorted sequence. At any stage we use the variable `runlength` to denote the length of the sorted sequences (initially 1). Also assume the length of the array is a power of 2, i.e.,  $n = 2^k$  for some  $k \geq 1$ . Complete the following function in Java that implements this strategy. Do NOT define any new variables. (9)

```
public static void mergesort(double [] A ) {

    int runlength = 2; // we need not start with runlength = 1 because
                       // sequences of length 1 are already sorted.

    int i, b ;

    while (runlength <= A.length) {

        for ( i = 0 ; _____ ; i = i+ runlength )

        {
            b = runlength/2 ;

            _____

        } // end for

        runlength = _____
    } // end while
} //end mergesort

public static void merge (int i1, int i2, int j1, int j2 , double [] A)
```

$A[i1] \dots A[i2]$  and  $A[j1] \dots A[j2]$  are two disjoint sorted subarrays where  $i1 \leq i2$  and  $j1 \leq j2$ . The function merges these two subarrays into a sorted sequence and stores the result in the same locations, i.e.,  $A[i1] \dots A[i2]$  and  $A[j1] \dots A[j2]$ . For example if  $A = \{10, 5, 6, 12, 15, 3, 1, 7, 9, 18\}$ ,  $i1 = 2$ ,  $i2 = 4$ ,  $j1 = 7$ ,  $j2 = 9$ , then after the function finishes,  $A = \{10, 5, 6, 7, 9, 3, 1, 12, 15, 18\}$ . You can use this function to merge two sorted portions of the array - you don't have to write the code.

- (ii) What is the number of comparisons used in the iterative mergesort ? (Show the analysis assuming that merging two sorted sequences takes time proportional to the sum of the lengths of the two sorted sequences). (3)

6. Complete the following program that copies one file to another except that consecutive blanks are replaced by a single blank. Recall that -1 is the equivalent integer (of character) for end-of-file. The blank character is denoted by ' '. Do NOT use the methods trim or split of the Java String class. Do NOT define any new variables. (12)

```
import java.io.* ;
class delblanks {

    public static void main (String[] args)
        throws IOException
    {

        FileReader fr = null;
        PrintWriter pr = new PrintWriter(new BufferedWriter(new FileWriter
            (args[1]))) ;

        fr = new FileReader(args[0]) ;

        int cc = 0 ;

        while ( _____ != -1 ) {
            cc = fr.read() ;

            if ( _____ == ' ' ) {

                pr.print( _____ ) ;

                while ( cc == ' ' ) {

                    cc = _____ ;

                }

            }
            if (cc != ' ' && cc != _____ ) {pr.print((char)cc) ; }
            //(char) cc converts the integer represented by cc to the corresponding ASCII
        }

        _____ ;
    } //main
} //class
```