DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI

MAJOR TEST 2013-2014 FIRST SEMESTER
MAL 230 (NUMERICAL METHODS AND COMPUTATION)

Time: 2 hours **Max. Marks: 36**

1a. Prove or disprove that Newton-Raphson method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ is a linear method with rate of convergence $1 - \frac{1}{p}$ when applied to find a root ξ of multiplicity p of $f(x) = 0$. (3)

1b. What happens if Newton's method is applied to system of linear equations $Ax = b$ where A is $n \times n$ non-singular matrix? (2)

2a. For the function f it holds that $| f^{(n+1)}(x) | \leq M$ for all $x \in [-1, 1]$. Then prove or disprove that there is a polynomial P_n of degree n such that

$$| f(x) - P_n(x) | \leq \frac{M}{2^n (n+1)!} \quad \text{for all } x \in [-1, 1].$$

(2)

2b. Approximate $f(x) = \sqrt[3]{x}$ by a straight line in the interval [0,1] in the maximum norm. Also, give the norm of the error function for this best approximation. (4)

3. The function $f(x)$ is supposed to be differentiable three times. Prove or disprove that

$$f(x) = \frac{-(x - x_1)(x - 2x_0 + x_1)}{(x_1 - x_0)^2} f(x_0) + \frac{(x - x_0)(x - x_1)}{x_0 - x_1} f'(x_0) + \frac{(x - x_0)^2}{(x_1 - x_0)^2} f(x_1) + E(x)$$

where

$$E(x) = \frac{(x - x_0)^2 (x - x_1)}{6} f'''(\xi), \qquad x_0 < \xi < x_1.$$

(4)

4. Find a quadrature formula

$$\int_0^1 f(x) \frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f(\frac{1}{2}) + \alpha_3 f(1)$$

which is exact for polynomials of highest possible degree. Use this formula to evaluate

$$\int_0^1 \frac{dx}{\sqrt{(x - x^3)}}.$$

(5)

5a. If A= tridiag $\{1, 4, 1\}$, find an upper bound for $\|A^{-1}\|_\infty$. (3)

5b. Find the ∞-norm of 3×3 Hilbert matrix

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}.$$

(2)

6. Find the optimal relaxation factor ω_{opt} if the following linear system is solved by Relaxation method:

$$\begin{aligned} 4x + 0y + 2z &= 4 \\ 0x + 5y + 2z &= -3 \\ 5x + 4y + 10z &= 2 \end{aligned}$$

(4)

7a. Using Given's method transform the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

to the tridiagonal form. Hence write the Strum sequence. Also find the number of eigenvalues lying in the interval $(-2, 2)$ and in the interval $(5, 6)$ using Sturm theorem. (5)

7b. Find the solution of the initial value problem $y' = -xy$, y(0)=1 at $x = 0.1$ using Taylor's second order method. (2)