Max Marks: 20

Time: 1 Hr

Each question carries equal marks.

Q1. A drunkard randomly remove two letters of the message **HAPPY HOUR** that is attached on a billboard of a pub creating two vacant spots. His drunk friend puts these two letters back on the vacant spots so created in the billboard but in a random order. The two friends then left the pub. What is the probability that this billboard again reads **HAPPY HOUR**?

Q2. Let the joint pdf of two continuous random variables X and Y be

$$f(x, y) = \begin{cases} e^{-(x+y)} & x \geq 0, \ y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find the conditional probability $P(X < Y \mid X < 2Y)$.

Q3. Suppose a noodle manufacturer stamps a gift coupon on packet the noodle with a probability 0.1, independently across the packets. Use the binomial distribution to answer the following:

(a) How many noddle packets do you have to buy to make sure with a 95% probability that you will find at least one coupon among the set?

(b) What probability should the manufacturer employ to print a coupon so that the chance that a customer finds at least one coupon in a set of 100 is reduced to 0.01?

Q4. Wisdom of crowds theory says that the opinion of the masses (or the mean of their opinions) is always more closer to the actual value (or the most probable) than that of a single person. It is asked to guess the weight of an ox. Eight people from the crowd give their guesses on the ox weight as 430, 530, 650, 900, 790, 250, 100, 390. (in kgs). Given that the weight of the ox is normally distributed with variance 2500 Kg, determine the probability that the ox weight lies between 450 Kg. and 600 Kg.

Q5. A company X buys an insurance policy for two years to insure its revenue in the event of major snowstorms that shut down the business. The insurance policy, after it comes into force, pays nothing for the first such major snowstorm. Afterwards, the policy pays Rs 10000 for each such major snowstorm until the end of policy term with company X. The number of major snowstorms per year that shut down the business of the company X is assumed to have a Poisson distribution with mean 0.75. What is the expected amount to be paid to the company X under this insurance policy?