APL100 MECHANICS MAJOR

5th May 2018

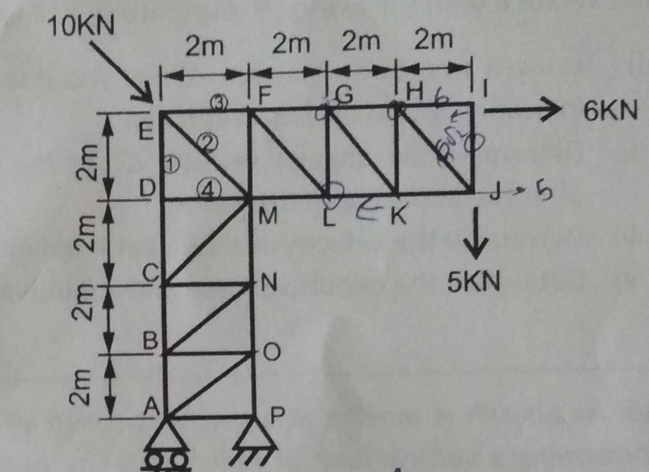
PART B (90 marks)

11.00 am to 12.30 pm

Questions are in order of increasing difficulty (according to the instructors).

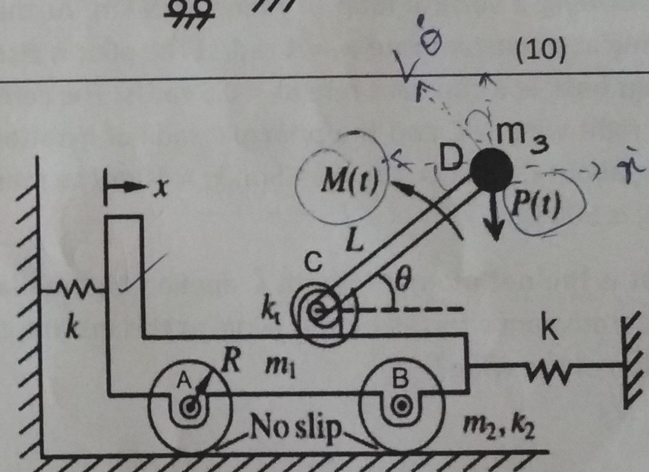
Q1 Consider a n d.o.f system with generalized co-ordinates $q_1, q_2, q_3, \ldots\ldots q_n$. A set of m external forces $F_1, F_2, F_3, \ldots\ldots F_m$ act at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots\ldots \vec{r}_m$ respectively. Using the definition of virtual work, δW, obtain an expression for the generalized force Q_i corresponding to the generalized co-ordinate q_i. (5)

Q2: In the truss shown all the members are light. Find the forces in members ①, ②, ③ and ④. Clearly indicate whether the members are in tension or compression.



(10)

Q3: For the system shown the generalized coordinates are x and θ. The bearings at A and B and the hinge at C are frictionless. **The rod CD is light** and m_3 may be treated as a point mass. The springs are un-deformed for $x = 0$, $\theta = 0$.



i) Determine the kinetic energy T and potential energy V for the system.

ii) Determine the generalized non-conservative forces Q_1^{nc} and Q_2^{nc}.

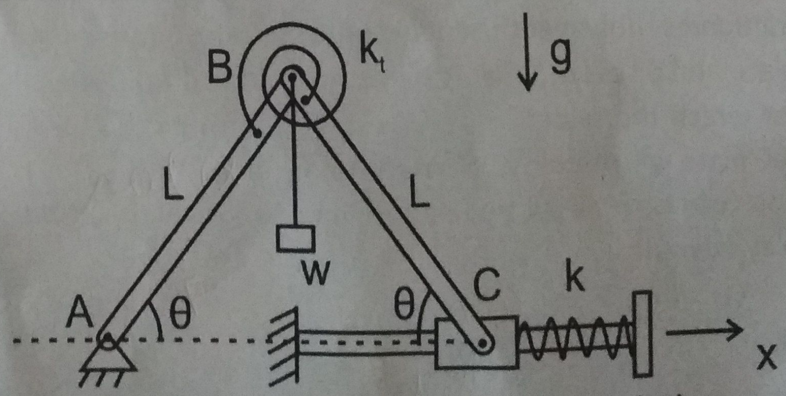
iii) Determine $\frac{\partial T}{\partial x}$; $\frac{\partial T}{\partial \dot{\theta}}$; $\frac{\partial V}{\partial \theta}$ and $\frac{\partial V}{\partial \dot{x}}$.

iv) Determine $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right)$.

Note: Each set of wheels (with centres at A and B) has a mass m_2 and k_2 is the radius of gyration about the wheel axis.

(16)

Q4: In the system shown, rods AB and BC are light. The springs are un-deformed for $\theta = \pi/2$.



i) Show that $\theta_1 = \pi/2$ is an equilibrium configuration for all values of k_t and k.

ii) For $k_t = 0$, determine the 2nd equilibrium solution θ_2. What is the condition for a distinct 2nd equilibrium solution to exist?

iii) For $k_t = 0$, determine the stability of θ_1.

(12)

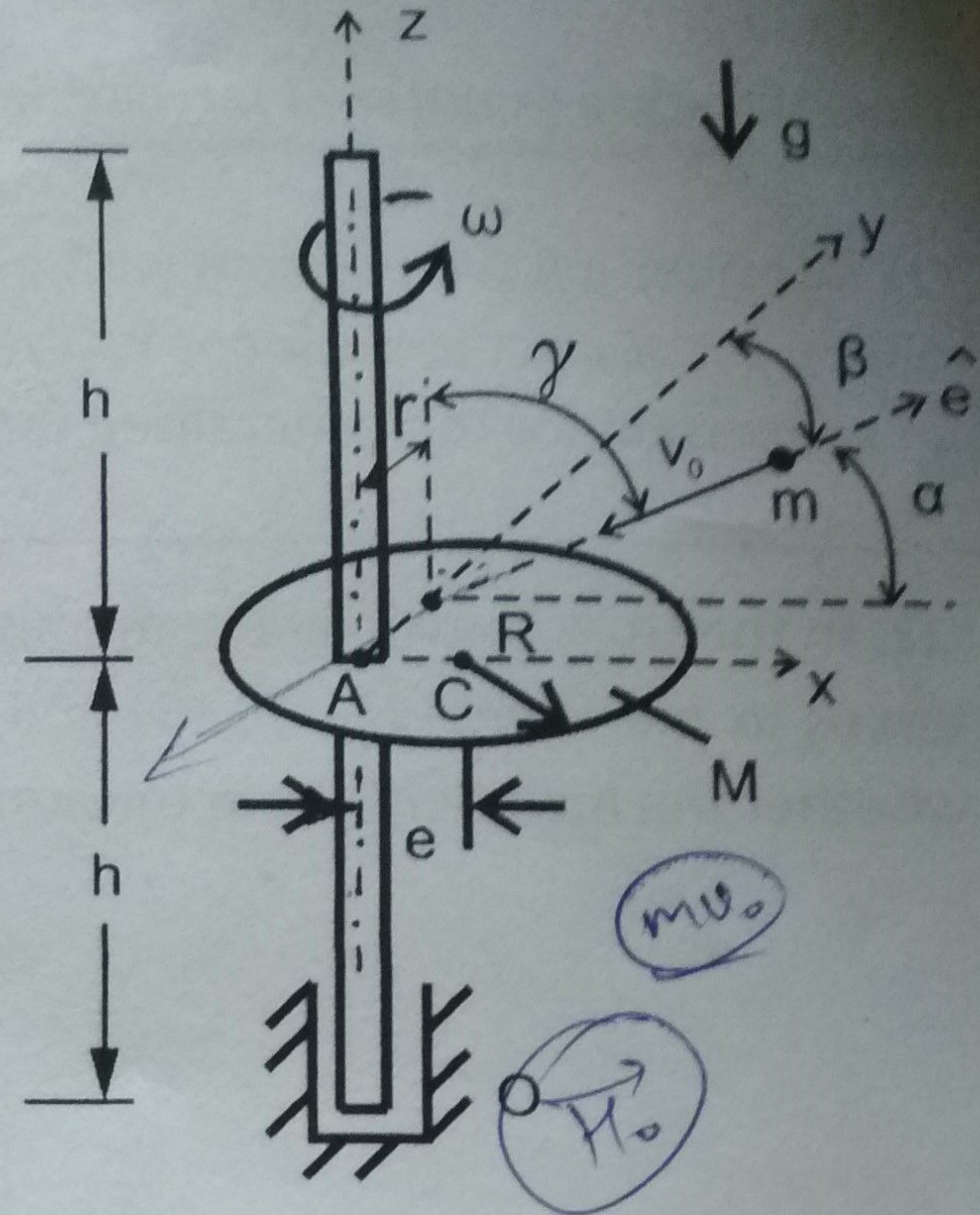
Q5: A disc of mass M is eccentrically and **rigidly** mounted on a light vertical rod at A as shown. The rod is supported on a long bearing at O which permits rotation **only about the z-axis**. The centre of mass of the disc is C and its radius is R. A small friction torque M_f acts at the bearing. The angular speed of the rod at this instant (t_0) is ω. At $t = t_0$, a bullet of mass m and velocity V_0 along ($-\hat{e}$ direction as shown) strikes the disc at location $r\hat{j}$ $(r < R)$ and gets embedded in it. The angles made by the unit vector $\hat{e}$ with the axes x, y and z are α, β and γ respectively.

i) Draw a free body diagram of the rod-disc-bullet system clearly indicating all the external reactions.

ii) Determine the angular velocity $\vec{\omega}'$ of the rod/disc immediately after the impact with the bullet.

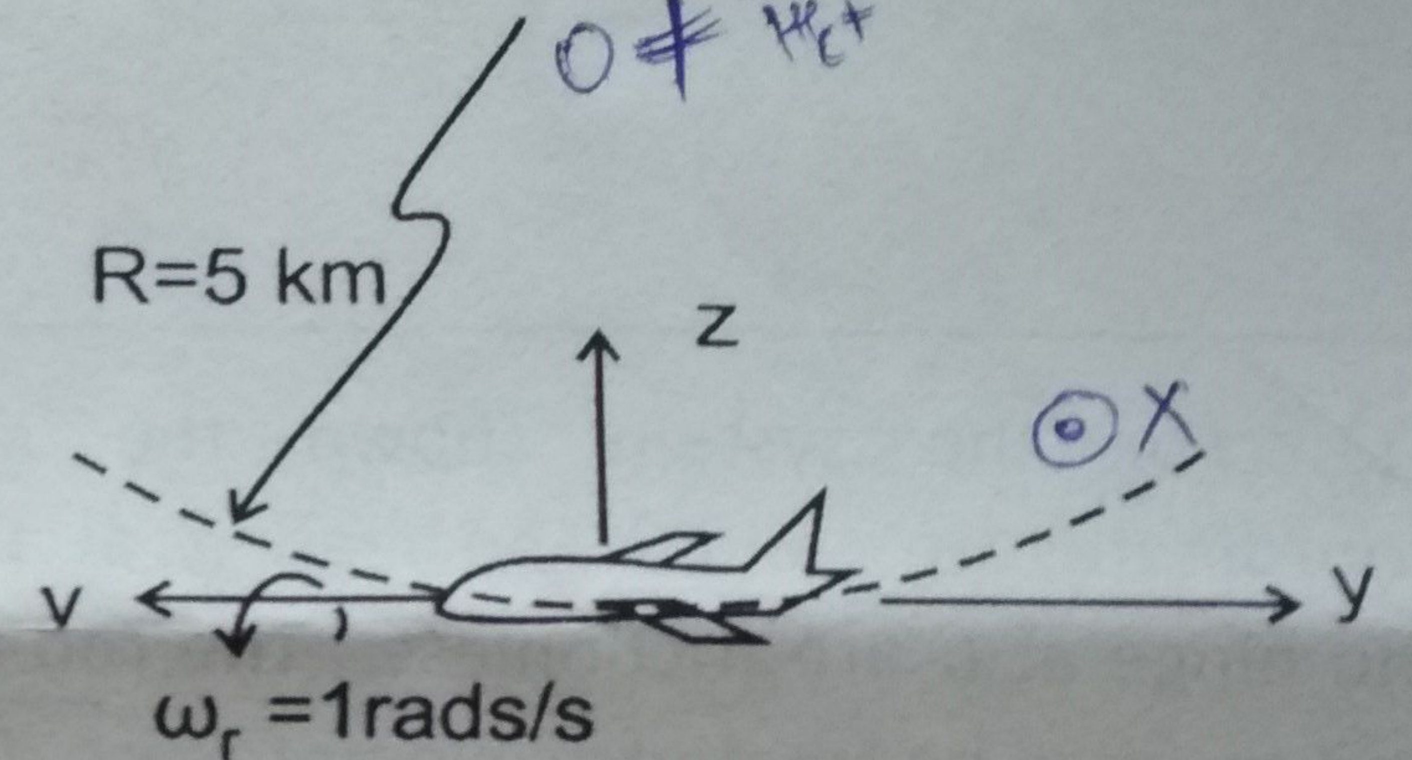
iii) Determine the velocity of the bullet immediately after the impact.

iv) Determine the impulsive force and impulsive couple reaction at O.

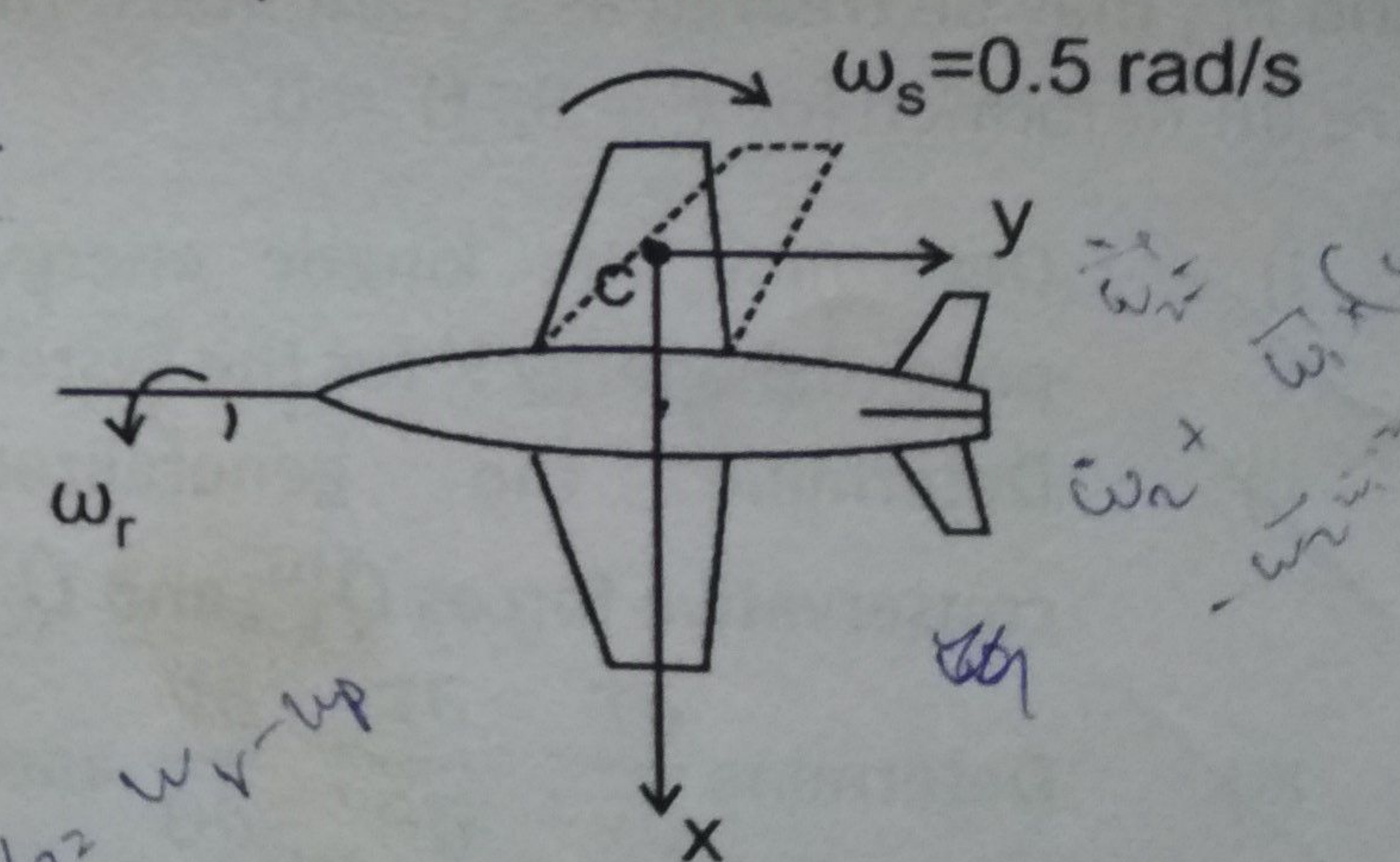


(16)

Q6: An aircraft is moving at a constant speed V = 500 m/s. It is performing a vertical loop of radius R=5 km. At this instant it is rolling at a constant rate ω_r = 1 rad/s. The pilot is also swinging his wings back at a constant rate ω_s = 0.5 rad/s. The centre of mass of the right wing is C and the principal radii of gyration of the right wing about C are k_x = 1m, k_y = 5m, k_z = 10m. The mass of the right wing is 500 kg.

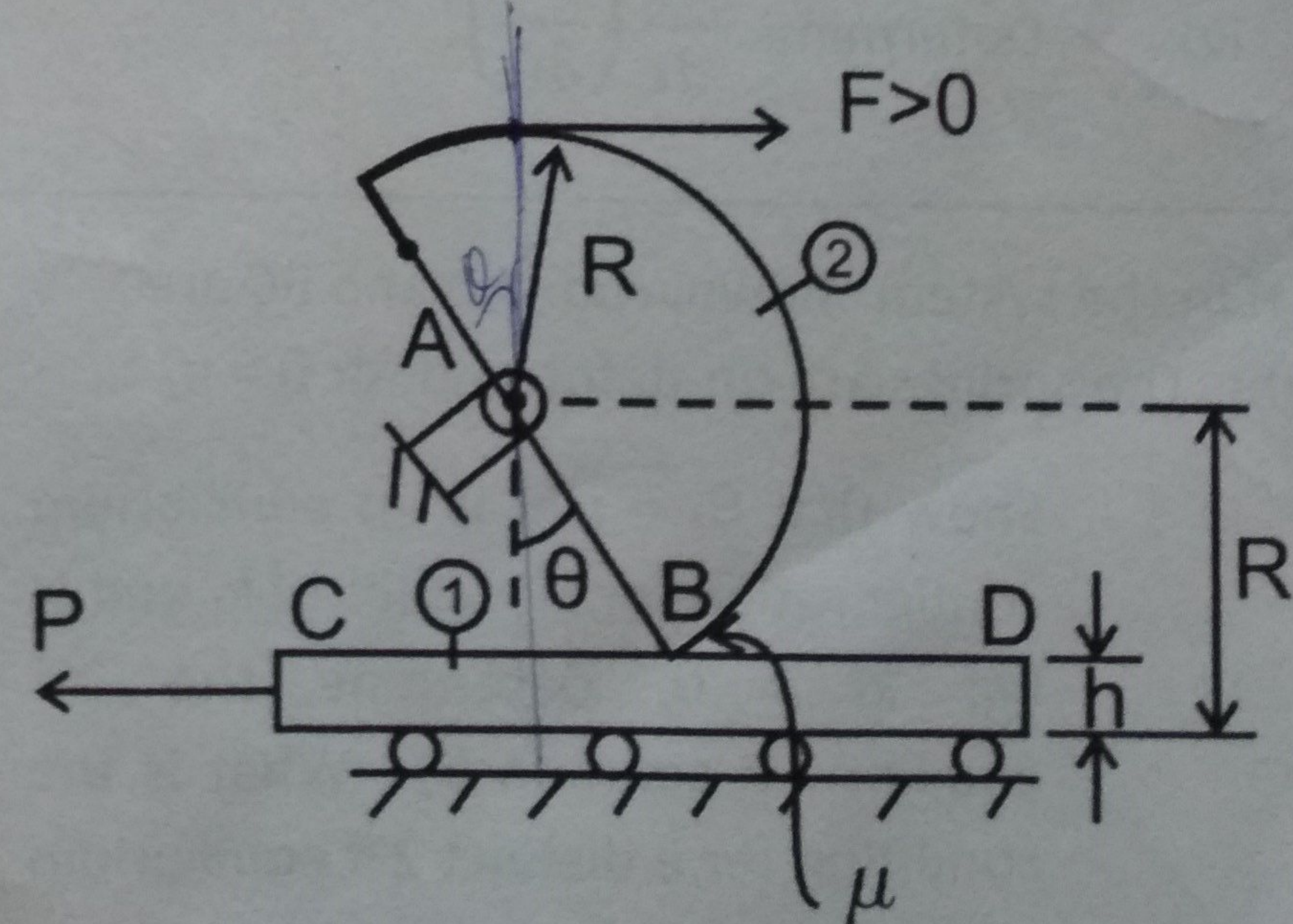


What is the net moment about C applied by the plane body and the aerodynamic forces on the wing, at this instant, to enable this motion of the wing?



(16)

Q7: A plate CD (body ①) is being pulled to the left by a force P as shown. The plate's movement is resisted by the semi-circular disc ②. The hinges and rollers are frictionless however the coefficient of friction between the plate and disc is μ. What is the largest value of thickness h for which the system is self-locking? i.e. for given $F > 0$, the plate will not move no matter how large P is.
You may assume all bodies are light so gravity does not play any role here.



(15)