PHY113: MINOR II

Max. Marks: 25

Attempt all questions.

1. The one-dimensional neutron diffusion equation with a (plane) source is

$$-D\frac{d^2\phi(x)}{dx^2} + K^2D\phi(x) = Q\delta(x),$$

where $\phi(x)$ is the neutron flux, $Q\delta(x)$ is the source at x=0, and Dand K^2 are constant. Obtain the solution of the differential equation using the Fourier transform technique.

2. Use the method of Frobenius to find one solution near x = 0 of the ordinary differential equation

$$x^2y'' + (x^2 + 2x)y' - 2y = 0$$

for any positive integer n.

3. The Legendre polynomials can be shown to satisfy the orthonormality relation

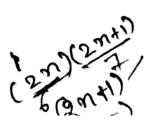
$$\int_{-1}^{1} P_m(x) P_n(x) dx = \frac{2}{2n+1}.$$

Expand the function

$$G(x) = \begin{cases} 1, & 0 \le x \le 1, \\ -1, & -1 \le x \le 0, \end{cases}$$

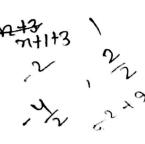
in a Legendre series.

4. We derived the the Bessel functions in class and found them to be



$$J_{\nu}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\Gamma(n+\nu+1)} \left(\frac{x}{2}\right)^{\nu+2n}.$$

$$2^{\nu+2n}$$



(8)

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Using the equation

$$\frac{d}{dx}\left[x^{\nu}J_{\nu}(x)\right] = x^{\nu}J_{\nu-1}(x),$$

and given that $J_{1/2}(x) = (2/\pi x)^{1/2} \sin x$ and $J_{-1/2}(x) = (2/\pi x)^{1/2} \cos x$, express $J_{3/2}(x)$ in terms of trigonometric functions. (4)

5. Using the generating function for the Legrendre polynomials

$$g(x,h) = (1 - 2xh + h^2)^{-1/2},$$

express the Coulomb potential in terms of the Legendre polynomials. This is very useful in multipole expansions and boundary value problems in electrostatics. (4)