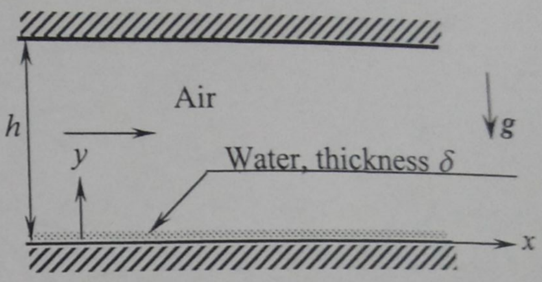
AML160 Mechanics of Fluids **MAJOR II** **PART B**

(*Max marks 90*) 24th November 2013 1.30 pm to 3.00 pm

Please attempt all questions. Marks for each question are given alongside.

Data: $\rho_w = 1000$ kg/m^3 $\rho_{air} = 1$ kg/m^3 $\mu_w = 10^{-3}$ Kg/(ms) $\mu_{air} = 1.5 \times 10^{-5}$ kg/(ms) and $g = 10$m/s^2.

Q1) Air flows steadily in a horizontal 2D channel due to a pressure gradient dP_o/dx. There is a layer of water of thickness δ over the lower plate as shown. You may assume that there is no slip between the water and the air and that the velocity at the interface is U_1. The flow is fully developed.



i) Show that $v = 0$ in both the air and water, by applying the continuity equation in air and water separately.

ii) Assume that $\delta << h$ and that U_1 is much smaller than the maximum velocity in the air. Hence for the air flow $u(0) \approx 0$. Using this determine the velocity profile for the air flow.

iii) Determine dP/dy in the water and show that the variation of static pressure is hydrostatic.

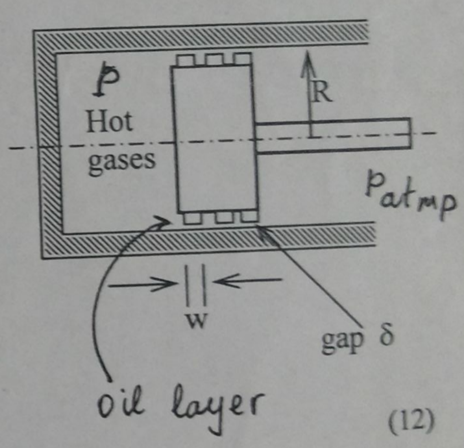
iv) Assume that the pressure gradient is too small to drive flow in the water and that it effectively sees a constant external pressure. Determine U_1 by matching appropriate quantities at the air-water interface.

v) Determine the flow rate of water per unit depth, q.

vi) If $-dP_o/dx = 10^{-2}$ N/m^3; $h = 10$cm; and $\delta = 1$mm; determine the maximum speed of air and the values of U_1 and q.

vii) Are the assumptions used in sections ii) and iv) justified? (18)

Q2) The figure shows a view of a car engine. The region to the left of the piston is filled with hot gases at a pressure of $p = 5\times10^5$ N/m^2 (gauge). The piston is forced out of the cylinder (of radius $R = 3$cm) due to this pressure. The gap between the cylinder and the piston rings is $\delta = 0.02$mm (this gap is uniform). This gap is lubricated with oil of viscosity $\mu = 10^{-2}$ Ns/m^2. There are 3 rings of width, $w = 3$mm each. The mass of the piston is, m=100gm. **The cylinder is horizontal.**



If the piston starts from rest, how far does it travel before its speed reaches 20 m/s. ***The pressure, p, may be assumed constant during this process.***
(Note: The acceleration of the piston is not constant.) (12)

Q3) A scaled down model of an aircraft is to be tested in **a water tunnel**. The length of the actual aircraft, L_P, is 30m while the model is 1m long. The aircraft is expected to fly at a speed U_P of 30m/s and is powered by a propeller of diameter, $D_P = 2$m, running at ω_P =1000 rpm. The drag force on the aircraft is known to depend on $(L, D, \rho, \mu, U, \omega)$.

i) Choose L, ρ, and U as repeating variables and determine all the relevant non-dimensional groups.

ii) Hence write down all the conditions required for complete dynamic similarity in this test.

iii) For complete dynamic similarity what should be the fluid velocity in the water tunnel and at what rpm should the model propeller run?

iv) Dye released near the wing body junction is found to reach the tail of the model in 10^{-2} s. How long would a dust particle take to travel between corresponding points with respect to the actual aircraft?

v) The drag force on the model is measured to be 1kN. What is force expected on the actual aircraft? (14)

Q4) Using an appropriate elemental control volume show that if there is no internal motion in a fluid then the governing equation is: (6)

$$-\nabla p + \rho \vec{g} = \rho \vec{a}.$$

Q5) Consider the simplified momentum equation and the continuity equation for a thin boundary-layer a over a flat plate (2 dimensional incompressible flow) with no external pressure gradient:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}; \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$$

Integrate both sides of the above momentum equation with respect to y from 0 to ∞ and show that this equation can be reduced to the momentum integral:

$$\frac{d\theta}{dx}=\frac{\tau_w}{\rho U_\infty^{\ 2}}$$

Here, the momentum thickness, θ, is defined as $\theta=\int_0^\infty \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy$.

Hint: You will need to use the continuity equation in different ways to eliminate v. (8)

Q6) Consider the simplified boundary-layer equations given in problem 5 above. Assume that a similarity solution exists and define the stream function as:

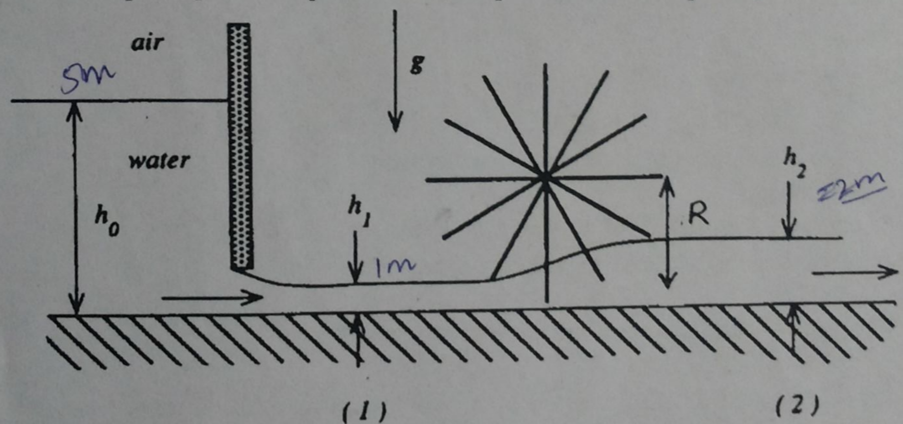
$$\psi = U_\infty \delta\, f(\eta), \quad where, \eta \equiv y/\delta \quad and \quad \delta=\sqrt{\frac{\nu x}{U_\infty}}.$$

Show that f satisfies the following ordinary differential equation:

$$f''' + \frac{1}{2} f f'' = 0.$$

What are the boundary conditions that must be satisfied by f? (8)

Q7) The paddle wheel of an old mill is located in a water stream of width, $W = 2$m. The water driving the mill is supplied by a reservoir whose water surface is at a (constant) height $h_o = 5$m above the stream bed as shown. The stream has a depth $h_1 = 1$m upstream of the paddle and a depth $h_2 = 2$m downstream of the paddle as shown.



The flow from the reservoir to station (1) may be treated as inviscid, but between sections (1) and (2) the fluid flow is disturbed by the paddle and it must be considered as viscous. However, the shear stress at the bed is negligible even between sections (1) and (2).

i) Determine the water speeds V_1 and V_2 present at sections (1) and (2) respectively.
ii) Determine the peizometric pressures P_1 and P_2 present at sections (1) and (2) respectively.
iii) Determine the net horizontal force F_x exerted by the flow on the paddle wheel.
iv) The effective radius of the paddle wheel, R = 3m. Assuming an efficiency of 50% determine the angular speed of the paddle wheel. (14)

Q8) The speed of water at the exit of a garden hose is seen to be 1m/s when the tap is fully open. The roof tank that feeds the tap is at a height, 20m above ground level. Keeping everything else the same it is found that by squeezing the end of the hose the speed can be increased to 10m/s.

i) Explain why the speed increases.
ii) Let A_1 be the exit area of the hose and A_2 be the area after squeezing. Estimate A_1/A_2. (10)

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