EEL 781 Minor II Marks: 100 Time: 60 Minutes

Q. 1. Figure Q2 shows the decision boundary for a SVM based classifier, in the **input** space (x_1, x_2). Suggest what might have been a suitable nonlinear map ϕ Hint: think *very* simple ! Note that the map need not be invertible, but the classifier is a linear one in the image (ϕ) space. Determine

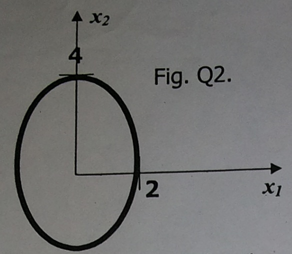

Fig. Q2.

(a) the locus of support vectors of class 1
(b) the locus of support vectors of class (-1), and
(c) the Kernel function K(y, z) for two arbitrary vectors p and q in terms of the 2D components of p and q.

$K(p,q)$

Hint: the locus of support vectors should be a sensible real function !

(20 marks)

Q. 2. A SVM primal problem for determining the hyperplane in feature space is given by

$$\underset{q,w}{\text{Minimize}} \quad \frac{1}{2}w^T w + \frac{A}{2}b^2 + Ce^T q \qquad (1)$$

subject to the constraints

$q \geq 0$

$$y_k\left[w^T \varphi(x^k) + b\right] \geq 1 - q_k, \; k = 1, 2, \ldots, M. \qquad (2)$$

1. Write the Lagrangian for the problem.
2. Determine the K.K.T. conditions.
3. Determine the Dual. Use a Kernel Function - do not leave the result in terms of φ.

(40 marks)

Q. 3. A dataset consists of Class 1 patterns : (-1, 2), (1, -2), (-1, -2), (1, 2); and Class (-1) patterns: (-2, 4), (2, -4), (-2, -4), (2, 4). Use a SVM to classify the patterns using a RBF kernel of the form $\exp(\beta * \|p-q\|^2)$

(a) Choose and indicate the values of C and β.
(b) Determine the discriminant function.
(c) Derive an expression for the margin. Find the margin in the image (ϕ) space.

(40 marks)