ELL – 212, Sem II (2017-2018) Minor 2
Module- Antennas: Retarded potential, radiation field and power for short dipole antenna

Total Points: 10

1. a) The Laplacian operator in two dimensions can be defined as:
$\nabla_{2D} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ (in Cartesian coordinates)
$= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$ (in polar coordinates)

Show that $\frac{1}{2\pi}\ln(r)$ is the Green's function for the Laplacian operator in two dimensions. *(Points: 2)*

b) Consider an infinitely long charged wire in three dimensions lying along z-axis with charge density of μ per unit length. The wire has infinitesimal width, so you can write the charge distribution as:

$$\rho(\vec{r}) = \rho(x, y, z) = \delta(x)\delta(y)\mu$$

Since the potential at any point in space is independent of z direction in this case, reduce the problem to two dimensions and use Green's function from (a) to calculate potential as function of position. *(Points: 1)*

c) Use Gauss's law and definition of potential to obtain an expression for potential due to the same infinitely long charged wire as a function of position. Compare your result with that of part (b).
(Note – do not worry about the limits of the integral, which determine the reference potential. It can take any constant value. So just do an indefinite integral and take the constant of integration to be 0.) *(Points: 2)*

d) Show that Green's function for Laplacian operator in two dimensions is the integral of Green's function in three dimensions with respect to the third dimension as shown below:

$$G_{2D}(x, y, x', y') = \int_{-Z_1}^{Z_2} G_{3D}(x, y, z, x', y', z')\, d(z - z')$$

,where G_{2D} is the Green's function in 2 dimensions, G_{3D} is the Green's function in 3 dimensions, and Z_1 and Z_2 are large positive numbers.

(Note: Don't assume specific Green's functions like log function (2D) or 1/r function (3D). Just assume general functions which are Green's function for Laplacian operator and use the definition of Green's function, along with the assumption that derivative of Green's function in 3D with respect to (z-z') is 0 for high values of Z_1 and Z_2.) *(Points: 1)*

2. a) A 2 m long dipole antenna is excited by 5 MHz alternating current with an amplitude of 5 A. Express the "1/r" component of electric field and magnetic field radiated by the antenna as a function of radial distance *(r)* from the antenna, angle with respect to dipole axis *(θ)* and time *(t).*
Don't leave your answer as a complex number, express it as a real measurable field. Specify the direction of the fields, angular frequency *(ω)* and wave number *(k).* Use SI units only. *(Points: 2)*

b) What is the power density at the distance of 2 km from the antenna at a direction 45^o with respect to the dipole axis? *(Points: 1)*

c) If the dipole antenna is 50 m long can the same expressions for fields and power of part (a) and (b) be used? Explain. *(Points: 1)*

($\mu_0 = 4\pi \times 10^{-7}$ SI unit, $\varepsilon_0 = 8.85 \times 10^{-12}$ SI unit)