Department of Mathematics
MAL 250 (Introduction to Probability Theory and Stochastic Processes)
Minor 1 Test (I Semester 2013 - 2014)

Time allowed: 1 hour

Max. Marks: 25

1. (a) Let $\Omega = \{a, b, c, d\}$. Find three different σ-fields $\{F_n\}$ for $n = 0, 1, 2$ such that $F_0 \subset F_1 \subset F_2$.

    (3 marks)

    (b) The first generation of particles is the collection of off-springs of a given particle. The next generation is formed by the off-springs of these members. If the probability that a particle has k off springs (splits into k parts) is p_k, where $p_o = 0.4$, $p_1 = 0.3$, $p_2 = 0.3$. Assume particles act independently and identically irrespective of the generation. Find the probability that there is only one particle in second generation.

    (2 marks)

2. Consider the random variable X that represents the number of people who are hospitalized or die in a single head-on collision on the road in front of IIT Delhi main gate in a year. The distribution of such random variables are typically obtained from historical data. Without getting into the statistical aspects involved, let us suppose that the cumulative distribution function of X is as follows:

x	0	1	2	3	4	5	6	7	8	9	10
F(x)	0.250	0.546	0.898	0.932	0.955	0.972	0.981	0.989	0.995	0.998	1.000

    Find (a) $P(X = 10)$ (b) $P(X \le 5/X > 2)$ (c) $E(X)$. 1.514

    (1 + 2 + 2 marks)

3. Assume that, taxis are waiting in a queue for passengers to come. Passengers for these taxis arrive according to a Poisson process with an average of 60 passengers per hour. A taxi departs as soon as two passengers have been collected or 3 minutes have expired since the first passenger has got in the taxi. Suppose you get in the taxi as first passenger. What is the distribution of your waiting time for the departure? Also, find its variance.

    (2 + 2 marks)

4. State **True** or **False** with valid reasons for the following statements. Without **valid reasons**, marks will NOT be given.

    (a) The probabilities that a student in MAL 250 will expends a high, medium or low amount of effort in studying are 0.50, 0.30 and 0.20, respectively. Given that the student expends a high, medium or low amount of effort, the respective conditional probabilities of getting grade A in this courses are 0.90, 0.40 and 0.05. Then the probability that the student will get grade A in the course is 0.58.

    (b) Consider a parallel system with identical components each with reliability 0.8. If the reliability of the system is to be at least 0.99, then the minimum number of components in this system is 3.

    (c) Define the $(100p)$th percentile of a random variable X is the smallest value of x such that $F(x) = P(X \le x) \ge p$. Then, 50th percentile is called the _mode_ of X.

    (d) Let X be a continuous random variable with pdf $f(x) = \dfrac{1}{\pi(1 + x^2)}$, $-\infty < x < \infty$. Then, $\text{Var}(X) = \pi$.

    (1 + 1 + 1 + 1 marks)

5. Pick the **odd one out** with valid reasons for the following statements. Without **valid reasons**, marks will NOT be given.

    (a) (1) Bernoulli distribution (2) Binomial distribution (3) Poisson distribution (4) uniform distribution

    (b) (1) Gamma distribution (2) Exponential distribution (3) Poisson distribution (4) Erlang distribution

    (1 + 1 marks)

6. A point X is chosen at random in the interval $[-2, 1]$. Find the pdf of $Y = X^2$.

    (5 marks)