

Q.1: (a) Let  $f(t) = e^{-\alpha t} (-\alpha t)$  where  $\alpha > 0$ . Find unilateral LT of  $f(t)$ .  
 Now, consider  $g(t) = \int_{-\infty}^t f(\tau) d\tau$ . Find unilateral LT of  $g(t)$ .

(b) Consider rotational mech. system shown in Fig. 1 where  $T(t)$  is input torque and  $\theta(t)$  output displacement. Fig. 1 shows a rotational system with a moment of inertia  $J = 1 \text{ kg-m}^2$ . It consists of three gears:  $N_1 = 25$ ,  $N_2 = 5$ , and  $N_3 = 10$ . Gear  $N_2$  has damping  $D$ . Gear  $N_4 = 5$  is connected to a spring with stiffness  $K = \frac{1}{4}$ . The output is the angular displacement  $\theta(t)$ .

The gear  $N_2$  is lossy with damping  $D$ . Find  $D$  such that for unit step torque input, the output angular displacement has 30% max overshoot. (5)

Q.2: For the unity feedback system shown in Fig. 2, construct a parameter plane  $k_1 - k_2$  where  $k_1$  is horizontal axis and show the following regions in  $k_1 - k_2$  plane: (5)

- Stable and unstable region.
- Region in which system is overdamped ( $\zeta > 1$ ).
- Region in which system is underdamped ( $\zeta < 1$ ).

Q.3: A controlled process is represented by following equations:

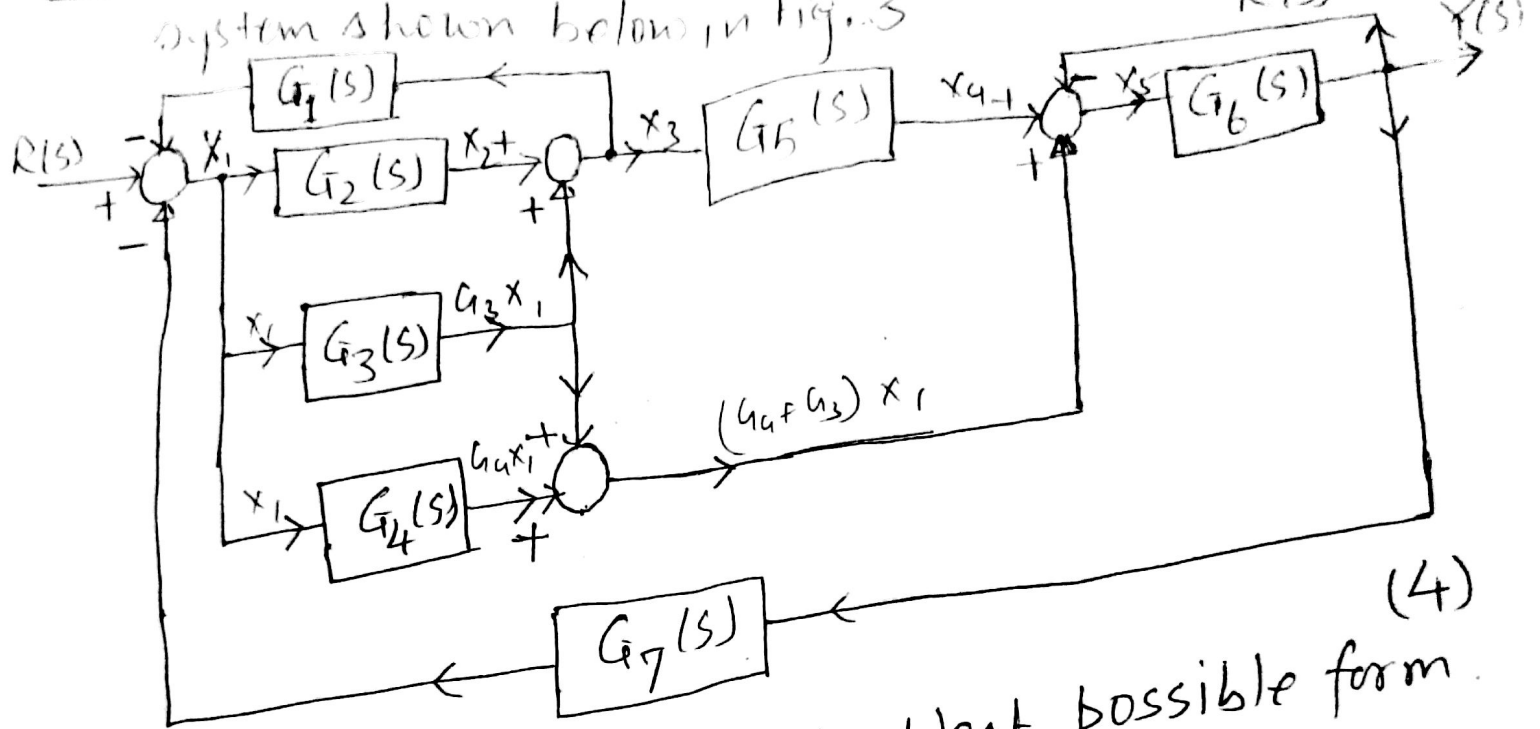
$$\begin{aligned} \frac{dx_1}{dt} &= -x_1(t) + 5x_2(t) \\ \frac{dx_2}{dt} &= -6x_1(t) + u(t) \\ y &= x_1(t) \end{aligned}$$

$$k_1 = \frac{K_2 N_1}{N_2}$$

where control input is obtained from  $u(t) = -k_1 x_1 - k_2 x_2 + r(t)$ ,  $r(t)$  is reference input.

- Find the region in  $k_1 - k_2$  plane ( $k_1 = \text{vertical axis}$ ) such that overall system has damping ratio  $0 < \zeta < 0.707$ .
- Find values of  $k_1$  and  $k_2$  such that  $\zeta = 0.707$  and  $\omega_n = 10$ .
- Find the locus in  $k_1 - k_2$  plane such that  $\lim_{t \rightarrow \infty} y(t) = 1$  when  $r(t)$  is a unit step signal. (6)

Q 4:- Find the overall transfer function  $T(s) = \frac{Y(s)}{R(s)}$  for system shown below in Fig. 3



Give answer in the simplest possible form using block diagram reduction technique. (4)