MCL735
MAJOR

CAD & FEA
Nov 18, 2017

MAX MARKS: 120

MAX TIME: 120 MINS.

Note:

1. Answers should be **brief and to the point.**
2. **Marks shall be deducted for unnecessarily long answers.**

[Marks vector: 15+15+15+15+15+15+10+10+10]

Q 1 Derive the stiffness matrix for a 3 noded 1D isoparametric quadratic element.

Q 2 Proove that the B-spline curve defined with n points and a parameter 'k' is a curve of degree 'k-1, and is controlled by k points.

Q 3 Given a cubic Bezier curve $p(u)=U_3M_3B$ (B is the matrix of control points), find the control points for the Bezier curve represented by the portion of the curve from u=0.3 to 0.6 in this curve.

Q 4 For a Bezier curve prove that the kth derivative at the starting point depends on the first k+1 points.

Q 5 Given the blending functions for B-spline curve with n=5, k=3 as follows:

$$B_{0,3}(u) = (1-u)^2 \quad \text{for } 0 \le u < 1 \qquad B_{1,3}(u) = \begin{cases} \frac{1}{2}u(4-3u) & \text{for } 0 \le u < 1 \\ \frac{1}{2}(2-u)^2 & \text{for } 1 \le u < 2 \end{cases}$$

$$B_{2,3}(u) = \begin{cases} \frac{1}{2}u^2 & \text{for } 0 \le u < 1 \\ \frac{1}{2}(-2u^2+6u-3) & \text{for } 1 \le u < 2 \\ \frac{1}{2}(3-u)^2 & \text{for } 2 \le u < 3 \end{cases} \qquad B_{3,3}(u) = \begin{cases} \frac{1}{2}(u-1)^2 & \text{for } 1 \le u < 2 \\ \frac{1}{2}(-2u^2+10u-11) & \text{for } 2 \le u < 3 \\ \frac{1}{2}(4-u)^2 & \text{for } 3 \le u < 4 \end{cases}$$

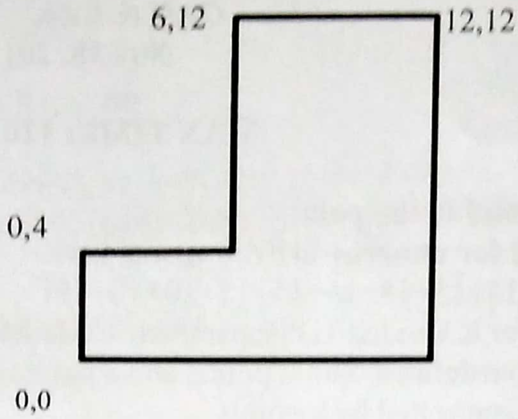
$$B_{4,3}(u) = \begin{cases} \frac{1}{2}(u-2)^2 & \text{for } 2 \le u < 3 \\ \frac{1}{2}(-3u^2+20u-32) & \text{for } 3 \le u < 4 \end{cases}$$

$$B_{5,3}(u) = (u-3)^2 \quad \text{for } 3 \le u \le 4$$

Express the equations of each curve segment in a parametric form with a parameter ranging from 0 to 1. What would be the curve if the B-spline curve was uniform in nature (reasons REQUIRED for the answer).

Q 6. If the viewing direction is given by (4 7 4), and a point (10 10 10) is to appear at the origin of the viewing coordinate system, such that the vector (1 1 0) appears vertical in it. Find the transformation matrix required to convert the World Coordinate System to the Viewing Coordinate system.

Q 7. Consider the planar object shown below with nodes as indicated.



a) Sketch the object and the continuous medial axis of the object on the same figure.
b) Generate the discrete medial axis over a 1 by 1 grid using the Taxicab (Manhattan) metric. Sketch the object and the medial axis and indicate the values at grid points belonging to the medial axis.
c) Comment on the differences between the two medial axis.

Q 8. a) Obtain the surface of the object generated by rotational sweeping about the x axis of the area represented parametrically as : $x = cos^3\theta,\ y = sin^3\theta\ ; 0 \leq \theta \leq \frac{\pi}{2}$ by using, $S = \int 2\pi y ds$

Q 9. Neatly sketch the volume $S(u,v) = f(\gamma\ (u),v)$, parametrised by :
$\gamma : [0,1] \rightarrow \mathbf{R}^3$, $\mathbf{u} \rightarrow (\cos^3 2\pi u, \sin^3 2\pi u)$
$f(\mathbf{p}, v) = \left(1 - \frac{v}{6}\right) R_v(\mathbf{p}) + \left(v, 0, -\frac{1}{3}(x-3)^2 + 3\right)$ with R_v being the rotation about the y-axis in $\mathbf{R}^3$ through an angle of $\pi v/6$ and $v : [0,1]$