IIT-Delhi Dept. of Electrical Engineering
ELL803: Model Reduction in Control

May 2016 *Instructor: M. Nabi*

Major (Time: 2 Hr)

1. Use Gram-Schmidt Orthogonalization to obtain an orthogonal (orthonormal) basis of $\mathbf{R}^3$, beginning from the given basis

$$(\bar{v}_1, \bar{v}_2, \bar{v}_3) = (\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 13 \\ 4 \\ 3 \end{bmatrix} \begin{bmatrix} 7 \\ 5 \\ 10 \end{bmatrix})$$

Sketch clearly the orthogonal basis vectors obtained. 6+2=8

2. The Grammian W_C^2 of a system has eigenvalues 7, 3, 1 and respective eigenvectors $v_1 = [0\ p\ p]^T, v_2 = [1\ 0\ 0]^T, v_3 = [0\ -p\ p]^T$ where $p = 1/\sqrt{2}$. Now find (numeric value) a transformation P_1 which transfers the Grammians to $W_C^2 = I$ and $W_O^2 = \text{diag}[9, 4, 1]$. 5

3. A state space system is given as

$$A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 \\ 1 & 0 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} ; b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} c = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

   (a) Find the 0-th Moment about $s = 0$ of the corresponding transfer function of this (unreduced) system.

   (b) Form a Krylov matrix $K_2(A, b)$ of order two,

   (c) Then form an orthogonalized basis V.

   (d) Use this to obtain a 2nd order reduced order model(ROM)

$$\dot{z} = \tilde{A}z + \tilde{B}u; y = \tilde{C}z$$

   from the 4th order

$$\dot{x} = Ax + Bu; y = Cx$$

   (e) Find the 0-th Moment of this reduced system. Does it agree with part (a)

   Important Advice: First obtain the ROM using symbols only, then at the end put numerical values 4+2+6+6+2=20

4. The large non-linear system given by

$$\dot{x} = f(x) + Bu \quad (1)$$
$$y = cx \quad (2)$$

is to be reduced using the TPWL method.

(a) With the non-linearity given in (1), what is the system called ?

(b) For any one Linearization Point (LP) x_i, the nonlinearity is linearized using the first-order Taylor Series approximation. Write the resultant linearized version of (1) & (2) ?

(c) Now assuming the dynamics is projected on the space spanned by V, write the reduced sub-model obtained at the LP x_i.

(d) Assuming the weightages w_i given to each of the submodel at LP x_i, find the expression for the aggregate reduced model of the system. 1+2+2+2=7
