IIT Delhi

Department of mathematics

MAL 260: Boundary Value Problems

Minor Exam-2

Maximum Marks:20 **Time:1 Hour**

---

1. Sketch the solution of the vibrating string problem, at times $ct = 0, 0.1a, 0.3a, 0.4a, 0.5a, 0.6a$, if $g(x) = 0$ and [3 Marks]

$$f(x) = \begin{cases} 0, & 0 < x < 0.4a, \\ 10h(x - 0.4a), & 0.4a < x < 0.5a, \\ 10h(0.6a - x), & 0.5a < x < 0.6a, \\ 0, & 0.6a < x < a. \end{cases}$$

2. Let $u(x,t)$ be a solution of [3 Marks]

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, 0 < x, 0 < t, u(0,t) = 0, 0 < t, u(x,0) = f(x), \frac{\partial u}{\partial t}(x,0) = g(x), 0 < x.$$

Sketch the solution $u(x,t)$ as a function of x at times $t = 0, a/6c, a/2c, 5a/6c, 7a/6c$. Use $g(x) = 0$ and

$$f(x) = \begin{cases} \frac{3hx}{2a}, & 0 < x < \frac{2a}{3}, \\ \frac{3h(a-x)}{a}, & \frac{2a}{3} < x < a, \\ 0, & a < x. \end{cases}$$

3. Let $u(x,t)$ be the solution of the wave equation on the semi-infinite interval $0 < x < \infty$, with both initial conditions equal to zero but with the time-varying boundary condition

$$u(0,t) = \begin{cases} \sin\left(\frac{ct}{a}\right), & 0 < t < \frac{\pi a}{c}, \\ 0, & \frac{\pi a}{c} < t. \end{cases}$$

Sketch $u(x,t)$ as a function of x at various times. [4 Marks]

4. Solve the potential equation on unit square with boundary conditions: $u(x,1) = 100, 0 < x < 1; u = 0$ on the other three sides of the square. [3 Marks]

5. Solve the potential equation on unit disk with boundary conditions [3 Marks]

$$u(1,\theta) = \begin{cases} 1, & 0 < \theta < \pi, \\ 0, & -\pi < \theta < 0. \end{cases}$$

6. Solve the potential problem in the upper half-plane, $-\infty < x < \infty, 0 < y$, with boundary condition: $u(x,0) = \exp(-a|x|)$. [4 Marks]