Indian Institute of Technology Delhi
Department of Electrical Engineering

Control Engineering I

ELL225, Major Test, **2015-16/II**

Total Time: 2 hour. **Maximum marks: 40; Q1:5, Q2:6, Q3:5, Q4: 5, Q5:4, Q6:4, Q7:5, Q8: 3, Q9: 3**

- **Write answer all the parts of a question in same place.**

Q1. Consider the following set of equations for an electrical system where E_i and E_d are two inputs, Y is the output and k is a constant.

$$I_1 = E_i - E_1,\; E_1 = I_1 - I_2,$$
$$I_2 = E_1 - E_2,\; E_2 = E_d + I_2 - I_3,$$
$$I_3 = E_2 - Y + kI_2,\; Y = I_3$$

(a) Draw the signal flow graph of this system. (You must use all **the variables** and clearly show the node variables and branch gain. Hint: Use I_1, E_1, I_2, E_2, I_3 ... as node variables).

(b) Find the ratio $\frac{I_3}{E_i}$ and $\frac{I_3}{I_2}$ using Mason's gain formulae? (Assume $E_d = 0$ in this case).

Q2. The open loop transfer function of a unity feedback system is given as

$$G(s) = \frac{k(1+\tau_1 s)}{s^2(1+\tau_2 s)},\quad \text{where } k>0,\ \tau_1>0, \tau_2>0$$

(a) Draw the Nyquist plot of the system in G(s) plane for the following two cases: Case (i): $\tau_1 > \tau_2$ and Case (ii) $\tau_1 < \tau_2$. Show all the intermediate computations.

(b) Verify the stability of the system in each case **using Nyquist stability criteria.** Clearly state your arguments.

Q3. Consider that the open loop transfer function of a unity feedback system is

$$G(s) = \frac{e^{-Ts}}{s(s+1)(s+2)},\quad T \ge 0$$

(a) Suppose for delay T=0, gain-cross-over frequency is 0.446 rad/sec. Find the phase-cross over frequency, the gain margin and phase margin?

(b) Determine the **critical/maximum** value of delayT for stability. (Compute it without using any approximation of e^{-Ts})

Q4. Consider a SISO (single-input single-output) controllable and observable system (A,B) described as

$$\dot{x} = Ax + Bu$$

where x is n-dimensional state vector.

(a) Show that the system (A+BK, B) is controllable where K is an non-zero (1xn) row vector?

(b) A state vector z = Mx (where M is a non-singular matrix) is used to define a new state variable model (A_C, B_C). Show that the transformed system (A_C, B_C) is controllable.

Q5. Consider a second order state variable model (A,B,C)

$$\dot{x} = Ax + Bu,\quad x(0) = x_0,\ A = \begin{bmatrix} 0 & a_1 \\ 1 & a_2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$y = Cx,\quad \text{Initial time } t_0 = 0$$

(a) ~~Suppose~~ Find the range of scalars a_1, a_2 such that the system is observable.

(b) Suppose the state response $x(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} = x_u(t)$ for $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and an unit impulse input.

Further it is known that $x(t) = \begin{bmatrix} f_3(t) \\ f_4(t) \end{bmatrix} = x_z(t)$ for $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and zero input. Based on these two information, find the state transition matrix e^{At}?

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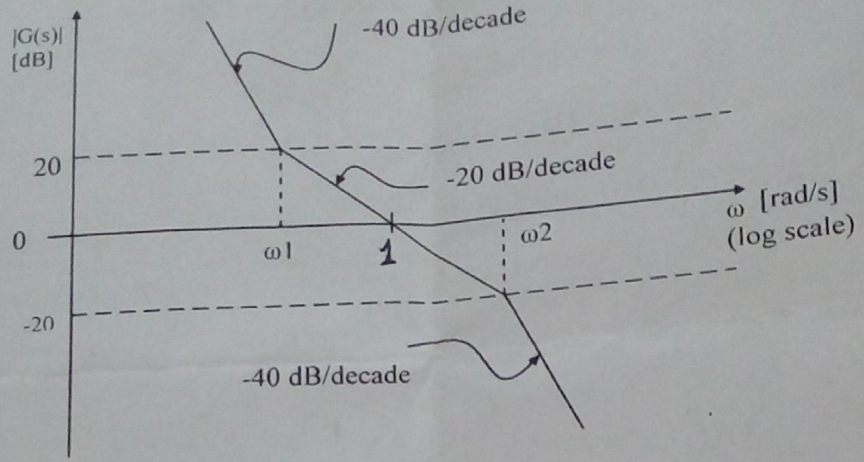
Q6. The Bode plot of a minimum phase system is shown in the figure below.



(a) Determine the transfer function G(s) of the system?

(b) Find a constant K such that the gain cross-over frequency of KG(s) will be 10 rad/sec?

Q7. Suppose in figure-2, **D(s)=1** and G(s) is given as

$$G(s)=\frac{k(s+1)}{s(1+Ts)(1+2s)},\quad k>0, T>0$$

(a) Using Routh-Hurwitz criterion, determine the region of k-T plane in which the closed loop system is stable. Use **k** as th x-axis and **T** as the Y-axis.

(b) Suppose the system in figure-2 is stable (with D(s) =1) and R(s) is an unit step input. Express $J=\int_0^\infty e(t)dt$ in terms

static velocity error constant K_v?

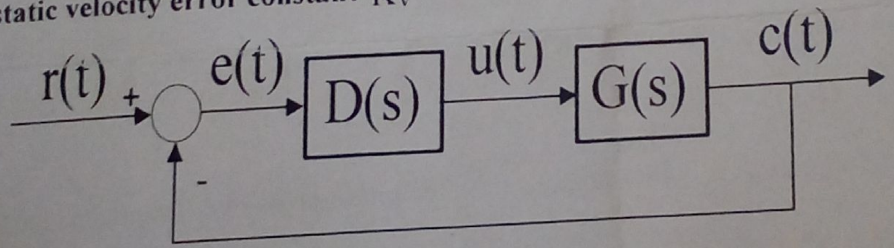


Figure 2 (**For question 7 & 8**)

Q8. In figure-2, D(s) is the compensator and G(s) is given by

$$G(s)=\frac{k}{s^2}$$

Using the results of root locus, design a first order lead compensator **D(s)** and **k** such that dominant pole will be located at $-1+j2$. It is assumed that the zero of D(s) is located at $s=-1$.

Q9 To realize a compensator, a R-C circuit is to be designed as shown in figure-3.

(a) Find conditions on R_1 and R_2 such that it will act as (i) a **lead compensator** and (ii) **lag compensator**.

(b) What are the differences between (i) lead compensator and PD controller (ii) Lag compensator and PI controller.