Max Marks: 35

All questions are compulsory.

Time: 2 Hrs

 $\sqrt{1}$ . Let Q be an n imes n symmetric matrix. Consider the following nonlinear program

$$(P) \qquad \min \ \frac{1}{2} x^t Q x \quad \text{ subject to } \quad \frac{1}{2} x^T x \leq \frac{1}{2}, \ \ x \in \mathbb{R}^n.$$

Show that (P) and its Lagrange dual have same optimal values.

(Wint: use idea of minimum eigen-value of Q)

[6]

Consider the nonlinear programming problem

$$\begin{array}{lll} & \min & 4x_1x_2 - x_1 - 2x_2 \\ \text{subject to} & & \\ & -x_1 + x_2 & \leq & 0 \\ & & x_1 & \leq & 1 \\ & & x_2 & \leq & 1 \\ & & x_1, x_2 & \geq & 0. \end{array}$$

(a) all KKT points of the problem.

- Does the constraint qualification met at any of these KKT points?
- (c) What is the global solution to the problem? Justify your answer.

[6=3+2+1]

Let A and B be an  $m \times n$  and  $m \times \ell$  matrices respectively. Prove, by using separation theorem of convex sets, that exactly one of the following two systems is consistent.

(I) 
$$Ax + By < 0, x \in \mathbb{R}^n, y \in \mathbb{R}^\ell$$
  
(II)  $A^T p + B^T p = 0, p, p \ge 0, p \ge \mathbb{R}^m$ 

[6]

Consider the following linear program

$$\begin{array}{rcl} & \min & -x_1 - 3x_2 \\ \text{subject to} & & x_1 + x_2 & \leq & 6 \\ & -x_1 + 2x_2 & \leq & 6 \\ & & x_1, x_2 & \geq & 0. \end{array}$$

It is desired to find an optimal solution and optimal basis when right hand side vector  $b = (6,6)^T$  is perturbed along the direction  $d = (-1,1)^T$ , that is, b is changed to  $b + \lambda d$ ,  $\lambda \ge 0$ . An optimal solution for  $\lambda = 0$  is given below.

$v_B$	$x_B$	$y_1$	$y_2$	$y_3$	<i>y</i> <sub>4</sub>	
$x_1$	2	1	0	2/3	-1/3	
$x_2$	4	0	1	1/3	1/3	
		0	0	-5/3	- 2/3	$\leftarrow z_i - c$

(a) Find the range of  $\lambda$  for which the present basis remains optimal.

- (b) Describe the optimal simplex table for the range of  $\lambda$  obtained in (a).
- (a) Ing the optimal table in (b), find an optimal solution when  $\lambda = 2$ .

(d) Is an optimal solution in (c) unique? If not, find an another alternate optimal solution.

[7=2+2+1+2]

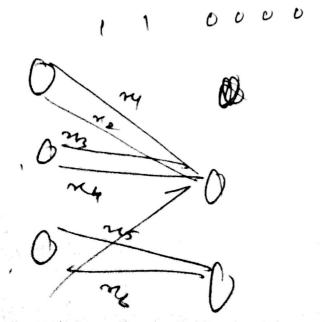
Q3. Find an optimal solution and optimal value of the dual to the following linear program

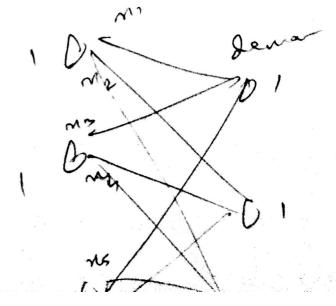
$$\begin{array}{rll} \max & 4x_1+6x_2+2x_3+x_4+7x_5+5x_6\\ \text{subject to} & & x_1+x_2&\leq&1\\ & & x_3+x_4&\leq&1\\ & & x_5+x_6&\leq&1\\ & & x_1+x_3+x_5&=&1\\ & & x_2+x_4+x_6&=&1\\ & & x_1,x_2,x_3,x_4,x_5,x_6&\geq&0. \end{array}$$

Q6. Consider the problem

$$\begin{array}{lll} \max & x_1-2x_2+x_3\\ \text{subject to} & & x_1+2x_2+4x_3-x_4 & \leq & 6\\ & & 2x_1+3x_2-x_3+x_4 & \leq & 12\\ & & x_1+x_3+x_4 & \leq & 4\\ & & x_1,x_2,x_3,x_4 & \geq & 0. \end{array}$$

Find a basic feasible solution with basis variables  $x_1$ ,  $x_2$ ,  $x_4$ . Can you suggest a better feasible solution than this one? Why and how?





[5]

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