Minor 1: MTL103
February 17, 2015

Max Marks 20

Max Time 1 hr

1. This question contains multiple choice questions. Each question may contain one or multiple correct answers. [2+2+2+2+2]
(*) Write all the correct choices with justification.
(*) One mark will be awarded if only partially correct answer(s) with justification and no wrong choice is answered.
(*) No mark will be awarded if partial correct choices and partial wrong choices are answered.
(*) No mark will be awarded if all choices are marked wrong.

(I) Which of the following is possible in a maximizing linear program? (You can think of examples/counter examples.)
(a) An unbounded non-empty feasible set with finite optimal objective value.
(b) An unbounded problem with a polytope feasible set.
(c) Infinitely many optimal supporting hyperplanes to the non-empty feasible set.
(d) Optimal solution in the interior of the feasible set which is having a non-empty interior.

(II) If the simplex method cycles then which of the following are true for two consecutive degenerate iterations?
(a) The objective function value does not change.
(b) The values of variables do not change.
(c) The values of variables may change but the objective function value does not change.
(d) The set of basic variables change.

(III) Consider the simplex tableau of some maximizing linear programming problem:

x_B	y_1	y_2	y_3	y_4	y_5	y_6	
0	-1	1	-3	0	1	2	
1	2	0	-5	1	-1	1	
$z=-1$	1	-1	0	0	-2	0	$\leftarrow z_j - c_j$

(a) Since the objective function is negative, at least one of the variable in a problem must be the free variable.
(b) The problem is unbounded since all entries in y_3 are negative with zero opportunity cost.
(c) There is an error in the tableau because of a negative value in the opportunity cost row.
(d) The problem has alternate solution because y_6 has positive entries with zero opportunity cost.

(IV) Consider the following three functions

$$\begin{aligned} f(x_1,x_2) &= \sqrt{x_1x_2}, \quad (x_1,x_2)\in\mathbb{R}^2_+\setminus\{(0,0)\} \\ g(x_1,x_2) &= \max\{x_1,x_2\}, \quad (x_1,x_2)\in\mathbb{R}^2 \\ h(x) &= x\ell n(x), \quad x\in\mathbb{R}_+\setminus\{0\}. \end{aligned}$$

(a) f, g are convex functions and h is a concave function.
(b) g, h are convex functions and f is a concave function.
(c) g is a convex function, h is a concave function, and nothing can be concluded about f.
(d) f is a concave function, h is a convex function, and nothing can be concluded about g.

(V) A maximization linear program with is solved using two-phase simplex method (with lexicographic rule for leaving variable, if required). Which of the following is/are **NOT** observed in the second phase of the method.
(a) The problem is infeasible.
(b) The problem is unbounded.
(c) The iterations iterate, i.e. cycles.
(d) An optimal basic feasible solution containing artificial variable with zero value.

2. Consider the following linear program in which the objective involves a parameter t: [5]

$$\begin{aligned} \min\ & 2x_1 + (3+t)x_2 \\ \text{subject to}\ & \\ & x_1 + 2x_2 \geqslant 1 \\ & x_1 - 3x_2 \geqslant -3 \\ & x_1,\ x_2 \geqslant 0. \end{aligned}$$

Solve this problem for all values $t \in (-\infty,\infty)$. Tabulate your solutions $x(t)$ and the corresponding objective function values $z(t)$ on each interval of t values.

3. Find the basic feasible solution with (x_1, x_2, x_4) in basis for the following LPP. Is this solution optimal? Give reason(s). [5]

$$\begin{aligned} \min\ & 2x_1 - x_2 - 5x_3 - 3x_4 \\ \text{subject to}\ & \\ & x_1 + 2x_2 + 6x_3 - x_4 = 18 \\ & 2x_1 - 3x_2 - 10x_3 + x_4 = 35 \\ & x_1 + 2x_3 + x_4 = 36 \\ & x_1,\ x_2,\ x_3,\ x_4 \geqslant 0. \end{aligned}$$