Department of Mathematics

Indian Institute of Technology Delhi

MTL 101 Linear Algebra and Differential Equations: Major

Total marks: ~~20~~ 40

Time: 1 hour

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1. Every question is compulsory

2. No marks will be provided if appropriate justification is not provided

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1. In the month of January the consumption of LPG, electricity, petrol and water of four individuals P1, P2, P3 and P4 working in a company is shown in the following chart.

January	LPG	Electricity	Petrol	Water
P1	10 unit	20 unit	40 unit	50 unit
P2	20 unit	25 unit	30 unit	40 unit
P3	30 unit	35 unit	20 unit	30 unit
P4	40 unit	40 unit	10 unit	40 unit

The company pays fully for some (but not all) of these commodities (and therefore the employees do not have to pay any charges for them). If total expenditure of each of P1, P2, P3 and P4 turned out to be equal in January then for which of the commodities among these four is (are) paid by the company? [5]

2. Let $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{pmatrix}$. Find the characteristic polynomial of A. Is A diagonalizable (give reasons)? Use the Cayley-Hamilton Theorem to compute its inverse. [5]

3. Solve the ODE: [5]

$$y^{(4)} - 4y''' + 6y'' - 4y' + y = e^x.$$

4. Solve the following nonhomogeneous system of ODEs using the method of variation of parameters. [5]

$$\begin{aligned} x_1' &= 8x_1 - x_2 + t \\ x_2' &= 4x_1 + 4x_2 + 2t - 1 \end{aligned}$$

5. Find the Laplace transform of [5]

$$\sin t \cos(2t) + t^2 \sin t + t^3 * t^4.$$

6. Find the inverse Laplace transform of [5]

$$\ln\left(\frac{s+2}{s+3}\right) + \tan^{-1}\left(\frac{4}{s}\right).$$

7. Solve the IVP: $y'' + 4y' + 5y = \delta(t-1)$, $y(0) = 0$, $y'(0) = 3$. [5]

8. Discuss existence and uniqueness of solution of

$$\left(\frac{y}{x^2+y^2} + x(x^2+y^2)^n\right)dx + \left(y(x^2+y^2)^n - \frac{x}{x^2+y^2}\right)dy = 0, \quad y(0) = 1,$$

where n is a natural number. Solve this IVP in implicit form. [3+2=5]