EEL 316 Minor Test II Semester II 2013-2014

Answer all questions (Q.1: 20 marks, Q.2: 20 marks) Full Marks: 40

1. A binary PAM signal is given by $y(t) = \sum_k a_k\, p(t - kT_b)$, where $a_k \in \{-1, 1\}$, and $p(t)$ is a real-valued bandlimited pulse with bandwidth W. The pulse $p(t)$ is given by

$$p(t) = \frac{4\sin^2\left(\frac{\pi W t}{2}\right)\left[\cos^2\left(\frac{\pi W t}{2}\right) - \alpha\right]}{\pi^2 W^2 t^2},$$

where α is a real constant, and $P(f)$, the spectrum of $p(t)$, satisfies the condition

$$P(f) = \text{constant} \quad \text{for } |f| \le \frac{W}{2}.$$

The bandwidth W is related to the bit duration T_b by $W = \frac{1}{T_b}$.

(a) Calculate the value of α. [6]

(b) Sketch the plot of $P(f)$, labeling the relevant portions in terms of f and W. [4]

(c) Find, in terms of T_b, the first six consecutive positive values of t for which $p(t) = 0$. [4]

(d) The signal $y(t)$ is passed through a tapped delay line equalizer with impulse response

$$h_{eq}(t) = w_{-1}\,\delta(t + T_b) + w_0\,\delta(t) + w_1\,\delta(t - T_b).$$

Find the tap weights w_{-1}, w_0, w_1 for which $p_{eq}(t) = p(t) \star h_{eq}(t)$ approximately satisfies the Nyquist criterion. Assume $p_{eq}(0) = 1$. [6]

2. Equicorrelated signals $y_1(t), y_2(t), y_3(t)$ over a signaling interval $[0, T)$, each having energy E, and satisfying

$$\int_0^T y_i(t) y_j(t)\,dt = \frac{E}{2} \quad \text{for } i \neq j,$$

are converted to another set of signals $\{s_1(t), s_2(t), s_3(t)\}$ by the transformation

$$s_i(t) = y_i(t) - \frac{1}{3}\sum_{k=1}^{3} y_k(t), \quad i = 1, 2, 3.$$

(a) What is the dimension N of the signal space $\{s_1(t), s_2(t), s_3(t)\}$? Starting with $s_1(t)$, obtain an orthonormal basis $\{\phi_1(t), \ldots, \phi_N(t)\}$ for the signal space using Gram-Schmidt orthogonalization. [10]

(b) Find the vectors $\underline{s}_1$, $\underline{s}_2$, and $\underline{s}_3$ for the orthonormal basis found in (a). [10]

Some Formulae

- $\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \text{if } |t| \le \frac{T}{2}, \\ 0 & \text{if } |t| > \frac{T}{2}, \end{cases} \qquad \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$
- Fourier Transform pairs: $\text{rect}\left(\frac{t}{T}\right) \leftrightarrow T\,\text{sinc}(fT)$, $\exp(j2\pi f_0 t) \leftrightarrow \delta(f - f_0)$, $G(t) \leftrightarrow g(-f)$
- Matrix inverse:

$$\begin{bmatrix} 1 & a & b \\ a & 1 & a \\ b & a & 1 \end{bmatrix}^{-1} = \frac{1}{(1-b)(1+b-2a^2)} \begin{bmatrix} (1-a^2) & -a(1-b) & (a^2-b) \\ -a(1-b) & (1-b^2) & -a(1-b) \\ (a^2-b) & -a(1-b) & (1-a^2) \end{bmatrix}$$