IIT Delhi

Department of Mathematics

MAL 260: Boundary Value Problems

Major Exam

Maximum Marks:30 **Time:2 Hour**

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1. Find the Laplace transform $U(x,s)$ of the solution of: [4]

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} - 1, 0 < x < 1, 0 < t,$$

$$u(0,t) = 0, u(1,t) = 0, 0 < t, u(x,0) = 0, 0 < x < 1.$$

2. Solve the below potential problem in a cylinder: [5]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2} = 0, 0 < r < a, 0 < z < b,$$

$$u(a,z) = 0, 0 < z < b, u(r,0) = 0, u(r,b) = U_0, 0 < r < a.$$

3. Solve the 2-dimensional heat conduction problem in a rectangle if there is insulation on all boundaries and the initial condition is $u(x,y,0) = x + y$. [5]

4. Solve the potential equation in the sphere $0 < \rho < 1, 0 < \phi < \pi$ with the boundary condition

$$u(1,\phi) = \begin{cases} 1, & 0 < \phi < \pi/2, \\ 0, & \pi/2 < \phi < \pi, \end{cases}$$

together with appropriate boundedness condition. [5]

5. Using the infinite series representations for the Bessel functions, prove the following: [5]

$$\frac{d}{dx}(x^{-\mu}J_\mu(x)) = -x^{-\mu}J_{\mu+1}(x),$$

6. Prove the following: [6]

(a) $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}[(x^2-1)^n]$ satisfies the Legendre differential equation $\left((1-x^2)y'\right)' + n(n+1)y = 0.$

(b) $\int_{-1}^{1} P_n(x)P_m(x)dx = 0 \quad n \neq m.$

(c) $(n+1)P_{n+1}(x) + nP_{n-1}(x) = (2n+1)xP_n(x).$