Department of Mathematics
II Semester 2015-2016
MAL 656/ MAL 468/ MTL 768 Graph Theory
Major Examination
Date: 7. 6 .2016 Weightage: 40%+ 5% bonus Time: 8 A.M. - 10 A.M.

Q1. Let G be a graph of order at least 2. If G has exactly two non-cut vertices, then prove that G is the path on n vertices. [4]

Q2. Let G be a connected graph with a cut vertex, say v. Prove that $\chi(G) \leq \Delta(G)$. [3]

Q3. Let N=(V,A.c,s,t) be a network and let f be a maximum flow in N. Construct a minimum cut in N. Justify your answer. [5]

Q4. Let G be a connected bipartite graph with bipartition (X,Y) with $|X| \geq 2$ and $|Y| \geq 2$. Prove that the following statements are equivalent. [6]

(a) Each edge of G is contained in a perfect matching of G.
(b) $|X|=|Y|$ and $|S| < |N(S)|$ for every proper non-empty subset S of X.
(c) G-{x,y} has a perfect matching for every $x \in X$ and every $y \in Y$.

Q5. Let G be a connected graph such that x and y be the only vertices of G of odd degree. Prove that there is an Euler trail from x to y (an open x-y walk that contains each edge exactly once). [2]

Q6. Let G be a non-bipartite graph and v be a vertex of G such that every odd cycle of G contains the vertex v. What is the chromatic number of G? Justify your answer. [2]

Q7. Let G be a graph with n vertices, m edges and has chromatic number k. Prove that $k(k-1) \leq 2m$. [2]

Q8. Let N=(V,A.c,s,t) be a network and let f be a maximum flow in N. Prove that the valueof(f) = $f(P,\overline{P}) - f(\overline{P},P)$, $(P,\overline{P})$ is an s-t cut of G. [4]

Q9. The Cartesian product $G_1 \times G_2$ of two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is the graph G=(V,E), where $V=V_1\times V_2$ and E={(a,b)(c,d)| either a=c and $bd \in E_2$ or $ac \in E_1$ and b=d }. Prove that the Cartesian product of two Hamiltonian Graphs G_1 and G_2 is Hamilton.[6]

Q10. Let u and v be two non-adjacent vertices of a connected graph G. A subset S of vertices of G is called a u-v separator if u and v lies in different components of G-S. S is called a minimal u-v separator if S is a u-v separator but no proper subset of S is a u-v separator. Let G be a connected bipartite graph in which every minimal u-v separator for non-adjacent vertices u and v induces a complete sub graph. Prove that G must be a tree. [6]

Bonus Section

Q11. Prove that if $o(G-S) \leq |S|$ for every proper subset S of V(G), where o(G-S) is the number of odd components of G, then G has a perfect matching. [5]

The End