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ELL 703 OPTIMAL CONTROL

MAJOR TEST

Time: 2 hrs. Marks: 80

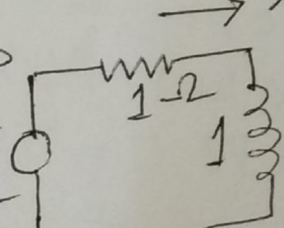
Q1(a): Consider minimization of PI $L(x,u)$ subject to constraint $f(x,u)=0$. Let (x^*, u^*) be optimal sol^n. Now, constraint is changed by df. If we are required to remain at optimal sol^n, then DERIVE the expressions for dx and du (in terms of df) which represent the change in optimal solution.

Q.1(b): Find the extremal of functional

$$J=\int_{-2}^{0}[12t\,x(t)+\dot{x}^2(t)]\,dt \text{ with } x(-2)=3 \;\&\; x(0)=0$$

Find optimal $x^*(t)$ and nature of extrema. (18)

Q.2(a): It is desired to change up the inductor to $x(T)=2$ Amp at $T=1$ if $x(0)=0$ while minimizing $J=\int_0^1 u^2(t)\,dt$. Find optimal control and optimal state trajories.



Q.2(b): Let $\dot{x}_1=x_2$ and $\dot{x}_2=u(t)$ with PI

$$J=\frac{1}{2}\int_0^{\infty}(x_1^2+2x_1x_2+2x_2^2+u^2(t))\,dt$$

(i) Find the solution to ARE (ii) Find optimal control and optimal closed loop system. (18)

Q.3: ⊛ Let $\dot{x}=x-u$ where $x\in\mathbb{R}$. It is desired to drive any initial state $x(0)$ to ZERO in MINIMUM TIME if $|u(t)|\le 1$. ⊛ Formulate and solve the problem using Pontryagin's Principle. (i) Find solution for costate eq^n in terms of unknown $\lambda(T)$. Sketch $\lambda(t)$. (ii) Express $u^*(t)$ in terms of $\lambda(T)$ for all possible cases to find possible values for $u^*(t)$. (iii) Solve state eq^n

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⊛ P.T.O.

for all possible cases to find possible values for $u^*(t)$ if $x(T)=0$. (iv) Sketch switching curve and sample trajectories in phase plane. (v) Find optimal cost J^* in terms of $x(0)$ and optimal feedback control (vi) In terms of $x(0)$, when does this optimal control problem have a solution. (14)

Q.4:— Using the results (on extremization of functional on extremization of functional with Terminal Cost, DERIVE the complete solution for optimal control <u>using Hamiltonian</u> when plant is

$$X(k+1) = A(k)X(k) + B(k)U(k) \quad , \quad x(k=k_0) = X(k_0)$$

and PI $J = J(X(k_0), U(k_0), k_0) = \frac{1}{2} X'(k_f) F(k_f) X(k_f)$. (10)

$$+ \frac{1}{2} \sum_{k=k_0}^{k_f-1} [X'(k)Q(k)X(k) + U'(k)R(k)U(k)]$$

Q.5:—(a) Consider plant $\dot{X}(t) = f(X(t), U(t), t)$ with PI $J(X(t_0), t_0) = S(X(t_f), t_f) + \int_{t_0}^{t_f} V(X(t), U(t), t)\,dt$.

DERIVE the H-J-B equation.

(b) For 1st order system $\dot{x}(t) = -2x(t) + u(t)$ with

PI $J = \frac{1}{2} x^2(t_f) + \frac{1}{2} \int_0^{t_f} [x^2(t) + u^2(t)]\,dt$

find the optimal control <u>using H-J-B framework</u>. (20)

<u>SHOW ALL STEPS IN ALL PROBLEMS</u>

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(*) **Q.3:**—(0) Find state & costate eq^ns, boundary cond^ns and Pontryagin's ~~stationarity~~ optimality condition.