Department of Mathematics
Indian Institute of Technology Delhi
MTL 108 – Introduction to Statistics
Major Test – 02-05-2015

Time: Two Hour **Total Marks: 40**

Q1. Consider the following bi-variate data table:

X	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Y	4	4	6	7	8	7	9	12	12	13

a) Find the linear regression coefficients of X on Y and Y on X by forming and solving the Normal Equations.
b) Determine the angle between these two lines.
c) Obtain the Normal equations if you like to fit a 2nd degree polynomial.

[2+ 1+ 3 = 6]

Q2. Consider the following data on two groups (M and P) of students of 24 each.

M	26	27	29	31	32	35	36	39	40	41	42	43
	47	48	50	51	52	53	58	61	62	66	70	71
P	28	30	33	34	37	38	44	45	46	49	54	55
	56	57	59	60	63	64	65	67	68	69	72	73

Test Median = 50 Median = 50

(a) Use Signed Rank test for testing whether both the groups have the same central tendency. You may use Normal tables with appropriate justification
(b) Discuss how the Wald-Wolfowitz Run test can be used for the same.

[4 + 3 = 7]

Q3. (a) Prove or disprove: If T, a function of n samples, is consistent estimator for λ, the unknown parameter of a Poisson distribution, the T^2 is also a consistent estimator of λ^2.

(b) If 10 Observations from a Poisson population are 1, 2, 3, 2, 3, 4, 1, 4, 3, 2, then find the MVUE for λ. Justify your answer.

[3 + 4 = 7]

Q4. (a) State and Prove the Neyman- Pearson Lemma.

(b) Use Neyman-Pearson Lemma to construct the Most Powerful Critical Region for testing $H_0 : \theta = \theta_0$ vs. H1: $\theta = \theta_1$ where $\theta_0 < \theta_1$, based on a sample of size n taken from N(θ, 1), at 95% level of confidence.

[3 + 4 = 7]

Q5. (a) Explain the term *Stochastically Smaller* in comparing two populations X and Y.

(b) What is the Mann-Whitney U-statistic? Explain how it helps in comparing two populations.

(c) Compute the Mean Variance of the U-statistic when there are 2 X and 4 Y observations.

[1 + 3 + 3 = 7]

Q6. (a) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population characterized by the pdf:

$$f(x) = \frac{1}{x^2} \quad 1 < x < \infty$$

Find the density function of the ***minimum*** of n samples drawn from this population

(b) Two random samples gave the following results:

Sample	Size	Sample Mean	Sum of Sq of deviation from Mean
A	10	15	90
B	12	14	108

Test at 5% level of significance whether the samples are from the same Normal population

[2 + 4 = 6]