

IIT Delhi, Department of Mathematics

MAL335: Differential Equations

Max Marks 25

Minor-2,

Max. Time: 1 hour

1. Consider the equation $y'' + \sin(x)y' + y = 0$. Let ϕ be a non-trivial solution, and let $\psi(x) = \phi(x + 2\pi)$.

(a) Prove that ψ is also a solution.

(b) Show that ϕ is a periodic solution of period 2π if and only if, $\phi(0) = \phi(2\pi), \phi'(0) = \phi'(2\pi)$. [6]

2. Using the *method of reduction of order*, find the second linearly independent solution $\phi_2(x)$ when the first solution $\phi_1(x)$ of the following equations are given:

(a) $4x^2y'' + y = 0, (x > 0)$; $\phi_1(x) = x^{1/2} \log x$. [6]

(b) $y'' - 4xy' - (2 - 4x^2)y = 0, (x > 0)$; $\phi_1(x) = xe^{x^2}$. [6]

3. Find all solutions of $x^2y'' + xy' + x^2y = 0$ for $(x > 0)$. [7]