Department of Mathematics

MTL107: Numerical Methods and Computations

Major | Max Marks 50 | Max Time 2 Hours

Answer ALL questions ONLY by the methods indicated.
No marks for using Graphical Methods and No marks for using different methods.

1. (a) Complete the missing lines in the below MATLAB code for finding the solution accurate to within 10^{-5} of the equation $x - 2^{-x} = 0$ for $0 \le x \le 1$: [2 Marks]

```
clc; close all; clear all; a=0; b=1; if func(a)*func(b) > 0 break else n=1;
c=(a+b)/2; while abs(func(c))>epsilon if (func(a)*func(c) < 0) b=c else a=c end
n=n+1; c=(a+b)/2; end end
```

(b) Modify the below code for finding the solution accurate to within 10^{-4} of the equation $x - \cos x = 0$ for $0 \le x \le \frac{\pi}{2}$ using the False Position Method: [2 Marks]

```
clc;clear all;close all; x(1)=0; x(2)=1.57; eps=.0001; if new(x(1))*new(x(2))> 0
break end for i=2:100
x(i+1)=x(i)-(new(x(i))*(x(i)-x(i-1)))/(new(x(i))-new(x(i-1)))
if abs(new(x(i+1)))<eps break end end
```

(c) Add the lines of code concerning the back-substitution for implementing the Gaussian Elimination Method [3 Marks]

```
clc; clear all; close all; disp('Solution of N-equation "[A][X]=[r]"') n=input ('Enter number of Equations :'); A=input ('Enter Matrix [A]:'); r=input ('Enter Matrix
[r]:'); D=A;d=r; s=0;
for j=1:n-1 for i=1+s:n-1 L=A(i+1,j)/A(j,j); A(i+1,:)=A(i+1,:)-L*A(j,:);
r(i+1)=r(i+1)-L*r(j); end s=s+1; end
```

2. Compute two iterations $\underline{x}^{(1)}, \underline{x}^{(2)}$ using Conjugate Gradient Method for the linear system:

$$3x_1 - x_2 + x_3 = 1$$
$$-x_1 + 6x_2 + 2x_3 = 0$$
$$x_1 + 2x_2 + 7x_3 = 4,$$

by taking initial guess $\underline{x}^{(0)} = (0, 0, 0)^t$. [7 Marks]

0.153
-0.169

$x + tv$

$t = \frac{\langle \ldots \rangle}{\langle v, Av \rangle}$

$x' = x + tv$
$r' = r$

3. (a) Let $f(x) = \sqrt{x - x^2}$ and $P_2(x)$ be the Lagrange interpolation polynomial on $x_0 = 0$, x_1 and $x_2 = 1$. Find the largest value of x_1 in $(0, 1)$ for which $f(0.5) - P_2(0.5) = -0.25$. [4 Marks]

(b) Derive the formula (inequality) for the total error $E(h)$ (both round-off and truncation) for the numerical differentiation formula [3 Marks]

$$y'(x_0) = \frac{1}{2h}\Big[y(x_0 + h) - y(x_0 - h)\Big] - \frac{h^2}{6} y^{(3)}(\xi_0), \quad x_0 - h < \xi_0 < x_0 + h.$$

1/3

$\left(\frac{3\epsilon}{M}\right)$

(c) By using the above error formula $E(h)$ compute the optimal h to minimize $E(h)$ for the given initial value problem [4 Marks]

$$y' = -y + 2, \qquad y(0) = 0, \qquad 0 \leq x \leq 1.$$

0.25

(Assuming that the maximum round-off error is bounded by $\epsilon = 10^{-2}$.)

4. (a) Prove that there exists a $\mu \in (a, b)$ for which the Composite Simpson's rule for n subintervals can be written with its error term as [4 Marks]

$$\int_a^b f(x)dx = \frac{h}{3}\Big[f(a) + 2\sum_{j=1}^{\frac{n}{2}-1} f(x_{2j}) + 4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1}) + f(b)\Big] - \frac{(b-a)}{180} h^4 f^{(4)}(\mu).$$

where $f \in C^4[a, b]$, n is even, $h = \frac{(b-a)}{n}$, and $x_j = a + jh$, for each $j = 0, 1, \cdots, n$.

(b) Write the statement of the theorem of Composite Trapezoidal rule for n subintervals along with the error term. Determine the values of n and h required to approximate using Composite Trapezoidal rule, the $\int_0^2 \frac{dx}{x+10}$ to within 10^{-6}. [7 Marks]

(c) Approximate the improper integral $\int_0^\infty \frac{dx}{1+x^4}$ using the Gaussian quadrature with $n = 2$ (given the roots 0.57735, -0.57735 and coefficients 1,1.) [5 Marks]

5. Compare the approximate solutions $y(0.5), y(1.0)$ with the exact solutions by tabulating the values and the corresponding absolute errors for the initial value problem [9 Marks]

$$y' = \sin x + e^{-x}, \quad 0 \leq x \leq 1, \quad y(0) = 1$$

with $h = 0.5$, using

(a) the Euler Method,

(b) the Taylor Series Method of order two,

(c) and any of the Runge-Kutta Methods.