PHY113 : MINOR II

Max. Marks : 25

Attempt all questions.

1. The one-dimensional neutron diffusion equation with a (plane) source is

$$-D\frac{d^2\phi(x)}{dx^2} + K^2 D\phi(x) = Q\delta(x),$$

where $\phi(x)$ is the neutron flux, $Q\delta(x)$ is the source at $x = 0$, and D and K^2 are constant. Obtain the solution of the differential equation using the Fourier transform technique. (5)

2. Use the method of Frobenius to find one solution near $x = 0$ of the ordinary differentail equation

$$x^2y'' + (x^2 + 2x)y' - 2y = 0$$

for any positive integer n. (8)

3. The Legendre polynomials can be shown to satisfy the orthonormality relation

$$\int_{-1}^{1} P_m(x)P_n(x)\mathrm{d}x = \frac{2}{2n+1}.$$

Expand the function

$$G(x) = \begin{cases} 1, & 0 \le x \le 1, \\ -1, & -1 \le x \le 0, \end{cases}$$

in a Legendre series. (4)

n^2+n+2n

n^2+3n

4. We derived the the Bessel functions in class and found them to be

$$J_\nu(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\Gamma(n+\nu+1)}\left(\frac{x}{2}\right)^{\nu+2n}.$$

~~n+3~~ $n+1+3$

$\frac{(2n)(2n+1)}{6(2n+1)}$

Using the equation

$$\frac{d}{dx}\left[x^{\nu} J_{\nu}(x)\right] = x^{\nu} J_{\nu-1}(x),$$

and given that $J_{1/2}(x) = (2/\pi x)^{1/2}\sin x$ and $J_{-1/2}(x) = (2/\pi x)^{1/2}\cos x$, express $J_{3/2}(x)$ in terms of trigonometric functions. (4)

5. Using the generating function for the Legrendre polynomials

$$g(x,h) = (1 - 2xh + h^2)^{-1/2},$$

express the Coulomb potential in terms of the Legendre polynomials. This is very useful in multipole expansions and boundary value problems in electrostatics. (4)