MAL 245 MINOR - II

# TOPOLOGY AND FUNCTIONAL ANALYSIS

Maximum Credit : 25 October 9, 2014

The numbers on the right indicate maximum credit for the corresponding problems. JUSTIFY YOUR ANSWERS.

Q.1. Let $(X, \tau)$ be a Hausdorff topological space, A be a non-empty compact subset of X and $x \in X - A$. Show that there exist open sets U and V in $(X, \tau)$ such that $x \in U$, $A \subseteq V$ and $U \cap V = \phi$. [6]

Q.2. Show that a compact metric space $(X, d)$ is separable. [6]

Q.3. Let $f : (X, d) \rightarrow (Y, \rho)$ be a continuous function between two metric spaces $(X, d)$ and $(Y, \rho)$. Suppose $(X, d)$ is compact. Show that $f$ is uniformly continuous. [7]

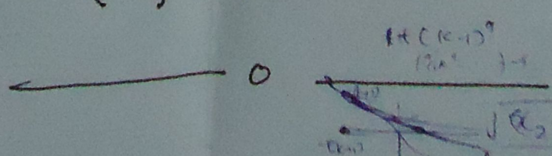
Q.4. (a). Show that the set $A = \{ (1 - \frac{1}{n}) : n \in \mathbb{N} \} \cup \{1\}$ is compact in $(\mathbb{R}, |\cdot|)$. [3]

(b) Show that the set $B = \{ (x, y) \in \mathbb{R}^2 : x^9 + y^9 = 1 \}$ is unbounded in $(\mathbb{R}^2, |\cdot|)$. [3]

$(-(k+1))^9 + y^9 = 1$

$y^9 = 1 + (k+1)^9$

$y = 1 + (k+1)^9$



$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} < M$

$\sqrt{(x_2 - x_1)^2 + ((1 - x_1^9)^{1/9} - (1 - x_2^9)^{1/9})}$

$(x_2 - x_1)^2 +$ $|x_2 - x_1| < M$

$|y_2 - y_1| < M$

$-k < 1 - x_1 < k$

$|1 - x_1| < k$

$|y_1| < k$

$1 - k < x_1 < k + 1$

$-k + 1 < x_1 < k + 1$