

Prob. 1: Let $x(t)$ be USB-SSB signal (message $m(t)$ & carrier ω_c). Further let $\tilde{x}(t) = A_c \cos \omega_c t + x(t)$. Prove/disprove that $m(t)$ can be recovered from $\tilde{x}(t)$ using envelope detector. (3)

$$x(t) = m(t) \cos \omega_c t + m_h(t) \sin \omega_c t$$

$$\tilde{x}(t) = A_c \cos \omega_c t + m(t) \cos \omega_c t + m_h(t) \sin \omega_c t$$

$$= [A_c + m(t)] \cos \omega_c t + m_h(t) \sin \omega_c t$$

$$= E(t) \cos(\omega_c t + \theta)$$

2 1/2

$$E(t) = \sqrt{[A_c + m(t)]^2 + [m_h(t)]^2}$$

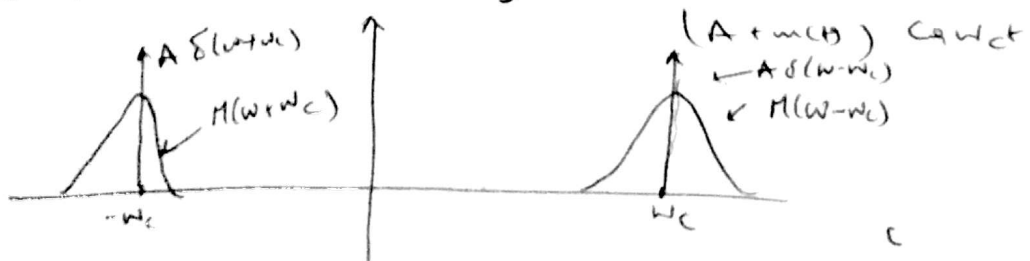
$$= |A_c + m(t)| \sqrt{1 + \left(\frac{m_h(t)^2}{(A_c + m(t))^2}\right)} \ll 1$$

$$|E(t)| \approx |A_c + m(t)|$$

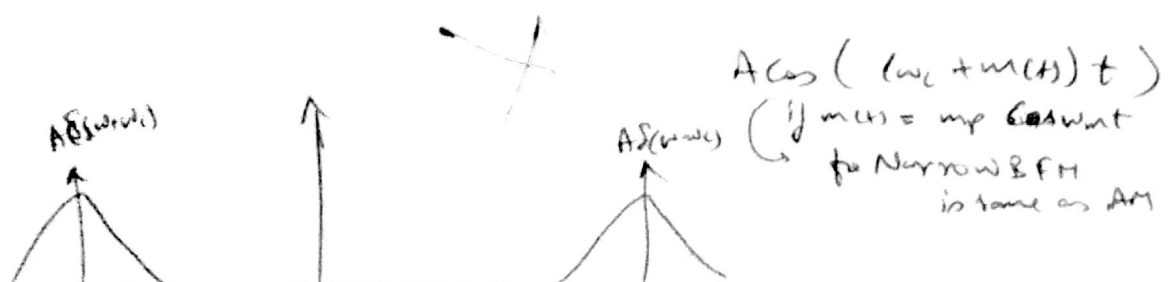
Passing $E(t)$ through a DC removal circuit will give $m(t)$.

Prob. 2 Draw the phasor diagram for (i) AM signal; (ii) NBFM signal. (3)

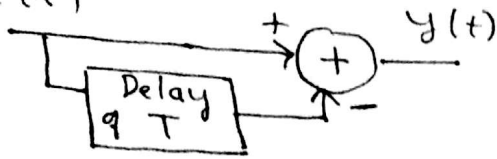
(i)



(ii)



Prob. 3 An FM signal $x(t) = A \cos(\omega_c t + k_f \int_{-\infty}^t m(\lambda) d\lambda)$ is passed through the following system $y(t)$, where $T \ll \frac{1}{\omega}$ for all ω of interest.



Can one recover $m(t)$ from $y(t)$? How? (2)

$$y(t) = m(t) - m(t-T)$$

$$= A \cos(\omega_c t + k_f \int_{-\infty}^t m(\lambda) d\lambda) - A \cos(\omega_c (t-T) + k_f \int_{-\infty}^{t-T} m(\lambda) d\lambda)$$

Multiplying $y(t)$ with $\cos \omega_c t$

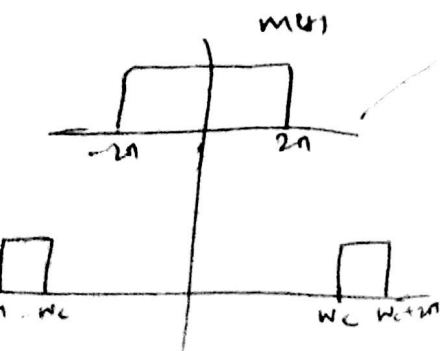
$$\begin{aligned} & A \cos(\cdot) \cos \omega_c t = A \cos(\omega_c (t-T) + k_f \int_{-\infty}^{t-T} m(\lambda) d\lambda) \cos \omega_c t \\ & = \cos(2\omega_c t + k_f \int_{-\infty}^t m(\lambda) d\lambda) + \cos(\int_{-\infty}^t m(\lambda) d\lambda) - \cos(2\omega_c (t-T) + k_f \int_{-\infty}^{t-T} m(\lambda) d\lambda) + \cos(\omega_c T + \int_{-\infty}^{t-T} m(\lambda) d\lambda) \\ & = \cos(\int_{-\infty}^t m(\lambda) d\lambda) + \cos(\int_{-\infty}^{t-T} m(\lambda) d\lambda) \\ & = \cos(2\int_{-\infty}^t m(\lambda) d\lambda) + \cos(\omega_c T) \end{aligned}$$

[No focus on Amplitude of signals see frequencies]

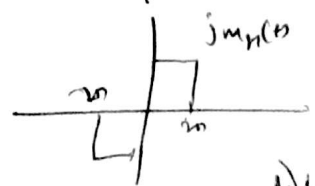
Yes ~~we~~ we can recover the $m(t)$ from $y(t)$, by multiplying it by $\cos \omega_c t$ and then a low pass filter and then \cos^{-1} circuit.

Prob. 4 Let $m(t) = \frac{\sin 2\pi t}{\pi t}$. Determine (3)

the time domain expression for corresponding USB-SSB signal with carrier freq $\omega_c = 40\pi$

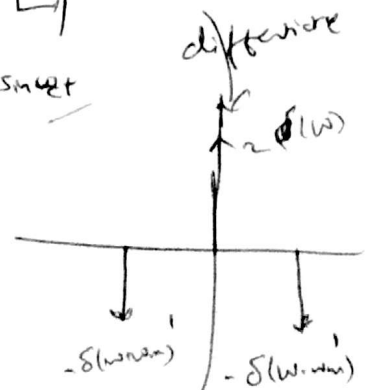


$$\begin{aligned} m_+(t) &= \frac{1}{2} (m(t) + j m_H(t)) \\ m_-(t) &= \frac{1}{2} (m(t) - j m_H(t)) \end{aligned}$$



$$\text{USB-SSB: } m(t) \cos \omega_c t + j m_H(t) \sin \omega_c t$$

$$j m_H(t) \rightarrow j \delta(\omega - \omega_c) + j \delta(\omega + \omega_c) + 2j \delta(\omega)$$



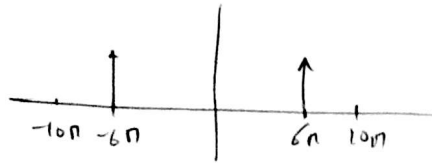
(1/2)

Q.D. > Let $m(t) = 2 \cos \omega_m t$. This signal is LSB-SSB modulated with carrier frequency ω_c . Take $\omega_m = 4\pi$ & $\omega_c = 10\pi$. Find & plot the time-domain SSB signal. ③

$M(\omega)$



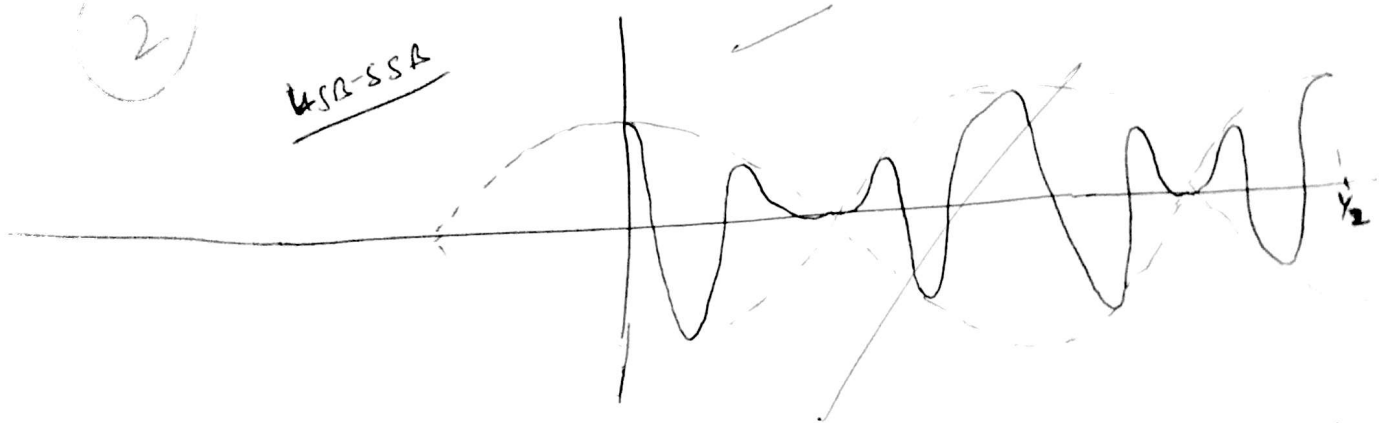
LSB-SSB-SC



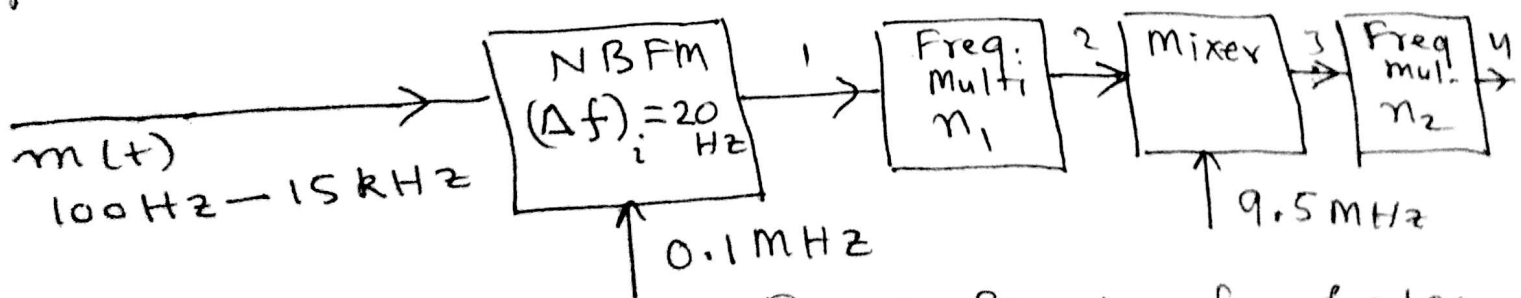
$$\rightarrow 2 \cos (\omega_c - \omega_m) t$$

②

LSB-SSB



Prob. 6 Consider the block diagram for Armstrong method for FM generation. Determine n_1 & n_2 such that they could be realized by multiplication of 2, 3 & 5 only. ④



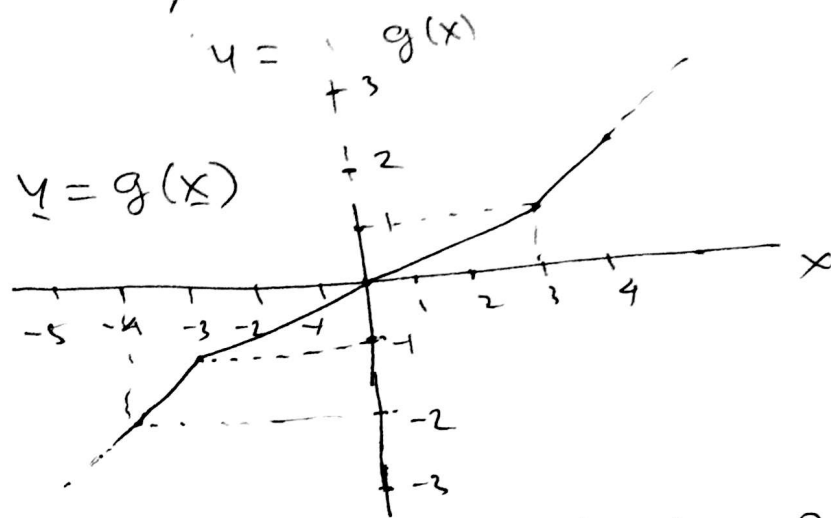
Final $\Delta f = 75 \text{ kHz}$ & Final carrier freq $f_c = 100 \text{ MHz}$

- | | f_c | Δf |
|---|---------|------------|
| ④ | 100 MHz | 75 kHz |
| ③ | 10 MHz | 7.5 kHz |
| ② | 0.5 MHz | 7.5 kHz |
| ① | 0.1 MHz | 1.5 kHz |

$$\begin{matrix} n_2 = 10 \\ n_1 = 5 \end{matrix}$$

①

Prob. 7



(5)

Let a random variable X , having pdf $f_X(x) = ke^{-2|x|}$ is transformed as given in fig. Find (i) k ; (ii) $f_Y(y)$, i.e. pdf of Y .

$$y = g(x) = \begin{cases} x+2 & x < -3 \\ x & -3 \leq x < 3 \\ x+1 & x \geq 3 \end{cases}$$

$$(i) \int_{-\infty}^{\infty} f(u) du = 1 \quad \Rightarrow \quad 2 \int_0^{\infty} ke^{-2u} du = 1 \quad k=1$$

$$(ii) \left| \frac{d}{dy} (u(y)) \right| = 1$$

$$f_Y(y) = \frac{1}{2}$$

$$e^{-2(y+2)}$$

$$e^{-2(y+1)}$$

$$e^{-2y}$$

$$e^{-2(y-1)}$$

$$y+2 > 3$$

$$-3 < y+2 < 3$$

$$y+2 < 3$$

no steps