$\frac{2 k_B T}{m} x$. $dx \cdot \frac{k_B T}{m} \cdot \sqrt{\frac{2k_BT}{m}}\, x^{1/2}$

$v^2 dv = \frac{k_B T}{m} \cdot v\, dx$

10th May 2016

Total Maximum Marks: 35

PYL202/Major Examination, Physics Department, (IIT, Delhi)

All questions are compulsory, maximum marks for each question are given in **bold** numericals. Symbols carry their usual meaning unless otherwise stated.

(Q1) Consider the velocity distribution function $\rho(v)$ for a **one dimensional** gas of molecules of mass m in thermal equilibrium with a heat bath at temperature T.

$$\rho(v) = \sqrt{\frac{m}{2\pi k_B T}}\, exp\left(-\frac{mv^2}{2k_B T}\right),$$

where k_B is the Boltzmann constant.

(a) **(1 marks)** Calculate $< v >$.

(b) **(2 marks)** Calculate $< v^2 >$ and show that the equipartition theorem is valid.

(b) **(3 marks)** How will this distribution modify in **three dimensions**? Is the equipartition law still valid in this case? Justify.

(Q2) ***(4 marks)*** Consider a random walk consisting of equi-probable $p = q = 1/2$ steps in left or right directions. However the step length at i^{th} step is given by $e^{-\lambda i}$, $i = 1, 2, 3, ..., N$, with $\lambda > 0$ a constant. Calculate the **mean displacement** and **mean-squared displacement**, after N steps. What happens when $N \to \infty$?

$\langle x \rangle$
$\langle x^2 \rangle$

$e^{-\lambda e}$

(Q3) (a) **(3 marks)** Consider a system of three particles each of them can be in any of the two states 0 and 2ϵ. The system is in contact with a heat reservoir at temperature T. Find the canonical partition functions in Maxwell-Boltzmann and Bose-Einstein statistics.

(b) **(2 marks)** Now consider two spinless particles and three states with energy 0, ϵ and 2ϵ find the canonical partition function in Fermi-Dirac statistics. What is the Fermi energy ϵ_F for this system?

(P.T.O.)

ABC ABC $n-1 = \frac{1}{2}$

C

AB BC $n = \frac{3}{2}$ $\frac{1}{2}\Gamma\pi$

$\Gamma n = (n-1)\Gamma n-1$ $e^{(\epsilon - \epsilon_F}$

$2\frac{1}{2}\Gamma\frac{1}{2}$

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(Q4) (10 marks) Consider a gas of N zero spin Bosons confined in a three dimensional volume V in contact with a reservoir at temperature T. Energy of this system is given by:

$\epsilon > \mu$ $\mu > 0$

$$\epsilon(p) = \gamma|\vec{p}|^j,$$

where γ and j are positive constants. Find the condition on j for which the Bose-Einstein condensate takes place. Treating average energy $E \approx N_{\epsilon>0}\, k_B T$ calculate the specific heat C_v and the entropy $S = \int C_v/T\, dT$. (*Hint: It would be necessary to calculate $N_{\epsilon>0}$ that is number of particles in the excited state when $T < T_c$ with $\mu \approx 0$.*)

Solve any one question from (Q5) or (Q6)

(Q5) (10 marks) Consider N classical non-interacting molecules enclosed in a three dimensional volume V and in contact with a heat reservoir at temperature T. Single molecule Hamiltonian is given by:

$$\mathcal{H}(\vec{r}_1, \vec{r}_2, \vec{p}_1, \vec{p}_2) = \frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} + \frac{k}{2}|\vec{r}_1 - \vec{r}_2|^2 .$$

Calculate Helmholtz free energy F, average energy E, the entropy S, heat capacity C_V. How will you go about calculating the Grand-Canonical partition function for this system?

(Q6) (10 marks) Some molecule possesses net unit magnetic spin $\vec{S}$ such that z−component S_z is quantized to values $-1, 0, 1$. Some N molecules of such a gas are confined in a volume V kept in contact with a reservoir at temperature T. A magnetic field of magnitude B is applied in z−direction. Hamiltonian for this system is given by:

$$\mathcal{H} = \sum_{i=1}^{N}\left(\frac{\vec{p}_i^{\,2}}{2m} - \mu B S_z^i\right)$$

$e^{-\mu B}$ e^x

Treating $\{\vec{q}_i, \vec{p}_i\}$'s classically and spin degrees of freedom quantized. Calculate the Canonical partition function $Z(T, N, V, B)$, average magnetic moment $< M >$ such that $M = \mu \sum_i S_z^i$, the susceptibility $\chi = \frac{\partial <M>}{\partial B}|_{B=0}$. How will you go about calculating the Grand-Canonical partition function for this system?

$\frac{V}{h^3} 4\pi \cdot 2m\epsilon \cdot \frac{m\, d\epsilon}{\sqrt{2m\epsilon}}$

$4\pi p^2 dp$

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