IIT Delhi, Department of Mathematics

MAL335: Differential Equations

Minor-1 | Max. Marks: 25 | Max. Time: 1hour.

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1. Let a_1, a_2 be two continuous functions on an interval I containing the point x_0. Let $b_1, b_2 \geq 0$ such that for all x in I,

$$|a_j(x)| \leq b_j, (j = 1, 2),$$

and define $k = 1 + b_1 + b_2$. If ϕ is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on I, then prove that for all $x > x_0$ in I

$$|\phi(x)|^2 + |\phi'(x)|^2 \leq \{|\phi(x_0)|^2 + |\phi'(x_0)|^2\}e^{2k(x-x_0)}.$$

[6]

2. Let a_1, a_2, a_3 be three continuous functions on an interval I containing the point x_0. If ϕ_1, ϕ_2, ϕ_3 are three solutions of $L(y) = y^{(3)} + a_1(x)y'' + a_2(x)y' + a_3(x)y = 0$ on I, then prove that $W(x) = W(x_0)\exp[-\int_{x_0}^{x} a_1(t)dt]$, where W is the Wronskian of ϕ_1, ϕ_2, ϕ_3. [5]

3. Given that ϕ and its derivative ϕ' are continuous on the interval $[-3, 3]$, with $\phi(0) = 0$ and $\phi'(0) = 1$. Find such a function ϕ satisfying the differential equations:

$$y'' - 9y = 0 \quad \text{on} \quad [0, 1]$$

and

$$y'' - 16y = 0 \quad \text{on} \quad [1, 2]$$

[8]

4. Consider the Initial Value Problem:

$$\frac{dy}{dx} = x^2y - 1.1y, \quad y(0) = 1.$$

Use Euler and 4th order Runge-Kutta numerical methods to solve the above problem with step size $h = 0.5$ on the interval $[0, 1]$. Compare the accuracy of those two methods. Give the results in a table along with the percentage errors

$$\epsilon_t = \frac{\text{True value} - \text{Approximate value}}{\text{True value}} \times 100\%.$$

[6]