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Name: ADITI  Entry: 2014CS10205  Gp:

# CSL703: Logic for Computer Science

Fri 31 Aug 2017 | Minor 1 | 60 minutes | Max Marks 40

1. Answer only in the space provided on the question paper.
2. No sharing of notes is allowed.
3. You are allowed to refer to the slides. But no communication or sharing of devices is permitted.

1. **[4+10 = 14 marks]** Boolean expressions used in programming can sometimes be undefined as the following boolean-valued function shows (it is undefined for all non-positive values of $n$).

```
fun gtz n = if (n=1) then true else gtz(x-1)
```

Hence it is necessary to consider a third truth value viz. the undefined. Assume that $\mathfrak{tfu} = \{t, f, u\}$ is a 3-valued set (containing "true", "false" and "undefined" respectively) with the ordering $\sqsubset$ such that $u \sqsubset t$, $u \sqsubset f$ and $t \not\sqsubset f \not\sqsubset t$. For any $x, y \in tfu$, $x \sqsubseteq y$ iff $((x = y)$ or $(x \sqsubset y))$. The algebra $\mathbf{tfu} = \langle \mathfrak{tfu}, \{t, f, u, ?:\}, \{\sqsubseteq, =\}\rangle$, besides the constants $t, f$ and $u$ contains a single ternary operator $?:$ defined as

$$x?y:z \stackrel{df}{=} \begin{cases} u & \text{if } x = u \\ y & \text{if } x = t \\ z & \text{if } x = f \end{cases}$$

(a) Prove that for all values in $\mathfrak{tfu}$, $\boxed{x_1 \sqsubseteq x_2}$, $y_1 \sqsubseteq y_2, z_1 \sqsubseteq z_2$ implies $x_1?y_1:z_1 \sqsubseteq x_2?y_2:z_2$.

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Given, $x_1 \sqsubseteq x_2$, $y_1 \sqsubseteq y_2$, $z_1 \sqsubseteq z_2$.

LHS: $x_1 ? y_1 : z_1$

RHS: $x_2 ? y_2 : z_2$

we know $u \sqsubset t$, $u \sqsubset f$, $f \not\sqsubset t \not\sqsubset f$.

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(b) Let $x \&\& y \overset{df}{=} x?y : f$ and $x \,\|\, y \overset{df}{=} x?t : y$. Prove that

$$x \&\& (y \,\|\, z) = (x \&\& y) \,\|\, (x \&\& z)$$

$x \&\& (y \,\|\, z) = x \,?.\, (y \,\|\, z) \,:.\, f$

$(x \&\& y) \,\|\, (x \&\& z) = (x \&\& y) \,?.\, t \,:.\, (x$

$x \&\& y) \,\|\, (x \&\& z)$

Lets take cases on value of $x$:

$\boxed{x = u}$: LHS = ~~[illegible]~~ $x \,?.\, (y \,\|\, z$

$= u.$

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2. [10 marks] The following ROBDD uses the variable ordering $a < b < c < d < e$. Construct the ROBDD for the same expression using the variable ordering $e < c < d < a < b$. Don't simply draw the resulting ROBDD. Show the steps in your construction. The dashed arrows represent the low arm and the solid arrows represent the hight arm.

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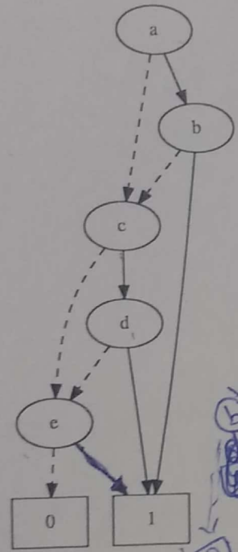
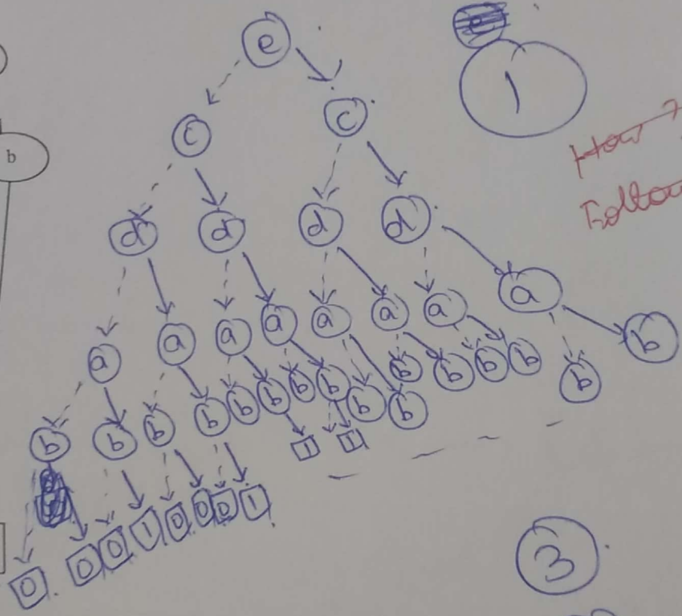
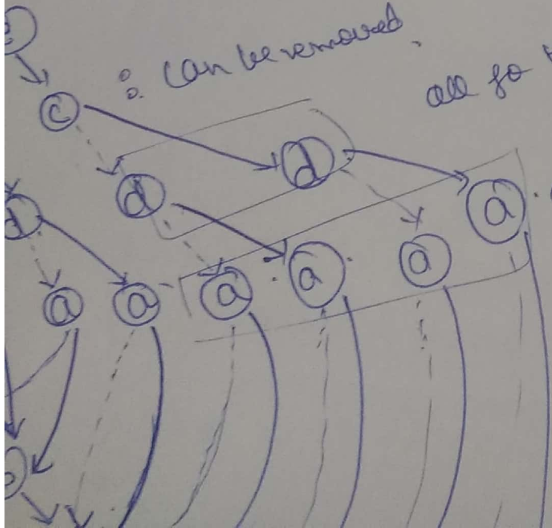
Lets draw most basic all go b

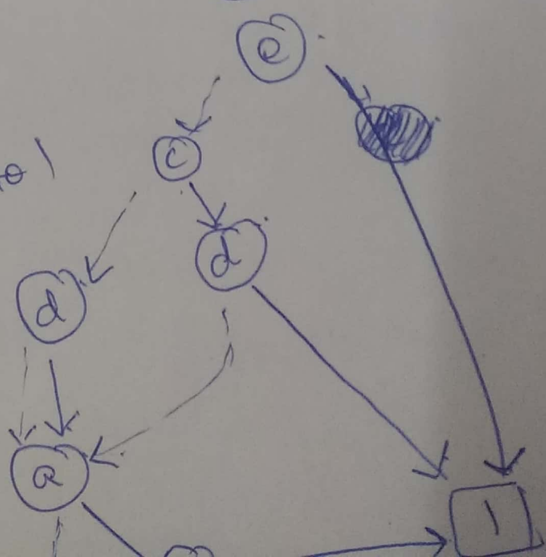


~~How?~~
~~Following what rules~~




3.



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3. [8+8 = 16 marks] Prove the absorption law $\vdash_{\mathcal{H}_0} \phi\wedge(\phi\vee\psi)\leftrightarrow\phi$ using the sequent form of the Hilbert-style proof system. Justify each step. You are allowed to use any of the derived inference rules given in the notes and exercises provided you reference them in your justification.

$\vdash \phi \wedge (\phi \vee \psi) \leftrightarrow \phi$ : To prove.

$\vdash \neg(((\phi\wedge(\phi\vee\psi)) \to \phi) \to \neg(\phi \to (\phi\wedge(\phi\vee\psi)))$ : To prove

$\phi\wedge(\phi\vee\psi) = \neg(\phi \to \neg(\phi\vee\psi))$    $\phi\to\neg($

$X = \boxed{\neg(\phi \to \neg(\neg\phi \to \psi))} = \neg Y$

prove : $\vdash \neg(\overset{\neg(\phi\to\neg(\neg\phi\to\psi))}{(X \to \phi)} \to \neg(\phi \to X))$