Department of Mathematics
MTL 108 (Introduction to Statistics)
Major Test (II Semester 2016 - 2017)

Time allowed: 2 hours Max. Marks: 50

1. Consider the random variable X that represents the number of people who are hospitalized or die in a single head-on collision on the road in front of a particular spot in a year. The distribution of such random variables are typically obtained from historical data. Without getting into the statistical aspects involved, let us suppose that the cumulative distribution function of X is as follows:

x	0	1	2	3	4	5	6	7	8	9	10
$F(x)$	0.250	0.546	0.898	0.932	0.955	0.972	0.981	0.989	0.995	0.998	1.000

Find (a) $P(X = 10)$ (b) $P(X \leq 5/X > 2)$. (2 + 2 marks)

2. Suppose Alwar district newspaper reported that for families in their circulation area, the distribution of weekly expenses for food consumed away from home has an average of Rs.258.485 and a standard deviation of Rs.45.00 . An economist randomly sampled 100 families for their outside-home food expenses for a week. What is the probability that the sample mean weekly expenses will be at most Rs. 248? (4 marks)

3. The temperature at which a thermostat goes off is normal distributed with mean μ and variance σ^2. Let S^2 be the sample variance of the five data values. If the thermostat is to be tested five times, find $P(S^2/\sigma^2 \leq 2.37)$. (4 marks)

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population X having uniform distribution on an interval $(0, \theta)$.

   (a) Find an estimator of the paratmeter θ using maximum likelihood method.

   (b) Use this estimator to find the estimators for the mean and the variance of X.

(3 + 2 marks)

5. The distribution of a variable is supposed to be normally distributed in two independent biological populations. The two population variances must be compared. After gathering information through simple random samples of sizes $n_1 = 11$, $n_2 = 10$, respectively, we are given the value of the estimators

$$S_1^2 = \frac{1}{n_1 - 1}\sum_{i=1}^{n_1}(x_i - \overline{x})^2 = 6.8; \; s_2^2 = \frac{1}{n_2}\sum_{i=1}^{n_2}(y_i - \overline{y})^2 = 7.1$$

For $\alpha = 0.1$, test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1"\sigma_1^2 \neq \sigma_2^2$. (7 marks)

6. (a) Consider two Bernoulli distributed populations with the same parameter. Prove that the pooled sample proportion is an unbiased estimator of the population proportion.

(b) Consider two normal distribued populations with the same variance. Prove that the pooled sample variance is an unbiased estimator of the population variance.

(2 + 3 marks)

7. The lifetimes (in hours) of samples from three different brands of batteries were recorded with the following results:

Y_1	Y_2	Y_3
40	60	60
30	40	50
50	55	70
50	65	65
30		75
		40

Test whether the three brands have different average lifetimes using ANOVA method. Use the 5% level of significance.

(7 marks)

8. The population is normally distributed. A random sample of 50 has a correlation coefficient of $r = 0.297$. Test the hypothesis that the population correlation coefficient, $\rho = 0$ against $\rho > 0$ at 5% level of significance. (7 marks)

9. Consider the following data satisfy a linear regression model $Y_i = \alpha + \beta x_i + \epsilon_i$.

x	0	1	2	3	4	5
y	0.475	1.007	0.838	-0.618	1.0378	0.943

Test the null hypothesis that $\beta = 0$ against $\beta \neq 0$ at the signficance level 0.05.

(7 marks)

Table Values

P(**Z** is a standard normal distribution $\geq Z_\alpha$) $= \alpha$

P(χ^2 r.v. with n d.f. $\geq \chi^2_{n,\alpha}$) $= \alpha$; P(**t** r.v. with n d.f. $\geq t_{n,\alpha}$) $= \alpha$

P(**F** r.v. with n_1 and n_2 degrees of freedom $\geq F_{n_1,n_2,\alpha}$) $= \alpha$

$Z_{0.025} = 1.96$; $Z_{0.05} = 1.645$; $Z_{0.0764} = 1.43$; $Z_{0.01} = 2.33$; $Z_{0.035} = 1.81$

$\chi^2_{2,0.05} = 5.99$; $\chi^2_{3,0.05} = 7.81$; $\chi^2_{4,0.05} = 9.48$; $\chi^2_{5,0.05} = 11.1$; $\chi^2_{6,0.05} = 12.6$

$t_{4,0.025} = 2.776$; $t_{8,0.025} = 2.31$; $t_{9,0.025} = 2.26$; $t_{10,0.025} = 2.22$

$F_{9,11,0.025} = 0.1539$; $F_{10,9,0.95} = 0.33$; $F_{10,9,0.05} = 3.14$; $F_{10,15,0.025} = 3.5217$; $F_{15,10,0.025} = 3.0602$; $F_{2,12,0.05} = 3.89$

Note: If above table values are not matched, please leave the answer without numerical.