

Department Of Mathematics  
Indian Institute Of Technology, Delhi  
MTL 108 - Introduction to Statistics  
Minor II Test - 22-03-2015

Time: One Hour

Total Marks: 21

Q1. a) What do you mean by the Critical region for testing of a Statistical Hypothesis? Explain its significance.

b) Suppose 9 samples are taken from Normal distribution with parameters mean =  $\mu$  and variance = 1. to test  $H_0: \mu = -1$  against  $H_1: \mu = 1$ . Suppose the Critical Region is:

$$X_1 - 2X_2 + 3X_3 - 4X_4 + 5X_5 - 6X_6 + 7X_7 - 8X_8 + 9X_9 > 0$$

Compute the size of the Critical Region and Power of the test.

[3 + 4 = 7]

Q2. a) A manufacturer produces chocolate bars in two different machines. Random samples of size 12 and 8 from the two populations show that the sample means of their weights are 101 and 99 grammes, respectively. Assuming the variance of weight distributions of the chocolate bars produced by the two machines are same and equal to 9 gramme<sup>2</sup>, test at 10% level of significance for  $H_0: \mu_1 = \mu_2$  vs.  $H_1: \mu_1 \neq \mu_2$ .

b) Given  $F_{(8, 16), \alpha} = 12.90$  and  $F_{(16, 8), \alpha} = 29.20$ , obtain  $F_{(8, 16), 1-\alpha}$ , where  $F_{(m, n), \delta}$  is such that  $P(X > F_{(m, n), \delta}) = \delta$ ,  $0 < \delta < 1$  where  $X \sim F_{m, n}$ . Justify your answer.

[4 + 3 = 7]

Q3. a) If X and Y are two independent random variables following  $\chi^2_{(8)}$  and  $\chi^2_{(10)}$ , respectively, find the pdf of  $X + Y$ .

b) A die is thrown 100 times and the outcomes are:

1	2	3	4	5	6
20	12	10	13	15	30

Test if the dice is unbiased by considering the following two cases separately:

- $\Pr(\text{even}) = \Pr(\text{odd})$
- $\Pr(1 \ \& \ 2) = \Pr(3 \ \& \ 4) = \Pr(5 \ \& \ 6)$

[3 + 2 + 2 = 7]