Q1. Model the following problems using Integer Linear Programming problem.

(a) s-t shortest path problem in an undirected graph.   [1]

(b) Minimum spanning tree problem in graphs using n-1 edges and acyclic.   [1]

(c) Minimum spanning tree problem in graphs using n-1 edges and connected.   [1]

(d) Maximum weight Perfect Matching problem.   [1]

(e) The travelling salesman problem is to find a tour of minimum cost given n cities and the cost matrix C such that C[i,j] is the cost of going from city i to city j. A tour is a sequence of distinct cities covering all the cities. The travelling salesman starts from one of the cities and follow the tour (sequence) to go to the next city and upon reaching to the last city of the sequence goes to the first city in the sequence. Every city is visited exactly once.   [2]

Q2. Find a dual basic feasible solution of the following primal.   [3]

$$\text{Minimize } x_2 + 5x_3 - x_4$$
$$\text{s.t.}$$
$$x_1 + 2x_2 - x_3 + x_4 = 4$$
$$3x_2 + 4x_3 - x_4 + x_5 = 3$$
$$x_i \geq 0 \text{ for } i = 1,2,3,4,5.$$

Q3. Let G be a graph in which the end vertices of every odd length path are adjacent. Design a polynomial time algorithm to find a maximum matching in G. Justify your claim.   [6]