MAL 245: *TOPOLOGY AND FUNCTIONAL ANALYSIS*

DEPARTMENT OF MATHEMATICS
MINOR 1
MAX. MARKS 25

Answer any five. All questions carry equal marks.

I. Let $\mathcal{T}$ be the topology on $\mathbb{Q}$ generated by subbasic open sets of the form $[a,b] \cap \mathbb{Q}$ for all **irrationals** a, b.

a) Determine the closure of the sets:

(1) $(e, \pi) \cap \mathbb{Q}$

(2) $\{.9, .99, .999, \ldots\}$

(3) $\{\frac{m}{2^n} : m, n \in \mathbb{N}\} \cap (0,1)$ (dyadic rationals in $(0,1)$)

b) Is the topology $\mathcal{T}$ second countable? Justify.

II. For every $n \in \mathbb{Z}$, let

$$a_n = \{2n-1, 2n, 2n+1\}$$

Let $\mathcal{T}$ be the topology on $\mathbb{Z}$ generated by basic open sets $\{a_n : n \in \mathbb{Z}\}$. Is the topological space $(\mathbb{Z}, \mathcal{T})$ T_1?, Hausdorff?, second category? Justify your answer.

III. Define when $f : X \to Y$ is continuous. Let $X = \{a, b\}$ and $Y = \{a, b, c\}$. Give topologies on X and Y so that

(a) every $f : X \to Y$ is continuous.

(b) no $f : X \to Y$, except the constant function, is continuous.

IV. Let X be a second countable, Hausdorff space. Show that the set of all isolated points in X is either empty or countable.

V. Let $f : X \to Y$ be continuous. Prove that X is homeomorphic to the graph of f, $G_f = \{(x, f(x)) : x \in X\}$, in $X \times Y$.

VI. Give examples of:

1. topological space with compact subsets that are not closed.
2. topological space with closed subsets that are not compact.
3. infinite topological space with every open set compact.

VII. Define "finite intersection property" and show that in every compact topological space any collection of closed sets with finite intersection property has non empty intersection.

Give an example of a topological space for which a collection of closed sets with finite intersection property has empty intersection.