CHL231 Fluid Mechanics for Chemical Engineers
Major Exam

Total Marks: 70

Date: 22/11/13

Use: g=9.8 m/s^2, ρ_{water} = 1000 kg/m^3, $\mu_{water} = 10^{-3}$ Pa-s.
ρ_{air} = 1.23 kg/m^3, $\mu_{air} = 1.8 \times 10^{-5}$ Pa-s.
Make suitable assumptions wherever necessary.

1. The creeping flow occurs around a spherical inviscid fluid (*e.g.* air bubble) of radius R. Let the bubble be stationary and flow of incompressible fluid is in z-direction. Assume the bubble maintains its shape and size and ignore the role of surface tension. State the boundary conditions. Obtain the velocity field in the fluid around the air bubble. State the equation to be used to obtain the pressure field. Write down the expression to calculate the drag force on the bubble. Simplify the drag force expression (integral). [Do not calculate the pressure and drag force]. [12]

2. An air bubble of radius $R(t)$ is immersed in an incompressible fluid. The bubble is expanding such that its radius is changing with time at non-uniform rate $\dot{R}(t)$. The expanding bubble causes flow in the surrounding fluid. If the flow is assumed to be potential flow, obtain the velocity field in the surrounding fluid. Let the pressure far away from the bubble be p_0. Obtain the governing equation for the pressure field in the fluid. Find the pressure on the surface of the bubble. [10]

3. Consider the squeezing flow generated by pushing a rigid sphere of radius R through a viscous fluid towards a flat plate at speed U. Find out the order of magnitude of non-zero velocity components and pressure. Calculate the force required for this motion when the gap between the sphere and the plate δ is very small compared to the sphere radius R. [Hint: In the thin film, the coordinates shown in the figure may be used. In these coordinates, the distance between the lower surface of the sphere and plate can be approximated as: $h(r) = \delta + r^2/(2R)$.] [13]

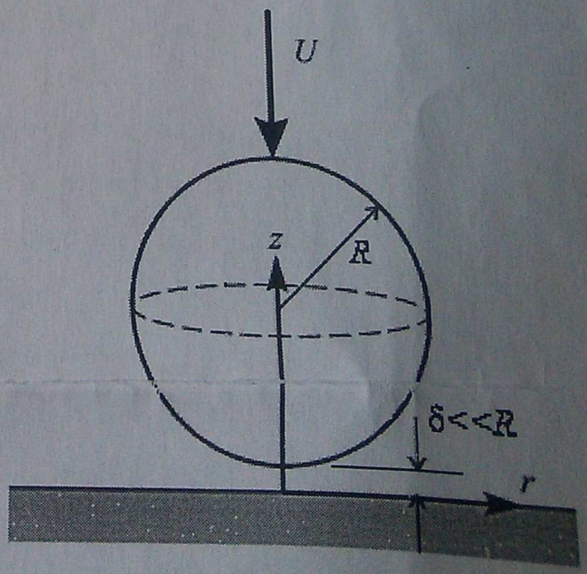

Figure 1: Problem 3.

4. Consider unbounded incompressible fluid with viscosity μ and density ρ undergoing circulatory motion described by ideal free vortex $v_\theta = \Gamma/(2\pi r)$. At time $t > 0$, the velocity at the center ($r = 0$) is forced to be zero. State the boundary and initial conditions. Obtain the transient velocity $v_\theta(r, t)$. [8]

5. Water flows through two sections of the vertical pipe shown in Figure 2. The bellows connections cannot support any force in the vertical direction. The pipe with diameter 0.12 m weighs 3 N/m. Let the length of pipe be L. The friction factor for the flow is assumed to be 0.02. At what velocity will the force required to hold the pipe be zero? [9]

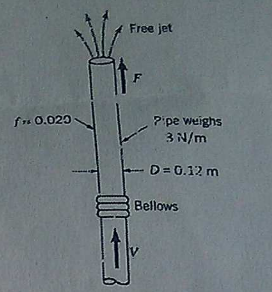

Figure 2: Problem 5.

6. Answer briefly [Explain your choices, no marks for guesswork]:

(a) A tank with re-entrant orifice called Borda mouthpiece is shown in Figure 3. Calculate the contraction coefficient A_j/A_0. [4]

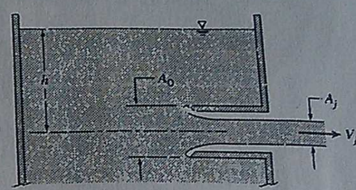

Figure 3: Problem 6(a).

(b) Very small micro-organism having a straight trail is suspended in a viscous fluid. The organism swishes (oscillates) its tail in order to swim in the fluid, as shown in Figure 4. Under Stokes condition (holds because $Re \ll 1$), which direction the organism will be able to swim? Explain your answer in detail. [4]

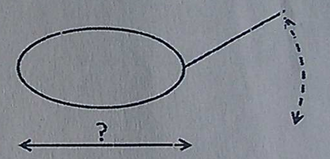

Figure 4: Problem 6(b).

(c) For flow around any immersed body, in the region near the stagnation point, the potential flow is described by velocity potential $\phi = (K/2)(x^2 - y^2)$. In reality, there will be a boundary layer near the solid surface. The x and y coordinates in ϕ expression are wall coordinates for the boundary layer flow. Write down the x-momentum balance equation to be solved for the boundary layer flow. Using the order of magnitude analysis, derive the scaling for boundary layer thickness $\delta(x)$. [5]

(d) For turbulence generated at length scale L with corresponding velocity scale U, using Kolmogorov's energy cascade arguments, find out the velocity scale associated with the eddy of size $l \ll L$. How does the kinetic energy per unit mass due to eddy motion scale with eddy size? Find out the size of the smallest eddy (Kolmogorov length scale). If turbulence is generated in agitated vessel by impeller of size D rotating with speed N, how does the smallest eddy size scale with N? [5]