MTL103: Mino -2 on March 22, 2015

Max Marks: 20 **Time: 1 Hr**

All questions are compulsory.

Q1. Prove that if the problem

$$\min\ c^T x \text{ subject to } Ax = b,\ x \geq 0$$

has a finite optimal solution, then the new problem

$$\min\ c^T x \text{ subject to } Ax = \hat{b},\ x \geq 0$$

can not be unbounded for any vector $\hat{b} \in \mathbb{R}^m$. [4]

Q2. Suppose the following is the optimal tableau of some maximizing LP problem with x_4, x_5 as slack variables and all variables are non-negative:

v_B	x_B	y_1	y_2	y_3	y_4	y_5
x_1	36	1	6	0	4	-1
x_3	6	0	-1	1	-1	1/2
$z_j - c_j \rightarrow$		0	9	0	11	1/2

Let a new constraint $x_1 + x_2 \geq 40$ be added to the LPP. Using the dual simplex method, determine an optimal solution and optimal value of the new LPP. [6]

Q3. The setting of this problem from William Shakespeare's play "The Merchant of Venice" written in 1597. In this play, a girl named Portia is the lead female character. Her life's ambition was to marry an extremely intelligent boy. For this, she purchased three caskets, made of gold, silver, and lead, and hid a stunningly beautiful portrait of herself in one of them. The suitor (prospective groom) was asked to identify the casket containing the portrait without opening the casket. If his choice is correct, he can claim Portia as his bride otherwise he will be permanently banished. Portia put inscriptions on the outer surfaces of these closed caskets as follows:

Gold Casket: The portrait is in this casket.

Silver Casket: The portrait is not in this casket.

Lead Casket: The portrait is not in Gold casket.

And she explained to the suitor that at most one of the three inscriptions is true. Formulate this description as a $0-1$ optimization problem (an optimization problem involving only binary variables). [5]

Q4. Solve the following problem by branch and bound method and also depict your working using a binary tree diagram.

$$\begin{aligned} \max\ & 4x_1 - x_2 \\ \text{subject to} & \\ 7x_1 - 2x_2 &\leq 14 \\ x_2 &\leq 3 \\ 2x_1 - 2x_2 &\leq 3 \\ x_1, x_2 &\geq 0, \text{ both integers.} \end{aligned}$$