PYL113 : MINOR I

Max. Marks : 25

Attempt all questions.

1. (a) Consider a real 2-D vector space. Define a scalar product $< a|b >= a_1b_1 + (a_2b_2)/2$. Verify the Cauchy-Schwartz inequality for this scalar product. (3)

   (b) Consider a linear vector space V of quadratic real polynomials in x. Choosing an appropriate basis, obtain a matrix representation for the linear operator $A \equiv d/dx$. (4)

   (c) Obtain the eigenvalues and eigenvectors of the following Hermitian matrix:

$$\mathbf{A} = \begin{pmatrix} \gamma & i\beta\gamma \\ -i\beta\gamma & \gamma \end{pmatrix}.$$

   with real constants β and γ. (This is a transformation matrix for the Lorentz transformation in 2-D Minkowski space (x, ict). (6)

2. Evaluate the integral (6)

$$\frac{1}{2\pi i}\oint_C \frac{dz}{\sin 1/z}, \qquad C \quad \text{being } |z| = 1/5,$$

substitution $w = \frac{1}{z}$

3. Draw a rectangular slit of width $2a$ centered about the origin. Obtain the corresponding Fourier transform (FT) and plot it clearly indicating the significant features. Argue that in the limit $a \to \infty$, this FT goes over to the Dirac-δ function. (6)

$f(w)$ $\frac{1}{w^2 \sin w}$

$\frac{1}{(m-1)}$