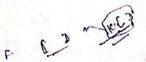
ALGEBRA - MAL516 MAJOR TEST

Maximum Credit: 40 May 2, 2015

The numbers on the right indicate maximum credit for the corresponding problems. JUSTIFY YOUR ANSWERS

- Q.1. Let R be a commutative ring with 1. Show that the following conditions are equivalent.
 - (i) R has a unique maximal ideal.
 - >(ii) All non-units of Recontained in some ideal M = R.
 - (iii) All mon-units of R form an ideal.
 - (v) For all 2,8ER, 1+3=1 implies either 2 is a unit or sis a unit.
- Q. 2. Let R be a ring such that for each sequence of ideals. A, A2, ... of R with A, CA2C..., there exists a V positive integer n (depending on the sequence) such that Am = An + mon. Let f: R -> R be a ring epimorphism. [5] Show that Kerf = fo3.
- Q.3. Let F be a finite field of characteristic 2 and 1 a be a non-zero element of F. Is the polynomial 22-a irreducible in F[x]?
- Q. 4. Prove that no finite field is algebraically closed

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(a) Every non-constant polynomial [FEX] has a next in F.

(b) Every non-constant polynomial f & F [r] splits over F.

(Every irreducible polynomial in FIX] has degree 1.

(d) There is no algebraic extension field of F (exept Fitself).

e) There exists a subfield K of F such that F is algebraic over K and every polynomial in K [X] splits in F [X]. [12]

G.6. Let F be an extension field of K, E an intermediate field and H a subgroup of Aut F. Show that

(i) H'= {v ∈ F: \(\sigma(v) = v \) \(\sigma \) \(\text{EH}\) is an intermediate field of the extension. [3]

(ii) E'= { $\sigma \in Aut_{K}^{F} : \sigma(u) = u + u \in E$ } is a subgroup of Aut_{K}.