Department of Mathematics

MTL 763 (Introduction to Game Theory)

Minor Exam 1

Time: 1 hour | Date: 30/08/17

Max. Marks: 25

Note: The exam is closed-book, and all the questions are compulsory.

Q.1 (*a*) Find a pure strategy saddle point equilibrium and value of the matrix game A given below:

$$A = \begin{pmatrix} 1 & 3 & 7 & 2 \\ 4 & 4 & 6 & 5 \\ 5 & 3 & 1 & 4 \end{pmatrix}.$$

(1 mark)

(*b*) Consider a matrix game A given below

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 1 \\ t & \frac{4}{3} & 1 \end{pmatrix},$$

where $-\infty < t < +\infty$.

(*i*) For a given strategy $x^* = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$ of player 1, find the optimal strategies of player 2 for all $-\infty < t < +\infty$.

(*ii*) For a given strategy $y^* = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$ of player 2, find the optimal strategies of player 1 for all $-\infty < t < +\infty$.

(*iii*) Using the information from (*i*) and (*ii*) can we say that (x^*, y^*) forms a saddle point equilibrium for a certain value of t.

(4 marks)

Q.2 Find a saddle point equilibrium and value of the game using graphical method

(*a*)

$$A = \begin{pmatrix} -1 & 2 \\ 3 & -4 \\ -5 & 6 \\ 7 & -8 \end{pmatrix}.$$

(2 marks)

(*b*)

$$A = \begin{pmatrix} 0 & 7 & 5 \\ 8 & 4 & 6 \\ 12 & -4 & 3 \end{pmatrix}.$$

(2 marks)

Q.3 Consider a matrix game

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 3 \\ \alpha & 5 & 1 \end{pmatrix},$$

where $\alpha \in (-\infty, 4]$. Using Linear programming method find the saddle point equilibria for all values of α.

(6 marks)

Q.4 (a) Let $I = \{A, B, C, D\}$ and $J = \{a, b, c, d, e, f\}$. Find all pure strategy Nash equilibria in following bimatrix game

$$\begin{pmatrix} (2,1) & (4,3) & (7,2) & (7,4) & (0,5) & (3,2) \\ (4,0) & (5,4) & (1,6) & (0,4) & (0,3) & (5,1) \\ (1,3) & (5,3) & (3,2) & (4,1) & (1,0) & (4,3) \\ (4,3) & (2,5) & (4,0) & (1,0) & (1,5) & (2,1) \end{pmatrix}.$$

Are Nash equilibria of above game also Pareto optimal?

(1 mark)

(b) Construct a 2×2 game which has two pure strategy Nash equilibria and both the Nash equilibria are also Pareto Optimal.

(1 mark)

(c) Find all the Nash equilibria of the following bimatrix game

$$\begin{pmatrix} (1,1) & (2,2) & (1,3) \\ (3,2) & (1,1) & (0,0) \end{pmatrix}.$$

(2 marks)

Q.5 (a) Consider an $m \times n$ matrix game A with value V. Let x and y be optimal mixed strategies of player 1 and player 2. Then

$$\sum_{j=1}^{n} a_{ij} y_j = V, \quad \text{for all } i \text{ for which } x_i > 0,$$

and

$$\sum_{i=1}^{m} a_{ij} x_i = V, \quad \text{for all } j \text{ for which } y_j > 0.$$

(2 marks)

(b) A matrix game is symmetric if A is a square matrix and $A^T = -A$. Show that the value of a symmetric matrix game is zero. If x is an optimal strategy of player 1, show that (x, x) is a saddle point equilibrium.

(2 marks)

(c) Let A be an $m \times n$ matrix game, and E be an $m \times n$ matrix whose entries are 1. Show that A and $k \cdot E + A$ have the same set of saddle point equilibria. Generalize this result for the case of bimatrix game (A, B).

(2 marks)