Minor Test-I

Course: MTL 105   Duration: 1 hour   M. Marks: 25

Note: All questions are compulsory.

I. Prove that S_3 is a homomorphic image of S_4.   [5 marks]

II. Prove or disprove the following statement: If H is a normal subgroup of G and K is a normal subgroup of H, then K is a normal subgroup of G.

[5 marks]

III. (a) Let H be a normal subgroup of G and N be a ~~normal~~ subgroup of H. normal in G

Show that H/N is a normal subgroup of G/N.

(b) If H' is a normal subgroup of G/N, then show that there exists a normal subgroup H of G such that $H' = H/N$.

[5 marks]

IV. If H and K are normal subgroups of a group G such that $H \subseteq K$, then show that

$$G/K \simeq \frac{G/H}{K/H}.$$

[5 marks]

V. The centre of a group G is defined as the set $Z(G) = \{g \in G : gx = xg \text{ for all } x \in G\}$. Determine $Z(S_n)$ for all $n \geq 1$.   [5 marks]