

7.12

inverse function of  $y_1, y_2 \Rightarrow x_1 = y_1 y_2, x_2 = y_1(1-y_2)$   
for  $y_1 > 0, 0 < y_2 < 1$

$$J = \begin{vmatrix} \partial x_1 / \partial y_1 & \partial x_1 / \partial y_2 \\ \partial x_2 / \partial y_1 & \partial x_2 / \partial y_2 \end{vmatrix} = \begin{vmatrix} y_2 & y_1 \\ 1-y_2 & -y_1 \end{vmatrix} = -y_1$$

$$\therefore g(y_1, y_2) = f(y_1 y_2, y_1(1-y_2)) |J| = y_1 e^{-(y_1 y_2 + y_1(1-y_2))} = y_1 e^{-y_1}$$

$$g(y_1) = \int_0^1 y_1 e^{-y_1} dy_2 = y_1 e^{-y_1}$$

$$g(y_2) = \int_0^\infty y_1 e^{-y_1} dy_1 = P(z) = 1$$

$$\therefore g(y_1, y_2) = g(y_1)g(y_2) \quad \therefore y_1, y_2 \text{ are independent } \times$$

7.14

when  $y = x^2, 0 < y < 1$

inverse function  $x_1 = y^{\frac{1}{2}}, x_2 = -(y^{\frac{1}{2}}), J_1 = \frac{1}{2\sqrt{y}}, J_2 = -\frac{1}{2\sqrt{y}}$

$$g(y) = f(y^{\frac{1}{2}}) \frac{1}{2\sqrt{y}} + f(-y^{\frac{1}{2}}) \frac{1}{2\sqrt{y}} = \frac{1+\sqrt{y}}{2} \frac{1}{2\sqrt{y}} + \frac{1-\sqrt{y}}{2} \frac{1}{2\sqrt{y}}$$

$$= \frac{2}{4\sqrt{y}} = \frac{1}{2\sqrt{y}}, \text{ for } 0 < y < 1 \quad \times$$

7.18

$$M_x(t) = E(e^{tx}) f(x) = \sum_{x=1}^{\infty} e^{tx} q^{x-1} p = \frac{p}{q} \sum_{x=1}^{\infty} e^{tx} q^x = \frac{pe^t}{1-qe^t}$$

$$M = M'_x(0) = \frac{(1-qe^t)pe^t + pqe^{2t}}{(1-qe^t)^2} \bigg|_{t=0} = \frac{(1-q)p + pq}{(1-q)^2} = \frac{p}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p}$$

$$\begin{aligned} M'_2 = M''_x(0) &= \frac{(1-qe^t)^2 pe^t + 2pqe^{2t}(1-qe^t)}{(1-qe^t)^4} \bigg|_{t=0} = \frac{(1-q)^2 p + 2pq(1-q)}{(1-q)^4} \\ &= \frac{p-pq+2pq}{(1-q)^3} = \frac{p(1+q)}{(1-q)^2} \\ &= \frac{2-p}{p^2} \end{aligned}$$



$$G^z = M_2' - M^z = \frac{z-p-1}{p^2} = \frac{q}{p^2} \quad \text{✗}$$

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$$M' = M'_x(0) = V(1-zt)^{-(V/2)-1} \Big|_{t=0} = V$$

$$M'' = M''_x(0) = V(V+2)(1-zt)^{-(V/2)-2} = V(V+2)$$

$$G^z = M'' - M^z = V^2 + 2V - V^2 = 2V \quad \text{✗}$$