7.12

$\because$ inverse function of $Y_1, Y_2 \Rightarrow x_1 = y_1 y_2$, $x_2 = y_1(1-y_2)$

for $y_1 > 0$, $0 < y_2 < 1$

$$J = \begin{vmatrix} \partial x_1/\partial y_1 & \partial x_1/\partial y_2 \\ \partial x_2/\partial y_1 & \partial x_2/\partial y_2 \end{vmatrix} = \begin{vmatrix} y_2 & y_1 \\ 1-y_2 & -y_1 \end{vmatrix} = -y_1$$

$$\therefore g(y_1, y_2) = f(y_1 y_2, y_1(1-y_2))|J| = y_1 e^{-(y_1 y_2 + y_1 - y_1 y_2)} = y_1 e^{-y_1}$$

$$g(y_1) = \int_0^1 y_1 e^{-y_1} dy_2 = y_1 e^{-y_1}$$

$$g(y_2) = \int_0^\infty y_1 e^{-y_1} dy_1 = \Gamma(2) = 1$$

$\because g(y_1, y_2) = g(y_1) g(y_2)$ $\therefore Y_1, Y_2$ are independent ※

7.14

when $y = x^2$, $0 < y < 1$

inverse function $x_1 = y^{\frac{1}{2}}$, $x_2 = -(y^{\frac{1}{2}})$, $J_1 = \frac{1}{2\sqrt{y}}$ $J_2 = \frac{-1}{2\sqrt{y}}$

$$g(y) = f(y^{\frac{1}{2}})\frac{1}{2\sqrt{y}} + f(-y^{\frac{1}{2}})\frac{1}{2\sqrt{y}} = \frac{1+\sqrt{y}}{2}\frac{1}{2\sqrt{y}} + \frac{1-\sqrt{y}}{2}\frac{1}{2\sqrt{y}}$$

$$= \frac{2}{4\sqrt{y}} = \frac{1}{2\sqrt{y}}, \text{ for } 0 < y < 1$$ ※

7.18

$$M_x(t) = E(e^{tx}) f(x) = \sum_{x=1}^{\infty} e^{tx} q^{x-1} p = \frac{p}{q} \sum_{x=1}^{\infty} e^{tx} q^x = \frac{pe^t}{1-qe^t}$$

$$\mu = M'_x(0) = \left.\frac{(1-qe^t)pe^t + pqe^{2t}}{(1-qe^t)^2}\right|_{t=0} = \frac{(1-q)p + pq}{(1-q)^2} = \frac{p}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p}$$

$$\mu'_2 = M''_x(0) = \left.\frac{(1-qe^t)^2 pe^t + 2pqe^{2t}(1-qe^t)}{(1-qe^t)^4}\right|_{t=0} = \frac{(1-q)^2 p + 2pq(1-q)}{(1-q)^4}$$

$$= \frac{p - pq + 2pq}{(1-q)^3} = \frac{p(1+q)}{(1-q)^2}$$

$$= \frac{2-p}{p^2}$$

$$\sigma^2 = \mu_2' - \mu^2 = \frac{2-p-1}{p^2} = \frac{q}{p^2} \text{ 㓂}$$

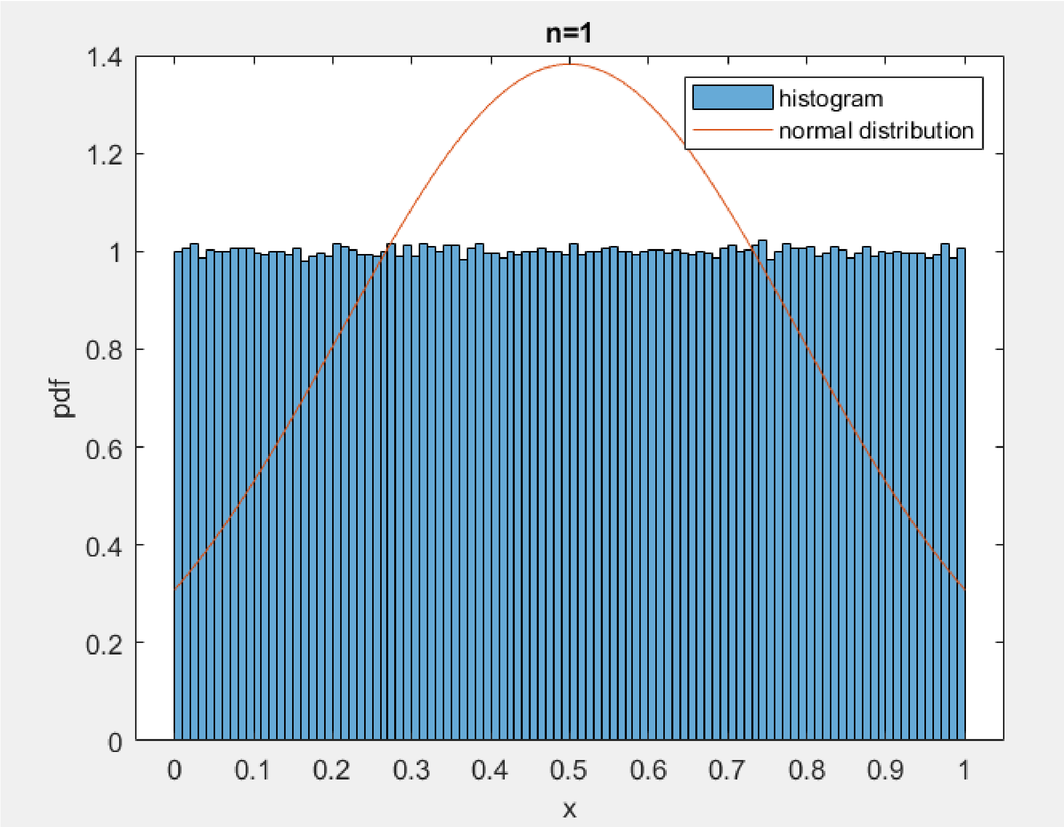
7.22

$$\mu' = M_x'(0) = v(1-2t)^{-(v/2)-1}\Big|_{t=0} = v$$

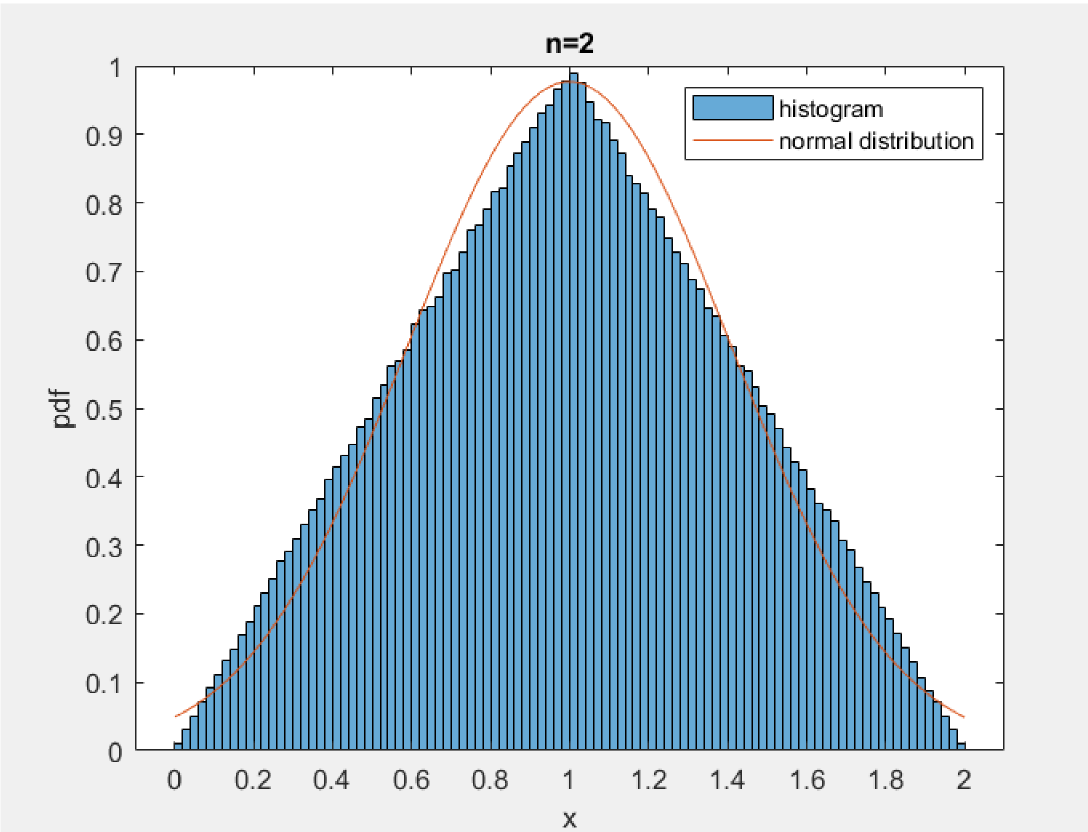
$$\mu'' = M_x''(0) = v(v+2)(1-2t)^{-(v/2)-2} = v(v+2)$$

$$\sigma^2 = \mu'' - \mu^2 = v^2+2v-v^2 = 2v \text{ ※}$$

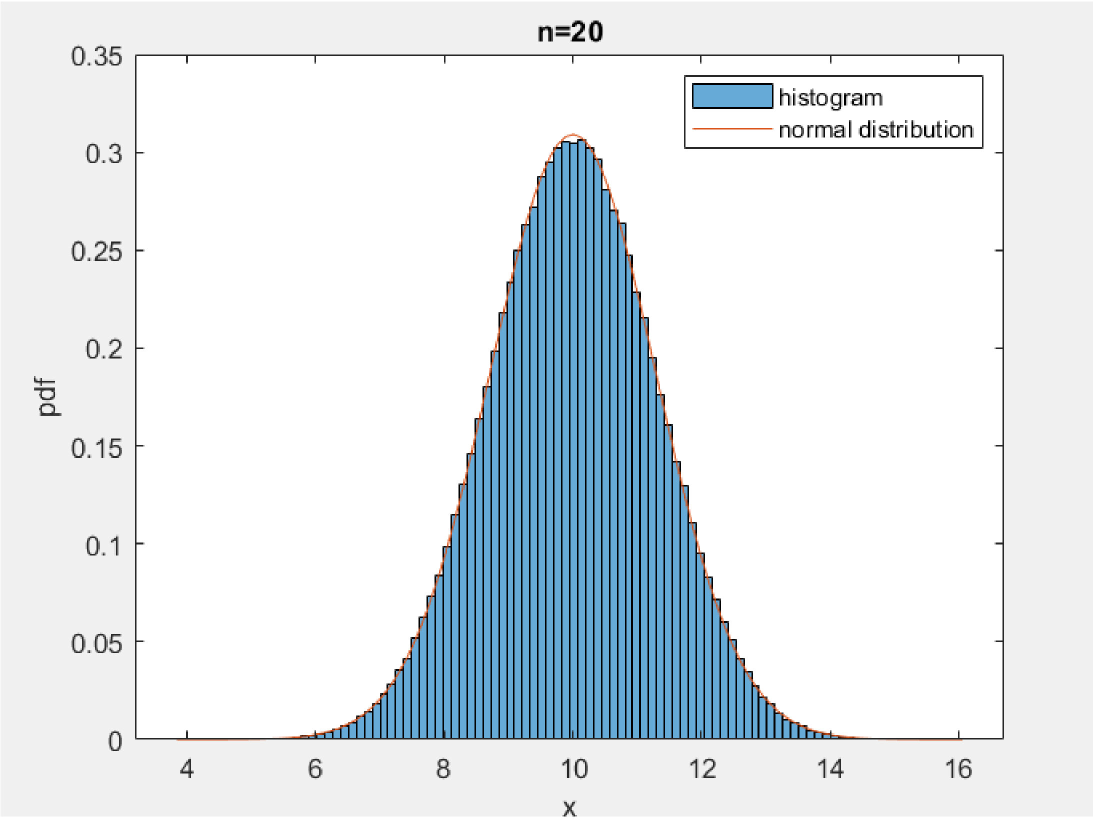
1.(b)



For n=1 , the histogram looks like square



For n=20, the histogram looks like triangle



For n=20 , the histogram is very close to the curve of normal distribution