Variables

$$
\begin{aligned}
x &= \frac{r}{R_*} \\
y_1 &= x^{2-\ell}\,\frac{\xi_r}{r} \\
y_2 &= x^{2-\ell}\,\frac{1}{gr}\left(\frac{p'}{\rho} + \Phi'\right) \\
y_3 &= x^{2-\ell}\,\frac{\Phi'}{gr} \\
y_4 &= x^{2-\ell}\,\frac{1}{g}\frac{\mathrm{d}\Phi'}{\mathrm{d}r} \\
y_5 &= x^{\ell}\,\frac{\delta S}{c_p} \\
y_6 &= x^{-1-\ell}\,\frac{\delta L_{\mathrm{rad}}}{L_*}
\end{aligned}
$$

Differential Equations

Adiabatic

$$
\begin{aligned}
x\frac{\mathrm{d}y_1}{\mathrm{d}x} &= \left(\frac{V}{\Gamma_1} - 1 - \ell\right) y_1 + \left(\frac{\ell(\ell+1)}{c_1\omega^2} - \frac{V}{\Gamma_1}\right) y_2 + \frac{V}{\Gamma_1} y_3 \\
x\frac{\mathrm{d}y_2}{\mathrm{d}x} &= (c_1\omega^2 - A^*) y_1 + (3 - U + A^* - \ell) y_2 - A^* y_3 \\
x\frac{\mathrm{d}y_3}{\mathrm{d}x} &= (3 - U - \ell) y_3 + y_4 \\
x\frac{\mathrm{d}y_4}{\mathrm{d}x} &= A^* U y_1 + \frac{V}{\Gamma_1} U y_2 + \left[\ell(\ell+1) - \frac{V}{\Gamma_1} U\right] y_3 - (U + \ell - 2) y_4
\end{aligned}
$$

Non-Adiabatic

$$
\begin{aligned}
x\frac{\mathrm{d}y_1}{\mathrm{d}x} &= \left(\frac{V}{\Gamma_1} - 1 - \ell\right) y_1 + \left(\frac{\ell(\ell+1)}{c_1\omega^2} - \frac{V}{\Gamma_1}\right) y_2 + \frac{V}{\Gamma_1} y_3 + \delta x^2 y_5 \\
x\frac{\mathrm{d}y_2}{\mathrm{d}x} &= (c_1\omega^2 - A^*) y_1 + (3 - U + A^* - \ell) y_2 - A^* y_3 + \delta x^2 y_5 \\
x\frac{\mathrm{d}y_3}{\mathrm{d}x} &= (3 - U - \ell) y_3 + y_4 \\
x\frac{\mathrm{d}y_4}{\mathrm{d}x} &= A^* U y_1 + \frac{V}{\Gamma_1} U y_2 + \left[\ell(\ell+1) - \frac{V}{\Gamma_1} U\right] y_3 - (U + \ell - 2) y_4 - \delta U x^2 y_5 \\
x\frac{\mathrm{d}y_5}{\mathrm{d}x} &= V\left[\nabla_{\mathrm{ad}}(U - c_1\omega^2) - 4(\nabla_{\mathrm{ad}} - \nabla) + c_{\mathrm{kap}}\right] x^{-2} y_1 + V\left[\frac{\ell(\ell+1)}{c_1\omega^2}(\nabla_{\mathrm{ad}} - \nabla) - c_{\mathrm{kap}}\right] x^{-2} y_2 + \\
&\quad V c_{\mathrm{kap}} x^{-2} y_3 + V\nabla_{\mathrm{ad}} x^{-2} y_4 + \left[V\nabla(4 - \kappa_S) - \ell\right] y_5 - \frac{V\nabla}{c_{\mathrm{rad}}} x^{-2} y_6 \\
x\frac{\mathrm{d}y_6}{\mathrm{d}x} &= \left[\ell(\ell+1)\left(\frac{\nabla_{\mathrm{ad}}}{\nabla} - 1\right) c_{\mathrm{rad}} - \epsilon_{\mathrm{ad}} V c_{\mathrm{gen}}\right] y_1 + \left[\epsilon_{\mathrm{ad}} V c_{\mathrm{gen}} - \ell(\ell+1) c_{\mathrm{rad}}\left(\frac{\nabla_{\mathrm{ad}}}{\nabla} - \frac{3 + c'_{\mathrm{rad}}}{c_1\omega^2}\right)\right] y_2 + \\
&\quad \left[\ell(\ell+1)\frac{\nabla_{\mathrm{ad}}}{\nabla} c_{\mathrm{rad}} - \epsilon_{\mathrm{ad}} V c_{\mathrm{gen}}\right] y_3 + \left[\epsilon_S c_{\mathrm{gen}} - \frac{\ell(\ell+1) c_{\mathrm{rad}}}{\nabla V} - \mathrm{i}\omega c_{\mathrm{thm}}\right] x^2 y_5 - \left[1 + \ell\right] y_6
\end{aligned}
$$

Boundary Conditions

Adiabatic

Inner

$$\left.\begin{aligned} c_1\omega^2 y_1 - \ell y_2 = 0 \\ \ell y_3 - y_4 = 0 \end{aligned}\right\}$$

Outer

$$\left.\begin{aligned} y_1 - y_2 + y_3 = 0 \\ U y_1 + (\ell+1) y_3 + y_4 = 0 \end{aligned}\right\} \qquad \text{(Zero)}$$

$$\left.\begin{aligned} \left\{1 + V^{-1}\left[\frac{\ell(\ell+1)}{\omega^2} - 4 - \omega^2\right]\right\} y_1 - y_2 + \left\{1 + V^{-1}\left[\frac{\ell(\ell+1)}{\omega^2} - \ell - 1\right]\right\} y_3 = 0 \\ (\ell+1) y_3 + y_4 = 0 \end{aligned}\right\} \qquad \text{(Dziembowski)}$$

Non-Adiabatic

Inner

$$\left.\begin{aligned} c_1\omega^2 y_1 - \ell y_2 = 0 \\ \ell y_3 - y_4 = 0 \\ \left[(c_1\omega^2 - \ell)\nabla_{\rm ad} + (\ell - 3)\nabla\right] y_1 - \ell\nabla_{\rm ad} y_3 + \frac{\ell}{V} x^2 y_5 + \frac{\nabla}{c_{\rm rad}} y_6 = 0 \end{aligned}\right\}$$

Outer

$$\left.\begin{aligned} y_1 - y_2 + y_3 = 0 \\ U y_1 + (\ell+1) y_3 + y_4 = 0 \\ (2 - 4\nabla_{\rm ad} V) y_1 + 4\nabla_{\rm ad} V y_2 - 4\nabla_{\rm ad} V y_3 + 4 y_5 - y_6 = 0 \end{aligned}\right\} \qquad \text{(Zero)}$$

$$\left.\begin{aligned} \left\{1 + V^{-1}\left[\frac{\ell(\ell+1)}{\omega^2} - 4 - \omega^2\right]\right\} y_1 - y_2 + \left\{1 + V^{-1}\left[\frac{\ell(\ell+1)}{\omega^2} - \ell - 1\right]\right\} y_3 = 0 \\ (\ell+1) y_3 + y_4 = 0 \\ (2 - 4\nabla_{\rm ad} V) y_1 + 4\nabla_{\rm ad} V y_2 - 4\nabla_{\rm ad} V y_3 + 4 y_5 - y_6 = 0 \end{aligned}\right\} \qquad \text{(Dziembowski)}$$

Structure Coefficients

Mechanical

$$V = -\frac{\mathrm{d}\ln p}{\mathrm{d}\ln r} \qquad A^* = \frac{1}{\Gamma_1}\frac{\mathrm{d}\ln p}{\mathrm{d}\ln r} - \frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} \qquad U = \frac{\mathrm{d}\ln M_r}{\mathrm{d}\ln r} \qquad c_1 = \frac{r^3}{R_*^3}\frac{M_*}{M_r} \qquad \Gamma_1 = \left(\frac{\partial\ln p}{\partial\ln\rho}\right)_S$$

Thermal

$$\nabla = \frac{\mathrm{d}\ln T}{\mathrm{d}\ln p} \qquad \nabla_{\mathrm{ad}} = \left(\frac{\partial \ln T}{\partial \ln p}\right)_S \qquad \delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_p$$

$$c_{\mathrm{rad}} = x^{-3}\frac{L_{\mathrm{rad}}}{L_*} \qquad c'_{\mathrm{rad}} = \frac{\mathrm{d}\ln c_{\mathrm{rad}}}{\mathrm{d}\ln r} \qquad c_{\mathrm{gen}} = x^{-3}\frac{4\pi r^3 \epsilon \rho}{L_*} \qquad c_{\mathrm{thm}} = x^{-3}\frac{4\pi r^3 c_p T \rho}{L_*}\sqrt{\frac{GM_*}{R_*^3}}$$

$$c_{\mathrm{kap}} = (\kappa_{\mathrm{ad}} - 4\nabla_{\mathrm{ad}})\, V\nabla + \nabla_{\mathrm{ad}}\left(\frac{\mathrm{d}\ln \nabla_{\mathrm{ad}}}{\mathrm{d}\ln r} + V\right)$$

$$\kappa_{\mathrm{ad}} = \left(\frac{\partial \ln \kappa}{\partial \ln p}\right)_S \qquad \kappa_S = c_p T\left(\frac{\partial \ln \kappa}{\partial S}\right)_p$$

$$\epsilon_{\mathrm{ad}} = \left(\frac{\partial \ln \epsilon}{\partial \ln p}\right)_S \qquad \epsilon_S = c_p T\left(\frac{\partial \ln \epsilon}{\partial S}\right)_p$$