

Variables

$$\begin{aligned}
x &= \frac{r}{R_*} \\
y_1 &= x^{-\lambda_0} \frac{\xi_r}{r} \\
y_2 &= x^{-\lambda_0} \frac{1}{gr} \left(\frac{p'}{\rho} + \Phi' \right) \\
y_3 &= x^{-\lambda_0} \frac{\Phi'}{gr} \\
y_4 &= x^{-\lambda_0} \frac{1}{g} \frac{d\Phi'}{dr} \\
y_5 &= x^{-\lambda_0-2} \frac{\delta S}{c_p} \\
y_6 &= x^{-\lambda_0} \frac{\delta L_{\text{rad}}}{L_*}
\end{aligned}$$

Differential Equations

Adiabatic

$$\begin{aligned}
x \frac{dy_1}{dx} &= \left(\frac{V}{\Gamma_1} - 3 - \lambda_0 \right) y_1 + \left(\frac{\ell(\ell+1)}{c_1 \omega^2} - \frac{V}{\Gamma_1} \right) y_2 + \frac{V}{\Gamma_1} y_3 \\
x \frac{dy_2}{dx} &= (c_1 \omega^2 - A^*) y_1 + (1 - U + A^* - \lambda_0) y_2 - A^* y_3 \\
x \frac{dy_3}{dx} &= (1 - U - \lambda_0) y_3 + y_4 \\
x \frac{dy_4}{dx} &= A^* U y_1 + \frac{V}{\Gamma_1} U y_2 + \left[\ell(\ell+1) - \frac{V}{\Gamma_1} U \right] y_3 - (U + \lambda_0) y_4
\end{aligned}$$

Non-Adiabatic

$$\begin{aligned}
x \frac{dy_1}{dx} &= \left(\frac{V}{\Gamma_1} - 3 - \lambda_0 \right) y_1 + \left(\frac{\ell(\ell+1)}{c_1 \omega^2} - \frac{V}{\Gamma_1} \right) y_2 + \frac{V}{\Gamma_1} y_3 + \alpha_T x^2 y_5 \\
x \frac{dy_2}{dx} &= (c_1 \omega^2 - A^*) y_1 + (1 - U + A^* - \lambda_0) y_2 - A^* y_3 + \alpha_T x^2 y_5 \\
x \frac{dy_3}{dx} &= (1 - U - \lambda_0) y_3 + y_4 \\
x \frac{dy_4}{dx} &= A^* U y_1 + \frac{V}{\Gamma_1} U y_2 + \left[\ell(\ell+1) - \frac{V}{\Gamma_1} U \right] y_3 - (U + \lambda_0) y_4 - \alpha_T U x^2 y_5 \\
x \frac{dy_5}{dx} &= V \left[\nabla_{\text{ad}} (U - c_1 \omega^2) - 4(\nabla_{\text{ad}} - \nabla) + c_{\text{kap}} \right] x^{-2} y_1 + V \left[\frac{\ell(\ell+1)}{c_1 \omega^2} (\nabla_{\text{ad}} - \nabla) - c_{\text{kap}} \right] x^{-2} y_2 + \\
&\quad V c_{\text{kap}} x^{-2} y_3 + V \nabla_{\text{ad}} x^{-2} y_4 + [V \nabla (4 - \kappa_S) - 2 - \lambda_0] y_5 - \frac{V \nabla}{c_{\text{rad}}} x^{-2} y_6 \\
x \frac{dy_6}{dx} &= \left[\ell(\ell+1) \left(\frac{\nabla_{\text{ad}}}{\nabla} - 1 \right) c_{\text{rad}} - \epsilon_{\text{ad}} V c_{\text{gen}} \right] y_1 + \left[\epsilon_{\text{ad}} V c_{\text{gen}} - \ell(\ell+1) c_{\text{rad}} \left(\frac{\nabla_{\text{ad}}}{\nabla} - \frac{3 + c'_{\text{rad}}}{c_1 \omega^2} \right) \right] y_2 + \\
&\quad \left[\ell(\ell+1) \frac{\nabla_{\text{ad}}}{\nabla} c_{\text{rad}} - \epsilon_{\text{ad}} V c_{\text{gen}} \right] y_3 + \left[\epsilon_S c_{\text{gen}} - \frac{\ell(\ell+1) c_{\text{rad}}}{\nabla V} - i \omega c_{\text{thm}} \right] x^2 y_5 - [3 + \lambda_0] y_6
\end{aligned}$$

Boundary Conditions

Adiabatic

Inner

$$\left. \begin{aligned} c_1 \omega^2 y_1 - \ell y_2 &= 0 \\ \ell y_3 - y_4 &= 0 \end{aligned} \right\}$$

Outer

$$\left. \begin{aligned} y_1 - y_2 + y_3 &= 0 \\ U y_1 + (\ell + 1) y_3 + y_4 &= 0 \end{aligned} \right\} \quad (\text{Zero})$$

$$\left. \begin{aligned} \left\{ 1 + V^{-1} \left[\frac{\ell(\ell+1)}{\omega^2} - 4 - \omega^2 \right] \right\} y_1 - y_2 + \left\{ 1 + V^{-1} \left[\frac{\ell(\ell+1)}{\omega^2} - \ell - 1 \right] \right\} y_3 &= 0 \\ (\ell + 1) y_3 + y_4 &= 0 \end{aligned} \right\} \quad (\text{Dziembowski})$$

Non-Adiabatic

Inner

$$\left. \begin{aligned} c_1 \omega^2 y_1 - \ell y_2 &= 0 \\ \ell y_3 - y_4 &= 0 \\ [(c_1 \omega^2 - \ell) \nabla_{\text{ad}} + (\ell - 3) \nabla] y_1 - \ell \nabla_{\text{ad}} y_3 + \frac{\ell}{V} x^2 y_5 + \frac{\nabla}{c_{\text{rad}}} y_6 &= 0 \end{aligned} \right\}$$

Outer

$$\left. \begin{aligned} y_1 - y_2 + y_3 &= 0 \\ U y_1 + (\ell + 1) y_3 + y_4 &= 0 \\ (2 - 4 \nabla_{\text{ad}} V) y_1 + 4 \nabla_{\text{ad}} V y_2 - 4 \nabla_{\text{ad}} V y_3 + 4 y_5 - y_6 &= 0 \end{aligned} \right\} \quad (\text{Zero})$$

$$\left. \begin{aligned} \left\{ 1 + V^{-1} \left[\frac{\ell(\ell+1)}{\omega^2} - 4 - \omega^2 \right] \right\} y_1 - y_2 + \left\{ 1 + V^{-1} \left[\frac{\ell(\ell+1)}{\omega^2} - \ell - 1 \right] \right\} y_3 &= 0 \\ (\ell + 1) y_3 + y_4 &= 0 \\ (2 - 4 \nabla_{\text{ad}} V) y_1 + 4 \nabla_{\text{ad}} V y_2 - 4 \nabla_{\text{ad}} V y_3 + 4 y_5 - y_6 &= 0 \end{aligned} \right\} \quad (\text{Dziembowski})$$

Structure Coefficients

Mechanical

$$V = -\frac{d \ln p}{d \ln r} \quad A^* = \frac{1}{\Gamma_1} \frac{d \ln p}{d \ln r} - \frac{d \ln \rho}{d \ln r} \quad U = \frac{d \ln M_r}{d \ln r} \quad c_1 = \frac{r^3}{R_*^3} \frac{M_*}{M_r} \quad \Gamma_1 = \left(\frac{\partial \ln p}{\partial \ln \rho} \right)_S$$

Thermal

$$\begin{aligned}
\nabla &= \frac{d \ln T}{d \ln p} & \nabla_{\text{ad}} &= \left(\frac{\partial \ln T}{\partial \ln p} \right)_S & \alpha_T &= \left(\frac{\partial \ln \rho}{\partial \ln T} \right)_p \\
c_{\text{rad}} &= x^{-3} \frac{L_{\text{rad}}}{L_*} & c'_{\text{rad}} &= \frac{d \ln c_{\text{rad}}}{d \ln r} & c_{\text{gen}} &= x^{-3} \frac{4\pi r^3 \epsilon \rho}{L_*} & c_{\text{thm}} &= x^{-3} \frac{4\pi r^3 c_p T \rho}{L_*} \sqrt{\frac{GM_*}{R_*^3}} \\
c_{\text{kap}} &= (\kappa_{\text{ad}} - 4\nabla_{\text{ad}}) V \nabla + \nabla_{\text{ad}} \left(\frac{d \ln \nabla_{\text{ad}}}{d \ln r} + V \right) \\
\kappa_{\text{ad}} &= \left(\frac{\partial \ln \kappa}{\partial \ln p} \right)_S & \kappa_S &= c_p T \left(\frac{\partial \ln \kappa}{\partial S} \right)_p \\
\epsilon_{\text{ad}} &= \left(\frac{\partial \ln \epsilon}{\partial \ln p} \right)_S & \epsilon_S &= c_p T \left(\frac{\partial \ln \epsilon}{\partial S} \right)_p
\end{aligned}$$