Jardel_Metodos_Numericos_Unidade_1_Semana_2

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**

Disciplina:** Métodos Numéricos

**

Semestre:** 2020.2

**

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**

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**

2ª Unidade:** RAÍZES E OTIMIZAÇÃO

[1]:
```python
import scipy
import numpy as np
import scipy.optimize as opt
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
from mpmath import *
from sympy import *
from itertools import *
```

[2]:
```python
#funções auxiliares

erro_total = lambda valor_verdadeiro, aproximacao: valor_verdadeiro -␣
↪aproximacao

erro_relativo_fracionario = lambda valor_verdadeiro, aproximacao:␣
↪(valor_verdadeiro - aproximacao) / valor_verdadeiro

erro_relativo_percentual = lambda valor_verdadeiro, aproximacao:␣
↪(valor_verdadeiro - aproximacao) / valor_verdadeiro * 100

def imprimir_tabela(dados, index = None):
```

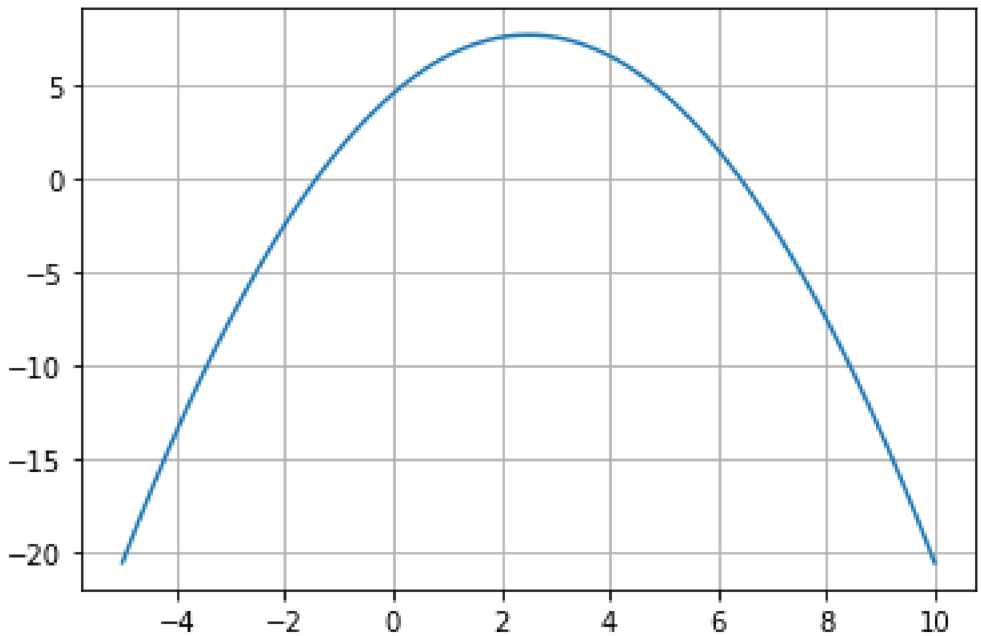
```
    pd.set_option("display.precision", 20)
    df = pd.DataFrame(dados, index)
    print(df.to_string())
```

0.0.1 Problema 1

$$f(x) = -0.5x^2 + 2.5x + 4.5$$

a)

[3]:
```
f = lambda x: -0.5 * x ** 2 + 2.5 * x + 4.5
x = np.linspace(-5, 10,10000)
plt.plot(x,f(x))
plt.grid(True)
```



b)

[4]:
```
raizes_quadratica = lambda a, b, c: ((-b + sqrt((b * b) - (4 * a * c))) / (2 *␣
 ↪a), (-b - sqrt((b * b) - (4 * a * c))) / (2 * a))
a = -0.5
b = 2.5
c = 4.5

print(raizes_quadratica(a, b, c))
PRIMEIRA_RAIZ, SEGUNDA_RAIZ = raizes_quadratica(a, b, c)
```

```
(-1.40512483795333, 6.40512483795333)
```

c)

[5]:
```python
def bissecao(funcao, x_inicial, x_final, ITERACOES = np.inf, TOLERANCIA = np.
 ↪NINF):
    ponto_medio = (x_inicial + x_final) / 2
    if abs(x_final - x_inicial) < TOLERANCIA or funcao(ponto_medio) == 0 or␣
 ↪bissecao.counter == ITERACOES:
            bissecao.counter = 0
            return [ponto_medio]
    else: bissecao.counter += 1

    if funcao(ponto_medio) * funcao(x_inicial) < 0:
        return [ponto_medio] + bissecao(funcao, x_inicial, ponto_medio,␣
 ↪ITERACOES, TOLERANCIA)
    else:
        return [ponto_medio] + bissecao(funcao, ponto_medio, x_final,␣
 ↪ITERACOES, TOLERANCIA)

bissecao.counter = 0
```

[6]:
```python
x_i = 5
x_u = 10
iteracoes = 3

valores = bissecao(f, x_i, x_u, iteracoes)

erros_totais = [erro_total(valor, SEGUNDA_RAIZ) for valor in valores]
erros_relativos_fracionarios = [erro_relativo_fracionario(valor, SEGUNDA_RAIZ)␣
 ↪for valor in valores]
erros_relativos_percentuais = [erro_relativo_percentual(valor, SEGUNDA_RAIZ)␣
 ↪for valor in valores]

tabela = {'Raiz maior': SEGUNDA_RAIZ,
          'valor_i':        valores,
          'Erro total':          erros_totais,
          'Erro Fracionario':   erros_relativos_fracionarios,
          'Erro Percentual':     erros_relativos_percentuais}

imprimir_tabela(tabela)
```

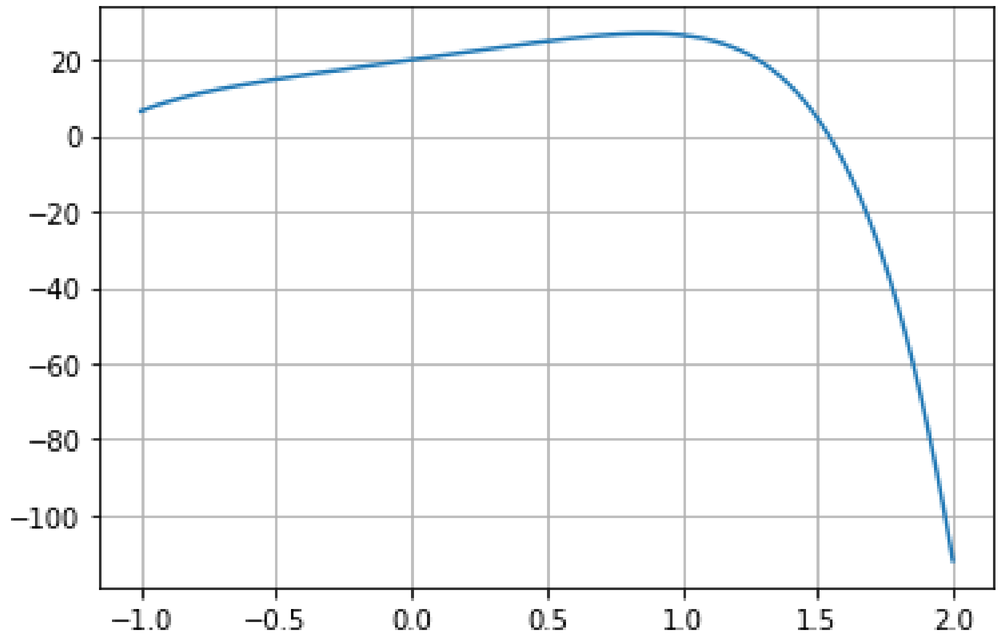
```
         Raiz maior   valor_i          Erro total      Erro Fracionario    Erro
Percentual
0   6.40512483795333   7.5000    1.09487516204667    0.145983354939556
14.5983354939556
1   6.40512483795333   6.2500  -0.155124837953327  -0.0248199740725323
-2.48199740725323
2   6.40512483795333   6.8750   0.469875162046673   0.0683454781158797
6.83454781158797
```

```
3  6.40512483795333    6.5625    0.157375162046673    0.0239809770737787
2.39809770737787
```

0.0.2 Problema 2

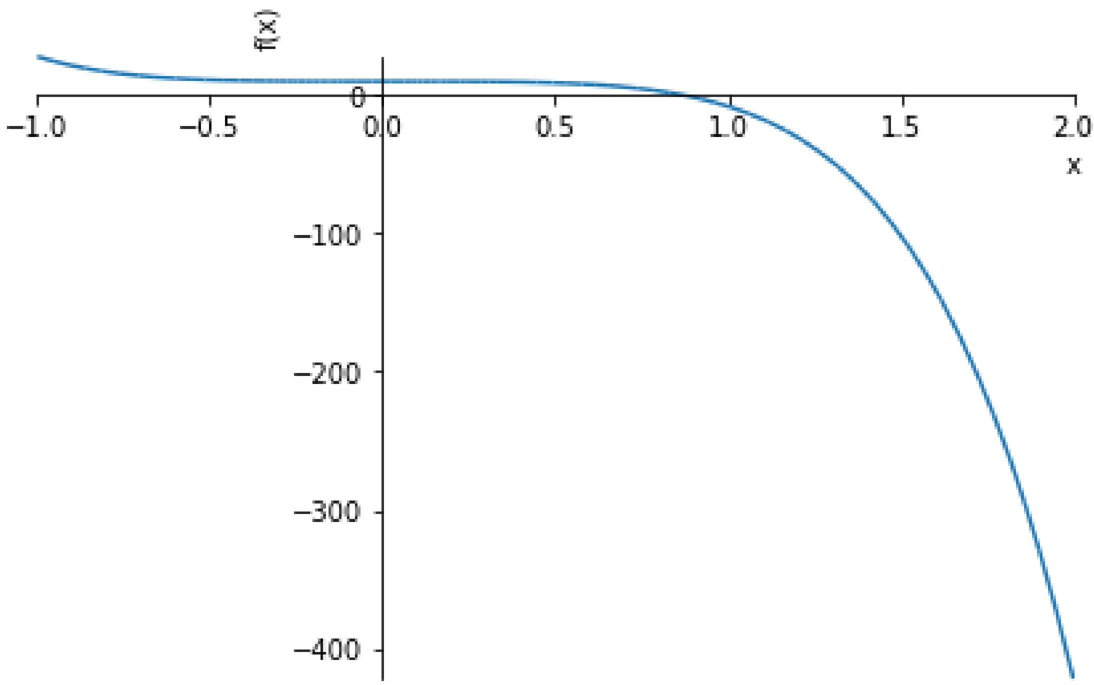
$$f(x) = -2X^6 - 1.5x^4 + 10x + 20$$

[7]:
```
f = lambda x: -2 * x ** 6 - 1.5 * x ** 4 + 10 * x + 20
x = np.linspace(-1, 2,10000)
plt.plot(x,f(x))
plt.grid(True)
```



[8]:
```
x = symbols('x')
f = lambda x: -2 * x ** 6 - 1.5 * x ** 4 + 10 * x + 20
derivada = diff(f(x))
print(derivada)
plot(derivada, (x, -1, 2))
```

```
-12*x**5 - 6.0*x**3 + 10
```



[8]: <sympy.plotting.plot.Plot at 0x221f4e95848>

[9]:
```
x_i = 0
x_u = 1

f = lambdify(x, derivada, 'numpy')
valores = bissecao(f, x_i, x_u, iteracoes, 0.05)
print(valores)
```

```
[0.5, 0.75, 0.875, 0.8125]
```

0.0.3 Problema 3

[10]:
```
def falsa_posicao(funcao, x_inicial, x_final, ITERACOES=np.NINF, TOLERANCIA=np.
 ↪NINF):
    funcao_ponto_inicial = funcao(x_inicial)
    estimativas = list()

    while falsa_posicao.counter != ITERACOES:
        estimativas.append((x_inicial * funcao(x_final) - x_final *␣
 ↪funcao(x_inicial)) / (funcao(x_final) - funcao(x_inicial)))
        funcao_ponto_estimado = funcao(estimativas[falsa_posicao.counter])
        if funcao_ponto_estimado == 0 and abs(funcao_ponto_estimado) <␣
 ↪TOLERANCIA :
```

```
            falsa_posicao.counter = 0
            return estimativas

        if(funcao_ponto_inicial * funcao_ponto_estimado > 0):
            x_inicial = estimativas[falsa_posicao.counter]
        else:
            x_final = estimativas[falsa_posicao.counter]
        falsa_posicao.counter += 1
    falsa_posicao.counter = 0
    return estimativas

falsa_posicao.counter = 0
```

0.0.4 Testando o método da falsa posição para o problema 1 letra c)

[11]:
```
f = lambda x: -0.5 * x ** 2 + 2.5 * x + 4.5

x_i = 5
x_u = 10
iteracoes = 3
tolerancia = np.inf

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
print(valores)

erros_totais = [erro_total(valor, SEGUNDA_RAIZ) for valor in valores]
erros_relativos_fracionarios = [erro_relativo_fracionario(valor, SEGUNDA_RAIZ)␣
 ↪for valor in valores]
erros_relativos_percentuais = [erro_relativo_percentual(valor, SEGUNDA_RAIZ)␣
 ↪for valor in valores]

tabela = {'Raiz maior': SEGUNDA_RAIZ,
          'valor_i':        valores,
          'Erro total':          erros_totais,
          'Erro Fracionario':   erros_relativos_fracionarios,
          'Erro Percentual':     erros_relativos_percentuais}

imprimir_tabela(tabela)
```

```
[5.9, 6.238532110091742, 6.351836734693878]
         Raiz maior                 valor_i           Erro total      Erro
Fracionario     Erro Percentual
0   6.40512483795333  5.900000000000035527   -0.505124837953327
-0.0856143793141231   -8.56143793141231
1   6.40512483795333  6.238532110091742133331   -0.166592727861585
-0.0267038343189893   -2.67038343189893
2   6.40512483795333  6.351836734693877777288   -0.0532881032594492
```

```
-0.00838940065452066  -0.838940065452066
```

Como pode-se observar, o metódo da falsa posição retornou um valor mais aproximado do real, com 1 repetição a menos, de acordo com a tabela de erros apresentadas

0.0.5 Testando o método da falsa posição para o problema 2

[12]:
```python
f = lambda x: -12 * x ** 5 - 6.0 * x ** 3 + 10

x_i = 0
x_u = 1
iteracoes = np.inf
tolerancia = 0.05

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
print(valores)
```

```
[0.5555555555555556, 0.7823501604757047, 0.8500227697940591, 0.8665353042945969,
0.8703139832359249, 0.8711651152704326, 0.8713561353559485, 0.8713989710953532,
0.8714085751329356, 0.871410728328975, 0.8714112110645322, 0.8714113192911553,
0.8714113435549503, 0.8714113489947548, 0.871411350214328, 0.8714113504877493,
0.8714113505490488, 0.8714113505627918, 0.871411350565873, 0.8714113505665637,
0.8714113505667186, 0.8714113505667533, 0.8714113505667611, 0.8714113505667628,
0.8714113505667632, 0.8714113505667633]
```

Como é possível observar, para atingir uma tolerância de erro de 5%, foram necessárias somente duas iterações, de tal forma que esse método para essa equação foi melhor que o método da bisseção

0.0.6 Testando o método da falsa posição para o exemplo 2 da nota de aula

$$e^x = x + 2$$

[13]:
```python
def f(x): return np.exp(x)-x-2
np.sign(f(-2)*f(0))
```

[13]:
```
-1.0
```

[14]:
```python
raiz = opt.root_scalar(f,method='bisect',bracket=[-2,0])
raiz
```

[14]:
```
      converged: True
           flag: 'converged'
 function_calls: 42
     iterations: 40
           root: -1.8414056604360667
```

[15]:
```python
x_i = -2
x_u = 0
```

```
valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
print(valores)
```

```
[-1.7615941559557646, -1.8402501723136198, -1.841389489184834,
-1.8414054342273545, -1.8414056572726762, -1.8414056603926976,
-1.8414056604363414, -1.841405660436952, -1.8414056604369604,
-1.8414056604369609]
```

0.0.7 Testando o método da falsa posição para o exemplo 3 da nota de aula

$$\sqrt{x} = \cos(x)$$

[16]:
```
def f(x): return np.cos(x) - np.sqrt(x)
```

[17]:
```
vp = opt.root_scalar(f,method='bisect',bracket=[0,1], xtol=1e-40).root
r = []
e = []

for i in range(1,7):
    x = opt.root_scalar(f,method='bisect',bracket=[0,1], maxiter=i).root
    r.append(x)
    e.append(np.around(np.absolute(x - vp) * 100 / 2, decimals=2))

import pandas as pd
pd.DataFrame({'Iterações': range(0,6), 'Raiz': r, 'Erro':e}).
 ↪set_index('Iterações')
```

[17]:

Iterações	Raiz	Erro
0	0.500000	7.08999999999999985789
1	0.500000	7.08999999999999985789
2	0.625000	0.83999999999999996891
3	0.625000	0.83999999999999996891
4	0.625000	0.83999999999999996891
5	0.640625	0.05000000000000000278

[18]:
```
x_i = 0
x_u = 1
iteracoes = 7
tolerancia = np.inf

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
print(valores)

erros_totais = [erro_total(valor, SEGUNDA_RAIZ) for valor in valores]
erros_relativos_fracionarios = [erro_relativo_fracionario(valor, SEGUNDA_RAIZ)␣
 ↪for valor in valores]
```

```
erros_relativos_percentuais = [erro_relativo_percentual(valor, SEGUNDA_RAIZ)␣
 ↪for valor in valores]

tabela = {'Raiz maior': SEGUNDA_RAIZ,
          'valor_i':        valores,
          'Erro total':          erros_totais,
          'Erro Fracionario':   erros_relativos_fracionarios,
          'Erro Percentual':     erros_relativos_percentuais}

imprimir_tabela(tabela)
```

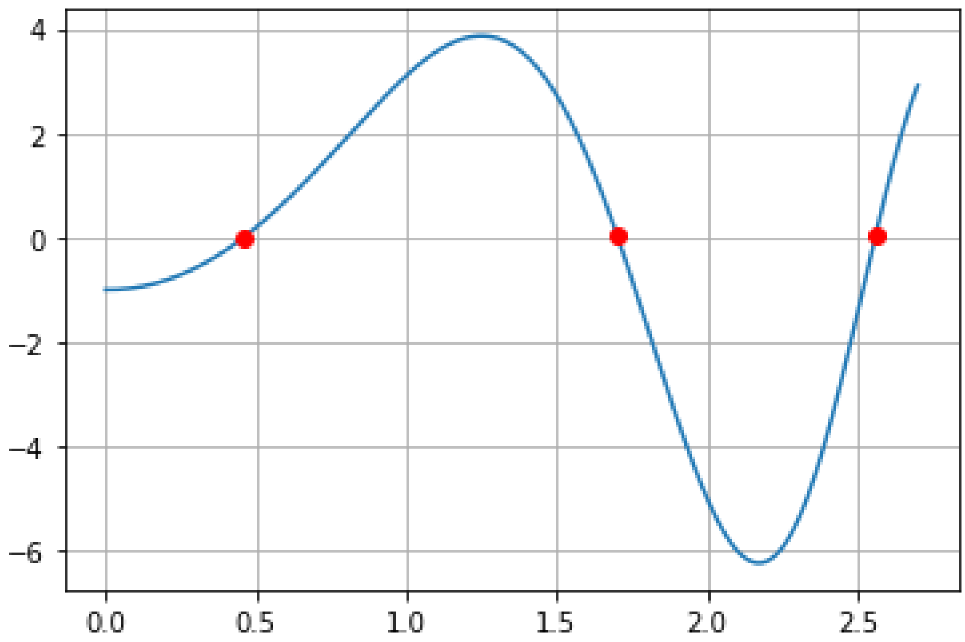
```
[0.6850733573260451, 0.6503949801283647, 0.6435565520339048, 0.6421098575945017,
0.6417994835090949, 0.641732697569524, 0.6417183174755584]
         Raiz maior                 valor_i         Erro total    Erro
Fracionario    Erro Percentual
0  6.40512483795333  0.68507335732604512923  -5.72005148062728
-8.34954595658718  -834.954595658718
1  6.40512483795333  0.65039498012836471919  -5.75472985782496
-8.84805392669111  -884.805392669111
2  6.40512483795333  0.64355655203390482733  -5.76156828591942
-8.95269928914636  -895.269928914636
3  6.40512483795333  0.64210985759450167620  -5.76301498035883
-8.97512304506346  -897.512304506346
4  6.40512483795333  0.64179948350909488131  -5.76332535444423
-8.97994701231722  -897.994701231722
5  6.40512483795333  0.64173269756952400211  -5.76339214038380
-8.98098563811985  -898.098563811985
6  6.40512483795333  0.64171831747555840852  -5.76340652047777
-8.98120929935475  -898.120929935475
```

0.0.8 Testando o método da falsa posição para o exemplo 4 da nota de aula

$$f(x) = 5sin(x^2) - exp\left(\frac{x}{10}\right)$$

[19]:
```
def f(x): return 5 * np.sin(x ** 2) - np.exp(x/10)
```

[20]:
```
x = np.linspace(0,2.7,200)
plt.plot(x,f(x))
plt.plot([0.46, 1.7, 2.56], [f(0.46), f(1.7), f(2.56)], 'o', color='r')
plt.grid(True)
```



[21]:
```
np.sign(f(0.4) * f(0.5))
np.sign(f(1.7) * f(1.8))
np.sign(f(2.5) * f(2.6))
```

[21]: -1.0

[22]:
```
print('1ª raiz positiva: ',opt.root_scalar(f,method='bisect', bracket=[0.4,0.
 ↪5],xtol=1e-5).root)
print('2ª raiz positiva: ',opt.root_scalar(f,method='bisect', bracket=[1.7,1.
 ↪8],xtol=1e-5).root)
print('3ª raiz positiva: ',opt.root_scalar(f,method='bisect', bracket=[2.5,2.
 ↪6],xtol=1e-5).root)
```

```
1ª raiz positiva:  0.459307861328125
2ª raiz positiva:  1.703570556640625
3ª raiz positiva:  2.558209228515625
```

[23]:
```
x_i = 0.4
x_u = 0.5
tolerancia = 1e-5

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
print(valores)

x_i = 1.7
x_u = 1.8
```

```
valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
print(valores)

x_i = 2.5
x_u = 2.6

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
print(valores)
```

```
[0.45679945731437466, 0.4592075958565898, 0.45930181111377394,
0.4593054847551961, 0.4593056279789401, 0.45930563356275533, 0.4593056337804496]
[1.703400048170842, 1.7035660686767091, 1.7035740592360833, 1.70357444354882,
1.7035744620320366, 1.7035744629209708, 1.703574462963723]
[2.5592348966927574, 2.5582261535765185, 2.5582128138589724, 2.558212641075543,
2.558212638838176, 2.5582126388092044, 2.558212638808829]
```

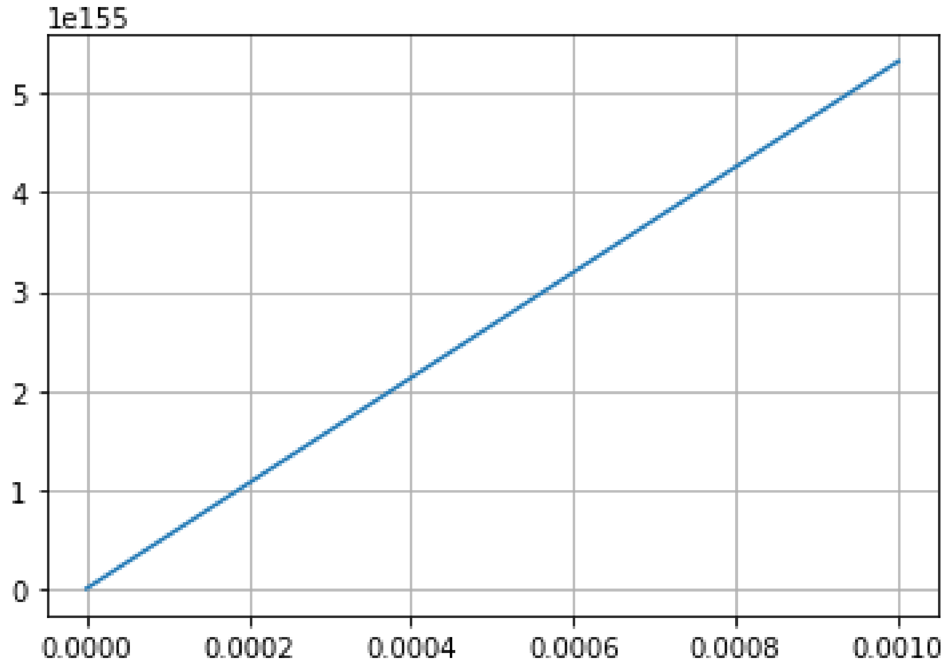
0.0.9 Testando o método da falsa posição para o exemplo 5 da nota de aula

$$I_d = I_R \left[exp\left(\frac{v_d}{v_d}\right) - 1 \right]$$

[24]:
```
def fa(vd, vs = 30, R = 1e+3): return 1e-12 * np.exp(((1.60217653e-19 * vd) /␣
↪(4.1419509e-25)) -1) - ((vs - vd)/(R))
```

[25]:
```
x = np.linspace(0,1,1000)
plt.plot(x,fa(x))
plt.grid(True)
```

```
C:\Users\jarde\Anaconda3\lib\site-packages\ipykernel_launcher.py:1:
RuntimeWarning: overflow encountered in exp
  """Entry point for launching an IPython kernel.
```



[26]:
```
opt.root_scalar(fa,method='bisect', bracket=[0,0.1]).root
```

```
C:\Users\jarde\Anaconda3\lib\site-packages\ipykernel_launcher.py:1:
RuntimeWarning: overflow encountered in exp
  """Entry point for launching an IPython kernel.
```

[26]: 6.495182024082167e-05

[27]:
```
def f(vd, vs, R): return 1e-12 * np.exp(((1.60217653e-19 * vd) / (4.
↪1419509e-25))-1) - ((vs - vd)/(R))
```

[28]:
```
vs = 300e-3
R = 1e+3
opt.root_scalar(f, args=(vs,R), method='bisect', bracket=[0,0.1]).root
```

```
C:\Users\jarde\Anaconda3\lib\site-packages\ipykernel_launcher.py:1:
RuntimeWarning: overflow encountered in exp
  """Entry point for launching an IPython kernel.
```

[28]: 5.304607184370981e-05

[29]:
```
vs = np.array([30, 3, 3, 300e-3, -300e-3, -30, -30])
R = np.array([1e+3, 1e+3, 1e+4, 1e+3, 1e+3, 1e+3, 1e+4])
```

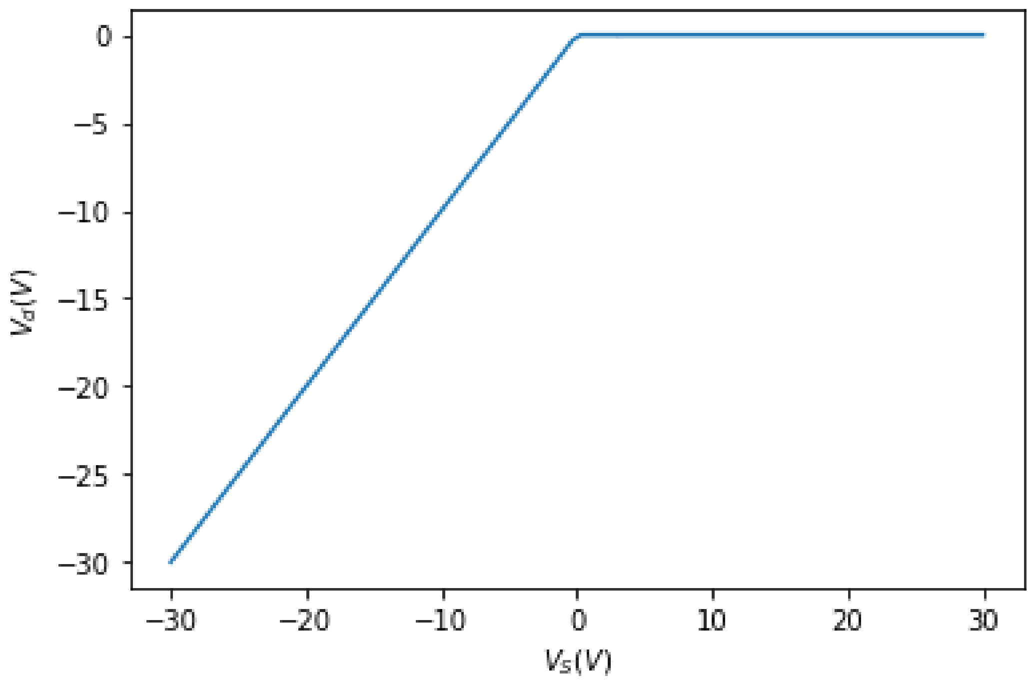
[30]:
```
vd = []
for i in range(len(vs)):
    Vs = vs[i]
    r = R[i]
    if Vs>0:
        a,b = 0,.1
    else:
        a,b = -32,.1
    x = opt.root_scalar(f, args=(Vs,r), method='bisect', bracket=[a,b]).root
    vd.append(x)
vd
```

```
C:\Users\jarde\Anaconda3\lib\site-packages\ipykernel_launcher.py:1:
RuntimeWarning: overflow encountered in exp
  """Entry point for launching an IPython kernel.
```

[30]:
```
[6.495182024082167e-05,
 5.899912648601458e-05,
 5.3046482207719243e-05,
 5.304607184370981e-05,
 -0.299999999999072,
 -30.00000000000098,
 -30.00000000000098]
```

[31]:
```
plt.plot(vs,vd)
plt.xlabel('$V_S (V)$')
plt.ylabel('$V_d (V)$')
```

[31]:
```
Text(0, 0.5, '$V_d (V)$')
```

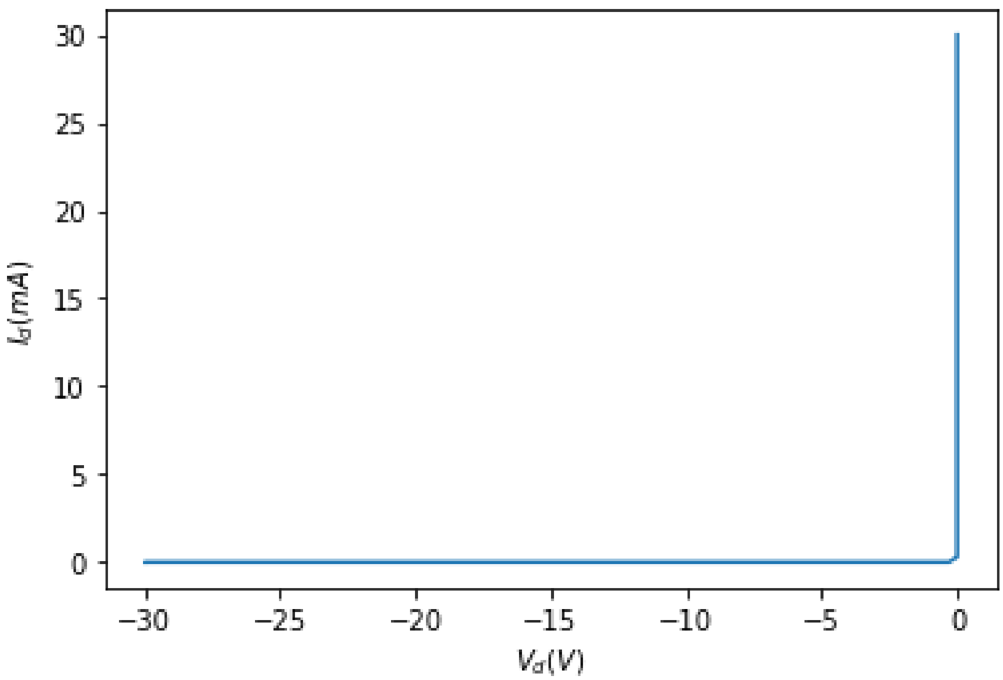


[32]:
```
id = (vs - vd) * 1000 / R
id
```

[32]:
```
array([ 2.99999350e+01,  2.99994100e+00,  2.99994695e-01,  2.99946954e-01,
       -9.27979915e-13,  9.80548975e-13,  9.80548975e-14])
```

[33]:
```
plt.plot(vd,id)
plt.xlabel('$V_d (V)$')
plt.ylabel('$I_d (mA)$')
```

[33]:
```
Text(0, 0.5, '$I_d (mA)$')
```



0.0.10 Problema 4

$$f(x) = 2sin(\sqrt{x}) - x)$$

[34]:
```
f = lambda x: 2 * np.sin(np.sqrt(x)) - x
```

[35]:
```
raiz = opt.fixed_point(f, 0.5, xtol=1e-2, method="iteration")
raiz
```

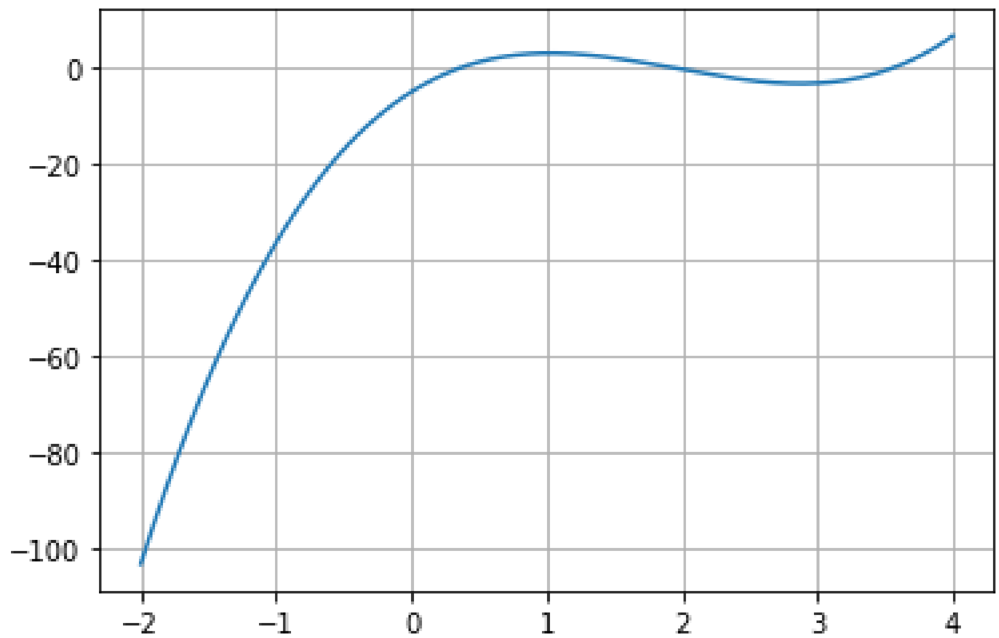
[35]: 0.768018090595916

0.0.11 Problema 5

$$f(x) = 2x^3 - 11.7x^2 + 17.7x - 5$$

a)

[36]:
```
f = lambda x: 2 * x ** 3 - 11.7 * x ** 2 + 17.7 * x - 5
x = np.linspace(-2, 4,10000)
plt.plot(x,f(x))
plt.grid(True)
```



b)

[37]:
```
# maxiter=3 -> Error
g = lambda x: (2 * x ** 3 - 11.7 * x ** 2 - 5) / -17.7
raiz = opt.fixed_point(g, x0=3)
raiz
```

[37]:
```
array(3.56316082)
```

c)

[38]:
```
x = symbols('x')
f = lambda x: 2 * x ** 3 - 11.7 * x ** 2 + 17.7 * x - 5
derivada = diff(f(x))
f_1 = lambdify(x, derivada, 'numpy')
opt.root_scalar(f, fprime=f_1, x0=3, method='newton', maxiter=3, xtol=1e-10)
```

[38]:
```
      converged: False
           flag: 'convergence error'
 function_calls: 6
     iterations: 3
           root: 3.7929344806432264
```

d)

[39]:
```
opt.root_scalar(f, x0=3, x1=4, xtol = 1e-10, maxiter=3, method='secant')
```

[39]:
```
      converged: False
           flag: 'convergence error'
 function_calls: 5
     iterations: 3
           root: 3.5862753847117346
```

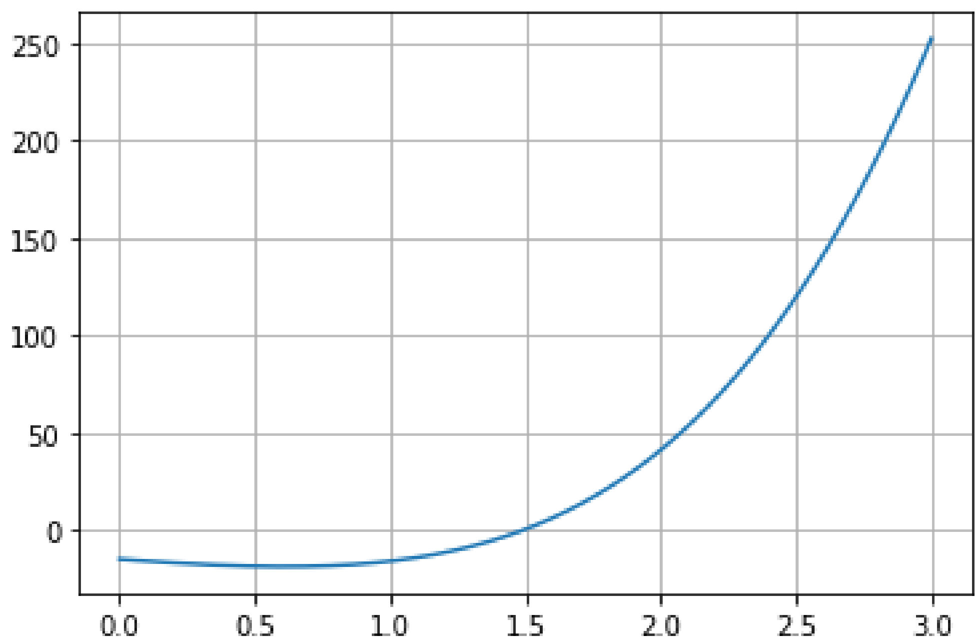
0.0.12 Problema 6

a)

$$f_1(x) = 2x^4 + 4x^3 + 3x^2 - 10x - 15, \text{com}\, x^* \in [0,3]$$

[40]:
```
f = lambda x: 2 * x ** 4 + 4 * x ** 3 + 3 * x ** 2 - 10 * x - 15
```

[41]:
```
x = np.linspace(0, 3,10000)
plt.plot(x,f(x))
plt.grid(True)
```



Método da bisseção

[42]:
```
%timeit opt.root_scalar(f,method='bisect', bracket=[0,3], maxiter=200,␣
 ↪xtol=1e-10)
opt.root_scalar(f,method='bisect', bracket=[0,3], maxiter=200, xtol=1e-10)
```

```
52.6 µs ± 2.2 µs per loop (mean ± std. dev. of 7 runs, 10000 loops each)
```

```
[42]:       converged: True
                 flag: 'converged'
       function_calls: 37
           iterations: 35
                 root: 1.4928787086100783
```

Método da falsa posição

```
[43]: x_i = 0
      x_u = 3
      iteracoes = 200
      tolerancia = 1e-10

      valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
      valores[-1]
```

```
[43]: 1.4928787086636037
```

Método do ponto fixo

```
[44]: g = lambda x:(2 * x ** 4 + 4 * x ** 3 + 3 * x ** 2 - 15) / 10
```

```
[45]: %timeit opt.fixed_point(g, 3, xtol=1e-10, maxiter=200)
      opt.fixed_point(g, 3, xtol=1e-10, maxiter=200)
```

```
17.7 ms ± 1.07 ms per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

```
[45]: array(1.49287871)
```

Método de Newton-Raphson

```
[46]: x = symbols('x')
      derivada = diff(f(x))
      f_1 = lambdify(x, derivada, 'numpy')
      opt.root_scalar(f, fprime=f_1, x0=3, method='newton', maxiter=200, xtol=1e-10)
```

```
[46]:       converged: True
                 flag: 'converged'
       function_calls: 14
           iterations: 7
                 root: 1.4928787086636037
```

```
[47]: %timeit opt.root_scalar(f, fprime=f_1, x0=3, method='newton', maxiter=200,␣
       ↪xtol=1e-10)
```

```
346 µs ± 50.4 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

Método da secante

[48]:
```
%timeit opt.root_scalar(f, x0=3, x1=0, xtol = 1e-10, maxiter=200,␣
↪method='secant')
opt.root_scalar(f, x0=3, x1=0, xtol = 1e-10, maxiter=200, method='secant')
```

```
500 µs ± 13.3 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

[48]:
```
      converged: True
           flag: 'converged'
 function_calls: 12
     iterations: 11
           root: -1.3003841326439196
```

[49]:
```
%timeit opt.root_scalar(f, x0=3, x1=1, xtol = 1e-10, maxiter=200,␣
↪method='secant')
opt.root_scalar(f, x0=3, x1=1, xtol = 1e-10, maxiter=200, method='secant')
```

```
448 µs ± 21.8 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

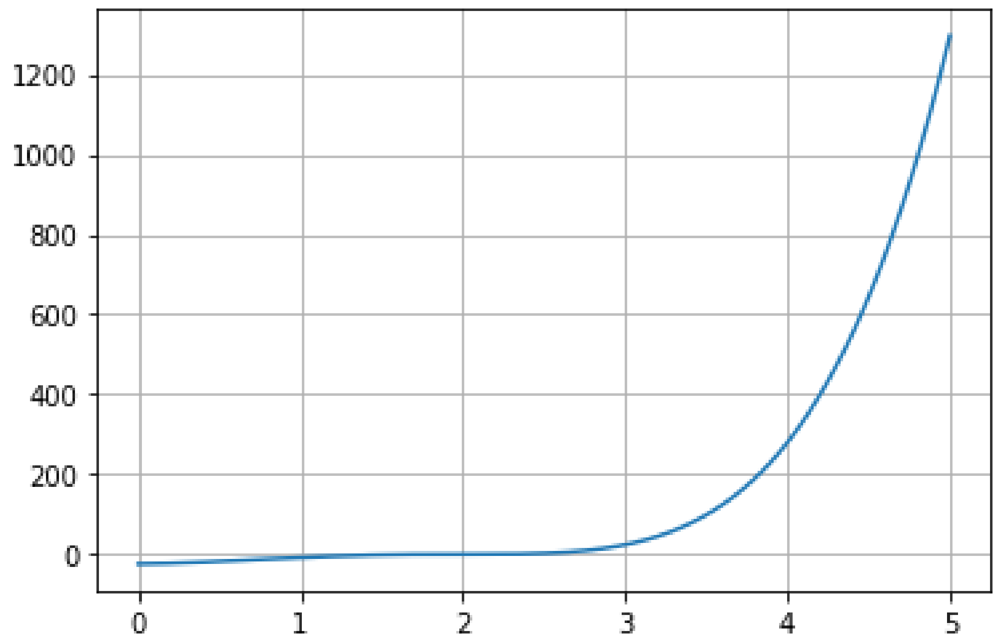
[49]:
```
      converged: True
           flag: 'converged'
 function_calls: 11
     iterations: 10
           root: 1.4928787086636035
```

b)

$$f_2(x) = (x+3)(x+1)(x-2)^3, \text{com } x^* \in [0,5]$$

[50]:
```
f = lambda x: (x + 3) * (x + 1) * (x - 2) ** 3
```

[51]:
```
x = np.linspace(0, 5,10000)
plt.plot(x,f(x))
plt.grid(True)
```



Método da bisseção

[52]:
```
%timeit opt.root_scalar(f,method='bisect', bracket=[0,5], maxiter=200,␣
↪xtol=1e-10)
opt.root_scalar(f,method='bisect', bracket=[0,5], maxiter=200, xtol=1e-10)
```

```
35 µs ± 934 ns per loop (mean ± std. dev. of 7 runs, 10000 loops each)
```

[52]:
```
      converged: True
           flag: 'converged'
 function_calls: 38
     iterations: 36
           root: 2.0000000000436557
```

Método da falsa posição

[53]:
```
x_i = 0
x_u = 5
iteracoes = 200
tolerancia = 1e-10

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
valores[-1]
```

[53]: 1.714409136227648

Método do ponto fixo $x^5 - 2x^4 - 9x^3 + 22x^2 + 4x - 24$

[54]:
```
g = lambda x: (x ** 5 - 2 * x ** 4 - 9 * x ** 3 + 22 * x ** 2 - 24) / -4
```

[55]:
```
%timeit opt.fixed_point(g, 3, xtol=1e-10, maxiter=200)
opt.fixed_point(g, 3, xtol=1e-10, maxiter=200)
```

```
364 µs ± 28 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

[55]:
```
array(-3.)
```

Método de Newton-Raphson

[56]:
```
x = symbols('x')
derivada = diff(f(x))
f_1 = lambdify(x, derivada, 'numpy')
opt.root_scalar(f, fprime=f_1, x0=3, method='newton', maxiter=200, xtol=1e-10)
```

[56]:
```
      converged: True
           flag: 'converged'
 function_calls: 112
     iterations: 56
           root: 2.0000000001726277
```

[57]:
```
%timeit opt.root_scalar(f, fprime=f_1, x0=3, method='newton', maxiter=200,␣
 ↪xtol=1e-10)
```

```
2.37 ms ± 93.6 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

Método da secante

[58]:
```
%timeit opt.root_scalar(f, x0=5, x1=0, xtol = 1e-10, maxiter=200,␣
 ↪method='secant')
opt.root_scalar(f, x0=5, x1=0, xtol = 1e-10, maxiter=200, method='secant')
```

```
3.06 ms ± 73.4 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

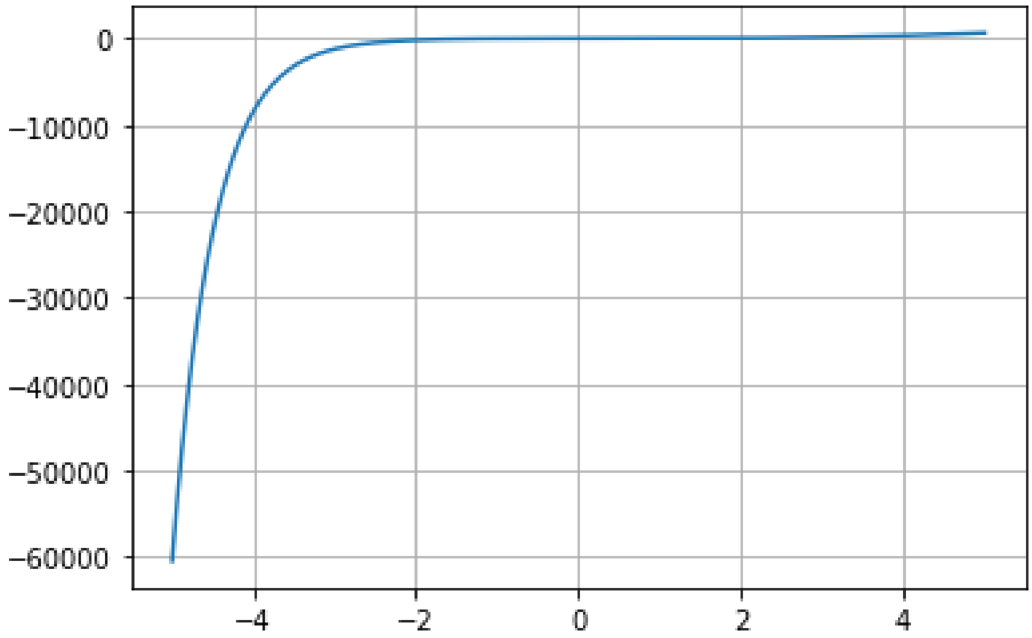
[58]:
```
      converged: True
           flag: 'converged'
 function_calls: 79
     iterations: 78
           root: 2.0000000002448965
```

c)

$$f_3(x) = 5x^3 + x^2 - e^{1-2x} + \cos(x) + 20, \text{com } x^* \in [-5, 5]$$

[59]:
```
f = lambda x: 5 * x ** 3 + x ** 2 - np.exp(1 - 2 * x) + np.cos(x) + 20
```

[60]:
```
x = np.linspace(-5, 5,10000)
plt.plot(x,f(x))
plt.grid(True)
```



Método da bisseção

[61]:
```
%timeit opt.root_scalar(f,method='bisect', bracket=[-5,5], maxiter=200,␣
↪xtol=1e-10)
opt.root_scalar(f,method='bisect', bracket=[-5,5], maxiter=200, xtol=1e-10)
```

```
174 µs ± 13.4 µs per loop (mean ± std. dev. of 7 runs, 10000 loops each)
```

[61]:
```
      converged: True
           flag: 'converged'
 function_calls: 39
     iterations: 37
           root: -0.9295604598446516
```

Método da falsa posição

[62]:
```
x_i = -5
x_u = 5
iteracoes = 200
tolerancia = 1e-10

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
```

```
valores[-1]
```

[62]: 1.568769261078004

Método do ponto fixo

[63]:
```
g = lambda x: (5 * x ** 3 - np.exp(1 - 2 * x) + np.cos(x) + 20) / -x
```

[64]:
```
%timeit opt.fixed_point(g, -5, xtol=1e-10, maxiter=200)
opt.fixed_point(g, -5, xtol=1e-10, maxiter=200)
```

```
C:\Users\jarde\Anaconda3\lib\site-packages\ipykernel_launcher.py:1:
RuntimeWarning: overflow encountered in exp
  """Entry point for launching an IPython kernel.
```

```
207 µs ± 6.41 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

[64]: array(-5.)

Método de Newton-Raphson

[65]:
```
x = symbols('x')
f = lambda x: 5 * x ** 3 + x ** 2 - exp(1 - 2 * x) + cos(x) + 20
derivada = diff(f(x))

f_1 = lambdify(x, derivada, 'numpy')
f = lambda x: 5 * x ** 3 + x ** 2 - np.exp(1 - 2 * x) + np.cos(x) + 20
opt.root_scalar(f, fprime=f_1, x0=-5, method='newton', maxiter=200, xtol=1e-10)
```

[65]:
```
      converged: True
           flag: 'converged'
 function_calls: 26
     iterations: 13
           root: -0.9295604598378413
```

[66]:
```
%timeit opt.root_scalar(f, fprime=f_1, x0=-5, method='newton', maxiter=200,␣
 ↪xtol=1e-10)
```

```
750 µs ± 8.59 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

Método da secante

[67]:
```
%timeit opt.root_scalar(f, x0=-5, x1=5, xtol = 1e-10, maxiter=200,␣
 ↪method='secant')
opt.root_scalar(f, x0=-5, x1=5, xtol = 1e-10, maxiter=200, method='secant')
```

```
1.2 ms ± 31.5 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

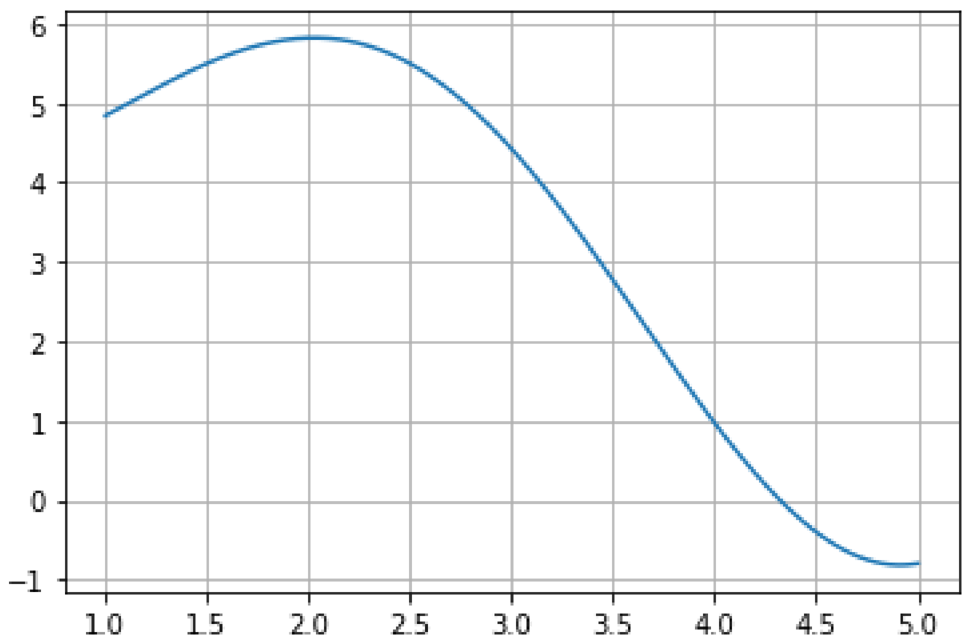
[67]:
```
       converged: True
            flag: 'converged'
  function_calls: 25
      iterations: 24
            root: -0.9295604598378412
```

d)

$$f_4(x) = \sin(x)x + 4, \text{com } x^* \in [1, 5]$$

[68]:
```
f = lambda x: np.sin(x) * x + 4
```

[69]:
```
x = np.linspace(1, 5,10000)
plt.plot(x,f(x))
plt.grid(True)
```



Método da bisseção

[70]:
```
%timeit opt.root_scalar(f,method='bisect', bracket=[1,5], maxiter=200,␣
↪xtol=1e-10)
opt.root_scalar(f,method='bisect', bracket=[1,5], maxiter=200, xtol=1e-10)
```

```
93 µs ± 9.71 µs per loop (mean ± std. dev. of 7 runs, 10000 loops each)
```

[70]:
```
       converged: True
            flag: 'converged'
  function_calls: 38
```

```
      iterations: 36
            root: 4.323239543766249
```

Método da falsa posição

[71]:
```
x_i = 1
x_u = 5
iteracoes = 200
tolerancia = 1e-10

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
valores[-1]
```

[71]:
```
4.323239543713714
```

Método do ponto fixo

[72]:
```
g = lambda x: -4 / np.sin(x)
```

[73]:
```
%timeit opt.fixed_point(g, 1, xtol=1e-10, maxiter=200)
opt.fixed_point(g, 1, xtol=1e-10, maxiter=200)
```

```
1.19 ms ± 25.1 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

[73]:
```
array(-4.32323954)
```

Método de Newton-Raphson

[74]:
```
x = symbols('x')
f = lambda x: sin(x) * x + 4
derivada = diff(f(x))

f_1 = lambdify(x, derivada, 'numpy')
f = lambda x: np.sin(x) * x + 4
opt.root_scalar(f, fprime=f_1, x0=-5, method='newton', maxiter=200, xtol=1e-10)
```

[74]:
```
      converged: True
           flag: 'converged'
 function_calls: 14
     iterations: 7
           root: -5.461308012588998
```

[75]:
```
%timeit opt.root_scalar(f, fprime=f_1, x0=-5, method='newton', maxiter=200,␣
↪xtol=1e-10)
```

```
357 µs ± 7.1 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

Método da secante

[76]:
```
%timeit opt.root_scalar(f, x0=5, x1=1, xtol = 1e-10, maxiter=200,␣
 ↪method='secant')
opt.root_scalar(f, x0=5, x1=1, xtol = 1e-10, maxiter=200, method='secant')
```

```
330 µs ± 6.47 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

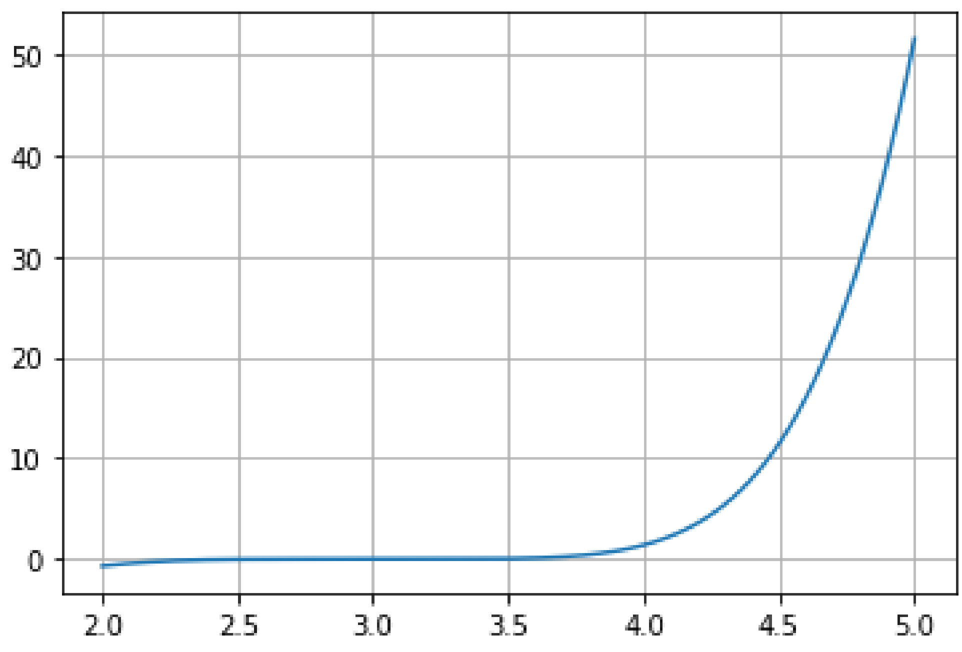
[76]:
```
      converged: True
           flag: 'converged'
 function_calls: 8
     iterations: 7
           root: 4.323239543713714
```

e)

$$f_5(x) = (x-3)^5 \ln(x) \mid, \text{com } x^* \in [2,5]$$

[77]:
```
f = lambda x: (x - 3) ** 5 * np.log(x)
```

[78]:
```
x = np.linspace(2, 5,10000)
plt.plot(x,f(x))
plt.grid(True)
```



Método da bisseção

[79]:
```
%timeit opt.root_scalar(f,method='bisect', bracket=[2,5], maxiter=200,␣
 ↪xtol=1e-10)
```

```
opt.root_scalar(f,method='bisect', bracket=[2,5], maxiter=200, xtol=1e-10)
```

```
99.9 µs ± 22.6 µs per loop (mean ± std. dev. of 7 runs, 10000 loops each)
```

[79]:
```
      converged: True
           flag: 'converged'
 function_calls: 37
     iterations: 35
           root: 3.000000000029104
```

Método da falsa posição

[80]:
```
x_i = 2
x_u = 5
iteracoes = 200
tolerancia = 1e-10

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
valores[-1]
```

[80]:
```
2.593909764205461
```

Método do ponto fixo

[81]:
```
g = lambda x: np.exp((x ** 5 * np.log(x) - 15 * x ** 4 * np.log(x) + 90 * x **␣
↪3 * np.log(x) - 270 * x ** 2 * np.log(x) + 405 * x * np.log(x)) / 243)
```

[82]:
```
# maxiter = 200 -> Error
%timeit opt.fixed_point(g, 5, xtol=1e-10, maxiter=9000)
opt.fixed_point(g, 5, xtol=1e-10, maxiter=9000)
```

```
1.65 s ± 46.5 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)
```

[82]:
```
array(3.0255312)
```

Método de Newton-Raphson

[83]:
```
x = symbols('x')
f = lambda x: (x - 3) ** 5 * ln(x)
derivada = diff(f(x))

f_1 = lambdify(x, derivada, 'numpy')
f = lambda x: (x - 3) ** 5 * np.log(x)
opt.root_scalar(f, fprime=f_1, x0=2, method='newton', maxiter=200, xtol=1e-10)
```

[83]:
```
      converged: True
           flag: 'converged'
 function_calls: 194
```

```
    iterations: 97
          root: 2.99999999964898
```

[84]:
```
%timeit opt.root_scalar(f, fprime=f_1, x0=2, method='newton', maxiter=200,␣
↪xtol=1e-10)
```

```
4.93 ms ± 221 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

Método da secante

[85]:
```
%timeit opt.root_scalar(f, x0=5, x1=2, xtol = 1e-10, maxiter=200,␣
↪method='secant')
opt.root_scalar(f, x0=5, x1=2, xtol = 1e-10, maxiter=200, method='secant')
```

```
6.16 ms ± 119 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

[85]:
```
     converged: True
          flag: 'converged'
function_calls: 139
    iterations: 138
          root: 2.999999999418057
```

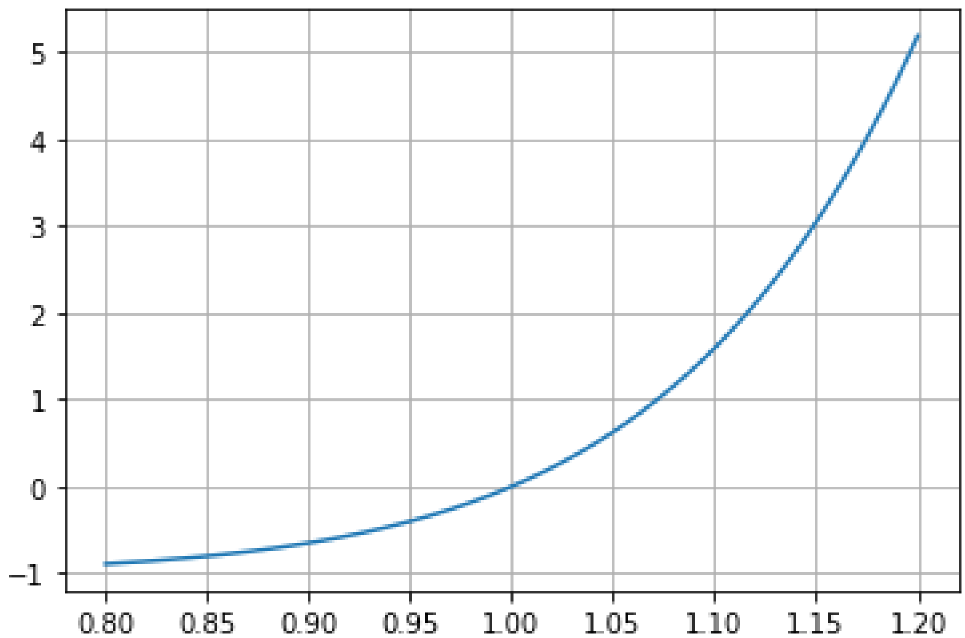
f)

$$f_6(x) = x^{10} - 1, \text{com } x^* \in [0.8, 1.2]$$

Método da bisseção

[86]:
```
f = lambda x: x ** 10 - 1
```

[87]:
```
x = np.linspace(0.8, 1.2,10000)
plt.plot(x,f(x))
plt.grid(True)
```



Método da falsa posição

[88]:
```
x_i = 0.8
x_u = 1.2
iteracoes = 200
tolerancia = 1e-10

valores = falsa_posicao(f, x_i, x_u, iteracoes, tolerancia)
valores[-1]
```

[88]:
```
0.9999999999999998
```

Método do ponto fixo

[89]:
```
g = lambda x: x ** 11
```

[90]:
```
%timeit opt.fixed_point(g, 0.8, xtol=1e-10, maxiter=200)
opt.fixed_point(g, 0.8, xtol=1e-10, maxiter=200)
```

```
455 µs ± 16.5 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

[90]:
```
array(0.)
```

Método de Newton-Raphson

[91]:
```
x = symbols('x')
derivada = diff(f(x))
```

```python
f_1 = lambdify(x, derivada, 'numpy')
opt.root_scalar(f, fprime=f_1, x0=0.8, method='newton', maxiter=200, xtol=1e-10)
```

[91]:
```
      converged: True
           flag: 'converged'
 function_calls: 20
     iterations: 10
           root: 1.0
```

Método da secante

[92]:
```python
%timeit opt.root_scalar(f, x0=1.2, x1=0.8, xtol = 1e-10, maxiter=200,␣
 ↪method='secant')
opt.root_scalar(f, x0=1.2, x1=0.8, xtol = 1e-10, maxiter=200, method='secant')
```

```
484 µs ± 16.4 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

[92]:
```
      converged: True
           flag: 'converged'
 function_calls: 12
     iterations: 11
           root: 1.0
```